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Optimal Carbon Tax Scheme under Uncertainty in an Oligopolistic Market of Polluters

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Abstract:

Carbon taxation is used by several countries to internalize the negative effects of carbon emissions to the emitters of carbon. In this article the effects of a carbon tax on an oligopolistic market of polluters are analyzed. With the help of a multi-criteria optimization model the optimal carbon tax rate is determined; first under certainty and then in presence of demand uncertainty. It is shown that demand uncertainty leads to a lower optimal carbon tax rate, while it simultaneously increases carbon emissions. Finally, the influence of a possible carbon emission reducing investment is analyzed with the help of a real option model.

Keywords: Carbon Tax; Climate Change; Real Option; Technological Progress; Uncertainty; Oligopolistic competition

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1. Introduction

On December 12th 2015 representatives of 195 countries agreed in Paris, France, to limit global warming to two degrees Celsius and to even strive for an increase of only 1.5 degrees Celsius.[1] Climate change is caused by the emission of greenhouse gases, including mainly carbon (dioxide). Consequently, the avoidance of carbon emissions is an urgent task. The Paris Climate Change Convention requires the ratifying states to set up, evaluate and, of course, meet individual targets for emissions reductions. From an economic viewpoint, carbon emissions are negative externalities from value-adding activities like electricity production or industrial production. Therefore, the task of the government is to price the emission of carbon and to internalize it for the polluter (Borchiellini et al., 2000).

By far the most important approaches to achieve such internalization are cap-and-trade and carbon taxation. Under a cap-and-trade system the government distributes or sells a fixed amount of emission allowance certificates, which can be traded on the exchange. Carbon emitters have to hand over to the government an amount of emission allowance certificates according to the quantity of carbon they have emitted. Instead of a cap-and-trade scheme governments can also implement a carbon tax. Here, the emitters of carbon have to pay the government a certain amount of tax per ton of carbon emitted. According to Carl and Fedor (2016) governments all over the world collect carbon revenues of roughly 28.3 billion US-dollars each year. While approximately 70% of the earnings from cap-and-trade have been invested into the support of green technologies, 70% of the

[1] https://treaties.un.org, retrieved January 7th, 2017

earnings from carbon taxes just increase the general funds of the governments or are reimbursed to taxpayers.

The question whether a cap-and-trade system or whether a carbon tax is preferable has already been hotly debated in research as well as in politics. In this article we do not want to enter into this discussion. Nevertheless, we will from now on only consider carbon taxation which has a clear advantage at least from a modeling perspective. In particular, following Andrew et al. (2010), the state is the central actor under a carbon tax scheme as he can directly establish a price of carbon, i.e. the carbon tax rate.

The first countries to introduce a carbon tax were Sweden, Norway, Denmark, Finland and the Netherlands (Baranzini et al., 2000). Currently, carbon taxes are implemented in a series of additional countries or at least parts of countries, including Japan, France, Switzerland and the Canadian province of British Columbia (Beck et al., 2015; Carl and Fedor, 2016). Please note, that carbon taxation and energy taxation are somehow different. Under a carbon tax the emission of carbon dioxide is taxed, under an energy tax, like the German Mineral Oil Tax, the taxation is based on the thermal value of energy products (see e.g. Al-Abdullah, 1999).

An extensive literature stream analyzes the economic effects of carbon taxation (see e.g. Proost and Regemorter, 1992; Zhang and Baranzini, 2004; Marron and Toder, 2014). Briefly summarized, carbon taxation has a negative effect on almost all economic variables, including GDP, consumption, international competitiveness, wages, social equality, and investment. Furthermore, it increases the prices of energy-intensive products while at least the relative prices of less energy-intensive products decrease. In return, carbon taxation is expected to

encourage innovations in low-carbon technologies. Additionally, carbon taxation should decrease the demand for energy and energy intensive products; consequently also decreasing the amount of carbon emission. This latter effect has been empirically analyzed by Lin and Li (2011) for the first five countries implementing a carbon tax. However, a significant negative influence of the carbon tax on carbon emissions could only be showed for one of the five countries. Thus, for governments the right design of the carbon tax is crucial to achieve the desired effects (Baranzini et al., 2000; Lin and Li, 2011).
According to Marron and Toder (2014) optimizing the carbon tax scheme consists of three tasks: Setting the tax rate, collecting the tax and using the resulting revenue. Likewise, Zhang and Baranzini, 2004 and Liang et al. (2007) show that not only the rate of a carbon tax is crucial, but also its use. For example as a subsidy for green competing technologies or as a production related re-payment into the market. However, for simplicity, in this paper we will only focus on the first task, i.e. optimizing the tax rate. Therefore, we assume that collecting the tax works flawlessly without creating any transaction cost and that all revenues of the carbon tax become part of the government’s general funds.

Some research has already done on the question how to optimize carbon tax rates. Many economists have argued that a carbon tax should strictly following the principles of a Pigovian tax, i.e. it should be exactly as high to prevent market failure by internalizing the negative externalities of carbon emissions to the emitters. For example, Roughgarden and Schneider (1999) develop a model to optimize the carbon tax rate from a global perspective. They show, that the optimal carbon tax rate critically depends on the strength of the relationship between carbon in the atmosphere and the resulting damage; a context that is still being investigated by many researchers (see e.g. Pearce, 2003).

However, Bovenberg and Goulder (1996) argue that as long as governments have more tasks to do than only preventing the climate change, it can be useful for them to deviate from the pure Pigovian principle and to use carbon taxation as a means of fiscal policy. Using a general-equilibrium approach they set up a model to determine the optimal carbon tax rate under the interaction with other means of taxation. Liski and Tahvonen (2004) set up a game-theoretic model to determine the optimal carbon tax rate. Interestingly they do not define optimality from an environmental view point but they view carbon taxes as a kind of import tariff for energy products that allows the importing countries to get back part of the monopoly rents of the OPEC cartel. For this purpose, the import countries must jointly select the optimal level of the carbon tax. Finally, Goulder and Mathai (2000) analyze the influence of the possibility of technological progress regarding carbon-emission-reducing technologies on the optimal carbon tax rate. In this regard Fahimnia et al. (2015) point out that in typical supply chains countless possibilities exist where investments can be made to reduce carbon emissions. With the help of their model decision-makers are able to filter out the most suitable investment options.

In this article we also deviate from the pure Pigovian principle and assume that governments use carbon taxation to meet several objectives. In particular, we set up a model of an oligopolistic market of producers that differ in production costs and in the degree of eco-efficiency, i.e. the ratio of production output and carbon emissions. The market price of the produced goods depends on the combined quantity offered by the oligopolistic companies and on the stochastic development of demand. At each point in time the oligopolistic companies non-cooperatively choose their optimal quantity offered. We analyze the effect of a carbon tax on the oligopolistic market and determine the optimal carbon tax rate from the governments' perspective. Finally, the possibility of one company in the oligopolistic market to invest in a carbon-emission-reducing project is introduced into the model. We use the real option approach to determine the optimal timing of this investment given the demand uncertainty. Furthermore, the influence of this investment possibility on the oligopolistic market and on the optimal carbon tax rate is analyzed.

Real options have already been used by several researchers in the context of carbon emissions; especially to account for the flexibility associated with the investments possibility under the presence of uncertainty (see e.g. Insley, 2003; Abadie and Chamorro, 2008; Lukas and Welling, 2014). In particular, following real option theory investors do not have to invest immediately but may wait with their investments until some uncertainty has resolved. The investment possibility thus contains a flexibility value that foregoes with investment. Real option methodology is also used to analyze company behavior in oligopolistic markets. For example, Bouis et al. (2009) analyze market entry decisions in an oligopoly of identical companies, Hackbarth and Miao (2012) analyze mergers and acquisitions in an oligopoly of companies that at each time-step optimize their produced quantity, and Huisman and Kort (2015) model the market-entry game of two (potential) duopolistic companies that have to decide on the time of market entry and the capacity they build.

