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Inventory related compensation in decentralized organizations

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Abstract:

We consider a principal agent problem in a decentralized organization. The agent holds private information with respect to an uncertain demand within a single selling season. As such his task is to determine the optimal order quantity. Being head of a profit center, however, he naturally focuses on maximizing profit of his particular unit while the principal aims at maximizing long run firm value. This goal incongruency results in a systematic shortfall of order quantity chosen by the agent as opposed to the strategically optimal level.

We show that a menu of contracts offered to the agent to pick from is suitable to solve the agency problem and to achieve first best outcomes. Each contract specifies a fixed payment and a bonus or a penalty, conditioned on the inventory level at the end of the selling season, along with a prescribed order quantity. An exogenously given profit share is added to reflect the assumed profit center structure. Omitting any of the contracting elements specified above, however, destroys first best. The paper not only demonstrates that first best can be achieved in the described setting, it also provides a theoretical explanation for the widely observed practice of using inventory related compensation elements in organizations.

Keywords: Newsvendor, Asymmetric Information, Incentive Design, Service Level

1. Introduction

Empirical evidence in line with personal experience from many discussions with operations managers (multi-national companies in the fast moving consumer goods industry, chemical industry, and wholesaling industry) suggests that service level based performance measures have become a frequently used element in compensation contracts (see e.g. Thomas 2005).

In this paper we offer an agency theory explanation for this observation. We analyze a setting in which compensation elements based on inventory levels turn out to be extremely useful to solve an incentive conflict immanent in an exogenously assumed profit center structure. In particular we presume that a principal, tantamount to headquarters or an upper level manager, considers what we call a *strategic underage cost* while this element is missing in the objective function of the agent.

To elaborate on the notion of strategic underage costs consider two examples:

First, super market chains as well as bakery chains usually operate various stores within a quiet narrow region. These businesses are often organized in a profit center structure. Running out of stock is not only likely to have an immediate effect on the profit of the specific store. One store's stock out might rather have an additional long run negative effect on other stores of the same chain. In fact customers suffering from stock out in one store do not only prefer another one for their future shopping activities but to some extent refrain from the entire chain. As such an interrelation is present that affects headquarters objective function in distinct fashion from those of each of the single store managers.

Second, consider a fast food or coffee shop chain that is steered by headquarters and organized in a profit center structure. Headquarters plans a campaign to sell promotional items at the chain stores for brand marketing reasons (e.g., branded coffee mugs or comic figures of a recent block buster). While headquarters favors high selling quantities for the promotion to be effective, managers may have little incentive to provide high availability. This is due to the fact that the profit margin for these items is likely to be low if the main goal is to reach many customers. Moreover, each chain store is only benefiting marginally from an increasing brand attractiveness.

Though further examples could be constructed easily, these two are meant to demonstrate that strategic underage cost in our setting is a cost of stock out that is long run in nature.[1] It arises from some kind of spillover effects. We assume the short term oriented manager does not care but the principal does.

[1] For further long run effects see e.g. Fitzsimons 2000, Corsten and Gruen 2004, Anderson et al. 2006, and Sieke et al. 2012.

The manager is equipped with private demand information in our setting. In particular he knows the distribution of demand while the principal does not. In order to use his superior knowledge, order choices should be delegated to the manager. With the incentive conflict present, however, the agent is tempted to systematically distort the order quantities downward as compared to the centrally optimal level.

To correct for this distortion we present a carefully designed compensation scheme. A menu of contracts is offered to the manager to pick from. The manager is paid an exogenously given profit share and each contract offered specifies a fixed wage, a bonus or penalty based on inventory holdings, and an order quantity to be chosen.

Contracts are specified in such a way that the agent optimally chooses a particular contract for each demand function he may face. Moreover, this contract prescribes the centrally optimal order quantity for the demand function given. As will be shown below the above mechanism solves the agency conflict and achieves first best. Further, we demonstrate that omitting either fixed pay or bonus/penalty elements prevents first best. It turns out to be necessary that the order quantity is an explicit part of the contract. Allowing the manager instead to optimize the order size with respect to the other contracting parameters again prevents first best.

The first best incentive scheme we derive considerably extends existing literature. In fact to our best knowledge all incentive schemes presented in previous papers assuming demand uncertainty resulted in either second best outcomes or at best asymptotically achieve first best when selling the shop to the agent (see Arya and Mittendorf 2004 and Babich et al. 2012, respectively). Moreover, we are able to achieve first best without distorting the profit margin of the considered product. This avoids problems from communicating different product costs or profitability measures to different parties. Finally the paper picks up on the fact that tools to measure inventory holdings on a day to day basis have been improved substantially in many organizations throughout the past decade (Akkermans et al. 2003). Based on that, the amount of inventory available at the end of a period has become a performance measure suitable for contracting which is increasingly used.

The remainder of the paper is organized as follows. Section 2 provides a literature review. Section 3 presents the model. We will analyze the benchmark solution in Section 4. Afterwards, we will show for the case of two agent types (low and high demand variance) that first best outcomes can be implemented when information is asymmetrically distributed and all of the above mentioned compensation elements are considered (see Section 5). We then show that first best outcomes will not occur if one compensation element is neglected. Section 5 also extends our simple two-type model to multiple types and continuous types. Section 6 discusses some further extensions and alternative interpretations of our stylized model.

2. Literature review

In this paper we derive optimal incentive contracts in a newsvendor context under asymmetric information. Even though we focus on an intra-firm coordination problem a supply chain interpretation of our model exists and will be discussed in Section 6.

Accordingly some of the existing supply chain literature is closely related to our work. Several contributions discuss so-called service level agreements for incentive alignments under symmetric information (see Cachon 2003, Lariviere 1999, Lee and Whang 1999, Thomas 2005, Katok et al. 2008, Sieke et al. 2012, Liang and Atkins 2013). Service level agreements condition the payments of the supply chain parties on the inventory position at the end of the selling season. A bonus (penalty) is paid (charged) when end-customer demand is satisfied (not satisfied). While this line of work shows that service level agreements can coordinate the supply chain to first best outcomes in case of symmetric information, we show below that inventory related compensation allows achieving first best outcomes even if asymmetric information is present.

Closely related to our newsvendor setup are Arya and Mittendorf 2004, Burnetas et al. 2007, and Babich et al. 2012. These papers consider incentive conflicts related to double marginalization.

A well-known fact in the coordination literature is that buy-back policies for unsold units can coordinate the supply chain in case of full information (see Pasternack 1985). Interestingly, the determination of the optimal contract parameters does not rely on the demand distribution and can be expressed as a function of the profit share each supply chain party is receiving. However, this profit share is the main obstacle when facing asymmetric demand, because the agent has an incentive to misrepresent his demand distribution in order to receive higher profit shares. In this context, Arya and Mittendorf 2004 analyze if incentive schemes that entail menus of quantities that are directly linked to pairs of wholesale prices and buy-back prices can coordinate the supply chain. Informational rents prevent first-best in this setup.

Babich et al. 2012 revisit this line of work and analyze an incentive scheme entailing menus of wholesale prices, buy back prices, and fixed payments. Babich et al. 2012 show that such an incentive scheme is asymptotically efficient, that is, when the wholesale-prices and the buy-back prices approach the exogenous end-customer price. This essentially translates to a situation where the principal sells the business to the agent which is not necessarily feasible in all situations. We instead present a first-best incentive scheme that is not only efficient in its limits but gives more leeway to set parameters while maintaining first best outcomes.

Burnetas et al. 2007 analyze to what extent frequently observed quantity discount schemes can resolve the double marginalization incentive conflict in supply chains. Such quantity discount schemes result in efficiency losses because the incentive compatibility requires informational rents that avoid first-best outcomes.

Compared to the large body of literature that analyzes the newsvendor problem in the supply chain context, relatively few studies concentrate on intra-firm coordination of operations management tasks under asymmetric information. One line of this research looks on sales and operations planning processes (see Oliva and Watson 2011). However, this line of research rather looks on the process to align sales forecast and production decisions but tends to neglect the inherent incentive conflicts. Celikbas et al. 1999 discuss how a fully informed corporate department can set penalties to limit over-forecasting by the marketing department. Atkinson 1979 analyzes a situation in which the manager's compensation is a profit share as in our setup. He discusses "standards-based contracts" that provide incentives to the manager to incorporate private information in the order size decision. Yet, the study does not analyze optimal incentive design.

3. Outline of model and basic assumptions

We consider a decentralized organization. A principal (P) employs an agent (A) as a head of a profit center. Both are risk neutral. The agent's only task is to choose the order quantity (q) for a single selling season in the presence of uncertain demand $\tilde{r}$. We assume that there are two types of agents: one faces a low demand variance (*l*) and the other a high demand variance (*h*). We extend the analysis to multiple and infinite types in Section 5.2.

Demand of the two types follows a continuous distribution function with density function $f_i(r)$ and cumulative distribution function (CDF) $F_i(r)$ with support on (a_i, b_i) with $a_i, b_i \in R_+$ and $i \in (l, h)$. We assume that both distributions have identical means μ (i.e., one is a mean-preserving spread of the other one) and $\sigma_l < \sigma_h$. Moreover, we assume that both functions satisfy second order stochastic dominance, i.e. $\int_{a_h}^{r} F_h(r)\,dr \geq \int_{a_l}^{r} F_l(r)\,dr \; \forall r$. Examples for such distribution functions are the uniform distribution or under certain conditions the truncated normal distribution (see Levy 2006 for an comprehensive discussion of stochastic dominance). We will refer to the agent facing a high standard deviation of demand as "high type" and to the agent facing a low one as "low type".

