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Flexible Time Window Management for Attended Home Deliveries

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Abstract In the competitive world of online retail, customers can choose from a selection of delivery time windows on a retailer’s website. Creating a set of suitable and cost-efficient delivery time windows is challenging, since customers want short time windows, but short time windows can increase delivery costs significantly. Furthermore, the acceptance of a request in a particular short time window can greatly restrict the ability to accommodate future requests. In this paper, we present customer acceptance mechanisms that enable flexible time window management in the booking of time-window based attended home deliveries. We build tentative delivery routes and check which time windows are feasible for each new customer request. We offer the feasible long delivery time windows as a standard and let our approaches decide when to offer short time windows. Our approaches differ in the comprehensiveness of information they consider with regard to customer characteristics as well as detailed characteristics of the evolving route plan. We perform a computational study to investigate the approaches’ ability to offer short time windows and still allow for a large number of customers to be served. We consider various demand scenarios, partially derived from real order data from a German online supermarket.

Keywords Time Window Management, Customer Acceptance, Attended Home Deliveries, Vehicle Routing with Time Windows, Route Flexibility, Online Supermarkets

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1 Introduction

Many of the products being sold online require the presence of the customer during delivery. Examples include groceries, appliances, furniture, and restaurant meal deliveries. To ensure the customer’s attendance and to avoid multiple and costly delivery attempts, it is common that retailers offer a selection of delivery time windows on their website and the customer chooses one. In this paper, we will use the time-window based delivery of groceries as an example of the challenges of attended home deliveries.

The fulfillment process for groceries, as well as for many other attended home deliveries, can be structured into the following steps (Campbell & Savelsbergh 2006): The first step is order acceptance, which involves customers filling their shopping baskets on the retailer’s website, revealing their ZIP code (or even their specific delivery address), and finally being offered a selection of delivery time windows. The resulting order is typically placed at least one day before the delivery occurs and is confirmed by the retailer immediately when the customer chooses a delivery time window. Next, the retailer assembles the order within the warehouse or store. Last, as part of the order delivery step, the retailer optimizes the delivery routes to ensure service at each customer within the promised time window at minimal cost.

The creation of customer-specific sets of delivery time windows in the order acceptance step is challenging. Since attended home deliveries are an expensive logistics service and e-grocers operate with low profit margins, the aim is to serve as many customers as possible with the lowest possible delivery costs to maximize overall profit. However, customers arrive dynamically on a retailer’s website, future customer requests are unknown, and offering a certain time window to a customer may prevent being able to accept more profitable future customer requests. Furthermore, the decision on which time windows to offer to a requesting customer has to be made instantaneously, which prevents the usage of some complex optimization methods during order acceptance (Ehmke & Campbell 2014).

For customers to accept one of the offered time windows, they must be suitable with regard to the customer’s high expectations. In particular, the offered time windows must fit with a customer’s day plan. As a consequence, demand for certain time windows is larger than others, e.g. there is higher demand for evening time windows, and not all customers will be offered time windows at the time they desire. Furthermore, customers want narrow time windows. Major e-grocers like *AllyouneedFresh* (Germany) and *Peapod* (United States) offer their customers a standard delivery window width of two hours. As e-commerce grows in popularity, so is the competition between retailers. This can result in businesses trying to compete through better delivery services and offering even shorter time windows, even as short as 30 minutes (Mintel 2014). However, the length of the offered time windows impacts the route plan’s efficiency, and short time windows can increase the retailers’ delivery costs (Lin & Mahmassani (2002), Ehmke & Campbell
Furthermore, short time windows can significantly restrict the ability to accept future requests by decreasing the route plan’s flexibility.

In this paper, we consider offering a mix of long and short time windows to some customers. We offer the feasible long delivery time windows as the default option and carefully decide when to offer short time windows to requesting customers as an additional service. Offering these short time windows could create a competitive advantage, since customers experience a higher service quality which may result in increased customer loyalty. We call this idea flexible time window management. We present several flexible time window management approaches that help in deciding when to offer short or long time windows based on customer characteristics as well as characteristics of the evolving route plan. We create customer-specific offer sets and offer short time windows when it does not restrict the retailer’s ability to accept future requests. We discuss different ways of measuring the current flexibility of a route plan in the creation of offer sets. We show that considering the route plan’s current flexibility can help to improve service quality in terms of offering short time windows to several customers while still preserving the retailer’s desire to serve as many customers as possible. We keep our approaches simple to enable easy implementation by online retailers.

The paper is organized as follows. In the next section, we present related literature in the areas of time window design and order acceptance. Then, we formalize our problem and introduce four flexible time window management approaches. We demonstrate and evaluate our approaches with a computational study. Many of our demand scenarios are inspired from the order data of a German e-grocer. We look at our results in many different ways to help build many useful insights. Finally, we conclude the paper with a summary and an outlook on future work.

2 Related Literature

Offering attended home deliveries can constitute a competitive advantage for online retailers, but it is also a challenging and costly logistics service to offer customers. It is well known that offering attended home delivery services with tight time windows creates the biggest potential risk of failure for e-grocers (Punakivi & Saranen 2001). We will focus on recent literature considering the design of time window offer sets as well as evaluating arriving customer requests to create time window offer sets. In particular, we address three major challenges for the creation of customer-specific offer sets: the feasibility, the profitability and the suitability of time windows.
2.1 Design of Time Windows

Before customer-specific offer sets can be created and orders can be accepted, tactical decisions on the design and availability of time windows have to be made. Since retailers are operating within an environment of fierce competition and low profit margins, the potential *profitability* of a time window has to be considered in the design of offer sets. To achieve maximal profit, as many customers as possible (given the available logistics capacities) need to be accepted. Shorter time windows can decrease the route plan’s efficiency and flexibility, which leads to higher costs and less customers that can be serviced. Hence, the retailer has to offer time windows that are short enough to be accepted by a customer, but also long enough to provide enough flexibility for creating efficient route plans.

Several studies have shown the impact of offering short time windows on the resulting route costs and profits. Lin & Mahmassani (2002) analyze the correlation between a decreasing time window size and increasing delivery costs. A loss in profit when offering shorter time windows of 18% (with long time windows of 180 minutes and short time windows of 30 minutes) is demonstrated by Campbell & Savelsbergh (2005). Also the total number of accepted customers decreases significantly as shown by Ehmke & Campbell (2014), who can accept 15% less customers when short instead of long time windows are offered. Gevaers et al. (2014) quantify logistics costs of customer acceptance and find that retailers have to pay around 3€ for deliveries when offering 4-hour time windows, and almost 6€ for deliveries within 1-hour time windows. Although these papers show the significant impact long and short time windows can have on the profitability of a retailer, flexibly adapting the time window length during the booking process has not been considered so far.

Another important design decision is which time windows to offer to each region within the delivery area. Several approaches forecast customer demand in an aggregated way, e.g. based on the ZIP code. This allows for solving cost-minimal route plans considering the estimated demand for each region. The resulting route plans can help create time window offer sets for each region. Corresponding approaches can be found in Agatz et al. (2011), Hernandez et al. (2017) and Bruck et al. (2017). A first attempt that considers revenues and delivery costs can be found in Cleophas & Ehmke (2014). These approaches can help companies decide on the fleet size and mix as well as which time windows to offer to serve the expected demand. However, the resulting offer sets are based on fixed patterns. We claim that especially for the competitive field of online retailing, more flexible solutions are needed.