The model of the oligopolistic market in this article builds on the model of Hackbarth and Miao (2012). The main advantages relevant to this article are that the companies may differ in production cost and can optimally choose their produced quantity at each time step. In particular, no capacity limit exists. Nevertheless, our model has slightly to deviate from the setting in Hackbarth and Miao (2012) as a reasonable modulation of the carbon tax requires to assign cost to companies that are proportional to their carbon emissions which is not possible in their setting. However, this deviation from Hackbarth and Miao (2012) leads to the disadvantage that the model in this article can only be solved numerically as it contains recursive calculations, because the optimal quantities offered by the companies in the oligopoly could otherwise become negative.

The remainder of the article is structured as follows. In Section 2 the model of the oligopolistic market is introduced and the optimal quantities of the oligopolistic companies are determined. Furthermore, the influence of several model parameters and of carbon taxation on the oligopolistic market is discussed. Section 3 analyzes the decision of the government. At first, the multi-criteria objective function is developed. Then, the optimal carbon tax rate is determined; in a setting with constant demand as well as under demand certainty. For both cases, the influence of several model parameters on the optimal carbon tax rate is analyzed. Section 4 discusses the influence of a company's possibility to invest in a carbon emission avoiding project. Using real option methodology the optimal investment timing is analyzed and the effect of this investment possibility is discussed. Finally Section 5 concludes and gives suggestions for further research.

2. The oligopolistic model

We consider $N \in \mathbb{N}$ oligopolistic companies that produce a homogenous good. We assume that at a time t the market price of the good linearly depends in the combined offered quantity $q(t)$. In particular,

$$P(Y(t), t) = aY(t) - bq(t) = aY(t) - b\sum_{j=1}^{N} q_j(t) \tag{1}$$

with $a, b, Y(t) > 0$ and $q_j(t) \geq 0$ as the offered quantity of company $j \in \{1, \dots, N\}$ at time t. The production cost $k_i > 0$ of one unit of the good differs between companies but are independent to the quantity produced. Likewise, the amount of carbon $c_i > 0$ that is emitted to produce one unit of the good differs between companies and is independent to the quantity produced. Given a carbon tax of $\psi > 0$ per unit carbon, the cost $C_i(t)$ of company i to produce the quantity $q_i(t)$ of the homogenous good can be expressed as

$$C_i(t) = q_i(t)(k_i + c_i\psi) \tag{2}$$

Hence, at time t the profit $\pi_i(Y(t), t)$ of company i is given by

$$\begin{aligned} \pi_i(t) &= P(t)q_i(t) - C_i(t) \\ &= \left(aY(t) - b\sum_{j=1}^{N} q_j(t) \right) q_i(t) - q_i(t)(k_i + c_i\psi) \end{aligned} \tag{3}$$

At each time t the companies compete on the oligopolistic market by choosing a optimal quantity to offer (Cournot-oligopoly). In particular, company i has to choose $q_i(t)$ in a way that it maximizes its profits $\pi_i(t)$ given the offered quantities $q_1(t), .., q_{i-1}(t), q_{i+1}(t), \dots, q_N(t)$ of the other companies in the market. Hence, for the optimal offered quantity of company i the necessary condition $\frac{\partial \pi_i\left(q_i^*(t)\right)}{\partial q_i} = 0$ has to hold.

Together, these N conditions of the form

$$0 = aY(t) - 2bq_i(t) - b \sum_{j=1, j \neq i}^{N} q_j(t) - k_i - c_i\psi \tag{4}$$

set up a linear equation system of N equations with the offered quantities $q_1(t), \dots, q_N(t)$ as variables. The vector $(q_1^*(t), \dots, q_N^*(t))$ of optimal offered quantities is the solution of the equation system. Please note, that this solution may contain negative quantities of several companies. If this is the case the respective companies are excluded from the market (just for the current time t) and their quantities $q_i^*(t)$ are set to zero. For the remaining $n \leq N$ companies the calculation is repeated. Let $M\big(Y(t)\big)$ denote the set of remaining companies at time t. Then, it can be easily shown that the optimal equilibrium production of company $i \in M\big(Y(t)\big)$ at time t equals

$$q_i^*\big(Y(t)\big) = \frac{1}{(n+1)b}\left(Y(t)a - n\psi c_i - nk_i + \sum_{j \in M(Y(t))/\{i\}} \big(\psi c_j + k_j\big)\right) \tag{5}$$

and that it solely depends on $Y(t)$ but not directly on t. The combined offered quantity $q(t)$ at time t of all N companies on the oligopolistic market is given by

$$q^*\big(Y(t)\big) = \sum_{j=1}^{N} q_j^*\big(Y(t)\big) = \frac{1}{b(n+1)}\left(naY(t) - \psi \sum_{j \in M(Y(t))} c_j - \sum_{j \in M(Y(t))} k_j\right). \tag{6}$$

After insertion of the optimal quantities in equation (1) we get the equilibrium market price at time t of

$$P^*\big(Y(t)\big) = \frac{1}{n+1}\left(Y(t)a + \psi \sum_{j \in M(Y(t))} c_j + \sum_{j \in M(Y(t))} k_j\right). \tag{7}$$

The profits of company $i \in M\big(Y(t)\big)$ are equal to

$$\pi_i^*(Y(t), c, \psi) = \frac{\left(Y(t)a - n\psi c_i + \sum_{j \in M(Y(t))/\{i\}} \big(\psi c_j + k_j\big) - nk_i\right)^2}{b(n+1)^2}. \tag{8}$$

The total revenue of the carbon tax a t time $\boldsymbol{t}$ can be obtained as

$$\xi^*(Y(t)) = \psi \sum_{j=1}^{N} c_j q_j^*(Y(t)). \tag{9}$$

Finally, we can calculate the eco-efficiency (i.e. the ratio of output and ecological impact) of the oligopolistic market by

$$\eta^*(Y(t)) = \frac{\sum_{j=1}^{N} q_j^*(Y(t))}{\sum_{j=1}^{N} c_j q_j^*(Y(t))}. \tag{10}$$

2.1 Benchmark Oligopoly

Due to the possible recursive calculation of the companies' optimal quantities hardly any results regarding the carbon tax can be obtained analytically. Instead, we use a certain class of oligopolies consisting of $N > 1$ companies to numerically analyze the effect of the carbon tax. While the first company always has the characteristics $c_1 = c_m > 0, k_1 = k_m > 0$, the other companies deviate from this first company by a distance $c_w > 0$ and $k_w > 0$, respectively. In particular, we define $c_i = c_m + c_w \cdot \cos\left((i-2)\frac{2\pi}{N-1}\right)$ and $k_i = k_m + k_w \cdot \sin\left((i-2)\frac{2\pi}{N-1}\right)$. Thus all the other companies lie at an even angular distance on an eclipse with the center formed by company 1. In particular, for $N = 5$ we get

$$c = \begin{pmatrix} c_m \\ c_m + c_w \\ c_m \\ c_m - c_w \\ c_m \end{pmatrix}, k = \begin{pmatrix} k_m \\ k_m \\ k_m + k_w \\ k_m \\ k_m - k_w \end{pmatrix}$$

which we will call the "benchmark oligopoly". Furthermore, unless stated otherwise we will assume the following remaining parameters: $Y = 1$; $c_m = 2, c_w = 1; k_m = 2; k_w = 1;\ a = 10;\ b = 1; \psi = 0.5$. Additionally, we assume the following values for parameters that will be introduced and defined in later sections of this article: $r = 0.1; \sigma = 0.2; \mu = 0; s = 0.5; K = 20$.

2.2 Influence of the demand parameter Y.

From equation (6) we can conclude that given a very low demand parameter none of the oligopolistic firms will produce a positive quantity. If the demand parameter increases to a certain level, the first company will start production. If it increases to another higher level a second company starts to produce. If it increases to an even higher level a third company will enter the market, et cetera (see Figure 1). As long as the number of active companies in the market stays constant, we can obtain from equation (5) that the offered quantity of each producing company increases in the demand parameter Y. However, nothing can be said for an increase in Y that leads to a higher number of active companies in the market. Anyhow, Figure 1 indicates that the quantities of active companies are continuous as well as monotonic in Y.