The distribution $f_i(r)$ occurs with probability θ_i. We assume that the distribution of types is common knowledge. By assumption the agent knows his type, but the principal does not.[2] The end-customer price is *p* and the per unit cost is *c*. Each unit that cannot be sold within the selling period has a salvage value of *s*. The expected operating profit[3] when agent type *i* orders *q* units is

$$\pi_i(q) = (p-s)\int_{a_i}^{q}(r-q)\,f_i(r)dr + (p-c)\cdot q\,. \tag{3.1}$$

The principal aims at maximizing long term firm value that includes strategic underage costs. Strategic underage cost, δ_u, is the cost associated to customer demand exceeding the order size of the period by one unit.[4] The principal's overall expected strategic underage cost when agent type i orders the quantity q is

$$C_i(q) = \delta_u\int_{q}^{b_i}(r-q)\,f_i(r)dr\,. \tag{3.2}$$

The principal's overall payoff per period Π_P equals operating profit less strategic underage cost less compensation paid to the agent. We assume the agent's compensation contains a share in annual profit $\gamma\cdot\pi_i(q)$ with $\gamma\in(0,1]$ in order to capture the profit center structure. The profit share is exogenous in the sense that γ is not subject to optimization throughout the analysis and is chosen identical for both types of managers. However, the presence of any non zero profit share, immediately creates a conflict of interest between the principal and the agent. In fact the agent becomes more short term oriented than the principal as he ignores strategic underage cost. In what follows we analyze if and how further compensation elements are capable of resolving this conflict and thus align the goals of the principal and the agent in the presence of asymmetric information.

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[2] Without loss of generality we assume that the type is known before the contract is signed. Alternatively, assuming the agent learns his type after signing the contract but before choosing the order quantity coupled with an option to drop out of the contract once the type is revealed would lead to identical results.

[3] In the following we will mostly omit to explicitly state that we are referring to expected values. Note, however, that uncertainty is present in two respects. Demand is uncertain and so is the agent type from the principal's perspective.

[4] Note that our model can easily incorporate strategic overage cost, δ_o. Strategic overage cost are the cost occurring when products are held in excess of the period's demand and must be salvaged (i.e., $r < q$). Those costs might be interpreted as costs related to reaching certain sustainability measures (e.g., reduction of waste). Yet, an extension of the model would not alter the basic insights and we therefore restrict our attention for the sake of clarity to the case of δ_o=0. Moreover, the model can also easily incorporate strategic underage cost of the agent. The key point here is that the principal has higher strategic underage cost than the agent and, therefore, wants to implement higher order sizes than the agent.

The agent is offered a contract $C = \{(q_i, B_i, T_i) : q_i \in \mathbb{R}_+, B_i \in \mathbb{R}, T_i \in \mathbb{R}, i \in \{l, h\}\}$. With q_i being the order size, B_i the inventory related compensation element and T_i being the fixed payment. For $B_i \geq 0$, a bonus is paid when all demand within the selling season has been satisfied. $B_i < 0$ represents a penalty charged whenever there are leftovers at the end of the selling season. This implies assuming that realized demand is not only verifiable but also not subject to any kind of distortions by the agent. Accordingly, it is used for contracting without doubt regarding the reported amount.[5] However, as we will show in Section 6, managerial discretion with regard to the demand realization can be avoided by incorporating an auditing procedure in contract design.

The probability that all demand is fulfilled depends on the order size and equals $F_i(q)$. It is typically referred to as α - service level. Thus, the bonus is an inventory related compensation element that can be interpreted as a service level agreement as outlined in our literature review. The penalty can be interpreted as a company's endeavor to avoid overstock and in turn waste of resources.

4. Benchmark setting: First best (FB)

In a setting with symmetric information the principal's expected profit maximizing compensation scheme for each type i results from:

$$\max_{q,B,T} \Pi_{P,i} = (1-\gamma) \cdot \pi_i(q) - C_i(q) - B \cdot F_i(q) - T \tag{4.1}$$

$$s.t. \quad \gamma \cdot \pi_i(q) + B \cdot F_i(q) + T \geq R \tag{4.2}$$

The principal maximizes his long term profits (4.1) including the operating profit, the strategic underage cost and the compensation to the agent. The participation constraint (4.2) ensures that each agent type i is willing to accept the offered contract. We assume that the agent faces a reservation pay *R*.

Proposition 1: In a first best setting the participation constraint (4.2) is binding for both agents. The principal determines $q = q_i^{FB}$ independently of *R*.

Proof: Minimizing expected payment to the agents requires that (4.2) holds as an equality. Solving (4.2) for T and inserting into (4.1) we obtain an unconstrained optimization problem for the principal.

$$\max_{q} \Pi_{P,i}(q) = \pi_i(q) - C_i(q) - R \tag{4.3}$$

[5] Managerial discretion regarding the demand realizations may be prevented by firm immanent organizational structures that separate incoming demand (e.g. customer service) from demand fulfillment (e.g. sales).

The solution to this problem is characterized by the familiar newsvendor solution (see, e.g., Ho et al. 2010), where CR denotes the critical ratio:

$$q = q_i^{FB} = F_i^{-1}(CR) = F_i^{-1}\left(\frac{p-c+\delta_u}{p-s+\delta_u}\right). \quad \square \tag{4.4}$$

Intuitively the principal chooses $q = q_i^{FB}$ outweighing the effect of the order quantity choice on profit $\pi_i(q)$ as well as on strategic underage cost $C_i(q)$. Independently from that choice, the principal has to offer an expected payment of R to each type of agent provided that the optimal order is placed. To do so, the contract offered to the agent needs to link the payment to the optimal order quantity. As profit sharing is exogenously assumed, part of the overall payment is covered by $\gamma \cdot \pi_i(q)$. Any difference between expected profit share and *R* can be paid in form of a bonus/penalty or fixed payment or a combination of both. Possible B,T - pairs can be easily derived from (4.2). Note, there are no restrictions on B, that is, the principal can either set a bonus or a penalty. Higher bonuses, however, result in lower fixed payments (which may even be negative and would then rather be called fixed charges). In turn, higher penalties result in higher fixed payments.

From the analysis above it becomes apparent that the principal has a maximum degree of freedom to set the contract parameters. For instance, first best outcomes can still be achieved if one of the compensation parameters is set to zero.

5. Asymmetric information setting (SB)

In what follows we assume that the agent's type, tantamount to the distribution of demand he is facing, is private knowledge. Accordingly the principal cannot assign a specific contract to each type as in the first best setting above. Rather, he offers a menu of (two) contracts for the agent to pick from. The contracts are to be set up such that a) both types are willing to agree to the contract and b) that none of the agent types has an incentive to imitate the other one. To achieve this, the principal solves the following program:

$$Max_{B_i,T_i,q_i}\ \Pi_P = \theta_l \cdot \Pi_{P,l} + \theta_h \cdot \Pi_{P,h} \tag{5.1}$$

$$\gamma \cdot \pi_i(q_i) + B_i \cdot F_i(q_i) + T_i \geq \gamma \cdot \pi_i(q_j) + B_j \cdot F_i(q_j) + T_j\ \forall i, j \in \{l,h\}\ and\ i \neq j \tag{5.2}$$

$$\gamma \cdot \pi_i(q_i) + B_i \cdot F_i(q_i) + T_i \geq R\ \forall i \in \{l,h\} \tag{5.3}$$

Constraints (5.2) are the incentive constraints ensuring that the agent type *i* will self-select into the compensation scheme $C = \{(q_i, B_i, T_i) : i \in \{l, h\}\}$. Constraints (5.3) are the participation constraints already introduced in the symmetric information setting.

The following lemma establishes a relation that turns out to be important for all of the following analysis.

Lemma 1: For a given order size that is sufficiently low (i.e., $q < b_l + \zeta$ with $\zeta \geq 0$), the operating profit is at least as high for the low variance type as for the high variance type.
Proof: All further proofs can be found in the appendix.

Solving the above program (5.1) - (5.3) and given lemma 1 we obtain proposition 2 and corollary 1.

Proposition 2: *First best outcomes can be achieved if all compensation elements are incorporated in the incentive scheme.*

With the profit sharing element complemented by a fixed payment and a bonus/penalty element, it is always possible to determine a B_i, T_i - combination that ensures that the participation constraints (5.3) are binding at q_i^{FB} while the self-selection constraints (5.2) are fulfilled. In fact we can identify thresholds for the bonus/penalty element B_i to ensure first best. Given B_i is above/below the threshold as defined below, T_i needs to be chosen as a function of B_i. Corollary 1 specifies the relevant thresholds for each type.

Corollary 1: The threshold value for B_i, $i \in l, h$, depends on the size of q_i^{FB} and the related distribution function values.

	$F_l\left(q_i^{FB}\right) < F_h(q_i^{FB}) \Leftrightarrow q_i^{FB} < \mu$	$F_l\left(q_i^{FB}\right) > F_h(q_i^{FB}) \Leftrightarrow q_i^{FB} > \mu$
$i = l$ (Low type)	$B_l \leq$ some non-negative threshold (bonus or penalty)	$B_l \geq$ some non-positive threshold (penalty or bonus)
$i = h$ (High type)	$B_h \geq$ some non-negative threshold (bonus)	$B_h \leq$ some non-positive threshold (penalty)

Table 1: Threshold values for first best inventory related compensation.

Note that we need to distinguish two cases as shown in the columns in Table 1. Case one covers all parameter settings that result in optimal first best order quantities for which $F_l\left(q_i^{FB}\right) < F_h\left(q_i^{FB}\right)$ holds while case two covers those implying $F_l\left(q_i^{FB}\right) > F_h(q_i^{FB})$. Recall from (4.4) that q_i^{FB} is the solution to the following equation:

$$F_i\left(q_i^{FB}\right)=\frac{p-c+\delta_u}{p-s+\delta_u} \qquad (5.4)$$

As both distribution functions by assumption have the same mean, they cross at $F_i\left(\mu\right)=0.5$. For any parameter constellation $\frac{p-c+\delta_u}{p-s+\delta_u}<(>)0.5$ we obtain $q_i^{FB}<(>)\mu \text{ for } i\in l,h$. Moreover, $q_i^{FB}<(>)\mu$ implies $F_l\left(q_i^{FB}\right)<(>)F_h(q_i^{FB})$ such that case one above in fact reflects optimal order quantities below mean demand and case two order quantities above mean demand. From (5.4) it is apparent that order quantities below (above) the mean are optimal if cost is high (low) in relation to salvage value.