Since logistics capacities such as the size of the delivery fleet are assumed to be fixed during order acceptance, it is important to utilize these resources efficiently. To balance the demand among the different time windows, incentives can be used to shift demand from popular to less popular time windows. Revenue management techniques as discussed
in Asdemir et al. (2009), Yang et al. (2014), Klein et al. (2017) analyze the customer’s willingness to pay as well as the likelihood of choosing a specific time window and determine prices for delivery time windows. We do not consider pricing approaches in this paper. However, flexible customer acceptance can likely be used to create better pricing decisions.

2.2 Evaluating Order Requests

The overall number and value of customer requests is usually not known beforehand, but becomes available incrementally as orders arrive during the booking process. To guarantee a delivery can occur within a promised time window, retailers have to check the feasibility of time windows before adding them to an offer set. After all customer requests have been accepted, the retailer can build cost-minimal delivery routes considering the customer’s time window choice by solving a Vehicle Routing Problem with Time Windows (VRPTW). However, at the order acceptance step, complete information about all customers requesting service is not yet available, and feasibility checks are needed in real-time. Campbell & Savelsbergh (2005) and Ehmke & Campbell (2014) present static and dynamic customer acceptance mechanisms that differ in the complexity of the information considered to decide on the feasibility of different time windows. Static customer acceptance mechanisms use a maximum number of customers per time window based on past delivery performance, which can be extracted from historical data or from tactical time window design. This simple approach allows for quick decisions on the offered time window sets, but it does not consider information about the customer location. For all promised time windows to be feasible, a conservative maximum of orders is often used and delivery capacities are possibly wasted (or routes become infeasible). The dynamic approach creates tentative route plans and considers the location of each delivery request to check whether the offering of a each particular time window is feasible. To keep run times manageable, insertion heuristics are used to create such route plans. Dynamic approaches can handle spatio-temporal information and hence utilize delivery capacities much better. However, they require the customer’s location as input, which is not always known. For our paper, we assume full knowledge of the customer’s location. The use of an insertion heuristic usually serves as fast feasibility check that considers the costs of inserting a customer request but does not consider the overall impact of the insertion. Lu & Dessouky (2006) present an adaption of the insertion heuristic that also considers the reduction of waiting times and slacks when a new customer is inserted in a route plan. Their idea is very similar to our approach since it tries to maintain flexibility within the route plans. However, Lu & Dessouky (2006) are considering given time window constraints where we can take advantage of a flexible offering of time windows to customers.

Since each accepted customer can restrict the ability to accommodate future requests, the deliveries accepted early in the booking process have a big impact on the remaining
deliveries that can be accepted. It is important to create offer sets that are attractive to customers, i.e. from which a customer is willing to accept at least one time window, during the full booking process. We call this the suitability of the time window offer set. The higher the probability a customer is willing to accept a time window from this set, the better the suitability. Casazza et al. (2016) name conditions for the suitability of time windows: First, the potential to accept a customer should have highest priority – an empty offer set would lead to disappointment of a customer and should be avoided. Second, the offer set should contain time windows that deviate as little as possible from the customer’s preferences for a specific time of the day. And last, the length of the offered time windows is crucial, since customers do not want to wait at home for a long period of time. As a consequence, longer time windows are perceived as worse customer service. So far, not much research has been done on the accordance of customers’ expectations and the time windows provided by retailers with the exception of Hungerländer et al. (2017), who present an approach that aims to create the largest possible selection of time windows within each customer’s offer set to achieve a higher probability of meeting the customer’s preferred time window. However, we assume that it is not the number of time window options that defines a good service level, but that even a single option would be accepted as long as it is suitable for the particular customer.

Operational time window management can also contribute to the overall profitability through using the information gathered during the booking process. A first approach that considers the interaction of incentives and routing costs on the overall profit during the step of order acceptance has been provided by Campbell & Savelsbergh (2006). Furthermore, cost approximations can be used to estimate the impact of each arriving customer on the overall routing costs. Ulmer & Thomas (2017) show in a same-day delivery setting that estimating the delivery costs in the beginning of the booking horizon is difficult, but becomes more accurate when the overall structure of the final delivery route plan has been determined. Within this paper, we use the utilization of a tentative route plan as a measure on how “settled” the tentative route plan already is and adapt the offered time window length to maintain sufficient flexibility.

Only a few approaches consider the time of commitment to a time window during the booking process and its impact on the route plan’s flexibility. Ulmer (2017) and Vareias et al. (2017) try to approximate the (optimal) arrival time at a customer first and derive a “self-imposed” time window to maintain flexibility. The latter paper also minimizes the time window length. However, the planned arrival times of customers can still change significantly as long as more customers will be accepted and the final route plan is evolving.
3 Flexible Time Window Management

We introduce the problem of flexible time window management formally in Section 3.1 and then describe our solution methodology in Section 3.2.

3.1 Problem Description

Our problem is motivated by the practice of many online retailers who want to maximize the number of attended deliveries made each day, but also offer many of their customers a high level of service. We assume that customers arrive in real-time and request an immediate time window offering. We consider two sets of time windows, $S$ and $L$, each containing time windows that are non-overlapping and consecutive. Set $S$ contains $m$ short time windows each of length $w_S$. Set $L$ contains $n$ long time windows each of length $w_L$, and we assume that this is the standard time window option of an online retailer.

The beginning and ending times of each time window are denoted as $a_S^m \in S, a_L^n \in L$ and $b_S^m \in S, b_L^n \in L$. The delivery fee is the same for all deliveries regardless of time window length or time of day. We want to decide which time windows from set $S$ and $L$ to offer when a request $j \in J$ arrives.

Requests are considered for acceptance until a specific cut-off time that is before the day of order delivery, i.e. same-day deliveries are not considered within this paper. We assume that we do not have any information on future incoming requests, and hence offer sets are created based on the set of accepted customers $C$ and the present request $j$ requesting service. We assume that we know the delivery location of each incoming request. In the case of online supermarkets, customers usually have to reveal their address before being offered delivery time windows. Each vehicle $v \in V$ can provide delivery service for a total duration $T$ on the day of order delivery. To create the offer set, the feasibility of delivery within the different time windows must be determined. Accepted requests including their promised time windows cannot be withdrawn or changed. Thus, an insertion of request $j$ during a particular time window is only allowed if the arrival at all accepted customers can still be guaranteed within their time windows, and the total delivery time for all drivers does not exceed $T$. If the request can be served feasibly within at least one time window, an offer set $O_j$ will be created for a requesting customer.

The offer set $O_j$ can contain short or long time windows or a mix of both. We assume that the retailer creates the offer set without knowing the customer’s time window preferences. If the offer set for a customer contains time windows during his or her preferred time of the day, short windows are always preferred over long ones. If the offer set contains only long time windows within the customer’s preferred time of the day, we model an acceptance rate for these long time windows with a probability of $a$. The value of $a$ can be as high as 100% to represent when customers are satisfied with the longer time
windows. We test our approaches also for acceptance rates of a less than 100% to model the risk of customers canceling their order process because only long time window options were offered. Acceptance rates may fall below 100% when there are competitors offering shorter windows than the default long windows, for example.

Our objective is to maximize the number of customers that can be accepted while offering short time windows to as many customers as possible. This means that short time windows are offered to customers when it will not restrict the ability to accept additional requests.

3.2 Offer Set Methodology

For a request \( j \), we create a request-specific offer set \( O_j \) that is based on spatio-temporal customer information as well as the tentative route plan. Our algorithm maintains a tentative route plan \( R_v \) for each vehicle \( v \in V \) considering the already accepted customers on vehicle \( v \) and the depot in the first and last positions. The algorithm creates the time window offer set \( O_j \) based on each potential insertion point in the tentative routes in three steps (Algorithm 1).