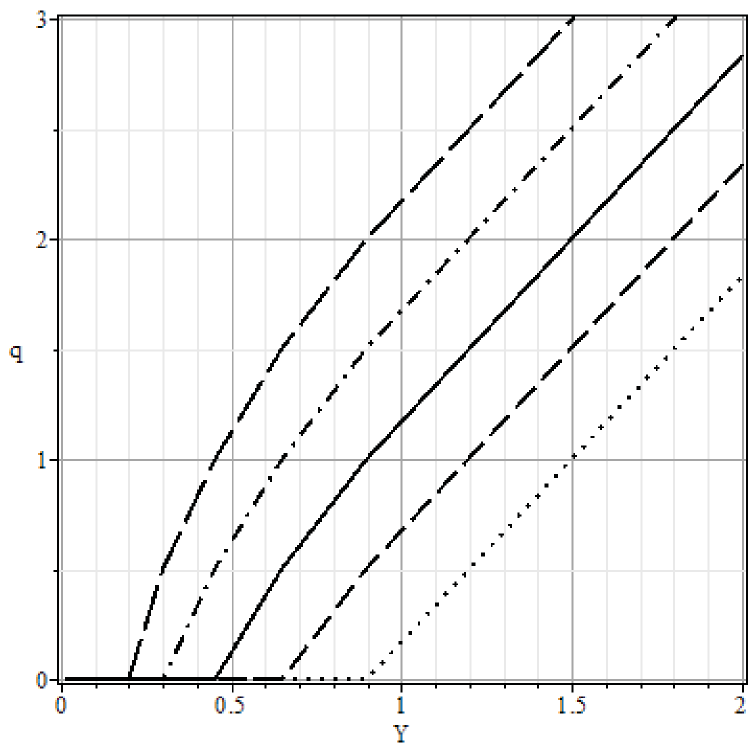

Figure 1: The optimal quantity q_i^* of the five oligopolistic companies in dependence of the demand parameter Y (solid line: company 1; dashed line: company 2; dotted line: company 3; dashdotted line: company 4; longdashed line: company 5).

2.3 Influence of the carbon tax

Following equation (6) we get

$$\frac{\partial q^*\big(Y(t)\big)}{\partial \psi} = \frac{1}{b(n+1)} \sum_{j=1}^{n} c_j < 0. \tag{11}$$

Thus, an increase in the carbon tax τ leads to lower the combined offer q^* - at least as long as the variation in the carbon tax does not lead to a variation in the number of active companies. Likewise, due to

$$\frac{\partial P^*\big(Y(t)\big)}{\partial \psi} = \frac{1}{n+1} \sum_{j=1}^{n} c_j > 0. \tag{12}$$

the market price of the homogenous good in the oligopoly is also increasing with the carbon tax if the number of active companies stays constant. According to Figure 2 this result also holds if an increase in the carbon tax leads to a decrease in the number of active companies.

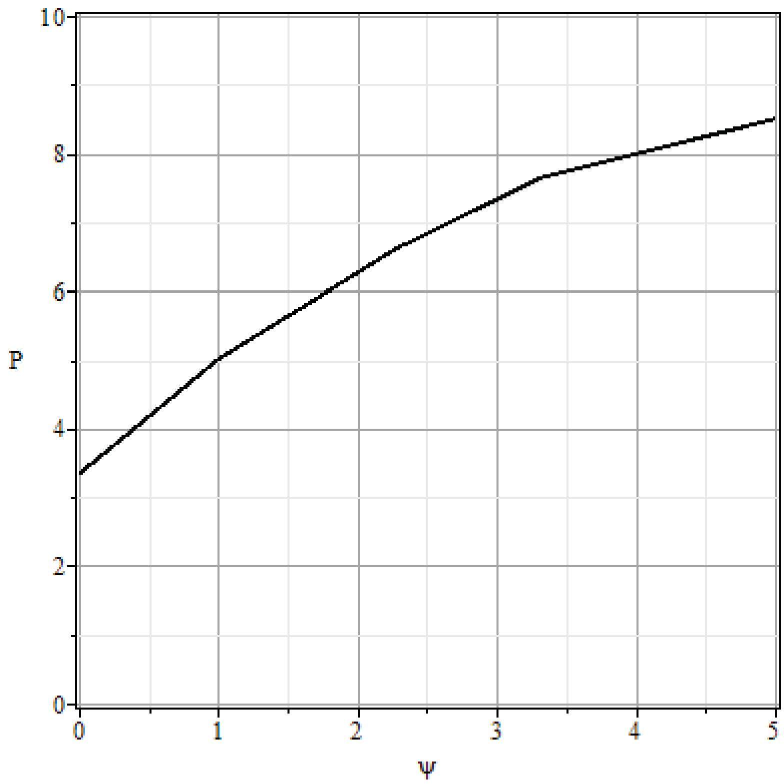

Figure 2: The market price of the homogenous good in dependence of the carbon tax rate.

However, the influence of the carbon tax on an individual company's offered quantity and profit is more complex. From

$$\frac{\partial q_i^*\big(Y(t)\big)}{\partial \psi} = -nc_i + \sum_{j=1, j\neq i}^{n} c_j \tag{13}$$

it follows that as long as the number of active companies stays constant the optimal offered quantity of company i is decreasing with the carbon tax if and only if $c_i > \frac{1}{n}\sum_{j=1,j\neq i}^{n} c_j$, i.e. if and only if the carbon emissions of company i per unit of the homogenous good are quite high compared to the other companies in the oligopoly. For example, in Figure 3 we can see that with the exemption of company 4 – which is the company with the lowest carbon emission per production unit – the quantity of all companies is decreasing with an increase in the carbon tax.

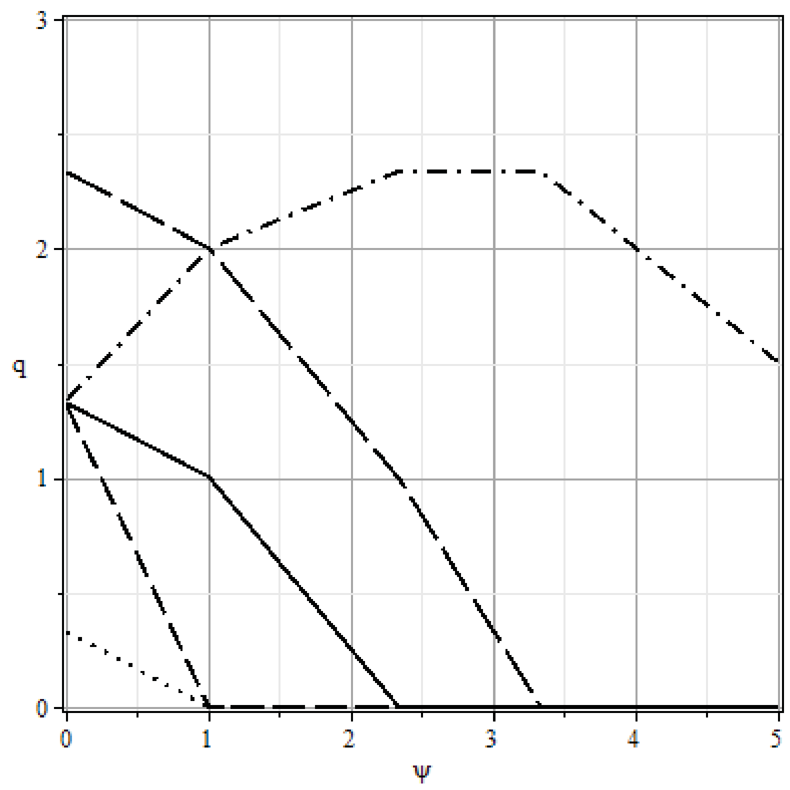

Figure 3: The optimal quantity q_i^* of the five oligopolistic companies in dependence of the carbon tax rate ψ (solid line: company 1; dashed line: company 2; dotted line: company 3; dashdotted line: company 4; longdashed line: company 5).

In contrast, the quantity of company 4 is increasing with the carbon tax if the carbon tax is low. Only if the carbon tax is as high that company 4 is the only remaining active company on the market an increase in the carbon tax leads to a lower optimal quantity of company 4.

Obviously carbon tax schemes are usually implemented to improve the environment. Indeed, according to Figure 4 the carbon tax succeeds in increasing the eco-efficiency. As long as more than one company is active on the market an increase in the carbon tax leads to a higher eco-efficiency. While the combined quantity on the market is decreasing with the carbon tax the quantity of the low emission company 4 is increasing. Hence, the ratio of homogenous goods produced and carbon emissions is improving. If the carbon tax is as high that only company 4 is active in the market an increase in the carbon tax cannot further increase the eco-efficiency.