Thus, corollary 1 shows that with q_i^{FB} below the mean order quantity, tantamount to a large difference between cost and salvage value, the low type will receive a bonus that must not be too high and can even be a penalty. The high type needs to receive a bonus above some lower bound. With q_i^{FB} above the mean order quantity, thus a product with small difference between cost and salvage value, first best requires to pay a bonus above some non-positive lower bound to the low type and a penalty to the high type that lies below some non-positive threshold value. Doing so ensures that neither type has an incentive to choose the compensation scheme that was designed for the other type. The fixed payment is used to extract all rents from the agent.

To add intuition to the above results we present a numerical example below. We use the parameter values depicted in Table 2.

p	s	δ_u	γ	R	θ_i
1	0	0.1	0.1	30	0.5

Table 2: Parameter values for numerical example.

In what follows we perform some comparative statics analysis by varying the unit cost *c*. Varying *c* affects the critical ratio (CR) and in turn q_i^{FB}. Thus, we are able to show the effect of different optimal order quantities and $F_i(q_i^{FB})$ values have on feasible bonus/penalty payments and the fixed payment. Assuming uniform distributions, $U(200,400)$ characterizes low demand variance and $U(100,500)$ high demand variance. Note that both types have identical means. The cumulative distribution functions cross at the mean of 300. It follows that $F_l(r>300)>F_h(r>300)$ and vice versa.

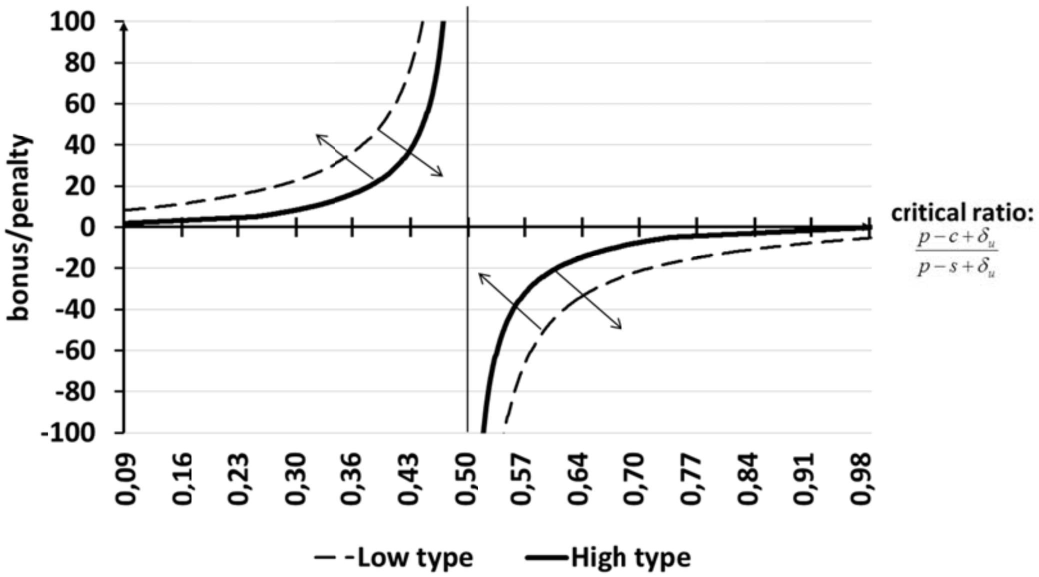

Figure 1: Bonus/penalty values in the optimal compensation scheme

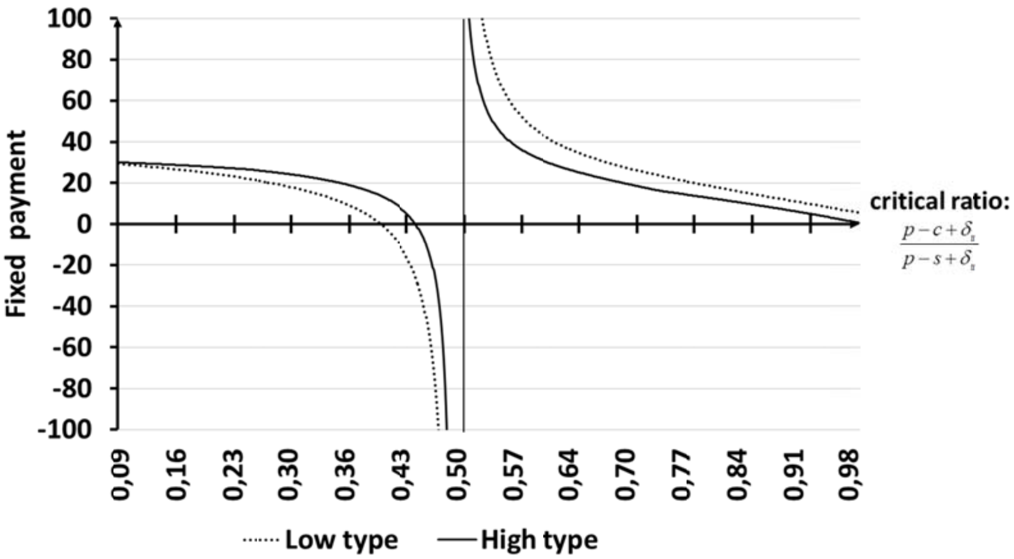

Figure 2: Fixed payments in the optimal compensation scheme

Figure 1 shows the upper and lower bounds for the bonus/penalty for critical ratios of $0.09 < CR < 0.99$. Figure 2 depicts related fixed payments at threshold bonus/penalty values. The arrows in Figure 1 indicate in which direction the bonus/penalty can be adjusted such that the compensation scheme is still incentive compatible.

For $CR < 0.5$ and $CR > 0.5$, a self-selection problem does exist and the principal optimally offers a menu of two different contracts. Let us consider exemplarily the case of $CR > 0.5$ for the moment. Using threshold values, an incentive compatible contract charges a higher penalty from the low demand type than from the high demand one. To ensure participation, this goes along with a higher fixed payment to the low type than to the high type. A low type imitating a high type results in a higher order quantity, as for $CR > 0.5$ it follows that $q_l^{FB} < q_h^{FB}$. This implies that with imitation of the high type, the probability of paying the penalty –which is lower in absolute terms- increases. In addition imitation would reduce the profit sharing element as $\pi_l\left(q_h^{FB}\right) < \pi_l\left(q_l^{FB}\right)$ which follows from the concavity of the agent's profit function. Both effects together suffice to keep the low type from selecting the high type contract. In contrast a high type imitating a low one would raise the penalty to be paid but decrease the probability for the payment to occur. He would receive an increased fixed payment. The effect on the profit share element is ambiguous.

Note that threshold values are not defined at $CR = 0.5$. In fact at this point we obtain $q_l^{FB} = q_h^{FB} = \mu$. Accordingly, the principal would require both agent types to choose identical order quantities. A separating incentive scheme would require an information rent to be paid to the low type which follows directly from Lemma 1. To cope with this problem, the order size of the high type can be distorted in an arbitrary manner, i.e. $q_h = \mu + \varepsilon$ with $\varepsilon \in \mathbb{R}$ *and* $\varepsilon \neq 0$. The bonus for the high type can then be set such that the low type has no incentive to mimic the high type according to the logic outlined in corollary 1. Accordingly, the agency costs are arbitrarily close to zero.

5.1 Agency costs resulting from reduced contracting

Proposition 2 has shown that asymmetric information in our setting does not prevent first best if the principal uses the full range of compensation elements. Compared to the symmetric information case, however, the principal has considerably less leeway to set the contracting parameters. In particular, we observed that under full information the principal may set one compensation element to zero while still avoiding efficiency losses. We will now discuss the effects on agency costs if the contract is reduced to two rather than three contracting parameters. Proposition 3 discusses the compensation scheme $C' = \left\{\left(q_i, T_i\right) : q_i \in \mathbb{R}_+, T_i \in \mathbb{R}, i \in \{l, h\}\right\}$, Proposition 4 the compensation scheme $C'' = \left\{\left(q_i, B_i\right) : q_i \in \mathbb{R}_+, B_i \in \mathbb{R}, i \in \{l, h\}\right\}$, and Proposition 5 the compensation scheme $C''' = \left\{\left(B_i, T_i\right) : B_i \in \mathbb{R}, T_i \in \mathbb{R}, i \in \{l, h\}\right\}$

Proposition 3: *First best order sizes cannot be implemented if the compensation scheme does not entail an inventory related element and if* $q_h^{FB} < b_l + \zeta$ *with* $\zeta \geq 0$.

In contrast to a situation with symmetric information, a fixed payment alone cannot achieve first best for a broad range of order quantities. The key insight is that the low type may have an incentive to imitate the high type given he is kept at reservation utility. To prevent imitation the principal needs to pay a rent to the low type which in turn renders first best impossible. The low type's incentive to imitate vanishes, however, if q_h^{FB} is sufficiently high. If the optimal order quantity for the high type is well beyond the maximal demand faced by the low type, it does not pay off any longer to imitate the high type. Doing so in fact implies to receive salvage value only for sure for the orders placed beyond b_l.

A qualitatively similar result occurs if the compensation contract is based on a bonus/penalty element only in addition to the share of profit, that is, $C" = \{(q_i, B_i): q_i \in \mathbb{R}_+, B_i \in \mathbb{R}, i \in \{l,h\}\}$.

Proposition 4: *First best order sizes cannot always be implemented if the compensation scheme does not entail a fixed payment. They cannot be implemented whenever* $\mu < q_i^{FB} < b_l + \zeta$ *with* $\zeta \geq 0$ *holds for* $i \in l,h$.

