



**Strategic Effects of Regulatory Capital  
Requirements in Imperfect Banking  
Competition**

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# Strategic Effects of Regulatory Capital Requirements in Imperfect Banking Competition

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## Abstract

This paper analyses the competitive effects of capital requirement regulation on an oligopolistic credit market. In the first stage, banks choose the structure of refinancing their assets, thereby making an imperfect commitment to a loan capacity as a function of the chosen degree of capitalization and the regulatory capital requirement. In the second stage, loan price competition takes place. It is shown that a capital requirement regulation may not only decrease the supply of credit through an increased marginal cost effect but can have an additional collusive enhancing effect resulting in even higher credit prices and increased profits for the banks.

JEL classification: G21, K23, L13

Keywords: equity regulation, oligopoly, capacity constraint

## 1. Introduction

The soundness of the banking system is fundamental for economic wealth and stability. Since the banking sector is particularly vulnerable to inefficient bank runs and contagion resulting in bank panics, the overall aim of banking regulation is to secure financial stability by minimizing, ex ante, the likelihood of bank runs, reducing ex post contagion, when banks fail. To reach this goal, most countries have introduced a governmental safety net including deposit insurances, lender of the last resort and bailout policies. The undesirable secondary effect of such a go-

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vernmental safety net is the destruction of the market discipline, which provides strong moral hazard incentives to exploit the option value of the safety net. Greenbaum and Thakor (1995) summarize this idea by stating: “The moral hazard engendered by one form of regulation, namely deposit insurance, creates the need for other forms of regulation such as capital requirements.” The intuition is that well capitalized banks have fewer incentives to increase asset risks. A bank endowed with more capital is less likely to exploit the option value of the deposit insurance and therefore, the probability of banking default is reduced.<sup>1</sup>

Yet, the actual impact of a regulative capital requirement on the individual behavior of banks and the individual incentives to take excessive risk is not undisputed in banking theory literature. Berger, Herring and Szegö (1995), Santos (2001), and Van Hoose (2007) offer comprehensive reviews of the theoretical literature on the impact of capital requirement regulation.

Regardless of the ambiguous theoretical predictions, there is a general consensus<sup>2</sup> that higher equity has a positive direct effect on the balance sheet structure. Banks facing a binding capital requirement have to either reduce their assets or increase their equity, thereby increasing the capital buffer for the case of asset default. Hence, a more stringent capital requirement reduces the set of states in which the bank defaults, and at the same time, it reduces the costs for the debt holder (the depositors or certain deposit insurance) given default of the bank.

In the short-run, an increase of equity to match the regulatory requirement may prove costly. Therefore, the immediate effect of increasing the capital requirements is likely to be a reduction in the total supply of loans and, accordingly, an increase in the credit interest rate. This effect is most often analyzed in a perfect competition environment, but would also stay valid in a Bertrand oligopoly. Thakor (1996), for instance, discusses that higher capital requirements increase the probability of each borrower being rationed by a bank competing in Bertrand competition. He argues that if the additional costs of raising equity are higher relative to other sources of raising money, then the bank may refrain from further lending and prefer to invest in marketable securities rather than in loans.

In addition to this cost effect, the introduction of a binding capital requirement regulation can have a second effect that has not yet been considered in banking literature. In fact, a binding capital requirement changes the sequence in which the strategic decisions are made, since it temporarily constrains the bank’s lending activities.<sup>3</sup> This idea goes back to Edgeworth (1988) who emphasizes that due to exogenous capacity constraints, Bertrand oligopolists may not be able to serve the whole market demand and therefore would not undercut each until the competitive equilibrium is reached. Kreps and Scheinkman (1983) generalized this idea for an endogenous capacity choice. In their two stage model, the oligopolists first compete in capacities, followed by a competition in prices, which is strictly constrained by the prior capacity decision. Kreps and Scheinkman (1983) conclude that when firms commit to a certain capacity of production before price competition takes place, the capacity and prices chosen in equilibrium are identical to the Cournot equilibrium. As a result, banks competing in a homogenous Bertrand competition can generate Cournot profits instead of zero profits. The question

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<sup>1</sup> See e.g. Furlong and Keeley (1989).

<sup>2</sup> Van Hoose (2008).

<sup>3</sup> This is in line with Brander and Lewis (1986) who analyze the strategic impact of leverage decisions on output decisions. They argue that increases in a firm’s leverage enhance the output level of the firm in a Cournot Oligopoly with random demand. In contrast, we concentrate on the effects of a strategic capital choice in a deterministic Bertrand competition and examine the impact of a capital commitment on the fierceness of the price competition.

that arises is: can such a rigid capacity constraint be applied to the case of lending competition among banks? Freixas and Rochet (1997) even state that a capacity constraint may not be feasible as a starting point for a theoretical analysis in the context of banking.

However, Freixas and Rochet (1997) may have overlooked that binding regulatory capital requirements<sup>4</sup> can affect the nature of strategic competition among banks. In particular, when short term recapitalization is costly, capital requirements temporarily constrain the bank's lending activities and thereby soften the price competition, as already mentioned by Gehrig (1995). In a static framework, a binding capital requirement regulation can, therefore, be abused to transform the Bertrand competition into a sequential game with a Cournot outcome.

In the first stage, the capital regulated banks decide on their refunding structure consisting of equity and deposits. In the second stage, the price competition takes place while the bank's ability to satisfy the demand resulting from the pricing decision is conditioned by the raised amount of equity and the capital requirement regulation. As recapitalization is assumed to be costly, the equity decision in the first stage is an imperfect commitment to capacity for bank loans. Notwithstanding the model of Kreps and Scheinkman (1983), the oligopolistic banks are able to extend their capacities in the second stage, but at additional costs. Such a flexible capacity constrained in Bertrand competition was already discussed by Güth (1995) and Maggi (1996) for differentiated product markets. The assumption of product differentiation thereby avoids one of the main shortcomings, for which the model of Kreps and Scheinkman (1983) has been often criticized. In their model, they assume that the firms compete in a homogenous product market, which necessitates the definition of a specific rationing rule on the customers that determines the specific demand addressed to each supplier. Yet, the derived results are not robust against changes in the specific rationing rule as it is formally proven by Davidson and Deneckere (1986). Assuming product differentiation is, thus, not only reasonable in a relationship bank lending context, but provides the means to well-define the demand of each firm in the second stage for any price pair and therefore avoids the dependency of the results on a specific rationing rule.