For each request \( j \) that arrives, we consider inserting it in each possible position in our tentative route plans. For each insertion position, based on the set of already accepted customers, we must determine which windows out of the predefined time window sets \( S \) and \( L \) are feasible (EVALUATEFEASIBLETIMEWINDOWS). The evaluation of the feasibility of time windows is implemented as described in Section 3.2.1. This first step results in the sets containing feasible time windows, \( S^v_{i,i+1} \) and \( L^v_{i,i+1} \).

Once the sets of feasible time windows are known, the retailer must decide which of the feasible short time windows to offer to a particular request (CHOOSETIMEWINDOWS). This decision is based on evaluating how accepting a request in a particular window impacts the flexibility of the current route plan. We propose four approaches for evaluating this flexibility, which vary in complexity and decision factors considered, in Section 3.2.2. Each approach creates the time window sets \( S^{\prime\prime}_{i,i+1} \) and \( L^{\prime\prime}_{i,i+1} \) that serve as input for the creation of the final customer-specific offer set.

After we conduct step one and two for all possible insertion positions, we combine all resulting time window options in the last step of the algorithm (CREATEOFFERSET) and create a single offer set \( O_j \) to display to the customer associated with request \( j \):

\[
O_j = S^{\prime\prime}_{i,i+1} \cup L^{\prime\prime}_{i,i+1}.
\]

Once the offer set \( O_j \) is created, it can be displayed to the customer, and the customer chooses a time window or cancels the booking process (SELECTTIMEWINDOW). We present the modeling of the customer decision in Section 3.2.3. Finally, if the customer selects a time window, the tentative route plans are updated (UPDATEROUTE).
for each Request $j \in C$ do
  for each Vehicle $v \in V$ do
    for each Insertion Position $i$ and $i+1$, $i \in R_v$ do
      EvaluateFeasibleTimeWindows
      \[ \rightarrow S^v_{i,i+1} \text{ and } L^v_{i,i+1} \]
      ChooseTimeWindows($S^v_{i,i+1}$, $L^v_{i,i+1}$)
      \[ \rightarrow S^v_{i,i+1} \text{ and } L^v_{i,i+1} \]
    end
  end
  CreateOfferSet ($S^v_{i,i+1}$, $L^v_{i,i+1}$)
  \[ \rightarrow O_j \]
  SelectTimeWindow($O_j$), UpdateRoute
end

Algorithm 1: Offer Set Creation for a Request $j$

3.2.1 Evaluation of Feasibility

We use an insertion-based heuristic as presented in Campbell & Savelsbergh (2005) to determine all feasible time windows for request $j$ within our tentative route plans. To evaluate if a time window is feasible at a particular insertion point, we compute a time span based on the concept of slack introduced in Savelsbergh (1992).

The time span reflects the range of time that service for request $j$ can start at the insertion position between customers $i$ and $i+1$ on vehicle $v$ while considering all time window constraints from the already accepted customers. We define the earliest possible time service can begin as $e^v_{i,i+1}$ based on the time service can begin at customer $i$ on vehicle $v$, the service time $u_i$ at customer $i$, and the time needed to travel from customer $i$ to request $j$ (please note that the service time is zero for the depot). The latest time service can begin, $f^v_{i,i+1}$, is the latest time service can begin at the next customer ($i+1$) minus the time needed for service $u_j$ at request $j$ and the travel time from request $j$ to customer $i+1$. The time span $s^v_{i,i+1}$ reflects the difference between these values and must be non-negative for the insertion of $j$ to be feasible for any time window.

The pseudo code of our feasibility evaluation is shown in Algorithm 2. For each feasible insertion position within the tentative route plans, we create the sets $S^v_{i,i+1}$ and $L^v_{i,i+1}$ that contain all feasible time windows. Set $S^v_{i,i+1}$ contains the feasible time windows from $S$ which are considered feasible if $e^v_{i,i+1}$ or $f^v_{i,i+1}$ are within a specific time window or when $e^v_{i,i+1}$ and $f^v_{i,i+1}$ are before and after the start and end time of a time window, respectively. The same conditions are checked to determine the feasible time windows from $L$ to create $L^v_{i,i+1}$.
EvaluateFeasibleTimeWindows

\[ e_{i,i+1}^v = e_i^v + u_i + t_{i,j} \]
\[ f_{i,i+1}^v = f_i^v - u_j + t_{j,i+1} \]
\[ s_{i,i+1}^v = f_i^v - e_i^{v+1} \]

if \( s_{i,i+1}^v \geq 0 \) then

for each time window \( m \in \mathcal{S} \) do

if \( a_m^S \leq e_{i,i+1}^v \leq b_m^S \) \( \land \) \( a_m^S \leq f_i^v \leq b_m^S \) \( \land \) \( (e_{i,i+1}^v \geq a_m^S \land b_m^S \leq f_i^v) \) then

add short time window \( m \) to \( \mathcal{S}_{i,i+1}^u \)

end

end

for each time window \( n \in \mathcal{L} \) do

if \( a_m^L \leq e_{i,i+1}^v \leq b_m^L \) \( \land \) \( a_m^L \leq f_i^v \leq b_m^L \) \( \land \) \( (e_{i,i+1}^v \geq a_m^L \land b_m^L \leq f_i^v) \) then

end

add long time window \( n \) to \( \mathcal{L}_{i,i+1}^v \)

end

end

Algorithm 2: Creation of Feasible Time Window Sets

3.2.2 Flexible Time Window Management Approaches

We introduce four time window management approaches to test if a short time window will not restrict the route plan’s flexibility. If the condition is true for a specific insertion position and time window approach, the corresponding short time windows are added to the offer set. If not, only long time windows are offered. Two of the presented approaches (LS and SL) are solely based on the route plan’s utilization and do not require the location of a request. The other two approaches (TT and TS) decide on the time window length by assessing the customer’s vicinity to already accepted customers within the tentative route plans. The approaches are as follows.

1. Long⇒Short (LS) In the course of the booking process, each accepted customer adds more information about what the final route plan will look like. With the LS approach, we try to maintain the high level of routing flexibility that is naturally available in the beginning of the booking process as long as possible. In particular, we offer long (standard) time windows to the early arriving customers and only switch to offering feasible short time windows when a large portion of the route plan has been defined. We determine the point of switching to short time time windows by measuring the current utilization of our service capacity. Specifically, we compute how much of the available service time \( T \) for the set of available delivery vehicles \( V \) has already been consumed by all accepted customers \( q \in \mathcal{R}^v \) in the current tentative route plans. The utilized time for delivery consists of service time at each customer and the travel times between the customers or depot. When a certain level \( x^{LS} \)
of the available service time has been utilized, we assume that the route plan is now well defined and begin offering short time windows. The corresponding rule for switching from long to short time windows is shown in Algorithm 3.

**Algorithm 3: LS Approach**

**ChooseTimeWindowLength**

```
if \( x^{LS} \leq (t_{v0} + \sum_{i=1}^{q-1} (t_{vi,i+1} + u_i) + t_{v0,i})/(|V| \cdot T) \) then
    | \( S_{i,i+1}^{no} = S_{i,i+1}^{no}, L_{i,i+1}^{no} = \emptyset \)
else
    | \( S_{i,i+1}^{no} = \emptyset, L_{i,i+1}^{no} = L_{i,i+1}^{no} \)
```

Figure 1a illustrates this approach. The first three customers were offered only long time windows reflected by the darker circles with the larger radius to maintain the flexibility of the route plan. Since these customers utilize a significant portion of the total available service capacity already, the offer sets for customer requests 4 and 5 contain only short time windows reflected by the lighter circles with the smaller radius.