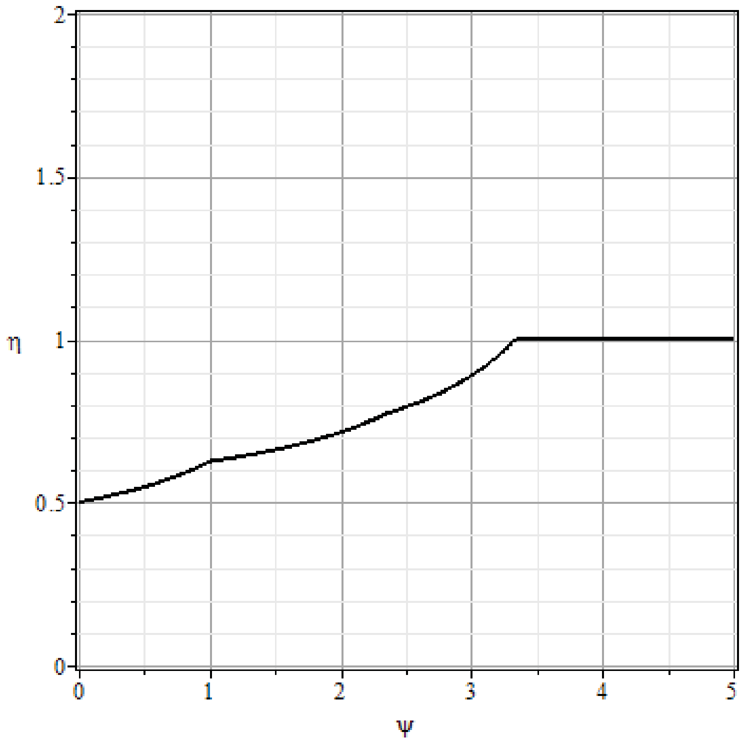

Figure 4: The influence of the carbon tax rate on the degree of eco-efficiency.

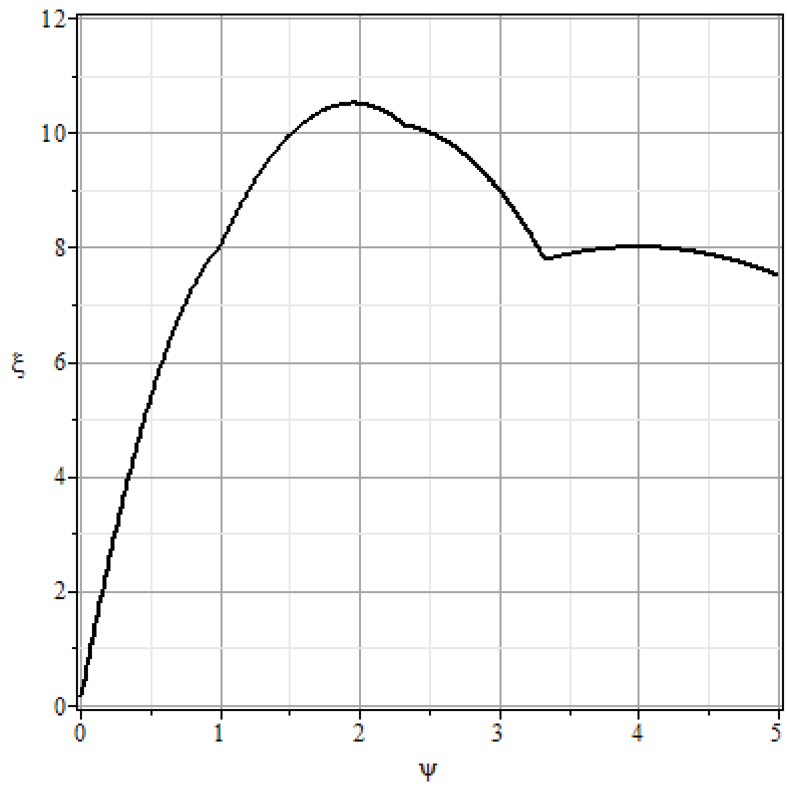

Figure 5: The influence of the carbon tax rate on the revenue of the carbon tax.

It can be easily shown that the influence of the carbon tax rate on the tax revenue is non-monotonic. Obviously, in absence of any carbon taxation, i.e. for $\psi = 0$, there is no tax revenue, i.e. $\xi = 0$. Likewise, according to equation (6) for a very high carbon tax rate no company will be active in the market and, thus, $\xi = 0$. Consequently, there will be at least one maximum in between.

As can be seen in Figure 5 also multiple maxima may exist. In particular, the tax revenue is first increasing with the carbon tax and reaches its global maximum for $\psi \approx 2$. Then it is decreasing with an increasing carbon tax rate until company 4 becomes the only active company on the market. If the carbon tax rate increases further the tax revenue slightly increases, too and for approximately $\psi = 4$ reaches a second local maximum. Though, a further increase in the carbon tax rate again leads to a decreasing tax revenue.

3. The decision of the government

The task of the government is to optimally choose the carbon tax rate ψ. Obviously, every government has to meet different targets at the same time. Consequently, the decision regarding the carbon tax is an example of multi-criteria decision making. We assume that at every point in time t the utility of the government is represented by the utility function

$$U\big(g_1(t), g_2(t), \dots, g_N(t)\big) = \prod_{j=1}^{N} g_j(t)^{\omega_j} \tag{14}$$

with $g(t) = (g_1(t), \dots, g_N(t))$ as realization of the different targets in t and $\omega = (\omega_1, \dots, \omega_N) \in (-1,1)^N$ as the weight of the targets. Please note, that the utility function deviates from the classic Cobb-Douglas-Function by the fact that also negative weights can occur. In these cases the government wishes to minimize the specific target value. However, it always applies that a target j is the more important to the government the higher $|\omega_j|$. To normalize the weights we assume that $\sum_{j=1}^{N} |\omega_j| = 1$.

In the following we will summarize the many different targets of the government in three groups:

1. Targets of environmental policy
2. Targets of economic policy
3. Targets of budgetary policy.

For each of the three groups we will now define a representing target the government in our model aims to achieve. The first group of targets is represented by the target to maximize eco-efficiency. In particular, $g_1(Y(t)) = \eta^*\big(Y(t)\big)$. The second group of targets is represented by the target to minimize the market price of the homogenous good. Thus, we get $g_2(Y(t)) = P^*(Y(t))$. Finally, the third group of targets is represented by the target to maximize the revenue of the carbon tax. Here we get $g_3(Y(t)) = \xi^*\big(Y(t)\big)$

Thus, we obtain the utility function of the government as

$$U\big(Y(t)\big) = \Big(\eta^*\big(Y(t)\big)\Big)^{\omega_1} \big(P^*(Y(t))\big)^{\omega_2} \Big(\xi^*\big(Y(t)\big)\Big)^{\omega_3}, \tag{15}$$

whereby $\omega_1 \geq 0, \omega_2 \leq 0, \omega_3 \geq 0$ and $|\omega_1| + |\omega_2| + |\omega_3| = 1$.

3.1 The government's decision (in a simplified economy)

For simplicity, let us first assume that the demand parameter is constant, i.e. $Y(t) = \bar{Y} > 0$. Hence, the government simply has to maximize $U(\bar{Y})$ by choosing the optimal carbon tax rate $\psi^* \geq 0$. In particular,

$$\begin{aligned} \psi^* &= argmax_{\psi \geq 0}\{U(\bar{Y})\} \\ &= argmax_{\psi \geq 0}\big\{\big(\eta^*(\bar{Y})\big)^{\omega_1}\big(P^*(\bar{Y})\big)^{\omega_2}\big(\xi^*(\bar{Y})\big)^{\omega_3}\big\}. \end{aligned} \tag{16}$$

As we have already discussed in the previous chapter an increase in the carbon tax rate will result in an increase in the market price of the homogenous good. Hence, we have $\frac{\partial (P^*)^{\omega_2}}{\partial \psi} < 0$. On the other hand, an increase in the carbon tax rate leads to an increase in eco-efficiency. That is $\frac{\partial (\eta^*)^{\omega_1}}{\partial \psi} > 0$. A low market price and a high eco-efficiency hence are mutually contradictory goals. Furthermore, the influence of the carbon tax rate on the tax revenue has been shown to be non-monotonic. Hence, in general the sign of $\frac{\partial U^*}{\partial \psi}$ is ambiguous and strongly depends on the relative weights ω_1, ω_2 and ω_3 of the different goals. As a consequence, we can only numerically determine the optimal carbon tax rate ψ^*.