Proposition 4 highlights that it is impossible to achieve first best when a fixed pay is missing and first best order quantity is higher than mean demand. The result is driven by the fact that with both participation constraints binding the low demand type has always an incentive to mimic the high demand type. To see this, recall that the participation constraint for the high demand type equals $\gamma \cdot \pi_h\left(q_h^{FB}\right) + B_h \cdot F_h\left(q_h^{FB}\right) = R$. From lemma 1 we know already that $\pi_l(q) \geq \pi_h(q)$. Moreover, an order quantity greater than mean demand implies $F_l(q) > F_h(q)$. To ensure that the high type receives R it follows that $B_h > B_l$ must hold. Now, if the low type imitates the high type, he would receive $\gamma \cdot \pi_l\left(q_h^{FB}\right) + B_h \cdot F_l\left(q_h^{FB}\right) > R$. It follows that the principal needs to pay an expected rent to the low type in order to keep him from imitating the high type in this case.

Yet, for $F_l(\text{q}) < F_h(q)$ we cannot draw general conclusions which type has an incentive to mimic the other type. First best can be implemented if $\pi_l(q_h^{FB}) - \pi_h(q_h^{FB})$ is sufficiently small and $\pi_l(q_l^{FB}) - \pi_h(q_l^{FB})$ is sufficiently large. If $\pi_l(q_h^{FB}) - \pi_h(q_h^{FB})$ is sufficiently small, then the inventory related compensation for the high type is not too high (compared to that for the low type) and the element can be set such that the high type receives reservation profit while it is not attractive for the low type to imitate. This is, because the higher bonus is offset by a lower probability of receiving it. The latter fact follows from $F_l(\text{q}) < F_h(q)$ and $q_l^{FB} > q_h^{FB}$.

In turn, if $\pi_l(q_l^{FB}) - \pi_h(q_l^{FB})$ is not sufficiently large, then the high type may even have an incentive to mimic the low type, because he has a higher probability to receive the bonus ($q_l^{FB} > q_h^{FB}$) while having relatively little cost of imitation. Thus, only for intermediate levels of differences in the profit functions between low type and high type at the first best order size level, first best outcomes might be achieved. Thus, a bonus alone will only coordinate the order size decisions in specific cases.

Finally we consider a contract that omits specification of the order size parameter, that is, $C''' = \{(B_i, T_i) : B_i \in \mathbb{R}, T_i \in \mathbb{R}, i \in \{l,h\}\}$. If the agent is free to choose any order quantity given that he has opted for a contract out of a menu of two, the principal's optimization problem changes as follows:

$$Max_{B_i,T_i} \Pi_P = \sum_{i\in\{l,h\}} \theta_i [(1-\gamma)\cdot \pi_i\left(q_i^*\right) - C_i\left(q_i^*\right) - B_i \cdot F_i\left(q_i^*\right) - T_i \tag{5.5}$$

$$\gamma \cdot \pi_i\left(q_i^*\right) + B_i \cdot F_i\left(q_i^*\right) + T_i \geq \gamma \cdot \pi_j\left(q_i\right) + B_j \cdot F_i\left(q_i\right) + T_j \quad \forall q_i, i, j \in \{l,h\} and\ i \neq j \tag{5.6}$$

$$\gamma \cdot \pi_i\left(q_i^*\right) + B_i \cdot F_i\left(q_i^*\right) + T_i \geq R \quad \forall i, j \in \{l,h\} \tag{5.7}$$

$$q_i^* \in argmax_{q_i}\ \gamma \cdot \pi_i\left(q_i\right) + B_i \cdot F_i\left(q_i\right) + T_i \tag{5.8}$$

Essentially an incentive compatibility constraint (5.8) is added to the original program. This constraint reflects that the agent, no matter which type, chooses the order quantity q_i^* in order to maximize his payoff.

Proposition 5: *For normal and uniform distributions, first best order sizes cannot be implemented when the order size is not prescribed in the contract.*

Proposition 5 shows that first best cannot be achieved anymore. Note that the principal's long term focus implies that the first best order quantity is higher than the one a purely profit maximizing agent would choose. Thus, to motivate this larger quantity a bonus ($B_i \geq 0$) needs to be offered. From corollary 1, however, we observed that a penalty might be necessary to achieve first best in some cases. Moreover, with non negative bonuses it turns out infeasible to construct a separating menu of compensation schemes. In fact the low type agent would always favor the compensation that was designed for the high type, as this leaves him with a rent beyond reservation profits.

5.2 Multiple and infinite agent types

We now extend our analysis to the case with multiple (i=1,…,n) agent types and to the case with infinite agent types. In the multiple agent types scenario, demand follows a continuous distribution function with density function $f_i(r)$ and cumulative distribution function (CDF) $F_i(r)$ with support on (a_i, b_i) with $a_i, b_i \in R_+$ and $i = 1,...,n$. We maintain the basic assumptions introduced earlier: In particular we assume that the distribution $f_i(r)$ arises with probability θ_i. All distributions have identical means μ (i.e., mean-preserving spread).[6] They can be ordered according to their standard deviation, i.e., $\sigma_i < \sigma_{i+1}\ \forall i = 1,...,n-1$. There exists only one crossing point of the CDFs (single-crossing property). Recall that $F_i(r) < F_j(r)\ \forall r < \mu \textit{ and } i < j$ and $F_i(r) \geq F_j(r)\ \forall r \geq \mu \textit{ and } i < j$ follows. Furthermore, second order stochastic dominance holds, i.e., $\int_{a_j}^{r} F_j(r)\,dr \geq \int_{a_i}^{r} F_i(r)\,dr\ \forall r$.

Proposition 6: *In a multiple type scenario, first best* outcomes result *if all compensation elements are incorporated in the incentive scheme.*

The multi type analysis reveals qualitatively similar results to the two type case. With all compensation elements available, first best outcome can be obtained. The proof relies on the fact that we can order the agents with respect to their CDFs and that we only have to consider adjacent types. Thus, if the inventory related scheme prevents type i+1 from choosing the compensation scheme for type i, then type i+2 will also never have an incentive to do so. In the two type case, we saw that we have either an upper or a lower bound for the inventory related compensation. We show in the proof to proposition 6 that we have simultaneously upper and lower bounds in case of multiple agent types. The reason is that the inventory related compensation must be set such that neither a lower type nor a higher type has an incentive to choose a compensation that was not designed for him.

Proposition 7: *In continuous type scenarios, first best outcomes result if all compensation elements are incorporated in the incentive scheme.*

With infinite agents types we once again receive the same result. The main difference with infinite agent types is that we will not observe ranges in which the bonus/penalty can be set. This is, because

[6] In fact, it is sufficient to show that all distribution functions have a single crossing point and can therefore be unambiguously ordered. The qualitative results remain identical. However, the mean and the crossing point do not necessarily fall together, which complicates the exposition without delivering additional insights.

the adjacent type is arbitrarily close, and we have to use calculus of variations. We assume that the standard deviation of the agent is continuously distributed in the interval $\left[\underline{\sigma},\overline{\sigma}\right]$. Moreover, we denote the actual standard deviation of a specific type with $\hat{\sigma}$. In the proof to proposition 7 we derive that the inventory related compensation element must satisfy

$$B\left(\hat{\sigma}\right)=\gamma\cdot\frac{\partial\pi_{\sigma}(q_{\hat{\sigma}}{}^{FI})}{\partial\sigma}\Bigg|_{\hat{\sigma}=\sigma}\cdot\left(-\frac{\partial F_{\sigma}(q_{\hat{\sigma}}{}^{FI})}{\partial\sigma}\Bigg|_{\hat{\sigma}=\sigma}\right)^{-1} \tag{5.9}$$

in order to prevent type $\hat{\sigma}$ to imitate any other type σ. As an example, we get for the uniform distribution the closed form solution

$$B\left(\hat{\sigma}\right)=-\frac{\sqrt{3}}{2}\cdot\gamma\cdot\left(p-s\right)\frac{\left[1-\left(2CR-1\right)^{2}\right]}{\left(1-2CR\right)}2\hat{\sigma}\,. \tag{5.10}$$

The fixed payment then results from (5.3). We see that the inventory related compensation is monotonically increasing in the standard deviation of the type. Thus a higher variance type is assigned a higher bonus (penalty) and a lower (higher) fixed payment. Moreover, it follows directly that $B\left(\hat{\sigma}\right)>0$ for $CR<0.5$ which is in line with our analysis of the two type case.

6. Extensions

6.1 First order stochastic dominance

Recent studies on supply chain contracting under asymmetric demand information assume first order stochastic dominance for the distribution functions of the agent types (see Cachon and Lariviere 2001 or Burnetas et al. 2007). Obviously, since first order stochastic dominance is a special case of second order stochastic dominance, our results also apply to this class of distribution functions. Consider, as an example, agent types with normally distributed demand, identical variance, and different means. The main difference is that we do not have to distinguish cases in which the CDFs of the agent types cross and, thus, a case wise analysis as in corollary 1 is not necessary.