Güth (1995) and Maggi (1996) both argue that capacity constrained Bertrand competition yields a Cournot outcome for sufficiently high additional costs of the capacity extension in the second stage. Applying the Maggi (1996) model to a capital regulated market for loans, we analyze the effects of a capital requirement regulation on the strategic behavior of oligopolistic banks. We will show that, if costs of recapitalization are above an identified threshold, the banks would no longer have an incentive to undercut each other in the second stage price competition. Under such an equity commitment, thus, the Bertrand price competition results in a Cournot-Nash equilibrium. Our comparative static analysis shows that an increase in capital requirement decreases the threshold that makes a first stage capacity decision binding. In other words, the higher the capital requirement, the more binding is a commitment to a credit capacity due to a certain choice of capital level.

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<sup>4</sup> Following Berger et.al. (1995), we define capital requirements to be binding if the capital ratio in the presence of regulatory capital requirements is greater than the bank's market capital requirement. Our crucial assumption in our model is, thus, that regulatory capital requirements constrain a significant portion of the banks in their optimal decision on the refinancing structure.

Similar to the discussed direct cost effect, the described collusive<sup>5</sup> effect tends to reduce the total lending resulting from less competitive prices. However, in contrast to the cost effect, the collusive effect is likely to generate positive profits. The generated profits in turn increase the bank's buffer against credit default risk on one hand and provide incentives against excessive risk taking by enhancing the "charter value" of the bank. An increased charter value reflects the higher anticipated future profits that would be lost in case of bankruptcy. The increased expected future losses in turn reduce the incentives to exploit the option value of the limited liability created by the governmental safety net, i.e. deposit insurance. This stabilizing effect has been discussed e.g. by Keeley (1990) and Chan, Greenbaum and Thakor (1992). Based on this charter value argumentation, Allen and Gale (2004) argue that regulators face a trade-off between increasing competition and reducing stability. In line with the charter value argumentation, the identified collusive effect of the capital regulation, thus, further enhances stability but to the cost of decreased competition and a reduction in lending.

The paper is organized as follows. In Section 2 we develop the model, including a detailed analysis of the capital regulated bank's cost structure. We introduce the competitive environment and derive the basic results. In Section 3 we discuss how the capital regulation rate affects the derived results.

## 2. The Model

### 2.1. Model Setup

We model a two stage price competition game among banks. In the first stage, banks can raise equity followed by imperfect price competition in the second stage. Each bank refinances its loan assets by equity and deposits. The refinancing structure of each bank is constrained by a minimum capital requirement rate. This rate is set by a regulator before the game starts. We will discuss the impact of the capital requirement rate on the outcome of our model in Section 3. We assume that all parties are risk-neutral and the bank management is acting in the best interest of the owner(s).

The timeline of our model is as follows. In stage zero, the regulator sets up a certain minimum capital requirement rate. Knowing this rate, the banks raise equity in stage one in order to take part in the pricing competition in stage two. The amount of equity then determines, based on the minimum capital requirement regulation, a capacity to provide loans to borrowers of a certain risk class. In the second stage, the banks compete in prices that turn into loan quantities demanded by the borrowers.

As Tirole (1988) mentions, the Bertrand and the Cournot model should not be seen as two exclusive models that predict contradictory outcomes of imperfect competition. The models rather describe the same markets with different cost structures. A change in the cost structure can transform a Bertrand competition into a two stage game with Cournot outcomes. In order to understand how capital requirement regulation influences the structure of a Bertrand competition among banks we therefore have a deeper look at the cost structure of capital regulated banks.

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<sup>5</sup> We call this effect collusive effect in the sense of spontaneous tacit collusion that results from a non-cooperative Nash equilibrium and does not require any coordination among the competitors.

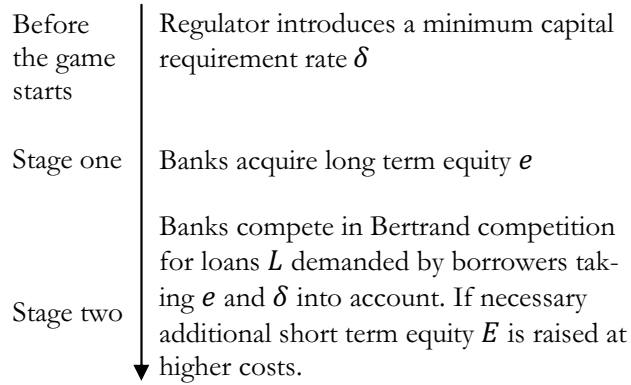


Figure 1: The timeline of decisions taken

Banks can invest solely in loans ( $L$ ) demanded by the representative borrowers of one risk class. The investment in assets is financed by deposits ( $D$ ) and equity raised in the first stage ( $e$ ) and equity raised in the second stage ( $E$ ). This implies the balance sheet constraint:

$$L \leq e + E + D. \quad (1)$$

This constraint is binding if banks maximize profits and cash yields no returns. We assume that depositors are fully insured against default at a premium normalized to zero. This assumption, which is often made in the literature,<sup>6</sup> allows us to neglect competition on the deposits market because insured depositors are insensitive to the bank's exposure to risk and ready to supply any amount of deposits at a deposit rate  $r_D$ .