2. **Short⇒Long (SL)** The second approach is quite similar to LS, but begins with offering short time windows. Following the example of many online retailers, customers that arrive early in the booking process are offered better service in terms of short time windows, and late arriving customers are offered long time windows. The decision on when to switch from short to long time windows can be formalized as follows (Algorithm 4).

**Algorithm 4: SL Approach**

```
ChooseTimeWindowLength
if \( x^{SL} \geq (t_{v0} + \sum_{i=1}^{q-1} (t_{vi,i+1} + u_i) + t_{v0,i})/(|V| \cdot T) \) then
    | \( S_{i,i+1}^{no} = S_{i,i+1}^{no}, L_{i,i+1}^{no} = \emptyset \)
else
    | \( S_{i,i+1}^{no} = \emptyset, L_{i,i+1}^{no} = L_{i,i+1}^{no} \)
```

Figure 1b shows an example where the first three arriving customers are offered a short time window, while customers 4 and 5 are offered only long time windows within their offer set to accommodate the flexible addition of remaining requests.

3. **Travel Time (TT)** With the TT approach, we want to examine offering sets of short time windows only to requests that are located in the vicinity of customers already contained in our tentative route plan. To this end, we check if a new request is in the vicinity of an existing customer. The vicinity is defined by the relative travel time from the location of an accepted customer to the location of the new request relative to the total time capacity. If the relative travel time is below the threshold value \( x^{TT} \), the new request should not impact the flexibility of the tentative route significantly and is hence offered a short time window (Algorithm 5):
Figure 1: Example of the mechanisms of the four time window management approaches. The customers are numbered according to their arrival order within the booking process.

4. Insertion Time and Span (TS) The TS approach uses two pieces of information:
(i) the relative insertion time of a request $j$ at a specific insertion position extending the idea of TT and (ii) the relative length of the time span that is connected to the insertion of request $j$ at a specific insertion position. With the latter, we want to quantify how much potential of including further requests the impacted part of the route plan has. The longer the time span is, the more likely are still major changes in the tentative route plan, and we may not want to offer a short time window to maintain flexibility.

Algorithm 6: TS Approach

Algorithm 6 summarizes this approach more formally. In particular, with TS, we
only offer short time windows for an insertion position if (i) the relative insertion time is below the threshold $x^{TS_{Time}}$ and (ii) if the relative impacted time span is below $x^{TS_{Span}}$. In the TS example of Figure 1d, requests 1, 2 and 3 have been accepted within a long time window. Although relative insertion time of request 5 is small due to the vicinity to customer 2, we do not offer a short time window, since the connected insertion time span is quite long. For request 4, the insertion time is relatively small and the connected relative time span of insertion is negligible, which allows us to offer a short time window in this case.

### 3.2.3 Customer Behavior and Update of Route Plans

After the creation of the offer set, it is displayed to the requesting customer who now can choose between the presented time windows or cancel the booking process. We assume that each customer requesting service belongs to one of two customer groups that differ in their preference for early or late time windows. With a probability of $p$, customers belong to the first group and accept only early time windows, and with a probability of $(1 - p)$, customers accept only late time windows. We define the early time windows as those that occur in the first half of the day (e.g. in the afternoon from 14:00–18:00) and the remaining as late time windows (e.g. in the evening from 18:00–22:00).

We model the customer’s choice for a specific time window option as displayed in Figure 2. A request with a preference for early time windows only confirms if the offer set contains time windows within the first half of the day. If not, the request cancels. If time windows are available during the preferred time of the day, a customer request will always favor short time window options over long ones. When only long early time window options are available during the preferred time of day, we assume that the requesting customer accepts one of these with probability $a$. If a requesting customer accepts one of the offered time windows, the customer is inserted in the tentative route plan within the selected time window at the minimal cost insertion point in the current set of routes. If needed, the beginning of service for the following customers on that route is updated (see Algorithm 7).
Algorithm 7: Insertion of a Customer and Update of Tentative Route Plans

\[ \text{if } \text{SelectTimeWindow}(O_j) = \text{true} \text{ then} \]
\[ \text{insert request } j \text{ into route } v, R^v \cup j \]
\[ \text{if } \text{Chosen Time Window} = a_m^S \in O^j \text{ then} \]
\[ e_j^v = \max(a_m^S, e_{i,i+1}^v) \]
\[ \text{else} \]
\[ e_j^v = \max(a_i^L, e_{i,i+1}^v) \]
\[ \text{end} \]
\[ e_{i+1}^v = \max(e_j^v + u_j + t_{i,i+1}^v, f_{i+1}^v) \]
\[ \text{for all subsequent customers do} \]
\[ e_k^v = \max(e_{k-1}^v + u_{k-1} + t_{k-1,k}^v, f_k^v), k = i + 2, ..., q \]
\[ \text{end} \]

4 Experimental Settings

To investigate the effectiveness of our flexible time window management approaches, we analyze the results from different demand patterns and varying customer behavior. We define our delivery area as a rectangle between longitude 13.3-13.5 and latitude 52.45-52.55, which overlays the inner city of Berlin and reflects a dense distribution of customers in an urban area. We use a road network for this area from OpenStreetMap. From this road network, we randomly select 400 nodes, which serve as potential customer locations and one fixed location for the depot (see Figure 3). The travel times in minutes between these nodes are determined with OSRM\(^1\), an OpenStreetMap routing service.

For each instance, we consider orders from 100 customers selected randomly from the

\(^1\)https://cran.r-project.org/web/packages/osrm/index.html
Our preliminary experiments showed 3 vehicles is sufficient to handle approximately 75 customers or 75% of demand. This is a realistic assumption for many online supermarkets which operate their own fleet for delivery (e.g. REWE in Germany). Retailers want to provide enough delivery capacity to ensure service for the majority of customers, so this represents a viable business model.

Each vehicle can service customers for a maximum of $T = 8$ hours. To enable easy comparisons, travel times from and to the depot are excluded, i.e., only the beginning of service for the last customer on each route has to be within the operating time. Since time window constraints are often more binding than capacity constraints for attended home deliveries, we do not specify the vehicles’ capacities. Our standard time window length is four hours, and the short time windows have a length of 30 minutes. We offer afternoon or evening time window options: $L = \{14:00–18:00, 18:00–22:00\}$ and $S = \{14:00–14:30, \ldots, 21:30–22:00\}$.

For our experiments, we begin with the assumption that demand from customers is spread evenly across the day. We call this pattern, shown at the top of Figure 4, equal demand. The long early and late time windows are each preferred with 50% probability, and the 50% is evenly divided among the short windows that comprise each long window. If not all short time window options are available, the probabilities are adjusted relative to the number of available short time window options. For example, if a customer prefers an evening time window and there are only two short windows feasible, both would have an equal chance of being selected. Equal demand serves as our base case to help us understand how the different flexible time window management schemes impact the results without including a more complicated demand pattern.

We also test our flexible time window management when the demand from the customers is concentrated in only a few of the offered time window options. We call this unequal demand. The demand pattern for unequal demand is described on the bottom of Figure 4 and is derived from the order data for Berlin of AllyouneedFresh, an online grocer operating in Germany. Late time windows exhibit a much higher popularity and
are requested by 90% of the customers. Here, we also consider larger popularity for some short time windows as derived from AllyouneedFresh. The highest demand for short time windows is between 18:00 and 20:00 (see right side of Figure 4). If not all short time window options are available, the probabilities are adjusted relative to the number of available short time window options. For the unequal demand, for one set of experiments, we increase the delivery fleet to $|V| = 5$ vehicles for the higher demanded time windows. We refer to delivery capacity of 3 as the *inhouse fleet* and the flexible logistics capacity with a larger number of delivery vehicles at peak times as the *flexible fleet*. Flexible fleet scenarios are especially relevant when online supermarkets have access to third party logistics service providers that can adapt to the number of requested delivery capacities more easily. For example, AllyouneedFresh can offer a larger number of deliveries in peak times by utilizing additional delivery vehicle from the DHL fleet.