6.2 Supply chain setting

In the recent past, channel conflicts between suppliers and buyer received an increasing attention. We show that our results can be applied in general also to these settings. Consider a supplier (s) delivering goods at wholesale price (w > c) to a buyer (b). Moreover, we assume that there is no exogenously defined profit sharing parameter and that there is no strategic underage cost. The remaining notation as previously introduced is used accordingly. The profits of the supplier (π_s) and buyer type i ($\pi_{b,i}$) are then

$$\pi_s(q) = (w-c)\cdot q \quad \text{and} \tag{6.1}$$

$$\pi_{b,i}(q) = (p-s)\int_{a_i}^{q}(r-q)f_i(r)dr + (p-w)\cdot q\ . \tag{6.2}$$

The supply chain optimal order size is

$$q_i^{FB} = F_i^{-1}\left(\frac{p-c}{p-s}\right) \tag{6.3}$$

and the buyer's individual optimal q_i^{ind} order size in the absence of a coordination scheme is

$$q_i^{ind} = F_i^{-1}\left(\frac{p-w}{p-s}\right) \tag{6.4}$$

We thus have a classical double marginalization problem in which the buyer orders less than the channel optimal profit, because he is only taking his own profit margin $(p-w)$ into consideration and not the channel wide profit margin $(p-c)$. As in our previous analysis, we consider only situations, in which the critical fractile is not changed, i.e., we assume that the wholesale price is exogenous. Then we get for the suppliers' optimal contract design:

$$Max_{B_i,T_i,q_i}\ \Pi_s = \sum\nolimits_{i\in\{l,h\}}\theta_i\cdot\left[\pi_s(q_i) - B\cdot F_i(q_i) - T_i\right] \tag{6.5}$$

$$\pi_{b,i}(q_i) + B_i\cdot F_i(q_i) + T_i \geq \pi_{b,i}(q_j) + B_j\cdot F_i(q_j) + T_j\ \forall i,j\in\{l,h\}\ and\ i\neq j \tag{6.6}$$

$$\pi_{b,i}\left(q_i\right) + B_i\cdot F_i(q_i) + T_i \geq R\ \forall i\in\{l,h\} \tag{6.7}$$

It can be easily shown that all of the previously derived results also apply for this case. The key property of our proofs is that the low type makes in a certain range higher profits than the high type. Yet, this does not change in the supply chain setup, because the profit comparison is independent of the wholesale price. Comparing the profit functions of a low type and the high type we get

$$(p-s)\int_{a_l}^{q}(r-q)f_l(r)dr + (p-w)\cdot q \geq (p-s)\int_{a_h}^{q}(r-q)f_h(r)dr + (p-w)\cdot q \tag{6.8}$$

and it occurs that

$$(p-s)\int_{a_l}^{q}(r-q)\,f_l(r)dr \geq (p-s)\int_{a_h}^{q}(r-q)\,f_h(r)dr \tag{6.9}$$

Formula (6.9) is identical to formula (8.2) in *Lemma 1*. Thus, in line with *Lemma 1* it can be easily verified that $\pi_{b,l}(q) \geq \pi_{b,h}(q)$ as long as second order stochastic dominance is satisfied and the order size is sufficiently low, i.e., $q_h^{FB} \leq b_l + \xi$. All other proofs follow accordingly.

6.3 Managerial discretion with regard to demand realization

Throughout our analysis we model realized demand as a measure that is not only verifiable but also not subject to any kind of distortions by the agent. Accordingly, it is used for contracting without doubt regarding the reported amount (along with order quantity). The principal either assigns a penalty for overstock or a bonus if demand is fully satisfied. In this section we allow for possible misrepresentations. For instance, in case of a bonus to be paid, the agent could reject to sell the last inventory unit in order to not forgo his bonus payment. Alternatively, if a penalty is present, the agent could possibly increase demand by buying the remaining inventory himself and, thus, circumventing the penalty payment.

In this context the distinction of the demand distribution and the actual demand realization is important. The demand distribution typically involves several qualitative aspects such as expert judgment that cannot be verified ex-post. In contrast a certain demand realization is verifiable in general. To deal with misrepresentation, however, some additional monitoring is possibly necessary. E.g. the principal may use specific auditing techniques such as requiring evidence of sale to an end-customer (see Taylor 2002) or employ mystery shoppers to detect stock outs.

We use the following simple amendment to our previous setting to include an auditing procedure.[7] At the beginning of the game the principal offers a contract that specifies, in addition to the contract parameters introduced in section 3, the probability for an audit to be carried out λ >0. In fact we assume that both parties agree on spot checks. Having accepted the contract, the agent learns whether an audit will indeed occur or not. If no audit takes place, no bonus at all will be paid to the agent. Rather, the agent will receive an extra fixed pay in order to compensate for the missing bonus. In that case the agent has no intention to misrepresent demand. In fact doing so would be costly for him as it would reduce his profit share. If an audit takes place, we assume this audit is perfectly reliable. Knowing that true demand will be revealed for sure, again the agent has no intention to

[7] Alternatively, the strategic interaction between the manager and the principal could be modeled as a manager-auditor game in the sense of e.g. Fellingham and Newman (1985) or Matsumura and Tucker (1992). This would allow to explicitly include audit costs and penalties for misrepresentation along with an endogenously derived audit probability.

distort demand information in the first place. We show in Proposition 8 below that allowing for such monitoring solves the incentive problem arising from discretionary reporting of demand realizations.

Proposition 8: If the demand realization is subject to managerial discretion, *first best order sizes can be implemented when random spot checks are performed and a bonus is part of a contract only if demand realization has been verified.*

If no audit takes place the manager's fixed pay is increased by $(1-\lambda)B_i$ in terms of our previous model. With an audit, demand is revealed and the agent receives (pays) the inventory related compensation only if $r < q$, which occurs with probability $F(q)$. Thus, in expectation the agent receives (pays) an inventory related compensation of

$$(1-\lambda)\cdot B_i + F_i(q_i)\cdot\lambda\cdot B_i \tag{6.10}$$

The term $(1-\lambda)\cdot B_i$ is simply a proportional adjustment of the fixed payment and has therefore no effect on incentive compatibility. Moreover, we can redefine $F_i(q_i)\cdot\lambda = \hat{F}_i(q_i)$. As this is a monotone operation, all assumptions regarding our distribution functions prevail (second order stochastic dominance and single crossing point) and all proofs follow accordingly.

7. Conclusion & Outlook

In this paper we show that inventory related compensation elements are a powerful coordination tool in organizations and supply chains if asymmetric demand information is present. When designed carefully, incentive schemes that condition the actual payment on the realization of demand can prevent rent payments to all types of superior informed agents and thus achieve first best.

Our first key insight is that the inventory position is valuable for contracting as it is informative about the agent's type. Technically, this results from the fact that a certain inventory position is (given a specific order size) more likely for one of the agent type than for another one. Thus adding the inventory position as a performance measure to the incentive contract allows the principal to exploit this information which is nowadays readily available in modern ERP systems. Yet, even if the demand realization may be misrepresented by the agent, spot checks (audits) would suffice to implement our proposed scheme. This insight might well be relevant to previous research in economics and supply chain management that considered asymmetric information settings and found agency costs inevitable.

The second key insight is that our incentive scheme does not require to change the agent's revenue and cost parameters that determine operating profit. Former studies (Babich et al. 2012, Arya and

Mittendorf 2004, Burnetas et al. 2007) aimed at changing the critical fractile such that the individual optimal critical fractile equals (or is as close as possible to) the supply chain optimal critical fractile. This is in fact the standard approach to design first best coordinating contracts under full information as discussed in the fundamental review by Cachon 2003. Yet, it appears that this approach hampers first best outcomes under asymmetric information, because essentially the supply chain optimal critical fractile is identical for all agent types.

While our incentive scheme leaves the individual optimal critical fractile untouched, we require the principal to specify the order quantity in the compensation scheme to achieve first best. If the agent would be allowed to optimize the order size with respect to a self-selected compensation scheme, agency costs arise. This result is in line with former studies on forecast information sharing in a setting where the supplier builds up capacity taking into account the buyer's demand forecasts. When signaling contracts are used and the buyer can dictate the capacity decision (=forced compliance), signaling the demand information comes at no cost and efficiency losses are avoided (Cachon and Lariviere 2001). If the capacity decision cannot be enforced (=voluntary compliance), signaling is costly.

Finally, we note that this contribution has limitations that are seemingly an interesting avenue for future research. First, we are only looking at a stylized one-period, one-product model. Second, we are assuming risk neutrality of all parties and risk aversion may reduce the benefits of using random elements in the incentive scheme. Third, the game-theoretic analysis is based on sequential rationality in the one-shot game. It is likely that the results convey to finitely repeated games where the private information varies randomly between periods. However infinitely repeated interaction may give opportunities to coordinate on the cooperative and efficient outcome without having complex incentive schemes in place (see Ren et al. 2010). Finally, we neglect the behavioral role of information sharing (Özer et al. 2011, Inderfurth et al. 2013) that may allow for even simpler compensation schemes.

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Online Appendix to

"Inventory related compensation in decentralized organizations"

Lemma 1: We need to show that $\pi_l(q) \geq \pi_h(q)$ holds as long as q is sufficiently low. That is using (3.1) the following inequality needs to hold.

$$\pi_l(q) = (p-s)\int_{a_l}^{q}(r-q)\,f_l(r)dr + (p-c)\cdot q \geq (p-s)\int_{a_h}^{q}(r-q)\,f_h(r)dr + (p-c)\cdot q = \pi_h(q) \quad (8.1)$$

Rearranging (8.1) gives

$$\int_{a_l}^{q}(r-q)\,f_l(r)dr \geq \int_{a_h}^{q}(r-q)\,f_h(r)dr \quad . \quad (8.2)$$

We have to consider 4 different cases:

- **Case a** - $q \leq a_h < a_l$
- **Case b** - $a_h < q < a_l$
- **Case c** - $a_l < q \leq b_l$
- **Case d** - $b_l < q \leq b_l + \xi$

Case a - $q \leq a_h < a_l$ **:** Both types sell per definition q units. It follows directly that $\pi_l(q) = \pi_h(q)$.

Case b - $a_h < q < a_l$: We know that the low type will sell all ordered units, because $q < a_l$. Thus, for $\pi_l(q) \geq \pi_h(q)$ to hold the following condition must hold:

$$(p-c)\cdot q \geq (p-s)\int_{a_h}^{q}(r-q)\,f_h(r)dr + (p-c)\cdot q\,. \quad (8.3)$$

Rearranging (8.3) gives

$$0 \geq (p-s)\int_{a_h}^{q}(r-q)\,f_h(r)dr \quad (8.4)$$

This is true because the r.h.s. of (8.4) is always smaller than 0 (i.e., *r* in its integral limits is always smaller than *q*).