We assume that the promised return rate on equity is higher than the return on insured deposits,<sup>7</sup> hence  $r_e = r_D + c$ , where  $c > 0$  reflects the promised risk premium. Moreover, we assume that it is costly to acquire additional equity in stage 2. Such additional costs may represent the dilution costs that a bank faces if it urgently needs to raise its equity.<sup>8</sup> We adopt this assumption by defining  $r_E = r_e + \theta$  with  $\theta > 0$ .

Under capital regulation, the minimum capital requirement rate  $\delta \in [0,1]$  forces banks to hold equity of at least  $\delta L$ . Thus, at the beginning of the second stage, holding equity  $e$  results in a capacity commitment: the maximum amount of loans that a bank can give out is  $e/\delta$ . This capacity commitment defines a constraint above which additional recapitalization costs have to be paid. If a bank plans to give out loans  $L > e/\delta$  the bank has to raise additional capital  $E$  at a promised rate  $r_E$ . However, the bank is forced to cover with equity only a share  $\delta$  of the additional loans ( $L - e/\delta$ ). Hence,  $E$  is determined by

<sup>6</sup> See, e.g., Boot and Marinc (2007), Wagner (2010).

<sup>7</sup> This is a common assumption in the literature (see, e. g., Hellmann, Murdock, and Stiglitz, 2000; Repullo, 2004). For explicit models that explain why the cost of capital might be higher than the return that depositors demand see Holmström and Tirole (1997) and Diamond and Rajan (2000).

<sup>8</sup> This cost may consist of to the price difference of urgently issued shares, as argued by Berger, Herring and Szegö (1995), or the cost of organizing an additional general assembly.

$$E = \delta \max(L - e/\delta; 0) = \max(\delta L - e; 0) \quad (2)$$

Note that with infinite recapitalization costs  $\theta$  the bank is unable to raise additional equity in the second stage. Such a setting reflects a rigid capacity constraint as presented in Kreps und Scheinkman (1983). An imperfect capacity constraint is described by finite recapitalization costs  $\theta > 0$ .

We now solve the two stage game by backwards induction.

## 2.2. The Second Stage Bertrand Competition

In the second stage, the capacity  $e/\delta$ , defined by the first stage equity decision, is an exogenous condition of the pricing decision. We first determine the cost and marginal cost function of a bank that provides loans  $L$ .

**Lemma 1:** A capital regulated bank that raises equity at  $r_e$  during the first stage faces, at the beginning of the second stage, the following piecewise defined cost function:

$$C(L) = \begin{cases} r_e e & \text{if } L \leq e, \\ r_D L + ce & \text{if } e < L \leq e/\delta \\ (r_D + c\delta)L + \theta(\delta L - e) & \text{if } L > e/\delta \end{cases} \quad (3)$$

**Proof:** As the balance sheet constraint (1) is binding, we can write  $D$  as a function of loans and equity  $D = L - E - e$ . If the bank plans to give out loans  $L \leq e$ , it finances its assets solely out of the equity raised in the first stage at cost  $r_e$ . Hence,  $D = 0$  and  $E = 0$ .

Intending to provide loans  $L > e$ , the bank needs additional funds. As long as  $e < L < e/\delta$ , deposits are sufficient and no additional equity is required ( $E = 0$ ) in the second stage. Thus, the costs of providing such a loan amount  $L$  are given by  $r_e e + r_D(L - e)$ . As  $r_e - r_D = c$ , providing the total loan amount costs the deposit rate plus the risk premium  $c$  for  $e$ .

The third piece of the cost function depicts a loan amount beyond the capacity constraint  $L > e/\delta$ . In this case, the bank is forced to raise additional equity in the second stage in order to comply with the minimum capital requirement. According to (2), the additional equity amounts to  $E = \delta L - e$ . Refinancing costs can therefore be summarized as  $r_D D + r_e e + r_E E$ . Using the balance constraint (1) leads to  $r_D(L - \delta L) + (r_D + c)e + (r_D + c + \theta)\delta(L - e/\delta)$ , which can be simplified to the third piece of the cost function. ■

Figure 2 illustrates the cost structure for such a capital constrained bank, where the light grey area depicts the fixed costs resulting from the first stage equity choice. From **Lemma 1** we can easily derive the piecewise defined marginal cost function by differentiation:

$$MC(L) = \begin{cases} 0 & \text{if } L \leq e, \\ r_D & \text{if } e < L \leq e/\delta \\ r_D + (c + \theta)\delta & \text{if } L > e/\delta \end{cases} \quad (4)$$

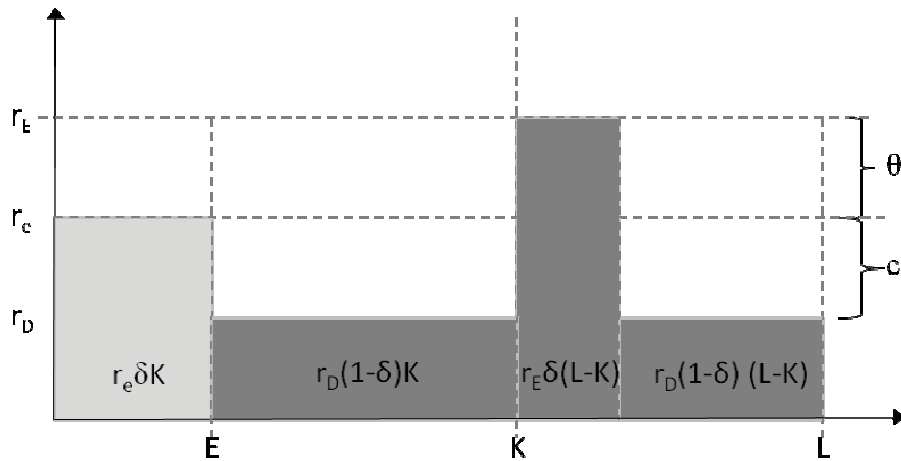


Figure 2: The cost of refunding assets for lending above the capacity

If the loan demand is low  $L(p) \leq \hat{E}$  no marginal costs arise, since the cost of equity is sunk at the second stage. If  $e < L \leq e/\delta$ , the marginal cost equal the cost of deposits. Lending above capacity requires the banks to increase its equity. Thus, the marginal cost of providing additional loans consist of marginal cost of equity plus the recapitalization cost.