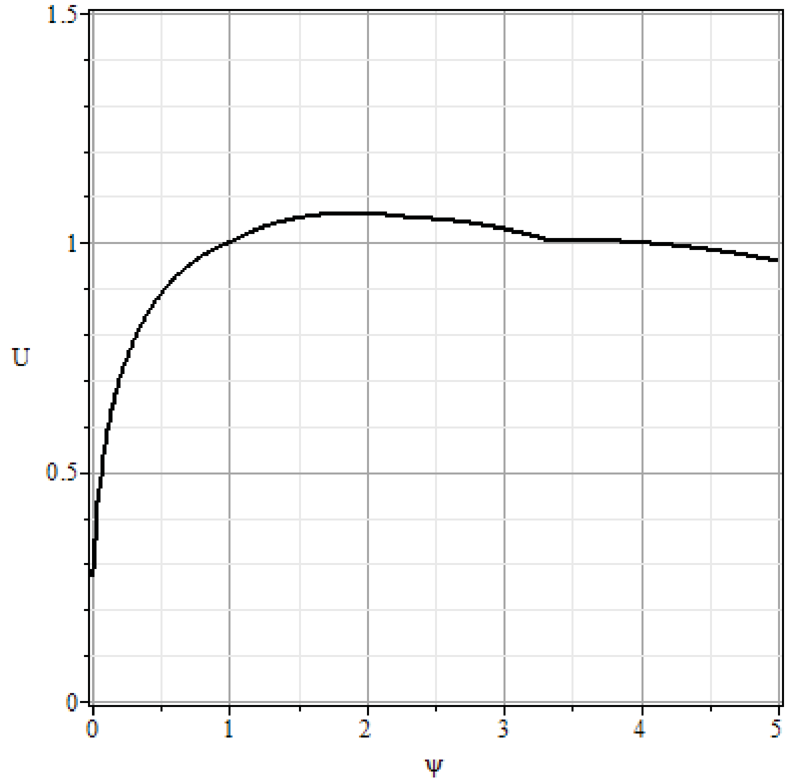

Figure 6: The influence of the carbon tax rate on the utility of the government.

For the benchmark case, Figure 6 depicts the influence of the carbon tax rate on the utility of the government. As can be seen, for the given parameters of the benchmark case a carbon tax of roughly 2 maximizes the government's utility. From Table 1 we get the exact optimal carbon tax rate $\psi^* = 1.9$. While for lower values of ψ an increase of ψ leads to a strong increase in utility, for higher values of ψ a further increase of ψ only leads to a slight decrease of utility.

Table 1: The government's decision in a simplified economy.

	ψ^*	U^*	η^*	P^*	ξ^*	q^*	$\lvert M^*\rvert$
$a = 5.00$	0.985	0.714	0.664	3.652	2.000	1.348	2
$a = 7.50$	1.375	0.910	0.691	4.844	5.285	2.656	3
$a = 10.0$	1.900	1.063	0.701	6.125	10.50	3.875	3
$a = 12.5$	2.415	1.186	0.706	7.394	17.47	5.106	3
$a = 15.0$	2.930	1.289	0.709	8.663	26.20	6.338	3
$b = 0.50$	1.900	1.340	0.701	6.125	21.00	7.750	3
$b = 0.75$	1.900	1.170	0.701	6.125	14.00	5.167	3
$b = 1.00$	1.900	1.063	0.701	6.125	10.50	3.875	3
$b = 1.25$	1.900	0.987	0.701	6.125	8.398	3.100	3
$b = 1.50$	1.900	0.929	0.701	6.125	6.998	2.583	3
$c_m = 1.00$	4.995	1.494	1201	5.998	0.017	4.002	2
$c_m = 1.50$	6.985	1.267	2.000	7.746	7.871	2.254	1
$c_m = 2.00$	1.900	1.063	0.701	6.125	10.50	3.875	3
$c_m = 2.50$	1.460	0.974	0.500	6.123	11.33	3.878	3
$c_m = 3.00$	1.185	0.909	0.392	6.120	11.72	3.880	3
$c_w = 0.50$	1.725	1.030	0.555	6.122	12.06	3.878	3
$c_w = 0.75$	1.810	1.046	0.610	6.126	11.50	3.874	3
$c_w = 1.00$	1.900	1.063	0.701	6.125	10.50	3.875	3
$c_w = 1.25$	4.655	1.107	1.333	7.746	7.871	2.254	1
$c_w = 1.50$	6.985	1.267	2.000	7.746	7.871	2.254	1
$k_m = 1.00$	1.980	1.178	0.690	5.475	12.98	4.525	3
$k_m = 1.50$	1.950	1.120	0.696	5.813	11.72	4.188	3
$k_m = 2.00$	1.900	1.063	0.701	6.125	10.50	3.875	3
$k_m = 2.50$	1.840	1.008	0.706	6.425	9.320	3.575	3
$k_m = 3.00$	1.765	0.953	0.709	6.706	8.201	3.294	3
$k_w = 0.50$	1.715	1.046	0.700	6.001	9.796	3.999	4
$k_w = 0.75$	1.890	1.054	0.713	6.175	10.14	3.825	3
$k_w = 1.00$	1.900	1.063	0.701	6.125	10.50	3.875	3
$k_w = 1.25$	1.910	1.073	0.690	6.075	10.86	3.925	3
$k_w = 1.50$	1.915	1.082	0.680	6.019	11.22	3.981	3
$N = 3$	3.490	1.005	1.000	7.745	7.870	2.255	1
$N = 4$	1.860	1.026	0.632	6.325	10.82	3.675	3
$N = 5$	1.900	1.063	0.701	6.125	10.50	3.875	3
$N = 6$	1.815	1.090	0.718	5.839	10.52	4.161	4
$N = 7$	2.685	1.112	0.823	6.615	11.04	3.385	2
$N = 8$	2.385	1.147	0.800	6.127	11.55	3.873	3
$N = 9$	2.175	1.159	0.795	5.830	11.41	4.170	4
$\omega_1 = \frac{1}{3}, \omega_2 = -\frac{1}{3}, \omega_3 = \frac{1}{3}$	1.900	1.063	0.701	6.125	10.50	3.875	3
$\omega_1 = \frac{2}{3}, \omega_2 = -\frac{1}{6}, \omega_3 = \frac{1}{6}$	3.490	1.003	1.000	7.745	7.870	2.255	1
$\omega_1 = \frac{1}{6}, \omega_2 = -\frac{2}{3}, \omega_3 = \frac{1}{6}$	0.535	0.462	0.551	4.225	5.607	5.775	5
$\omega_1 = \frac{1}{6}, \omega_2 = -\frac{1}{6}, \omega_3 = \frac{2}{3}$	1.945	3.342	0.707	6.181	10.51	3.819	3

For the benchmark case Table 1 depicts the influence of the various model parameters on the optimal carbon tax rate, on the utility of the government, on the fulfillment of the governmental goals, on the combined quantity of the homogenous good produced and on the number of active companies in the oligopolistic market. An increase in demand, i.e. an increase in a, results in a higher optimal carbon tax rate. Furthermore, the three targets of the government, the combined quantity and the number of active companies increase with demand. Contrarily, an increase of the negative influence of the combined quantity offered on the market price of the homogenous good, i.e. an increase of b, has no influence on the optimal carbon tax rate (see Table 1). This can be explained by the fact that according to equation (7) b has no influence on the equilibrium market price P^*. An increase of b solely leads to a lower combined quantity and hence to a lower tax revenue. However, the market share of the companies is not influenced by an increase in b. Thus, an increase in b has no influence on the eco-efficiency. Consequently, an increase in b leads to a lower utility of the government.