Our flexible time window management approaches require input parameters which define when short time windows are to be offered. To make the results of the different approaches comparable, we define the parameters in relation to the total delivery time capacity available, which is given by $|V| \times T$. The range of parameters has been derived from a computational study and has been chosen such that the total number of accepted customers are of comparable size. The particular values are as follows:

- **Long⇒Short (LS):** We include only long time windows in offer sets until deliveries reach a certain utilization level, i.e., until a certain portion of the total delivery time capacity has been assigned to already accepted customers. The investigated utilization levels are:
  \[ x^{LS} = \{90\%, 80\%, 70\%, 60\%, 50\%, 40\%, 30\%, 20\%, 10\%\} \].

- **Short⇒Long (SL):** As with LS, we measure the utilization level in relation to the total delivery time capacity, but switch from offering only short to long time windows once a certain utilization level has been reached. The investigated utilization levels are:
  \[ x^{SL} = \{10\%, 20\%, 30\%, 40\%, 50\%, 60\%, 70\%, 80\%, 90\%\} \].

- **Travel Time (TT):** We investigate the proximity of incoming order requests according to their relative travel time to already accepted customers. The relative travel time is derived from the additional travel time required to add a request between already accepted customers relative to total delivery time capacity. Short time windows are only offered if the relative travel time is below the following thresholds:
  \[ x^{TT} = \{0.25\%, 0.50\%, 0.75\%, 1.00\%, 1.25\%, 1.50\%, 1.75\%, 2.00\%, 2.25\%\} \].

- **Insertion Time and Span (TS):** We now consider the relative insertion time a request causes as well as the relative time span impacted by insertion. We conducted
preliminary experiments to define reasonable pairs of thresholds for relative insertion time and relative time spans impacted by insertion. As a result, we scale relative insertion time and relative time span with a factor of 6. The considered threshold pairs \((x^{TS}_{Time}/x^{TS}_{Span})\) are: \((1%/6%)\), \((2%/12%)\), \((3%/18%)\), \((4%/24%)\), \((5%/30%)\), \((6%/36%)\), \((7%/42%)\), \((8%/48%)\), \((9%/54%)\). For the ease of presentation, we will only use the \(x^{TS}_{Time}\) threshold as a reference in the computational experiments, i.e. \(x^{TS} = \{1%, 2%, 3%, 4%, 5%, 6%, 7%, 8%, 9%\}\).

We compare our flexible time window management approaches with randomly offering all feasible long or short time windows to customers to validate the contribution of our approaches. For the creation of random offer sets, a given probability \(x^{random}\) with thresholds of \(x^{random} = 10%, 20%, 30%, 40%, 50%, 60%, 70%, 80%, 90%\) is used to decide if only long or short time windows are added to the offer set. For instance, at a probability of \(x^{random} = 10%\), a customer request would receive an offer set containing only short time windows with a chance of 10%, and, with a chance of 90%, an offer set in which only long time windows are included.

When an offer set has been created by a flexible time window management approach, the requesting customer selects an offered time window according to the choice procedure presented in Section 3.2.3. When only a long time window is offered within the preferred time of the day, the acceptance probability for the long time window is determined by the value \(a\). We test four different acceptance probabilities of long time windows between \(a = 100%\) (i.e. a long time window is always accepted) and \(a = 25%\) (a long time window is rarely accepted). In the end, if a customer chooses one of the offered time windows, we reserve 12 minutes of service time for each customer request, which is a common assumption for attended home deliveries in the literature (e.g. Klein et al. (2017)).

Whenever further 10 requests have been accepted, the tentative route plans are improved by an Iterated Local Search procedure following Vansteenwegen et al. (2009), which enables fast improvement of existing route plans.

5 Computational Results

In this section, we will analyze the effectiveness of our flexible time window management approaches for different demand scenarios. As our aim is to accept as many customers as possible with the best possible service, our key metrics are the total number of customers accepted as well as the number of customers accepted in a short time window. We will also analyze how the flexibility of tentative route plans evolves over the booking process for the different acceptance mechanisms. Each booking process contains 100 customer requests and can be completed by our algorithm in only a few seconds when implemented in Java.
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Table 1: Results for Equal Demand – Average # of Customers Accepted in Total/in Short Time Windows

8 on a Windows 10 machine with an Intel i5-3470 processor. All reported numbers are based on averages of 1000 independent booking processes.

5.1 Equal Demand

5.1.1 Overall Results

The results of the four acceptance mechanisms in a setting with equal demand and varying acceptance probability of long time windows are displayed in Figure 5. The acceptance probability varies from $a = 100\%$ (see Figure 5a) to $a = 25\%$ (see Figure 5d). Each y-axis denotes how many customers have been accepted on average in total (max: 100), and each x-axis reads how many of these are accepted on average within a short time window. Each point represents a result of a specific combination of acceptance mechanism and threshold, and all investigated thresholds for one approach are connected by a plotted spline. The specific values for each threshold can be found in Table 1. The gray curve describes the results of randomly offering long or short time windows. Each farthest left point for an acceptance mechanism within Figure 5 reflects the metrics resulting from applying the highest (LS) or smallest threshold value (SL, TT, TS, random), respectively.
Figure 5: Results for Equal Demand and Varying Probabilities of Long Time Window Acceptance

LONG⇒SHORT (LS), TRAVEL TIME (TT), SHORT⇒LONG (SL), INSERTION TIME & SPAN (TS), RANDOM
First, we take a closer look at Figure 5a, which shows the results for an acceptance probability of long time windows with $a = 100\%$. This is our simplest case, since a customer would always accept a long time window if the retailer offers it at the desired time of the day. Generally, we can see that offering more short time windows decreases the route plan’s flexibility since less customers can be accepted by all mechanisms. For all approaches, the largest number of customers can be accepted by offering many long time windows. By offering short time windows to different customer requests at different points of the booking processes, the resulting key metrics differ significantly as indicated by the spread of the colored lines. LS, TT and TS outperform the random offering for all tested thresholds, but SL provides the worst results. Offering short windows early in the booking process creates the biggest reduction in flexibility and thus yields the lowest total number of accepted requests. For up to 30 requests in short windows, LS is able to accept the most deliveries. This indicates that this simple rule is able to preserve flexibility within the routes as it accepts customers in short windows only after a significant number of requests have been accepted.

If the acceptance probability of long time windows is reduced to $a = 75\%$ (see Figure 5b), it is still true that accepting many customers in short time windows results in less customers accepted in total. However, the peak of the LS and TS curves is no longer when the most long time windows are offered, but when a few short time windows are offered. The reason is that long time windows are not always accepted when offered. On average, about 10 (LS) or 17 (TS) customers are accepted within a short time window at the peak of the two curves. Although the overall behavior of the approaches is still the same, the shape of the curves is more dispersed indicating that the choice of the appropriate time window management becomes more important when $a$ is below 100%.

As already noted, the results for $a = 100\%$ and $a = 75\%$ show that accepting few customers within short time windows leads to more customers accepted in total. However, this is not true when the acceptance probability drops to $a = 50\%$ or $a = 25\%$. In Figure 5c, we can now see a shift in the evolution of the key metrics. Since many customers cancel the booking process because they will not accept a long time window, offering more long time windows is no longer advisable, which is indicated by the shift of the peaks of the LS, TT and TS curves. Now, TS provides the maximum number of accepted customers (around 65), from which the largest share (about 48) can be serviced within a short time window. TT and TS seem to work better when many customers avoid long time windows. This indicates that when long time windows are not preferred, the methods of TT and TS do a better job of determining which deliveries should not receive short time windows. They effectively discourage requests that are not easy to accomplish within short windows. For $a = 25\%$, the differences between the approaches become negligible, and more customers can be accepted if more short time windows are offered. However,
TT and TS can still accept a few more customers than the other approaches.