We will continue our analysis with introducing the competitive environment and further specify the demand for loans. We assume that banks compete in imperfect price competition. For simplicity, we concentrate on the case of two banks, labelled with the indices  $i, j = 1, 2 ; i \neq j$ , even though the results could be generalized to the case for an arbitrary number of banks.<sup>9</sup>

First, the oligopolistic banks choose the equity amount and then compete in a second stage price competition. The game is solved by backwards induction.

Akin to the model of Maggi (1996), we use a linear representative consumer model with product differentiation to describe the borrowers' demand for loans. The generalized inverse demand function is given with.

$$p_i(L_i, L_j) = a - bL_i - dL_j \quad (5)$$

In (5),  $p_i$  represents the loan interest rate of bank  $i$  and  $L_i$  the total lending of bank  $i$  to the borrowers. Note that  $a > b > d \geq 1$ . If  $d$  were negative, the goods would be complements and if  $d = 0$  the two goods would be independent in demand. As  $d \rightarrow b$  the loans become perfect substitutes. In our analysis, we will concentrate on the more general case of a heterogeneous market, so we do not have to invoke a specific customer rationing rule. The heterogeneity could emerge from the reputation of the bank, the specific service offered to the borrower or relationship banking combined with switching costs to the borrower. Yet, one can argue that bank loans are rather homogenous goods. Therefore, we will also consider the particular case  $d \rightarrow b$  in our analysis.

To secure strictly positive profits we assume  $a > r_E$ . We further assume that banks only choose prices that result in non-negative profits. This assumption

<sup>9</sup> See Boccard and Wauthy (2000) for a generalization of Kreps and Scheinkman (1983) to the oligopoly case with  $n \geq 2$  competitors.



is taken into account by introducing non-negativity constraints into the profit maximization problem:  $(bp_i - dp_j \leq a(b-d))$  and  $(dL_j \leq a - bL_i)$ . Provided that these constraints are not violated, the inverse demand function can be reversed<sup>10</sup> to obtain the direct demand curve:

$$L_i(p_i, p_j) = a/(b+d) - bp_i/(b^2 - d^2) + dp_j/(b^2 - d^2) \quad (6)$$

Just as in Dixit and Norman (1980) we assume in our analysis that a firm is willing to meet any level of demand beyond its installed capacity provided that the price is above the additional costs of extending the capacity. Thus, rationing is excluded in our analysis.

Banks try to maximize their profits subject to the imperfect capacity constraint, implicitly defined by the first stage equity choice  $e_i/\delta$ . Applying **Lemma 1**, the total profit function of bank  $i$  is defined as:

$$\Pi_i(p_i, p_j) = \begin{cases} p_i L_i - r_e e_i & \text{if } L_i \leq e_i, \\ (p_i - r_D)L_i - ce_i & \text{if } e_i < L_i \leq e_i/\delta \\ (p_i - r_D - \delta(c + \theta))L_i + \theta e_i & \text{if } L_i > e_i/\delta \end{cases} \quad (7)$$

The positive last term in the third piece of the profit function reflects the saved costs from raising capital in the first stage. Note that the profit function is not differentiable at the points  $L_i = e_i$  and  $L_i = e_i/\delta$ .

The best price response function of each bank is a function of the opponent's price with different segments determined by the capacity levels chosen in the first round and the parameters of the model. The discontinuous points in the profit function thereby lead to a kinked reaction function. Hence, the best reaction function is characterized by different branches.

**Lemma 2:** In the second stage, the best response function  $R_i^*(p_j)$  can consist of seven branches, depending on the parameters and the chosen levels of  $e$

I	$p_i^{L \leq e} = \frac{(b-d)a + dp_j}{2b}$	If $L_i(R_i^*(p_j), p_j) < e_i$
II	$p_i^e = \frac{(b-d)a - (b^2-d^2)\bar{e}_i + dp_j}{b}$	If $L_i(R_i^*(p_j), p_j) = e_i$
III	$p_i^- = \frac{(b-d)a + dp_j + br_D}{2b}$	If $e_i < L_i(R_i^*(p_j), p_j) < e_i/\delta$
IV	$p_i^{e_i/\delta} = \frac{(b-d)a - (b^2-d^2)\bar{e}_i/\delta + dp_j}{b}$	If $L_i(R_i^*(p_j), p_j) = e_i/\delta$
V	$p_i^+ = \frac{(b-d)a + dp_j + b(r_D + \delta(c + \theta))}{2b}$	If $L_i(R_i^*(p_j), p_j) > e_i/\delta$
VI	$p_i^{L_j=0} = \frac{bp_j - (b-d)a}{d}$	If $L_j(R_i^*(p_j), p_j) = 0$
VII	$p_i^M = \frac{1}{2}(a + r_d + \delta(c + \theta))$	If $L_i(R_i^*(p_j), p_j) \leq L_i^M$

<sup>10</sup> The inversion is only allowed under the assumption that both firms always satisfy their demand, otherwise, the quantity demanded of one firm is rather a function of the residual demand left by the rationing opponent. See Boccard and Wauthy (1998) for an analysis of the robustness of the Güth-Maggi results when firms are allowed to ration their customers. Furthermore, these equations are only valid for  $b \neq d$ .

**Proof:** To prove Lemma 2, we proceed as follows. We first derive the response functions for each piece of the profit function. Then we consider the discontinuous points.