The influence of an increase in carbon emissions, i.e. an increase in c_m, on the optimal carbon tax rate is ambiguous (see Table 1). Same holds for the influence on the market price of the homogenous good, on the quantity produced and on the number of active companies in the oligopoly. Only if carbon emissions are low an increase in carbon emissions leads to an increase in the carbon tax rate. Paradoxically, if carbon emissions are high, an increase in carbon emissions would lead to a lower optimal carbon tax rate. Furthermore, the higher the carbon emissions the lower the utility of the government. Obviously, the degree of eco-efficiency decreases with increasing carbon emissions while the tax revenue increases.

The influence of the production cost of the oligopolistic companies, i.e. an increase in k_m, has a monotonic influence on the optimal carbon tax rate. The higher the production costs the lower the carbon tax. Likewise, the influence of the production cost on the government's utility is monotonic. The higher the production cost the lower is the utility. However, the influence of production cost on the three governmental targets does not correspond consistently to the decrease in utility. In particular, the higher the production cost the higher the market price of the homogenous good and the lower the tax revenue but the higher the eco-efficiency (see Table 1). Interestingly, any increase in the difference between the oligopolistic companies, i.e. an increase in c_w or k_w, has a positive influence on the optimal carbon tax rate as well as on the governments utility. Further, an increase in these differences leads to a reduced number of active companies in the oligopoly (see Table 1). All these effects can be easily explained by the fact that due to the special construction of the oligopolies (see section 3.1) an increase in the differences of the oligopolistic companies reduces carbon emissions or production cost of the already leading companies while simultaneously increasing carbon emissions and production costs of the non-leading companies.

The influence of the number of companies in the oligopoly seems to be totally arbitrary. In particular, its influence on any of the observed quantities is ambiguous. That is, its influence on the optimal carbon tax rate, on the government's utility, on the market price of the homogenous good, etc. (see Table 1). Paradoxically, even the influence of the number of companies in the oligopoly on the number of active companies in the oligopoly is non-monotonic! While we observe four active companies in an oligopoly of six companies, only two companies are active in an oligopoly of seven companies.

Finally, the influence of variations in the weights of the governmental targets is quite intuitive. An increase in the weight of eco-efficiency increases the optimal carbon tax rate and the degree eco-efficiency. In return the values of the other two targets deteriorate. Furthermore, the combined quantity is reduced and only the company with the lowest carbon emission stays active in the oligopoly. Likewise, an increase in the weight of the market price decreases the market price but has negative consequences with respect to eco-efficiency and tax revenue. To allow a decrease of the market price an increase in the weight of the market price further results in a lower carbon tax rate, a higher quantity produced and a greater number of active companies in the oligopoly. In particular, all companies become active in the benchmark case. Finally, an increase in the weight of the tax revenues has almost any influence on the quantities observed (see Table 1).

3.2 The government's decision under uncertainty

In the following we assume that the demand parameter $Y(t)$ is evolving stochastically over time. In particular we assume that it follows the geometric Brownian motion

$$dY(t) = \mu Y(t)dt + \sigma Y(t)dz(t), \qquad Y(t_0) = Y_0 > 0, \tag{17}$$

with $\mu \in \mathbb{R}$ as the exponential growth rate, $\sigma > 0$ as the volatility and $z(t)$ as the increment of a Standard Wiener process with mean zero and variance equal to $\sqrt{dt}$. At the initial point in time t_0 the government has to decide on the optimal carbon tax rate. We assume that the government has a time preference, i.e. it puts a higher weight on the present than on the future. Thus, future utility is discounted with the exponential discount factor $r > 0$. Consequently, by choosing the optimal carbon tax rate ψ^* the government aims to maximize the expected integral of future discounted utility, i.e.

$$\begin{aligned}\psi^* &= argmax_{\psi \geq 0} \mathbb{E} \int_{t_0}^{\infty} e^{-r(t-t_0)} U\big(Y(t)\big) dt \\ &= argmax_{\psi \geq 0} \mathbb{E} \int_{t_0}^{\infty} \frac{\Big(\eta^*\big(Y(t)\big)\Big)^{\omega_1} \Big(P^*\big(Y(t)\big)\Big)^{\omega_2} \Big(\xi^*\big(Y(t)\big)\Big)^{\omega_3}}{e^{r(t-t_0)}} dt.\end{aligned} \tag{18}$$

Given the optimal carbon tax rate ψ^* the expected discounted average produced quantity $\tilde{q}_i^*$ of a company i can be calculated by

$$\tilde{q}_i^* := \mathbb{E} \frac{\int_{t_0}^{\infty} e^{-r(t-t_0)} q_i^*\big(Y(t)\big) dt}{\int_{t_0}^{\infty} e^{-r(t-t_0)} dt} = r \cdot \mathbb{E} \int_{t_0}^{\infty} e^{-r(t-t_0)} q_i^*\big(Y(t)\big) dt. \tag{19}$$

Please note, that $\tilde{q}_i^*$ is defined in a way that $\tilde{q}_i^* = q_i^*$ if $Y(t)$ would be constant over time. Thus, it makes sense to compare $\tilde{q}_i^*$ with q_i^* of section 3.1 to determine the effect of uncertainty on the company's quantity. Likewise, we define

$$\tilde{q}^* := r \cdot \mathbb{E} \int_{t_0}^{\infty} e^{-r(t-t_0)} q^*\big(Y(t)\big) dt \,, \widetilde{U}^* := r \cdot \mathbb{E} \int_{t_0}^{\infty} e^{-r(t-t_0)} U^*\big(Y(t)\big) dt,$$

$$\tilde{P}^* := r \cdot \mathbb{E} \int_{t_0}^{\infty} e^{-r(t-t_0)} P^*\big(Y(t)\big) dt \,, \tilde{\eta}^* := r \cdot \mathbb{E} \int_{t_0}^{\infty} e^{-r(t-t_0)} \eta^*\big(Y(t)\big) dt,$$

$$\tilde{\xi}^* := r \cdot \mathbb{E} \int_{t_0}^{\infty} e^{-r(t-t_0)} q^*\big(Y(t)\big) dt \,, \ \big|\widetilde{M}^*\big| := r \cdot \mathbb{E} \int_{t_0}^{\infty} e^{-r(t-t_0)} \big|M^*\big(Y(t)\big)\big| dt.$$

For the benchmark case the model is solved numerically using Monte-Carlo simulation. In particular, thousand realizations of the stochastic processes $Y(t)$ have been simulated over a time span of fifty years with exactly one time step per year. For the time span after year 50 a constant exponential growth of Y(t) with the growth rate μ was assumed, starting from the simulated value $Y(t_0 + 50)$. The results of the Monte-Carlo simulation are given in Tables 2 and 3.

Table 2: The government's decision under demand uncertainty (part 1).