Case c - $a_l < q \leq b_l$: Rearranging (8.2) gives

$$\int_{a_l}^{q} rf_l(r)dr - \int_{a_l}^{q} qf_l(r)dr \geq \int_{a_h}^{q} rf_h(r)dr - \int_{a_h}^{q} qf_h(r)dr \ . \tag{8.5}$$

Integrating the second term and integration by parts of the first term on each side of (8.5) gives

$$\left[rF_l(r)\right]_{a_l}^{q} - \int_{a_l}^{q} F_l(r)dr - qF_l(q) \geq \left[rF_h(r)\right]_{a_h}^{q} - \int_{a_h}^{q} F_h(r)dr - qF_h(q) \, . \tag{8.6}$$

Since $\left[rF_i(r)\right]_{a_i}^{q} = qF_i(q)$, it follows from (8.6) after some rearrangements

$$\int_{a_l}^{q} F_l(r)dr \leq \int_{a_h}^{q} F_h(r)dr \ . \tag{8.7}$$

Thus, as long as there is a second-order stochastic dominance, it holds that $\pi_l(q) \geq \pi_h(q) \ \forall \ q \leq b_l$.

Case d - $b_l < q \leq b_l + \xi$:

Using (8.1) again we require

$$(p-s)\int_{a_l}^{b_l} r \cdot f_l\left(r\right)dr + (s-c)\cdot q \geq (p-s)\int_{a_h}^{q} \left(r-q\right)\cdot f_h\left(r\right)dr + (p-c)\cdot q \tag{8.8}$$

to hold.

Rearranging (8.8) gives

$$\int_{a_l}^{b_l} r \cdot f_l\left(r\right)dr \geq \int_{a_h}^{q} \left(r-q\right)\cdot f_h\left(r\right)dr + q \tag{8.9}$$

Proceeding as in case c we get

$$b_l - \int_{a_l}^{b_l} F_l(r)dr \geq q - \int_{a_h}^{q} F_h(r)dr \tag{8.10}$$

Solving for q we obtain :

$$q \leq b_l + \int_{a_h}^{q} F_h\left(r\right)dr - \int_{a_l}^{b_l} F_l\left(r\right)dr = b_l + \zeta \tag{8.11}$$

where $\zeta \geq 0$. *We discuss the first best compensation scheme for* $q_h^{FB} \geq b_l + \zeta$ in proposition 3. □

Proposition 2: We show that there always exists a compensation scheme in which the first best outcome (i.e., $q = q_i^{FB}$) is implemented at first best cost.

Assuming the participation constraints (5.3) are binding at $q = q_i^{FB}$ and solving for T_i we get

$$T_l = R - \gamma\pi_l\left(q_l^{FB}\right) - B_l F_l(q_l^{FB}) \tag{8.12}$$

and

$$T_h = R - \gamma\pi_h\left(q_h^{FB}\right) - B_h F_h\left(q_h^{FB}\right). \tag{8.13}$$

Inserting (8.12) and (8.13) into the self-selection constraints (5.2) and simplifying we obtain

$$\gamma \cdot \left(\pi_h\left(q_l^{FB}\right) - \pi_l\left(q_l^{FB}\right)\right) \le B_l \cdot \left(F_l(q_l^{FB}) - F_h(q_l^{FB})\right) \quad \text{and} \tag{8.14}$$

$$\gamma \cdot \left(\pi_l\left(q_h^{FB}\right) - \pi_h\left(q_h^{FB}\right)\right) \le B_h \cdot \left(F_h(q_h^{FB}) - F_l(q_h^{FB})\right). \tag{8.15}$$

B_l can be chosen such that (8.14) holds and B_h can be chosen such that (8.15) holds. Determining T_l and T_h according to (8.12) and (8.13) ensures first best. □

Corollary 1: We use (8.14) and (8.15) to solve for feasible B_i . We distinguish two cases:

Case a: $F_l(q_i^{FB}) > F_h(q_i^{FB})$

From (8.14) we get

$$B_l \ge \frac{\gamma \cdot \left(\pi_h\left(q_l^{FB}\right) - \pi_l\left(q_l^{FB}\right)\right)}{F_l(q_l^{FB}) - F_h(q_l^{FB})}. \tag{8.16}$$

As $\pi_h\left(q_l^{FB}\right) - \pi_l\left(q_l^{FB}\right) \le 0$ (see Lemma 1) it follows that

$$B_l \ge z_a^{\ l} \tag{8.17}$$

where $z_a^{\ l} = \frac{\gamma \cdot \left(\pi_h\left(q_l^{FB}\right) - \pi_l\left(q_l^{FB}\right)\right)}{F_l(q_l^{FB}) - F_h(q_l^{FB})} \le 0$. We can interpret $z_a^{\ l}$ as a non-positive lower bound for the inventory related compensation element of the low type.

In turn, we get from (8.15) for the case $F_l(q_l^{FB}) \ge F_h(q_l^{FB})$

$$B_h \le \frac{\gamma \cdot \left(\pi_l\left(q_h^{FB}\right) - \pi_h\left(q_h^{FB}\right)\right)}{F_h(q_h^{FB}) - F_l(q_h^{FB})} \tag{8.18}$$

Since $\pi_l\left(q_h^{FB}\right) \ge \pi_h\left(q_h^{FB}\right)$ for $q_h^{FB} \le b_l + \zeta$ (see Lemma 1) we get

$$B_h \le z_a^{\ h} \tag{8.19}$$

where $z_a^{\ h} = \frac{\gamma \cdot \left(\pi_l\left(q_h^{FB}\right) - \pi_h\left(q_h^{FB}\right)\right)}{F_h(q_h^{FB}) - F_l(q_h^{FB})} \le 0$. We can interpret $z_a^{\ h}$ as a non-positive lower bound for the inventory related compensation element of the high type. Note, for $q_h^{FB} \ge b_l + \zeta$ a simple fixed payment suffices to implement first best order sizes (see proposition 3)).

Case b: $F_l\left(q_i^{FB}\right) < F_h(q_i^{FB})$

The same analysis as in the former case can be applied. For the necessary conditions we get

$$B_l \le z_b^{\ l} \tag{8.20}$$

where $z_b^{\ l} = \frac{\gamma \cdot \left(\pi_h\left(q_l^{FB}\right) - \pi_l\left(q_l^{FB}\right)\right)}{F_l(q_l^{FB}) - F_h(q_l^{FB})} \ge 0$. We, thus, have a non-negative upper bound for the low type's inventory related compensation element. Moreover, we get

$$B_h \ge z_b^{\ h} \tag{8.21}$$

where $z_b^{\ h} = \frac{\gamma \cdot \left(\pi_l\left(q_h^{FB}\right) - \pi_h\left(q_h^{FB}\right)\right)}{F_h(q_h^{FB}) - F_l(q_h^{FB})} \ge 0$. We, thus, have a non-negative lower bound for the high type's inventory related compensation element.

Proposition 3: Below we proceed in two steps. (a) refers to $q_h^{FB} \le b_l + \zeta$ and (b) to $q_h^{FB} > b_l + \zeta$.

(a) Achieving first best requires that both participation constraints hold with equality at q_i^{FB} .

Accordingly with $B_i = 0$, (5.3) for the high type and the low type equals

$$\gamma \cdot \pi_h\left(q_h^{FB}\right) + T_h = R \tag{8.22}$$

$$\gamma \cdot \pi_l\left(q_l^{FB}\right) + T_l = R \tag{8.23}$$

Solving (8.22) for T_h and inserting into (5.2) for the low type we obtain

$$\gamma \cdot \pi_l\left(q_l^{FB}\right) + T_l \geq \gamma \cdot [\pi_l\left(q_h^{FB}\right) - \pi_h\left(q_h^{FB}\right)] + R \tag{8.24}$$

Since $\pi_l\left(q_h^{FB}\right) > \pi_h\left(q_h^{FB}\right)$ at least for some cases (see lemma 1, case b) and γ and R are positive, satisfying (8.24) in these cases implies that (8.23) is violated.

In fact if $\pi_l\left(q_h^{FB}\right) > \pi_h\left(q_h^{FB}\right)$ preventing the low demand type from imitating the high demand type requires to pay a rent to the low type, holding the high type at reservation level. Paying a rent to one type, however, prevents first best.

(b) Assume $q_h^{FB} > b_l + \zeta$. Replacing the l.h.s. of (8.24) by R we obtain

$$0 \geq \gamma\left(\pi_l\left(q_h^{FB}\right) - \pi_h\left(q_h^{FB}\right)\right) \tag{8.25}$$

As $\pi_l\left(q_h^{FB}\right) < \pi_h\left(q_h^{FB}\right)$ it that

Using (8.22) and inserting into the l.h.s of (5.2) for the high type we get

$$R \geq \gamma \cdot \pi_h\left(q_l^{FB}\right) + T_l \tag{8.26}$$

Solving (8.23) for T_l and inserting into the r.h.s. of (8.26) we get

$$0 \geq \gamma\left(\pi_h\left(q_l^{FB}\right) - \pi_l\left(q_l^{FB}\right)\right) \tag{8.27}$$

The r.h.s. of (8.27) is always negative since $q_l^{FB} < b_l$ follows from (4.4). Thus, the inequality in (8.27) holds. It follows that the self-selection constraints for both types are satisfied given (8.22) and $q_h^{FB} > b_l + \zeta$. □

***Proposition 4*:** For first best to be implemented the participation constraints (5.3) must hold with equality and the self-selection constraints (5.2) must be fulfilled at q_i^{FB} and $T_i = 0$.