From ( 7 ) we get bank i's piecewise defined objective function  $\max_{p_i} \Pi_i(p_i, p_j)$ . The best reaction function  $R_i^*(p_j)$  is implicitly defined by  $\Pi'_i(R_i^*(p_j), p_j) = 0$ . The first order conditions for the three pieces of the bank i's objective function can be summarized as  $(p_i - MC)L'_i + L_i = 0$  with  $L'_i = -\frac{b}{b^2 - d^2}$ . Substituting ( 6 ) and solving for  $p_i$  gives:

$$p_i = \left( (b-d)a - (b^2 - d^2)\hat{E}_i + dp_j + b(MC) \right) / 2b \quad (8)$$

Substituting ( 4 ) into ( 8 ) gives the branches I, III and V of bank i's best response function:

*Branch I:* If the best reaction to the price chosen by bank j results in a demand  $L_i(R_i^*(p_j), p_j) < e_i$ , the marginal costs in the second stage are  $MC(L(p)) = 0$ . Substituting MC and solving for  $p_i$  gives the best response  $p_i^{L \leq e} \equiv \frac{(b-d)a + dp_j}{2b}$ .

*Branch III:* Similarly, for  $e_i < L_i(R_i^*(p_j), p_j) < e_i/\delta$  the second stage marginal costs are  $MC(L(p)) = r_D$  resulting in the best reaction  $p_i^- \equiv \frac{(b-d)a + dp_j + br_D}{2b}$ . The intuition is, that a relatively low price of the opponent induces a high demand for loans from the opponent bank and only few loans demanded from bank i such that the demand is smaller than the capacity. Therefore, the optimal response  $p_i^*$  is the Bertrand price for producing below the capacity constraint.

*Branch V:* If the best reaction to the opponent's price results in a demand  $L_i(R_i^*(p_j), p_j) > e_i/\delta$  the optimal price response is  $p_i^+ = \frac{(b-d)a + dp_j + b(r_D + \delta(c + \theta))}{2b}$ . If the opponent chooses a high price, such that the residual demand for loans from bank i raises above the installed capacity, the optimal response is to expand capacity, taking recapitalization costs  $\theta$  into account.

*Branches II and IV* result from the discontinuous jumps in the marginal cost curve at the points where  $L_i(R_i^*(p_j), p_j) = e_i$  and  $L_i(R_i^*(p_j), p_j) = e_i/\delta$ . These equalities implicitly describe the best response price. Solving for the best reaction we get:

$$\begin{aligned} \text{Branch II: } p_i^e &\equiv \frac{(b-d)a - (b^2 - d^2)e_i + dp_j}{b} \quad \text{and} \\ \text{Branch IV: } p_i^{e_i/\delta} &\equiv \frac{(b-d)a - (b^2 - d^2)e_i/\delta + dp_j}{b} \end{aligned}$$

These two functions give the possible price combinations that make the loan demand of bank i just equal to the precommitted equity amount or the capacity respectively.

*Branch VI* results from the non-negativity constraints on demand in the optimization problem:  $0 \leq L_j = a/(b+d) - bp_j/(b^2 - d^2) + dp_i/(b^2 - d^2)$ . A price pair that results in a negative demand for the opponent makes the non-negativity constraint binding. Solving for  $p_i$  yields  $p_i^{L_j=0} \equiv \frac{bp_j - (b-d)a}{d}$ .

*Branch VII* is the monopoly price solving the maximization problem  $p_i^M \equiv \operatorname{argmax} (p_i - r_D - \delta(c + \theta)) L_i(p_i) + \theta \hat{E}_i$  that is solved by  $p_i^M = \frac{1}{2}(a + r_d + \delta(c + \theta))$ .

The intuition behind this result is that the increase in the opponent's price for loans by bank j raises the loans demanded from bank i and allows for further

increases in  $p_i$ . If the opponent's price choice is higher than the price determined by  $L_j(R_i^*(p_j), p_j) = 0$ , then bank  $i$ 's best response is to further increase its price until the best response price equals the monopoly price. Yet, bank  $i$  would only increase its price up to the monopoly price. For any further increase in the opponent's price, it would not be optimal to increase also  $p_i$  since the monopoly price is by definition, the profit maximizing price. The best response price is, thus, independent of the opponent's price and the best response function horizontal. ■

After we have identified all feasible parts of the best response function we categorize the critical opponent's price levels for which each function becomes a part of bank  $i$ 's best response function.

**Proposition 1:** The best response function of bank  $i$  is the piecewise defined function:

$$R_i^*(p_j) = \begin{cases} p_i^{L \leq e} & \text{if } p_j < p^l \\ p_i^e & \text{if } p^l \leq p_j \leq p^h \\ p_i^- & \text{if } p^h < p_j < p_j^L \\ p_i^{e_i/\delta} & \text{if } p_j^L \leq p_j \leq p_j^H \\ p_i^+ & \text{if } p_j^H < p_j \leq p_j^{L_j=0} \\ p_i^{L_j=0} & \text{if } p_j^{L_j=0} < p_j < p_j^{L_i=L^M} \\ p_i^M & \text{if } p_j^{L_i=L^M} \leq p_j \end{cases}$$

**Proof:** Bank  $i$  chooses the optimal price  $p_i$  along its residual demand curve for a given  $p_j$ . As  $p_j$  increases the optimal price reaction is thereby given by the identified response functions. The intersections of the respective response functions determine the individual threshold values for  $p_j$ . The closed forms of the threshold values are given in the Appendix. ■

The entire best reaction function of bank  $i$ , denoted as  $R_i^*(p_j)$ , is depicted in Figure 3.

**Proposition 2:** If  $d \neq b$  then the equilibrium price vector  $p^* = (p_i^*, p_j^*)$  in each second stage subgame is unique. This equilibrium is defined by the intersection of either branch I, II, III, VI or V.

**Proof:** Branches VI and VII are not feasible candidates for a Nash equilibrium since the firm earning zero profits could strictly increase its profits by offering a lower credit price and, thus, would be strictly better off by deviating from the high price decision. The remaining branches define a best reaction function that is kinked, but continuous and monotone increasing. The slope of branches I, III and V equals  $d/2b$  and the slope of branches II and IV is equal to  $d/b$ .