Three main insights can be derived from the experiments with equal demand. First, the point in the booking process customers are offered a short time window can greatly impact the total number of accepted requests. Second, the performance of flexible time window management depends heavily on the customers’ willingness to accept long time windows. We will examine this observation in more detail in Section 5.1.2. Third, techniques that carefully evaluate spatio-temporal information of additional requests are more successful when $\alpha$ is below 100%.

5.1.2 First Short Time Window Offering and Cancellation

Next, we want to understand how the flexible time window management approaches differ in terms of what they offer to the same customers at different points in the booking process. The first column, FirstShort, for each acceptance rate in Table 2 tells us which request, on average, is the first to receive a short time window offer. The right of the two columns, First Cancel, reports which request, on average, is the first to cancel the booking process. We can see that with $\alpha = 100\%$, the different approaches vary wildly in terms of when the first short window is offered. We also see that LS does not cause cancellations until the 46.7$^{th}$ request for low thresholds and not until the 77.6$^{th}$ for larger ones. For SL, TT, and TS, the first cancellations are at about the 70$^{th}$ customer (69.4, 73.0, and 78.2, respectively) for the lower thresholds and at about the 40$^{th}$ customer (43.6, 44.5, and 43.4) for the higher thresholds.

The patterns for cancellation change significantly for lower acceptance rates. When $\alpha = 75\%$, for example, LS causes the first cancellation very early for all thresholds (e.g. 4.0 at 90%). SL causes the first cancellation much earlier than with $\alpha = 100\%$, but the values here steadily increase (e.g. 43.1 at 90%). For TT and TS, the first cancellation can also be observed much earlier than when $\alpha = 100\%$. LS, TT, and TS all tend to create cancellations earlier than SL for equivalent thresholds. This is because they offer long time windows earlier, which now have a certain chance of being rejected. When $\alpha = 50\%$ and 25%, the patterns remain similar, but the first cancellation for LS, TT, and TS is again even earlier.

5.1.3 Reasons for Cancellations

We want to understand better what are the particular causes of these cancellations. Is it because only long time windows are offered which were simply not accepted or is it because some of the approaches could not offer time windows during the preferred time of day? Figure 6 presents an example of a booking process and how it would be handled by our flexible time window management for $\alpha = 75\%$ with thresholds that result in roughly half of the customers being accepted within a short time window. Each box represents a
| x   | LS 90 | LS 80 | LS 70 | LS 60 | LS 50 | LS 40 | LS 30 | LS 20 | LS 10 | SL 10 | SL 20 | SL 30 | SL 40 | SL 50 | SL 60 | SL 70 | SL 80 | SL 90 | TT 0.25 | TT 0.50 | TT 0.75 | TT 1.00 | TT 1.25 | TT 1.50 | TT 1.75 | TT 2.00 | TT 2.25 | TS 1.0 | TS 2.0 | TS 3.0 | TS 4.0 | TS 5.0 | TS 6.0 | TS 7.0 | TS 8.0 | TS 9.0 |
|-----|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
|     | 76.4  | 67.7  | 59.5  | 51.1  | 42.6  | 34.1  | 25.2  | 16.6  | 8.3  | 1.0  | 1.0  | 1.0  | 1.0  | 1.0  | 1.0  | 1.0  | 1.0  | 1.0  | 0.25  | 0.50  | 0.75  | 1.00  | 1.25  | 1.50  | 1.75  | 2.00  | 2.25  | 1.0  | 2.0  | 3.0  | 4.0  | 5.0  | 6.0  | 7.0  | 8.0  | 9.0  |
|     | 77.6  | 75.2  | 72.3  | 68.8  | 65.0  | 60.5  | 55.7  | 51.4  | 46.7 | 69.4 | 61.3 | 54.1 | 48.9 | 45.1 | 44.3 | 43.0 | 43.1 | 43.6 | 98.6  | 89.9  | 78.9  | 67.6  | 56.4  | 45.1  | 33.1  | 21.8  | 10.8  | 7.0  | 6.7  | 6.4  | 4.4  | 3.4  | 2.6  | 2.3  | 1.6  | 1.3  |
|     | 4.0   | 4.0   | 4.0   | 4.0   | 4.1   | 3.8   | 4.0   | 4.3   | 8.8  | 11.2 | 18.5 | 24.9 | 30.9 | 36.2 | 39.8 | 42.2 | 43.1 | 43.6 | 22.1  | 12.3  | 7.8   | 6.7   | 9.2   | 1.0   | 3.2   | 2.8   | 2.4   | 2.1   | 2.0  | 4.0   | 4.1   | 3.9   | 5.4   | 7.6   | 14.9  | 18.1  | 5.1   |
|     |       |       |       |       |       |       |       |       |      |      |      |      |      |      |      |      |      |      |      |       |       |       |       |       |       |       |       |      |      |      |      |      |      |      |      |
|     |       |       |       |       |       |       |       |       |      |      |      |      |      |      |      |      |      |      |      |       |       |       |       |       |       |       |       |      |      |      |      |      |      |      |      |
| %   |       |       |       |       |       |       |       |       |      |      |      |      |      |      |      |      |      |      |      |       |       |       |       |       |       |       |       |      |      |      |      |      |      |      |      |      |
| a=100% |      |       |       |       |       |       |       |       |      |      |      |      |      |      |      |      |      |      |      |       |       |       |       |       |       |       |       |      |      |      |      |      |      |      |      |      |
| a=75%  |      |       |       |       |       |       |       |       |      |      |      |      |      |      |      |      |      |      |      |       |       |       |       |       |       |       |       |      |      |      |      |      |      |      |      |      |
| a=50%  |      |       |       |       |       |       |       |       |      |      |      |      |      |      |      |      |      |      |      |       |       |       |       |       |       |       |       |      |      |      |      |      |      |      |      |      |
| a=25%  |      |       |       |       |       |       |       |       |      |      |      |      |      |      |      |      |      |      |      |       |       |       |       |       |       |       |       |      |      |      |      |      |      |      |      |      |

Table 2: Results for Equal Demand – Average # of First Customer Receiving a Short Time Window/Canceling

customer request with (1) being the first and (100) being the last request. If a customer accepted a time window offering, the box is colored white, and it is shown whether a long (L) or short (S) time window was chosen. We display gray boxes for requests that canceled the booking process because they were only offered a long time window (L) or the offer set contained only time windows that were not within their preferred time of the day (P). We use dark gray boxes for requests where no time windows could be offered at all (N).

For early requests (1 to 50), we can see for all mechanisms that customers only canceled the booking process because of the offered time window lengths. LS strictly offers only long time windows to these customers and hence loses the most requests early in the booking process. SL offers short time windows from the first customer until the 30th customer, and only five customers cancel after the switch to long time windows. Note that SL offers short time windows to customers that would also have accepted a longer time window. Thus, valuable flexibility is forfeited quite early in the booking process (e.g. customer 6). For TT and TS, short and long time windows are not limited to only first or last customers. Here, offer sets are quite mixed, and so is the occurrence of requests canceling because of the offered time window length.