Solving (5.3) for both types for B_i we obtain:

$$B_l = \frac{R - \gamma \cdot \pi_l(q_l^{FB})}{F_l(q_l^{FB})} \tag{8.28}$$

and

$$B_h = \frac{R - \gamma \cdot \pi_h(q_h^{FB})}{F_h(q_h^{FB})} \tag{8.29}$$

Inserting (8.28) and (8.29) into the self-selection constraints (5.2) for both types and rearranging terms, we get

$$\frac{R-\gamma\cdot\pi_l(q_l^{FB})}{F_l(q_l^{FB})}\leq\frac{R-\gamma\cdot\pi_h(q_l^{FB})}{F_h(q_l^{FB})} \quad \text{and} \tag{8.30}$$

$$\frac{R-\gamma\cdot\pi_h(q_h^{FB})}{F_h(q_h^{FB})}\leq\frac{R-\gamma\cdot\pi_l(q_h^{FB})}{F_l(q_h^{FB})} \tag{8.31}$$

Note that in (8.31) the numerator of the l.h.s. term is at least as large as the numerator of the r.h.s. term as $\pi_l\left(q_h^{FB}\right)\geq\pi_h\left(q_h^{FB}\right)$ for the range of order quantities determined in lemma 1. It follows that for (8.31) to hold $F_h\left(q_h^{FB}\right)\geq F_l\left(q_h^{FB}\right)$ is a necessary, but not sufficient condition. From this follows directly that first best can never be achieved for $\mu<q_i^{FB}<b_l+\zeta$. We now provide a case by case analysis under what circumstance (8.30) can be fulfilled, given the necessary condition from (8.31) must hold.

Case 1) $F_h\left(q_l^{FB}\right)<F_l\left(q_l^{FB}\right)$ → "high profit margin"

Relation (8.30) always holds. However, from the necessary condtion from (8.31) follows due to our mean preserving spread assumption $F_h\left(q_h^{FB}\right)\geq F_l\left(q_h^{FB}\right)\Rightarrow F_h\left(q_l^{FB}\right)\geq F_l\left(q_l^{FB}\right)$ which contradicts the assumption of case 1. Thus, first best cannot be implemented with this parameter combination.

Case 2) $F_h\left(q_l^{FB}\right)>F_l\left(q_l^{FB}\right)$ → "low profit margin"

With $F_h\left(q_l^{FB}\right)>F_l\left(q_l^{FB}\right)$, the necessary condition from (8.31) holds. However, (8.31) is only satisfied if $\pi_l(q_h^{FB})-\pi_h(q_h^{FB})$ is sufficiently small, that is, if the operating profit difference between the low type and the high type at q_h^{FB} is sufficiently small. In this case, the inventory related compensation for the high type is not too high (compared to that for the low type) and the element can be set such that the high type receives reservation profit while it is not attractive for the low type to imitate because the higher bonus is offset by the lower probability of receiving it. The latter fact follows from the definition of this case and $q_l^{FB}>q_h^{FB}$.

In turn, (8.30) is only satisfied as long as $\pi_l(q_l^{FB})-\pi_h(q_l^{FB})$ is sufficiently large. If this is not the case, then the high type may even have an incentive to mimic the low type, because he has higher probability to receive the bonus ($q_l^{FB}>q_h^{FB}$) while having relatively little cost of imitation.

Thus, only for intermediate levels of differences in the profit functions between low type and high type at the first best order size level, first best outcomes might be achieved. □

Proposition 5: The proof follows in three steps.

1. We show that only bonuses can implement first best order sizes.
2. We show that the low type must receive a lower bonus than the high type to implement the first best order size.
3. We show that this condition necessarily leads to incentive incompatible schemes.

Step 1: From the standard newsvendor solution we know that the supply chain optimum is achieved when the order size is set according to (4.4). Thus, the optimal bonus B_i can be determined by solving (5.8) such that it is optimal for the agent to order $q_i * = q_i^{FB}$:

$$\gamma \cdot \left.\frac{\partial \pi_i}{\partial q}\right|_{q=q_i^{FB}} + B_i \cdot \left.\frac{\partial F_i}{\partial q}\right|_{q=q_i^{FB}} = 0 \tag{8.32}$$

It follows that

$$B_i = -\gamma \cdot \left.\frac{\partial \pi_i}{\partial q}\right|_{q=q_i^{FB}} \cdot \left(\left.\frac{\partial F_i}{\partial q}\right|_{q=q_i^{FB}} \right)^{-1} = \frac{-\gamma \cdot \left.\frac{\partial \pi_i}{\partial q}\right|_{q=q_i^{FB}}}{f_i(q_i^{FB})} \tag{8.33}$$

From any standard textbook on the newsvendor problem (see, e.g., Silver et al. 1998) we know that

$$\frac{\partial \pi_i}{\partial q} \begin{cases} \geq 0 & , for\ q \leq F^{-1}\left(\frac{p-c}{p-s}\right) \\ < 0 & , else \end{cases} \tag{8.34}$$

Moreover, we know that

$$f_i(q) \geq 0 . \tag{8.35}$$

It follows directly from (8.33) and (8.34) that $B_i > 0$ must hold.

Step 2: Next we show that $B_l \leq B_h$ holds for the uniform and the normal distribution. The derivative of the profit function at the point $q = q_i^{FB}$ is

$$\left.\frac{\partial \pi_i}{\partial q}\right|_{q=q_i^{FB}} = -(p-s)F_i(q_i^{FB}) + (p-c) \leq 0 \tag{8.36}$$

We know that $F_l(q_l^{FB}) = F_h(q_h^{FB})$, because the critical fractile (and therefore the probability of satisfying all demand in the period) is independent from the demand distribution. It follows therefore from inserting (8.36) into (8.33)

$$B_l = \frac{\gamma \cdot \left((p-s)F_l(q_l^{FB}) - (p-c)\right)}{f_l(q_l^{FB})} \le B_h = \frac{\gamma \cdot \left((p-s)F_h(q_h^{FB}) - (p-c)\right)}{f_h(q_h^{FB})} \tag{8.37}$$

as long as $f_l(q_l^{FB}) > f_h(q_h^{FB})$. The relation $f_l(q_l^{FB}) > f_h(q_h^{FB})$ is trivial for the uniform distribution. For the normal distribution we have density

$$f(r) = \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-(r-\mu)^2}{2\sigma}}. \tag{8.38}$$

Thus, comparing two density functions with identical mean and different variance we get

$$f_l(q_l^{FB}) = \frac{1}{\sigma_l\sqrt{2\pi}} e^{\frac{-(q_l^{FB}-\mu)^2}{2\sigma_l}} \ge f_h(q_h^{FB}) = \frac{1}{\sigma_h\sqrt{2\pi}} e^{\frac{-(q_h^{FB}-\mu)^2}{2\sigma_h}} \tag{8.39}$$

We can replace on the l.h.s. σ_l with σ_h in the denominator, i.e., if we can show that the relation in (8.39) holds for this manipulation, we can be sure that it holds in general. We thus get

$$\frac{1}{\sigma_h\sqrt{2\pi}} e^{\frac{-(q_l^{FB}-\mu)^2}{2\sigma_l}} \ge \frac{1}{\sigma_h\sqrt{2\pi}} e^{\frac{-(q_h^{FB}-\mu)^2}{2\sigma_h}} \tag{8.40}$$

Taking the logarithm we get after some manipulations

$$\frac{(q_l^{FB}-\mu)^2}{\sigma_l} \le \frac{(q_h^{FB}-\mu)^2}{\sigma_h} \tag{8.41}$$

The optimal order size can be expressed by

$$q_i^{FB} = \mu + F^{-1}{}_{N(0,1)}(CR) \cdot \sigma_i \tag{8.42}$$

Inserting (8.42) into (8.41) gives

$$\frac{(F^{-1}{}_{N(0,1)}(CR) \cdot \sigma_l)^2}{\sigma_l} \le \frac{(F^{-1}{}_{N(0,1)}(CR) \cdot \sigma_h)^2}{\sigma_h} \tag{8.43}$$

And after some manipulations we get $\sigma_l \le \sigma_h$ which is true by assumption.

Step 3: Next we show that $B_l \le B_h$ directly translates to compensation schemes that are not incentive compatible. For the sake of clear exposition we define the order size of type *i* optimizing with respect to bonus B_j as

$$q_{ij}{*} = \arg\max_{q_{ij}} \gamma \cdot \pi_i(q_{ij}) + B_j \cdot F_i(q_{ij}) \tag{8.44}$$

From (5.3) we get

$$T_l = R - \gamma \cdot \pi_l(q_l^{FB}) - B_l \cdot F_l(q_l^{FB}) \text{ and} \tag{8.45}$$

$$T_h = R - \gamma \cdot \pi_h(q_h^{FB}) - B_h \cdot F_h(q_h^{FB}) \tag{8.46}$$

From (5.6) we get for $i = l, j = h$

$$\gamma \cdot \pi_l(q_{ll}*) + B_l \cdot F_l(q_{ll}*) + T_l \geq \gamma \cdot \pi_l(q_{lh}*) + B_h \cdot F_l(q_{lh}*) + T_h \tag{8.47}$$

Inserting (8.45) and (8.46) into (8.47) gives

$$0 \geq \gamma \cdot \left(\pi_l(q_{lh}*) - \pi_h(q_h^{FB})\right) + B_h \cdot \left(F_l(q_{lh}*) - F_h(q_h^{FB})\right) \tag{8.48}$$

Because $\pi_l(q_{lh}*) > \pi_h(q_h^{FB})$, it suffices to show for incentive incompatibility that $F_l(q_{lh}*) \geq F_h(q_h^{FB})$. This follows directly $B_h > B_l \rightarrow q_{lh}^* > q_l^{FB}$ and $F_l(q_{lh}^*) > F_l(q_l^{FB}) = F_h(q_h^{FB})$.

Proposition 6: We only proof this proposition for the case of $F_{j>i}(\cdot) < F_i(\cdot)$ and $F_{j<i}(\cdot) > F_i(\cdot)$. The proof for the alternative cases follows analogously.