With  $b > d > 0$  all branches of the best reaction function have a slope between 0 and 1. Hence, the intersection between the two banks' best response correspondences is unique. ■

For the particular case where  $b = d$  the slope of branch II and IV would  $\frac{\partial R_i^*(p_j)}{\partial p_j} = 1$ , such that an intersection of the reaction functions at these branches give a continuum of equilibria as illustrated in Figure 4.

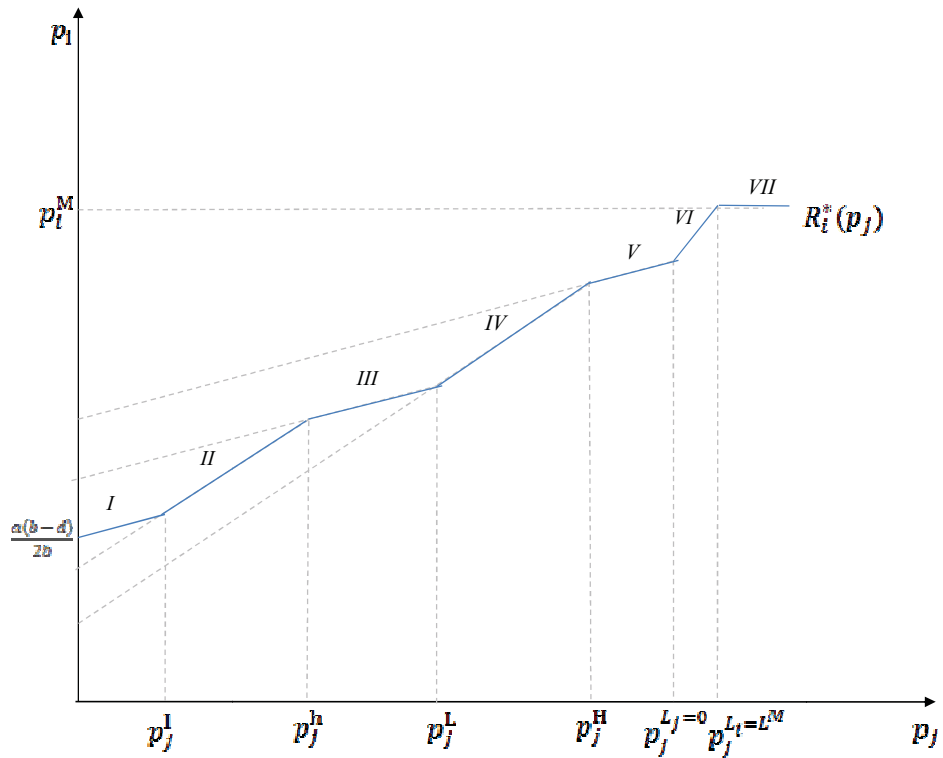


Figure 3: The best reaction function in the second stage

Yet, even a slight degree of differentiation results in a unique equilibrium solution. Since this is true for both agents, whenever  $d \neq b$  there must be a unique intersection of the best response functions that defines a pure strategy equilibrium in the price subgame for any pair of capacities installed in the first stage. Since the equilibrium is determined by a unique intersection of 5 branches, there are five possible second stage equilibrium types in our symmetric model.

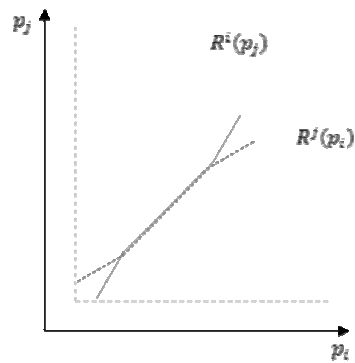


Figure 4: Continuum of equilibria for perfect homogenous loans

Knowing the possible intersections of the reaction functions in the second stage, it is possible to determine the first stage payoffs as a function of the respective capacity choices.

Note that, in the long run, Branch I and II would result in negative profits of both banks and thus a breakdown of the market. We, therefore, restrict our analysis to the cases where banks refinance their assets with deposits in order to make non-negative long-run profits.

In general, the optimal price choice is implicitly defined at the point where the marginal benefit, reflected by an increased demand resulting from lowering the price, equals the marginal costs of expanding the supply of loans. For very low additional costs ( $\theta \rightarrow 0$ ) the capacity choice in the first stage would not be binding and the equilibria of the two stage game would coincide with the Bertrand equilibrium in quantities and prices.

If the costs of adjusting equity are significantly different from zero ( $\theta > 0$ ) the highest price that can be sustained as equilibrium is, thus:

$$p_B^\theta = \frac{((b-d)a + b(r_D + (c+\theta)\delta))}{(2b-d)}. \quad (9)$$

With an increasing cost factor  $\theta$  as the highest attainable price, the symmetric Bertrand equilibrium price with recapitalization cost also increases. If the recapitalization costs are equal to or higher than the critical value  $\theta^H$ , the highest attainable Bertrand equilibrium price (with recapitalization costs) equals to the Cournot price for a supply within the capacity constraint. This critical value is implicitly defined at the point where the Bertrand best price response with marginal costs including the cost or recapitalization  $r_D + (c+\theta)\delta$  equals the Cournot best response price with marginal costs equal to  $r_D + c\delta$ :

$$\theta^H = \frac{d^2(a - r_D - c\delta)}{(2b-d)b\delta}, \quad (10)$$

Hence, if the short term expansion costs are above this critical level, it would not be profitable to provide loans above the capacity. In other words, the capacity commitment becomes strictly binding. As argued by Maggi (1996)  $\theta$  determines, thus, the irrevocability of the precommitment. The higher the  $\theta$ , the more effective the capacity commitment device is. The interesting implication for the regulation of banks is the impact of the regulatory requirement on the critical level of recapitalization costs.