In the second half of the booking process, from requests 51 to 100, a significant portion
of the total delivery capacity has already been assigned. Here, customers often cancel because the availability of time windows has become much more limited. LS benefits from the long time window offerings in the beginning since the first request to cancel because of not being presented a time window option within the preferred time of the day is request #84. In contrast, SL cannot offer any preferred time window option from the 64th request on. TT and TS maintain flexibility throughout the whole booking process, which results in more suitable time window options offered to customers compared to SL.

Following the above example, we present an aggregated analysis of cancellation reasons for all 1000 simulated booking processes. For \( a = 75\% \), Figure 7 displays the results for low, medium and high thresholds, respectively. The \( x \)-axis denotes the index of a customer requesting service during the booking process from 1 to 100. The \( y \)-axis represents the number of runs, in total 1000. Results are sorted according to cancellation reasons. The color of the resulting areas represents whether a customer was accepted within a long time window (white-dotted, L) or within a short time window (white, S). If a customer cancels the booking process, the three possible reasons are highlighted: (i) cancellation because of only being offered a long time window (gray-dotted, L), (ii) being offered a time window that is not within the customer’s preferred time of the day (gray, P), and (iii) no time window could be offered at all (dark gray, N). An overview of all simulated booking processes and all tested thresholds can be found in the Appendix.

For LS with a threshold of \( x^{LS} = 90\% \) (see upper left corner of Figure 7), reasons for cancellation be summarized quite easily: customers are either accepted within a long time window, or they cancel because they do not accept long time windows and no short time windows were offered. Limited availability is not a problem here. With smaller thresholds, however, short time windows tend to be offered earlier, and this reduces flexibility and hence the availability of time windows at the end of the booking process. As a consequence, the share of customers cancelling because of time windows not being offered at the desired time of day increases (P) or we run out of any time windows at all (N). SL accepts many customers within a long time window at a low threshold of \( x^{SL} = 10\% \) (see
upper right of Figure 7). Cancellations due to insufficient suitability and availability of time windows become a significant issue starting from \( x_{SL} = 50\% \) and higher thresholds, because flexibility cannot be exploited well (for \( x_{SL} = 50\% \)) or there is no flexibility to exploit at all any more (for \( x_{SL} = 90\% \)) due to the short time window promises.

For TT and TS, offering short time windows is not limited to only early or late customers, but can occur for nearly any customer during the booking process. However, for small thresholds (\( x_{TT} = 0.25\% \), \( x_{TS} = 1.0\% \)), the vicinity requirement becomes too restrictive, and customers cancel because only long time windows can be offered to them (as also observed for LS and SL with \( x_{LS} = 90\% \) and \( x_{SL} = 10\% \)). Medium thresholds seem to create a good compromise of offering many suitable time windows of different lengths, while large thresholds tend to cause availability problems as also observed for \( x_{SL} = 90\% \) and for \( x_{LS} = 10\% \).

In summary, offering many long time windows can come with a significant share of requests cancelling the booking process because of the time window length, and offering many short time windows can lead to a situation where no time window can be offered at all. Furthermore, the more customers are accepted within a short time window, the larger is the portion of customers canceling because their time window is not available within their preferred time of the day.
5.1.4 Evolution of Route Plans

Next, we want to understand how the different acceptance mechanisms impact the tentative route plans. To this end, we present examples of the route plans created by each approach. We consider only one vehicle operating in the second half of the delivery day (18:00–22:00) to enable a simple visualization. We consider 20 customer requests, all looking for a late time window option, and from which 5 requests do not accept long time windows (see Figure 8a). Similar to the previous section, Figure 8b presents the evolution of the booking process from request 1 to 20 for our four acceptance mechanisms. Figures 8c to 8f visualize the resulting route plans after the booking process has been finished. The number of the request reflects the order of arrival. We discuss the tentative route plans at the times when customers 1, 2, and 6 arrive and compare with the final routes.
**Request #1**, the first arriving request, is located close to the depot. LS assigns a long time window, but we can observe that in the final LS route plan, #1 is neither serviced directly after leaving or before arriving at the depot although closely located. SL and TT both accept #1 within a short time window, and the request is actually served directly from the depot in the final route. Note that all short time windows were available within the offer set of #1, since no other requests had been promised a time window so far, but not all short time window offerings would have been desirable to maintain the flexibility of the final route plan. Finally, TS accepts #1 in a long time window because of the long duration of the impacted time span, maintaining more flexibility for integrating future customers.

**Request #2** wants a short time window and would not accept a long time window. However, LS, TT and TS offer long time windows, so #2 cancels. Only SL offers short time windows, from which #2 chooses the one starting at 18:30. With the next arriving request, we can see that an acceptance of #2 would have probably been beneficial because the travel distance to #3 is low, and #3 is accepted by all mechanisms.

**Request #6** is not located close to any of the already accepted or later arriving requests. Only SL accepts #6 in a short time window because of its early appearance. SL’s final route plan reveals the rather long travel distances from requests #9 to #6 to #11 that are caused by the early short time window promise to #6. The other mechanisms handle #6 with a long time window and hence enable shorter travel distances.

Looking at the final route plans, it is obvious that TS does well in terms of number of customers accepted and number of customers accepted in a short time window. However, the resulting route plan shows many crossings, which can lead to long tours and indicate inferior route plan efficiency. SL’s final route plan also contains many crossings; the early accepted customers #1-6 make the route plan very inflexible and later customers with long time windows (e.g. customers #9 and #11) cannot adapt the structure of the route plan any more to avoid detours. On the contrary, the route plan created by LS only shows two crossings caused by the tight time window of #16. TT’s final route plan indicates that the vicinity condition works well for some customers (such as #10 which is close to customer #5 and #13 and #8 which are close to customer #3), while it does not for others (e.g. #1 and #7). The latter are located closely to the depot, but they are accepted within different short time windows causing costly detours.

### 5.2 Unequal Demand

#### 5.2.1 Overall Results

Inspired by order data provided by the online retailer AllyouneedFresh, in this section, we analyze the effectiveness of flexible time window management for unequal demand, i.e. when demand concentrates at peak times. In Figure 9, the results for the four mechanisms
**Table 3: Results for Unequal Demand – Average # of Customers Accepted in Total/in Short Time Windows**

<table>
<thead>
<tr>
<th></th>
<th>x</th>
<th>(a=100%)</th>
<th>(a=75%)</th>
<th>(a=50%)</th>
<th>(a=25%)</th>
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<td># Short</td>
<td># Accepted</td>
<td># Short</td>
<td># Accepted</td>
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As well as a random time window offering are shown. The detailed metrics can be found in Table 3.

For \(a = 100\%\) (Figure 9a) and \(a = 75\%\) (Figure 9b), the general behavior of flexible time window management is similar to equal demand, but the total number of customers accepted decreases to a total of about 56 and 53 on average. This reduction is caused by the demand concentration in popular time windows. SL still produces inferior results, and LS provides again the maximum number of accepted customers. TT and TS lose a bit in their performance as compared to equal demand distribution, but TT still yields the best combination of number of accepted customers and number of customers accepted within a short time window.

Significant differences to equal demand can be observed for an acceptance rate of \(a = 50\%\) (see Figure 9c). We recall that for equal demand, offering many long time windows was not advisable because of the customers’ low willingness to accept a long time window. Now, the largest number of customers can be accepted for small or large thresholds (i.e. accepting only a few or many customers in short time windows). For instance, SL reaches its minimum for a medium threshold with only 42 requests being accepted and 34 being accepted in a short time window. For equal demand, the same
Figure 9: Results for Unequal Demand and Varying Probabilities of Long Time Window Acceptance
threshold yields about the same number of requests accepted in short time windows, but a much higher total of 59. The lower totals reflect that all approaches have to deal with high demand and limited capacities at peak times as well as picky customers.