	$\tilde{\psi}^*$	$\tilde{U}^*$	$\tilde{\eta}^*$	$\tilde{P}^*$	$\tilde{\xi}^*$	$\tilde{q}^*$	$\lvert\tilde{M}^*\rvert$
$a = 5.00$	0.680	0.551	0.486	2.774	1.972	1.708	1.967
$a = 7.50$	0.940	0.730	0.543	3.737	4.925	3.135	2.654
$a = 10.0$	1.590	0.875	0.617	4.976	10.60	4.186	2.672
$a = 12.5$	1.650	0.989	0.628	5.711	15.95	5.895	3.130
$a = 15.0$	2.240	1.093	0.649	6.903	25.25	7.024	3.043
$b = 0.50$	1.590	1.103	0.617	4.976	21.20	8.372	2.672
$b = 0.75$	1.590	0.963	0.617	4.976	14.13	5.582	2.672
$b = 1.00$	1.590	0.875	0.617	4.976	10.60	4.186	2.672
$b = 1.25$	1.590	0.813	0.617	4.976	8.481	3.349	2.672
$b = 1.50$	1.590	0.765	0.617	4.976	7.067	2.791	2.672
$c_m = 1.00$	1.490	1.072	4.800	4.051	4.941	5.290	3.237
$c_m = 1.50$	3.360	1.013	1.406	5.605	10.02	3.558	1.760
$c_m = 2.00$	1.590	0.875	0.617	4.976	10.60	4.186	2.672
$c_m = 2.50$	1.100	0.798	0.423	4.796	10.49	4.366	2.971
$c_m = 3.00$	0.880	0.744	0.329	4.759	10.59	4.404	3.028
$c_w = 0.50$	1.280	0.847	0.467	4.730	10.77	4.432	3.227
$c_w = 0.75$	1.350	0.857	0.514	4.778	10.52	4.385	3.028
$c_w = 1.00$	1.590	0.875	0.617	4.976	10.60	4.186	2.672
$c_w = 1.25$	2.060	0.905	0.853	5.312	10.32	3.850	2.214
$c_w = 1.50$	5.170	0.982	1.606	6.235	9.230	2.730	0.940
$k_m = 1.00$	1.590	1.009	0.633	4.444	12.34	4.840	2.926
$k_m = 1.50$	1.390	0.939	0.619	4.554	10.75	4.731	2.963
$k_m = 2.00$	1.590	0.875	0.617	4.976	10.60	4.186	2.672
$k_m = 2.50$	1.380	0.813	0.601	5.072	9.191	4.091	2.702
$k_m = 3.00$	1.570	0.754	0.591	5.371	8.956	3.594	2.411
$k_w = 0.50$	1.610	0.863	0.652	5.061	10.35	4.101	2.838
$k_w = 0.75$	1.640	0.869	0.641	5.066	10.55	4.097	2.649
$k_w = 1.00$	1.590	0.875	0.617	4.976	10.60	4.186	2.672
$k_w = 1.25$	1.640	0.884	0.604	4.984	10.95	4.178	2.481
$k_w = 1.50$	1.580	0.894	0.570	4.872	11.02	4.291	2.522
$N = 3$	1.800	0.819	0.732	5.628	9.056	3.534	1.682
$N = 4$	1.570	0.845	0.548	5.184	10.53	3.978	2.381
$N = 5$	1.590	0.875	0.617	4.976	10.60	4.186	2.672
$N = 6$	1.680	0.902	0.635	4.905	10.95	4.257	3.039
$N = 7$	1.640	0.920	0.639	4.734	10.87	4.429	3.477
$N = 8$	1.780	0.934	0.667	4.753	11.29	4.409	3.204
$N = 9$	1.740	0.955	0.665	4.615	11.28	4.548	3.544
$\omega_1 = \frac{1}{3}, \omega_2 = -\frac{1}{3}, \omega_3 = \frac{1}{3}$	1.590	0.875	0.617	4.976	10.60	4.186	2.672
$\omega_1 = \frac{2}{3}, \omega_2 = -\frac{1}{6}, \omega_3 = \frac{1}{6}$	2.850	0.748	0.713	5.913	12.07	3.052	1.718
$\omega_1 = \frac{1}{6}, \omega_2 = -\frac{2}{3}, \omega_3 = \frac{1}{6}$	0.430	0.425	0.514	3.724	4.525	5.617	3.878
$\omega_1 = \frac{1}{6}, \omega_2 = -\frac{1}{6}, \omega_3 = \frac{2}{3}$	2.180	2.921	0.641	5.359	11.71	3.606	2.177

If we compare the results of the setting under demand uncertainty (Table 2) with the setting of constant demand (Table 1), we first see that demand uncertainty results in a lower optimal carbon tax rate, at least in the benchmark case. The same result also is valid for all other observed settings with the exemption of the case where the government puts a very high weight on tax revenue. Here, demand uncertainty slightly increases the optimal carbon tax rate. Furthermore, uncertainty has a clear negative effect on the government's ability to meet the objectives. In all settings the value of $\widetilde{U}^*$ is lower than the value of U^*. This result can be explained easily, as under certainty the government can adjust the tax rate perfectly to its objectives and to their weights. In contrast, under demand uncertainty the government has to choose a tax rate that has to fit as best as possible into a variety of future scenarios. Obviously, it cannot be perfect for all scenarios, i.e. all realizations of $Y(t)$. Likewise, with only one exemption, particularly a very high weight on the market price of the homogenous good, in all settings demand uncertainty leads to a lower eco-efficiency of the oligopolistic market. This result can be explained analogously.

However, if we analyze the effect of demand uncertainty on the market price of the homogenous good and on the revenue of the carbon tax, we find quite amazing results. In all settings under demand uncertainty the market price of the homogenous good is lower. Of course, simultaneously the combined offered quantity is higher. This effect can be explained by the lower carbon tax rate. With respect to the tax revenue the effect of uncertainty is ambiguous. In some settings, including the benchmark case demand uncertainty results in a higher tax revenue, but in other settings under uncertainty the tax revenue is lower. This difference in the effect of uncertainty can be explained by the fact that tax revenue multiplicatively consists of three parts: Tax rate, the reciprocal of eco-efficiency, i.e. the average carbon emissions per unit of the homogenous good produced, and the combined offered quantity of the oligopolistic companies. While the tax rate decreases with demand uncertainty, the reciprocal of eco-efficiency and the combined offered quantity increase in uncertainty. Further, demand uncertainty has neither a clear effect on tax revenue nor on the number of active companies.

A higher interest rate leads to a higher optimal carbon tax rate (see Table 3). Further, the higher the interest rate the higher the utility of the government. This effect is solely triggered by a strong increase in eco-efficiency. The market price is increasing with the interest rate and the tax revenue is decreasing with the interest rate. A higher growth rate of demand leads to a higher carbon tax rate, higher utility of the government and higher tax revenue. Obviously, higher growth in demand increases the market price of the homogenous good. Paradoxically, a higher growth rate of demand seems to be beneficial for the environment as it increases eco-efficiency. However, due to the increased combined quantity offered carbon emissions could nevertheless increase.

Table 3: The government‘s decision under demand uncertainty (part 2).

	$\tilde{\psi}^*$	$\tilde{U}^*$	$\tilde{\eta}^*$	$\tilde{P}^*$	$\tilde{\xi}^*$	$\tilde{q}^*$	$\lvert\tilde{M}^*\rvert$
$\mu = -0.050$	0.910	0.669	0.492	3.417	4.342	2.871	2.640
$\mu = -0.025$	1.500	0.765	0.571	4.283	7.565	3.223	2.284
$\mu = 0$	1.590	0.875	0.617	4.976	10.60	4.186	2.672
$\mu = 0.025$	1.820	0.993	0.650	6.003	18.70	6.201	2.880
$\mu = 0.050$	2.490	1.130	0.698	7.942	42.46	9.941	2.815
$r = 0.050$	1.560	0.820	0.586	4.725	11.25	4.386	2.515
$r = 0.075$	1.580	0.853	0.606	4.879	10.90	4.272	2.610
$r = 0.100$	1.590	0.875	0.617	4.976	10.60	4.186	2.672
$r = 0.125$	1.610	0.890	0.625	5.049	10.39	4.102	2.710
$r = 0.150$	1.710	0.899	0.639	5.181	10.39	3.942	2.523

4. Technological Progress

Now we assume that company 1 has the possibility to invest in a project to decrease its carbon emissions. In return of an investment of $K > 0$ the company can reduce its carbon emissions per produced unit of the homogenous good from c_m to $c_m - s$, whereby $0 < s < c_m$. Obviously, this investment does not only have consequences for company 1. Due to the new cost structure, the optimal production quantity of all oligopolistic companies is changing (as described in Section 2). Let $\hat{q}_i^*(Y(t))$ denote the gain of company i after company 1 has invested in its carbon emission reduction given a certain demand $Y(t)$. If company 1 would not have invested before a time τ, the expected present value $\varphi_1(Y(t))$ of investing in τ could be calculated as

$$\varphi_1(Y(t)) = -K + \mathbb{E}\left(\int_{\tau}^{\infty} e^{-r(t-\tau)}\left(\hat{\pi}_1^*(Y(t)) - \pi_1(Y(t))\right)dt\right). \tag{20}$$