**Lemma 3:** A higher minimum capital requirement ratio  $\delta$  reduces  $\theta^H$ .

**Proof:** Differentiating the critical cost level with respect to the capital requirement gives:

$$\frac{\partial \theta^H}{\partial \delta} = -\frac{d^2(a - r_D)}{(2b-d)b\delta^2} \quad (11)$$

Since  $a > r_D$  and  $b > d$ <sup>11</sup>, the partial derivatives clearly indicate a negative relation between the capital requirement and the critical recapitalization cost level that changes the Bertrand competition into Cournot equilibrium outcomes. ■

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<sup>11</sup> As  $d$  approaches  $b$  (the goods are nearly homogenous) this value approaches  $\left(-\frac{(a-r_D)}{\delta^2}\right)$ , which is clearly negative.

### 2.3. The First Stage Equity Choice

Anticipating the best reaction of the second stage, bank  $i$  will choose, in the first stage, an optimal level of equity that results in a capacity equal to the equilibrium demand of the second stage. It would not be profit enhancing to deviate from a capacity choice that equals demand in the second stage since equity is expensive. A profit-maximizing bank therefore takes no more equity than it is required by regulation to satisfy the loans demanded in the second stage. Reducing the equity to the amount the required loans demand, in the second stage, would not affect the demand and prices but saves costs. Similarly, a capacity below the anticipated equilibrium demand would not be optimal because raising additional equity in the first stage would save costs of recapitalization in the second stage without influencing the equilibrium prices or demand in the second stage. Hence, in the first stage equilibrium, a profit-maximizing bank will raise the exact amount of equity that satisfies the equilibrium demand in the second stage  $E_i/\delta = L_i$ .

Applying this argumentation to both agents, it becomes clear that only the interceptions of branches IV of the best response function qualify for a Nash-equilibrium of the whole game. This interception defines a capacity clearing equilibrium where the prices chosen in the second stage guarantee a demand that just clears the capacity defined by the equity raised in the first stage.

**Lemma 4:** When  $0 < \theta < \theta^H$ , the equilibrium equity choice is characterized by  $E_i = L_i (p_B^\theta)\delta$  and prices equal to  $p_B^\theta$ .

**Proof:** For intermediate recapitalization costs, the banks would still have incentives to revoke the capacity commitment. When installing Cournot capacities they would have the incentives to undercut the prices of the opponent. Thus, the Nash equilibrium for intermediate costs is exactly the capacity that allows to meet the demanded quantity of Bertrand prices for marginal costs equal to  $(r_D + \delta(c + \theta))$ , which was defined above as  $p_B^\theta$ . For a price that exactly equals the marginal costs with recapitalization, the banks have no incentives to undercut the opponent in prices and thus, no bank will deviate from the strategy. Anticipating this price pair, the optimal capacity chosen is, thus,  $e_i/\delta = L_i (p_B^\theta)^{12}$ . ■

In the second stage, the optimal prices chosen then generate a demand that exactly clears the capacity. Without any excess demand above capacity, no recapitalization costs occur to the bank. Hence, the imperfect capacity commitment in the first stage allows banks to set their prices higher (by  $\delta\theta$ ) than the marginal costs that actually occur to the bank and therefore raise profits.

Now consider the case where costs are  $\theta \geq \theta^H$ . Given that in equilibrium the capacity decision will equal the anticipated demand, firms know that the optimal price in the second stage will be the intersection of branch two of both agents. The symmetric anticipated equilibrium price will be defined by

$$p_i^{\theta \geq \theta^H} = a - e_i/\delta - e_j/\delta. \quad (12)$$

Both banks simultaneously maximize the objective function with respect to the constraint of the optimal equity level.

<sup>12</sup> Formally the optimal capacity is the quantity equal to the demand for the price pair that maximizes  $\max(p_i - r_D + c\delta)D(p_i, p_j)$  subject to  $p_j = R_j(p_i, j/\delta)$  and  $p_i^- \leq p_i^* \leq p_i^+$ .

$$\Pi_i = (p_i^{\theta \geq \theta^H} - r_D) e_i / \delta - ce_i \quad (13)$$

The First Order Condition then gives the optimal equity choice

$$e_i^* = \frac{a - (r_D + \delta c)}{(2b + d)\delta} \quad (14)$$

The second stage best reply to the installed capacity is then the Cournot prices:

$$p^* = \frac{(ab + (b + d)(r_D + \delta c))}{2b + d} \quad (15)$$

The optimal symmetric capacity and the according prices result in the following symmetric profits:

$$\Pi^* = \frac{b(a - (r_D + \delta c))^2}{(2b + d)^2} \quad (16)$$

If banks would compete in a Cournot competition on quantities with symmetric costs, they would select their optimal output as a function of the other bank's optimal loan supply.

**Lemma 5:** The symmetric Cournot equilibrium is given by the Cournot quantity  $L^C = \frac{a - (r_D + \delta c)}{2b + d}$ , leading to the Cournot equilibrium price  $p^C = \frac{ab + (b + d)(r_D + \delta c)}{2b + d}$  and equilibrium payoffs  $\pi^C = \frac{b(a - (r_D + \delta c))^2}{(2b + d)^2}$ .

**Proof:** Competing in a Cournot competition, bank  $i$  maximizes the profit function  $\pi_i = (p_i(L_i, L_j) - (r_D + \delta c))L_i$  with respect to the optimal loan amount. The first order condition for a maximum determines the best response function of bank  $i$  to any amount of loans supplied by bank  $j$ , which is:

$$R^C(L_j) = \frac{a - dL_j - (r_D + \delta c)}{b} \quad (17)$$

The banks' best response functions enable us to derive the Cournot equilibrium. ■

**Proposition 3:** For recapitalization costs  $\theta \geq \theta^H$  the equilibrium of the two stage game is characterized by the Cournot equilibrium prices and quantities.