The results of the two type case can be translated to the n type case. We get according to (8.14) and (8.15)

$$\gamma \cdot \left(\pi_j\left(q_i^{FB}\right) - \pi_i\left(q_i^{FB}\right)\right) \leq B_i \cdot \left(F_i(q_i^{FB}) - F_j(q_i^{FB})\right) \tag{8.49}$$

Rearranging (8.49) for $i < j$ gives

$$B_{j>i} \geq \frac{\gamma \cdot \left(\pi_{j>i}\left(q_i^{FB}\right) - \pi_i\left(q_i^{FB}\right)\right)}{F_i(q_i^{FB}) - F_{j>i}(q_i^{FB})} \tag{8.50}$$

Accordingly, we get for $i > j$

$$B_{j<i} \leq \frac{\gamma \cdot \left(\pi_{j<i}\left(q_i^{FB}\right) - \pi_i\left(q_i^{FB}\right)\right)}{F_i(q_i^{FB}) - F_{j<i}(q_i^{FB})} \tag{8.51}$$

Thus, the inventory related compensation element must lie in between

$$\frac{\gamma \cdot \left(\pi_{j>i}\left(q_i^{FB}\right) - \pi_i\left(q_i^{FB}\right)\right)}{F_i(q_i^{FB}) - F_{j>i}(q_i^{FB})} \leq B_i \leq \frac{\gamma \cdot \left(\pi_{j<i}\left(q_i^{FB}\right) - \pi_i\left(q_i^{FB}\right)\right)}{F_i(q_i^{FB}) - F_{j<i}(q_i^{FB})} \tag{8.52}$$

First of all, we show that it suffices to consider only the adjacent types, i.e., j=i+1 and j=i-1 when calculating the optimal inventory related compensation element.

From *Lemma 1* we get for $\sigma_{i+1} \geq \sigma_i \; \forall i = 1,...,n-1$

$$\pi_1(q) \geq ... \geq \pi_{i-1}(q) \geq \pi_i(q) \geq \pi_{i+1}(q) \geq ... \geq \pi_n(q) \tag{8.53}$$

Moreover we have by definition $F_{j>i}(\cdot) < F_i(\cdot)$ and $F_{j<i}(\cdot) > F_i(\cdot)$

For $j > i$ follows directly that

$$\frac{\gamma \cdot \left(\pi_{i+1}\left(q_i^{FB}\right) - \pi_i\left(q_i^{FB}\right)\right)}{F_i(q_i^{FB}) - F_{i+1}(q_i^{FB})} \geq \frac{\gamma \cdot \left(\pi_{i+2}\left(q_i^{FB}\right) - \pi_i\left(q_i^{FB}\right)\right)}{F_i(q_i^{FB}) - F_{i+2}(q_i^{FB})} \geq ... \geq \frac{\gamma \cdot \left(\pi_n\left(q_i^{FB}\right) - \pi_i\left(q_i^{FB}\right)\right)}{F_i(q_i^{FB}) - F_n(q_i^{FB})} \tag{8.54}$$

And analogously for $j < i$

$$\frac{\gamma \cdot \left(\pi_{i-1}\left(q_i^{FB}\right) - \pi_i\left(q_i^{FB}\right)\right)}{F_i(q_i^{FB}) - F_{i-1}(q_i^{FB})} \leq \frac{\gamma \cdot \left(\pi_{i-2}\left(q_i^{FB}\right) - \pi_i\left(q_i^{FB}\right)\right)}{F_i(q_i^{FB}) - F_{i-2}(q_i^{FB})} \leq ... \leq \frac{\gamma \cdot \left(\pi_1\left(q_i^{FB}\right) - \pi_i\left(q_i^{FB}\right)\right)}{F_i(q_i^{FB}) - F_1(q_i^{FB})} \tag{8.55}$$

Thus, to show that there exit an incentive compatible compensation scheme, it suffices to show that there exist according to (8.54) and (8.55) an inventory related compensation that satisfies

$$\frac{\gamma \cdot \left(\pi_{i+1}\left(q_i^{FB}\right) - \pi_i\left(q_i^{FB}\right)\right)}{F_i(q_i^{FB}) - F_{i+1}(q_i^{FB})} \leq B_i \leq \frac{\gamma \cdot \left(\pi_{i-1}\left(q_i^{FB}\right) - \pi_i\left(q_i^{FB}\right)\right)}{F_i(q_i^{FB}) - F_{i-1}(q_i^{FB})} \tag{8.56}$$

which is true, because $F_{i-1}(q_i^{FB}) > F_{i+1}(q_i^{FB})$ and $\pi_{i-1}\left(q_i^{FB}\right) \geq \pi_{i+1}\left(q_i^{FB}\right)$.

Proposition 7: We can assume without loss of generality that always the upper value from proposition 6, formula (8.54) for the inventory related compensation is taken. For an infinitesimal change of the variance we get

$$B(\hat{\sigma}) = \frac{\gamma \cdot \left.\frac{\partial \pi_\sigma(q_{\hat{\sigma}}^{FB})}{\partial \sigma}\right|_{\hat{\sigma}=\sigma}}{-\left.\frac{\partial F_\sigma(q_{\hat{\sigma}}^{FB})}{\partial \sigma}\right|_{\hat{\sigma}=\sigma}}. \tag{8.57}$$

The fixed payment can be computed from the participation constraint.

Example uniform distribution: We assume a uniform demand distribution with constant mean where the standard deviation is private information. We thus have $U(a_i, b_i)$ with $a_i = \mu - \sqrt{3}\sigma_i$ and $b_i = \mu + \sqrt{3}\sigma_i$. We do not have to make any assumptions regarding the a-priori distribution of types, since the optimal solution does not depend on the a-priori distribution. We get for the CDF's of type $\hat{\sigma}$

$$F_{\sigma}(q_{\hat{\sigma}}^{FB}) = \frac{q_{\hat{\sigma}}^{FB} - a_i}{b_i - a_i} = \frac{q_{\hat{\sigma}}^{FB} - \left(\mu - \sqrt{3}\sigma\right)}{\sqrt{12}\sigma} \quad \text{and} \tag{8.58}$$

$$\left.\frac{\partial F_{\sigma}(q_{\hat{\sigma}}^{FB})}{\partial \sigma}\right|_{\hat{\sigma}=\sigma} = \frac{\mu - q_{\hat{\sigma}}^{FB}}{\sqrt{12}\hat{\sigma}^2} \tag{8.59}$$

For the profit functions we get

$$\pi_{\sigma}(q_{\hat{\sigma}}^{FB}) = -\frac{(p-s)}{\sqrt{12}\sigma}\left(q_{\hat{\sigma}}^{FB} - \mu + \sqrt{3}\sigma\right)^2 + (p-c)q_{\hat{\sigma}}^{FB} \qquad \text{and} \tag{8.60}$$

$$\left.\frac{\partial \pi_{\sigma}(q_{\hat{\sigma}}^{FB})}{\partial \sigma}\right|_{\hat{\sigma}=\sigma} = -\left(p-s\right)\left[\frac{\sqrt{3}}{2} - \frac{\left(q_{\hat{\sigma}}^{FB} - \mu\right)^2}{\sqrt{12}\hat{\sigma}^2}\right] \tag{8.61}$$

For the optimal order size we get

$$q_{\hat{\sigma}}^{FB} = CR \cdot \sqrt{12}\hat{\sigma} + \mu - \sqrt{3}\hat{\sigma} \tag{8.62}$$

Inserting (8.59) ,(8.61), and (8.62) into (8.57) gives after some rearrangements

$$B\left(\hat{\sigma}\right) = -\frac{\sqrt{3}}{2} \cdot \gamma \cdot \left(p-s\right)\frac{\left[1-\left(2CR-1\right)^2\right]}{\left(1-2CR\right)}2\hat{\sigma} \tag{8.63}$$

The fixed payment then results from (5.3). □

Proposition 8: According to (8.12) and (5.2) we get

$$T_l = R - \gamma \cdot \pi_l\left(q_l^{FB}\right) - \left(1-\lambda\right) \cdot B_l - B_l \cdot \lambda \cdot F_l(q_l^{FB}) \le R - \gamma \cdot \pi_h\left(q_l^{FB}\right) - \left(1-\lambda\right) \cdot B_l - B_l \cdot \lambda \cdot F_h(q_l^{FB}) \tag{8.64}$$

It can be seen that the constant term $\left(1-\lambda\right) \cdot B_l$ crosses out. Thus, this term will be directly deducted from the fixed payment without having any impact on incentive compatibility. After some rearrangements of (8.64) we get

$$\gamma \cdot \left(\pi_h\left(q_l^{FB}\right) - \pi_l\left(q_l^{FB}\right)\right) \le B_l \cdot \lambda \cdot \left(F_h(q_l^{FB}) - F_l(q_l^{FB})\right) \tag{8.65}$$

and we get

$$B_l \ge \frac{\gamma \cdot \left(\pi_h\left(q_l^{FB}\right) - \pi_l\left(q_l^{FB}\right)\right)}{\alpha \cdot \left(F_h(q_l^{FB}) - F_l(q_l^{FB})\right)} \quad \text{for} \quad F_h(q_l^{FB}) \ge F_l(q_l^{FB}) \tag{8.66}$$

The other cases follow accordingly as in *Proposition 2*. For the multi-type situation we therefore get according to (8.56)

$$\frac{\gamma \cdot \left(\pi_{j<i}\left(q_i^{FB}\right) - \pi_i\left(q_i^{FB}\right)\right)}{\alpha \cdot \left(F_i(q_i^{FB}) - F_{j<i}(q_i^{FB})\right)} \leq B_i \leq \frac{\gamma \cdot \left(\pi_{j>i}\left(q_i^{FB}\right) - \pi_i\left(q_i^{FB}\right)\right)}{\alpha \cdot \left(F_i(q_i^{FB}) - F_{j>i}(q_i^{FB})\right)} \tag{8.67}$$

We see immediately that α crosses out. All other proofs follow accordingly.

□

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