Obviously, there is no good reason why company 1 would have to invest immediately. Instead, the company has the possibility to wait with the investment until the optimal investment time τ^*. Please note, $\varphi_1(Y(t))$ depends on the demand parameter Y but does not depend on time. From previous literature on real options (see e.g. Dixit and Pindyck, 1994; Lukas and Welling, 2014) it is well known that in these cases it is optimally to invest as soon as the stochastic variable, i.e. in this model $Y(t)$, reaches a certain threshold Y^*. Consequently, we get $\tau^* = \inf\{t \geq t_0 |\, Y(t) \geq Y^*\}$. Furthermore, it is known, that

$$\mathbb{E}\left(e^{-r(\tau^*-t_0)}\right) = \left(\frac{Y(t_0)}{Y^*}\right)^{\beta}, \tag{21}$$

whereby

$$\beta = \frac{1}{2} - \frac{\mu}{\sigma^2} + \sqrt{\left(\frac{1}{2} - \frac{\mu}{\sigma^2}\right)^2 + \frac{2r}{\sigma^2}}. \tag{22}$$

Hence, we can determine the optimal investment threshold Y^* via

$$Y^* = argmax_{y \geq Y(t_0)} \left(\varphi_1(y) \left(\frac{Y(t_0)}{y} \right)^{\beta} \right). \tag{23}$$

Obviously, Y^* depends on the carbon tax rate chosen by the government in t_0. Thereby, it makes sense to write $Y^*(\psi)$ and $\tau^*(\psi)$, i.e. to consider the optimal investment threshold and optimal investment time as a reaction (function) on the chosen tax rate ψ. Figure 7 depicts this reaction function of company 1. As can be seen, the optimal investment threshold is decreasing in the carbon tax rate. Hence, the government could use the carbon tax rate as an investment incentive.

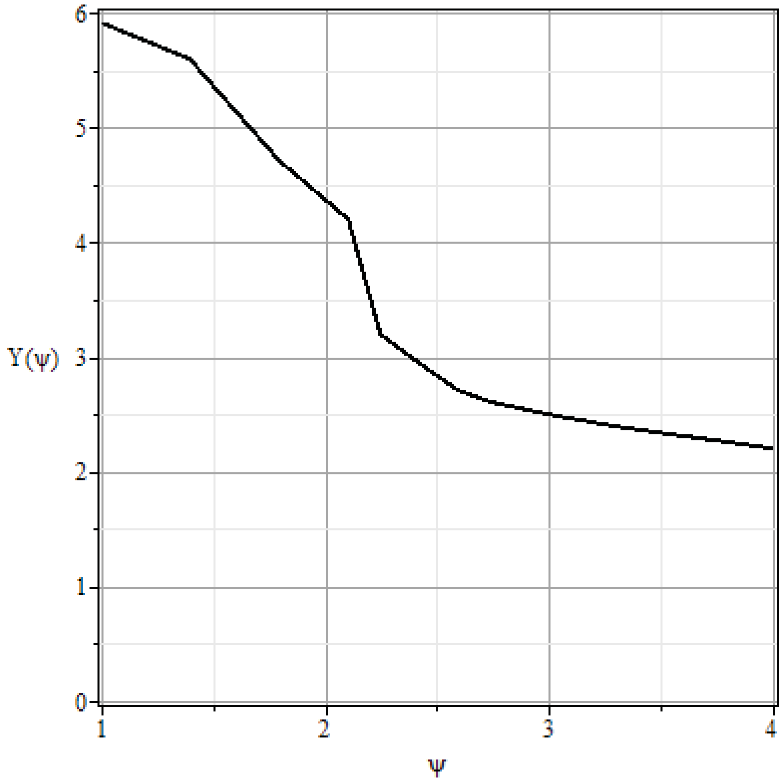

Figure 7: The reaction function of company 1.

Consequently, when optimizing the carbon tax rate the government should already take into account the reaction of company 1, i.e. its option to invest into the carbon emission reducing project. In particular, the government will choose

$$\psi^* = argmax_{\psi \geq 0} \mathbb{E}\left(\int_{t_0}^{\tau^*(\psi)} e^{-r(t-t_0)} U(Y(t),\psi) dt + \int_{\tau^*(\psi)}^{\infty} e^{-r(t-t_0)} \widehat{U}(Y(t),\psi) dt \right). \tag{24}$$

Again, the model is solved via Monte-Carlo-Simulation. Analogously to equation (19) we define for each variable $X \in \{q_i^*, q^*, P^*, U^*, \eta^*, \xi^*, |M^*|\}$ the variable

$$\breve{X}^* := r \cdot \mathbb{E}\left(\int_{t_0}^{\tau^*(\psi)} e^{-r(t-t_0)} X^*(Y(t),\psi) dt + \int_{\tau^*(\psi)}^{\infty} e^{-r(t-t_0)} \widehat{X}^*(Y(t),\psi) dt \right), \tag{25}$$

which again allows us tom compare the values with the settings in the previous Section. The results for the benchmark case are shown in Table 4. As we can see, the option of company 1 to invest in the carbon reducing technology leads to a higher optimal carbon tax rate. This can be explained by the fact that the government wants to give an incentive to invest in the project as it increases the eco-efficiency (as can be seen in Table 4). Furthermore, the higher carbon tax rate also leads to higher tax revenue. This effect dominates the effect of the higher carbon tax on the combined quantity offered on the oligopolistic market. However, the carbon emission reducing investment does not lead to an increased utility of the government. In particular, the market price of the homogenous good is increased due to the increased carbon tax rate.

Table 4: The option to invest in the carbon emissions reducing project.

	ψ^*	$Y^*(\psi^*)$	$\breve{U}^*$	$\breve{\eta}^*$	$\breve{P}^*$	$\breve{\xi}^*$	$\breve{q}^*$	$\lvert\breve{M}^*\rvert$
Benchmark	2.1	4.2	0.853	0.633	5.285	11.58	3.680	2.185

5. Conclusion

In this article we have set up a model of an oligopolistic market of polluters under a carbon tax scheme. We have analyzed the effect of the carbon tax on the oligopoly. In particular, the higher the carbon tax the lower the quantity offered, the lower the number of active companies in the oligopoly and the higher the market price of the homogenous good. Furthermore, the carbon tax succeeds in increasing the eco-efficiency and thereby in reducing carbon emissions.

In a second step we have determined the optimal carbon tax rate - on one hand in a setting of constant demand, on the other hand in presence of demand uncertainty. We have analyzed the influence of various parameters on the optimal carbon tax rate. Regarding the influence of demand uncertainty, we found that demand uncertainty is good for the customers as it results in a lower market price, and a higher quantity offered. But demand uncertainty is definitively bad for the environment as it also leads to a lower eco-efficiency and a higher production. Consequently, this results in higher carbon emissions. Paradoxically, demand uncertainty simultaneously leads to a lower optimal carbon tax rate. Finally, we introduced the possibility for a company to invest in a carbon emissions reducing project into the model. With the help of real option analysis we could determine the optimal tax rate under this setting. We found that the option to invest in the carbon emissions reducing project leads to an increasing optimal carbon tax rate. Thereby, the government can set a high incentive to invest early in the carbon saving technology. In particular, the option to reduce carbon emissions leads to a higher eco-efficiency of the oligopolistic industry. However, the influence of this option is bad for the consumer as it increases the market price of the homogenous good.

A weakness of the model and at the same time a possibility for future research lies in the fact that only one company in the oligopoly has the possibility to invest in reducing its carbon emissions. Of course, it would be more realistic if all companies or several companies had such an option. However, the complexity of the model would strongly increase with negative consequences for the possibility to still reasonably solve this model numerically. Nevertheless, it exists a research stream that successfully has modeled situations with competing real options of multiple players in oligopolies (see e.g. Pawlina and Kort, 2006; Mason and Weeds, 2010; Thijssen et al., 2012; Huisman and Kort, 2015); though usually under stronger assumptions regarding the oligopolistic companies. Maybe future research can successfully combine the setting of this article with parts of other model settings.

Further possibilities of future research are to allow the government to adjust the carbon tax rate at a certain time or at certain time steps to react on technological developments as well as on new data about carbon emissions. Likewise, the production and cost structure of the oligopolistic companies could be more realistic. In particular, fix costs could be included and carbon emissions per produced unit of the homogenous good should depend on the quantity produced. Finally, future research could apply the oligopoly-setting to the cap-and-trade scheme and maybe compare the influence of both schemes, i.e. cap-and-trade and carbon taxation, on oligopolies.

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