**Proof:** Substitute (1) into (14) and compare with **Lemma 5**. ■

### 3. Policy Implications and Discussion

A minimum capital requirement can turn a Bertrand competition in the loans market into a Cournot equilibrium resulting in lower credit output, higher credit prices and higher profits for oligopolistic banks. In contradiction to the literature arguing that regulatory capital requirements increase the cost of producing loans and thereby reducing the profits of the banks, this paper shows that the regulation can also increase the bank's profits, and thus, the charter value of the bank in excess of the increased cost of equity. Following the argumentation of the charter value hypothesis, such increased profits reduce the bank's risk taking incentives. This decrease in the bank's exposure to risk may enhance the stability in banking, however, to the social costs of an additional reduction in lending. These two effects may alter the decision on the optimal requirement rate by the regulator in a

stage zero. In such a zero stage, the regulator has to balance, on one hand, the aim of stabilizing the banking sector against the aim of enhancing competition and efficiency.

**Proposition 4:** A critical value of the minimum capital requirement  $\delta^H$  exists such that, whenever  $\delta \geq \delta^H$  then, ceteris paribus,  $L_i^* = L^C$  (resulting from  $p_i^* = p^C$ ). This leads to equilibrium profits  $\Pi_i^* = \Pi^C$ .

**Proof:** Holding the recapitalization costs fixed, equation ( 10 ) implicitly defines  $\delta^H$  which is the critical level of a minimum capital requirement that may be chosen by the regulator at a stage zero. This value leads to the critical regulatory capital requirement that induces collusive behavior among price competing banks. A requirement equal to or above this level, allows banks to gain Cournot profits by further raising prices and thereby decreasing lending. ■

When setting the minimum capital requirement for banks, the regulator, thus, not only needs to balance the cost effect but also the identified collusive effect against enhanced stability to find an optimal regulatory capital requirement.

The analysis has shown that a binding regulatory capital requirement reduces the incentives of competing banks to undercut in prices. This collusive effect results from the strategic complementarity of prices. The intensification of capital requirements restricts the market volume and thus results in higher interest rates. The banks have fewer incentives to engage in fierce Bertrand competition. The intensity of the price competition in the second stage thereby depends on the cost of recapitalization and the level of capital requirement. The higher the costs of recapitalization  $\theta$ , the higher the mark-up on the Bertrand price and the lower the loans demanded in the second stage. This holds until the cost of recapitalization reaches the critical level  $p_B^\theta = p^C$ . For any costs equal or above the critical level, it will be optimal to install Cournot capacities in the first stage and ask for Cournot prices in the second stage, which maximizes the non-cooperative equilibrium profits. Therefore, this paper offers a justification for the usage of the Cournot model in the context of banking.

In contrast to the literature on the impact of capital regulation, this analysis suggests that banks in fierce Bertrand competition may benefit from the introduction of a binding capital constraint due to regulatory capital requirements. This suggests a certain demand for regulation on the side of banks.

From the point of view of the regulator this result has ambiguous implications. On one hand, the regulator might prefer Bertrand competition among banks with lower prices and higher loan supply for the macroeconomic benefits of efficiency as described for example by Smith (1998). On the other hand, the higher profits resulting from a reduction in competitive fierceness may further stabilize the banking sector. Assuming that the regulator aims at an optimal trade-off between incentives for competitiveness of bank services and the solvency and stability of the industry, the effects on the strategic interaction among banks may be minor compared to other incentive effects of the requirement that influence the stability of banks. Nevertheless, they should be taken into account in the design of prudential banking regulation. The collusive effect should also be considered, especially in the discussion of an increase of capital requirements. Since banking regulation also tries to reduce the bank's exposure to risk, it will be important to analyze the capital constraint effects on a bank's portfolio risk decision.



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## Appendix

### Proof of Proposition 1

$$p_j^l: p_i^{L \leq e} = p_i^e$$

$$\frac{(b-d)a + dp_j}{2b} = \frac{(b-d)a - (b^2 - d^2)e_i + dp_j}{b}$$

$$p_j^l \equiv \frac{2(b^2 - d^2)e_i - (b-d)a}{d}$$

$$p_j^h: p_i^e = p_i^-$$

$$\frac{(b-d)a + dp_j + br_D}{2b} = \frac{(b-d)a - (b^2 - d^2)e_i + dp_j}{b}$$

$$p_j^h \equiv \frac{2(b^2 - d^2)e_i - (b-d)a + br_D}{d}$$

$$p_j^l: p_i^- = p_i^{e_i/\delta}$$

$$\frac{(b-d)a + dp_j + br_D}{2b} = \frac{(b-d)a - (b^2 - d^2)e_i/\delta + dp_j}{b}$$

$$p_j^l \equiv \frac{2(b^2 - d^2)e_i/\delta - (b-d)a + br_D}{d}$$

$$p_j^H: p_i^+ = p_i^{e_i/\delta}$$

$$\frac{(b-d)a + dp_j + b(r_D + \delta(c + \theta))}{2b} = \frac{(b-d)a - (b^2 - d^2)e_i/\delta + dp_j}{b}$$

$$p_j^H \equiv \frac{2(b^2 - d^2)e_i/\delta - (b-d)a + b(r_D + \delta(c + \theta))}{d}$$

$$p_j^{L_j=0}: p_i^+ = p_i^{L_j=0}$$

$$\frac{(b-d)a + dp_j + b(r_D + \delta(c + \theta))}{2b} = \frac{bp_j - (b-d)a}{d}$$

$$p_j^{L_j=0} = a - \frac{bd(a - r_D + \delta(c + \theta))}{b^2 - d^2}$$

$$p_j^{L_i=L^M}: p_i^{L_j=0} = p_i^M$$

$$\frac{bp_j - (b-d)a}{d} = \frac{1}{2}(a + r_a + \delta(c + \theta))$$

$$p_j^{L_i=L^M} = \frac{a(2b-d) + d(r_a + \delta(c + \theta))}{2b}$$