For the smallest probability of accepting a long time window \((a = 25\%)\), see Figure 9d), the effectiveness of the approaches is quite similar to an equal demand distribution.

In the following sections, we will investigate the reasons for the different behavior of acceptance mechanisms with unequal demand in more detail. We will look at how short time windows are distributed (Section 5.2.2), analyze the reasons customers cancel at times of peak demand (Section 5.2.3), and investigate the impact of flexible adaptations of a retailer’s delivery capacity on the behavior of our acceptance mechanisms (Section 5.2.4).

### 5.2.2 Distribution of Short Time Windows

In this section, we focus on the perspective of the customer in terms of how the chance of receiving a short time window can vary under different acceptance mechanisms when many customers request delivery at the same time of the day. We want to understand if there are patterns in times and locations of requests that create higher chances of receiving the desired short time window for the different acceptance approaches. We discuss sample results for \(a = 50\%\) and choose the thresholds such that the different approaches achieve a comparable number of customers accepted within a short time window (around 30 for \(x^{LS} = 20\%, x^{SL} = 50\%, x^{TT} = 1, 25\%, x^{TS} = 6.0\%\)). We analyze how the chance to receive a short time window varies across our acceptance mechanisms for the different customer locations. To this end, in Figure 10, we show how often (y-axis) a specific customer (x-axis) out of all possible 400 customer nodes was accepted within a short time window after 1000 booking processes have been completed.

For LS and SL, each customer is accepted between 50 and 100 times within a short time window, and short time windows are offered equally to all customers on average. For these approaches, the length of the offered time window mainly depends on the time of booking and not on the location of a customer. As expected, TT distributes short time windows less evenly. Some customers are accepted as often as almost 200 times, whereas other customers are accepted less than 10 times within a short time window. For example, the customer node that is accepted the most within a short time window is located at “Potsdamer Platz”, which is the most central location in the center of Berlin and hence easy to reach by a delivery vehicle. On the contrary, the customer node with the least short time window frequency is located in a remote neighborhood, and access is possible only via a few local roads. As a result, from a customer perspective, TT can lead to “unfair” allocation of short time windows and tends to discriminate customers that are not easy to access. TS seems to be a compromise: the allocation is more spread than with LS and SL, but still less heterogeneous than with TT.
In summary, we find that TT provides good results in terms of number of customers accepted and numbers of short time windows offered, but at the expense of discriminating some customers systematically through not offering them short time windows.

### 5.2.3 Reasons for Cancellation

Following the analysis of equal demand, we present the evolution of offer sets and reasons for cancellations based on a particular booking process and from an aggregated perspective at all runs. In Figure 11, we present an example for unequal demand with $a = 50\%$ and compare this to the example for equal demand presented in Figure 6. Three main differences can be found: first, since the acceptance rate is lower, many customers cancel the booking process because of the time window length (gray “L”). Furthermore, although all requests receive an offer set with at least one time window, many customers cancel, mainly because of not being offered a preferred (late) time window option (gray “P”). Given the high demand for late time windows, the available time window capacity is quite limited. At this point, LS clearly shows its disadvantages: since the threshold of $x^{LS} = 50\%$ is not met before the 98$^{th}$ customer, short time windows are only offered very late in the booking process, and many customers cancel because they do not want long time windows at all (gray “L”). However, time windows can be offered to all customers within the preferred time of the day, and no customers cancel because of their preferences for late time windows with LS.

An aggregated perspective on the investigated booking processes is shown in Figure 12, which summarizes the reasons for cancellations for all 1000 runs for low, medium and high thresholds. Throughout all booking processes, at least one time window could be offered to each customer (i.e. there are no dark gray areas as observed in Figure 7, for example). The impact of the unequal demand scenario is significant: although offered at least one time window, many customers cancel because their preferred time window was not available. For LS, on the one hand, for a very high threshold of 90%, customers are only offered long time windows and cancel because of the insufficient time window length. On the other
Figure 11: Unequal Demand – \( a = 50\% \), Example Booking Process, \( x^{LS} = 50\% \), \( x^{SL} = 50\% \), \( x^{TT} = 1.25\% \), \( x^{TS} = 5\% \)

Figure 12: Unequal Demand, \( a=50\% \), Detailed Overview Booking Process
hand, this has a positive impact on the availability of time windows as well as on routing flexibility and leads to an overall high number of accepted customers (see Figure 9). For SL, TT and TS, there are multiple reasons for cancellations that interact. For small thresholds, the largest portion of request cancels because of the insufficient time window length. For large thresholds, many short time windows are offered, but most of the customers cancel because of the insufficient temporal availability of their desired time window. For medium thresholds, both issues interact and cause the insufficient effectiveness of flexible time window management as presented in Figure 9.

In summary, with unequal demand and small acceptance probabilities of long time windows (e.g. $a = 50\%$), retailers need to create their offer sets very carefully and should either focus on the availability or the length of the offered time windows to achieve a large number of accepted customers. Trying to consider both can reduce the total number of customers significantly – only LS can handle this quite fairly and is hence the most advisable mechanism for this setting.

5.2.4 Flexible Fleet

Unequal demand can lead to unused delivery capacity during times of low demand and an inability to serve all of the customers wanting delivery during periods of high demand. Since retailers would want to serve more than 50\% of their customers, in this section, we assume that we can increase the delivery capacities for times of high demand. We add two more delivery vehicles to our fleet in times of high demand and thus operate a fleet of $|V| = 3$ vehicles between 14:00–18:00 and $|V| = 5$ between 18:00–22:00. We next analyze the outcomes of our flexible time window management with this flexible fleet.

Figure 13 presents the key metrics for this setting. With the unequal demand distribution, increased capacities obviously help to accept more customers. Furthermore, adapting the fleet size at peak times leads to similar behavior of our flexible time window management as observed with equal demand distribution. LS creates the best results in terms of customers accepted and customers accepted within a short time window for acceptance probabilities of $a = 100\%$ and $a = 75\%$. Surprisingly, SL as well as the random time window offering provide good results for high acceptance probabilities of long time windows now. A concentrated demand to only a few time windows and a larger number of vehicles seem to enable better partitioning of requests. However, the more customers are accepted within a short time window, SL’s weaknesses are revealed, and compared to the other approaches, SL becomes inferior. For the lower acceptance probabilities of $a = 50\%$ and $a = 25\%$, we can again see similar results as obtained for the equal demand: offering many long time windows is not advisable due to the low willingness of customers to accept these. If many short time windows are offered, the differences between the approaches become smaller, and TT tends to provide the best results.
Figure 13: Results for Unequal Demand, Varying Probabilities of Long Time Window Acceptance and a Flexible Fleet with $|V| = 3$ between 14:00–18:00 and $|V| = 5$ between 18:00–22:00
6 Discussion and Future Work

In this paper, we investigated the idea of flexible time window management to enable accepting as many customers as possible while providing good service with limited delivery resources. We wanted to develop simple ways to preserve the flexibility of the route plan such that the number of accepted customers would not be compromised by offering short time windows to customers that make route plans inflexible. We presented four approaches that consider the utilization of the route plan or the location of customer requests relative to already accepted customers in the decision of whether to add short time windows to an offer set. We tested the approaches for different demand patterns and customer behaviors to evaluate their effectiveness from a retailer’s perspective and also analyzed potential systematic discrimination of customers. Results showed that the more customers are willing to accept long time windows, this can help maintain flexibility and increase time window availability for later customers, especially when long time windows are offered in the beginning of the booking process. However, the more restrictive the acceptance of long time windows becomes, the more beneficial become approaches that explore information about the location of customer requests relative to already accepted customers. The presented approaches are easy to adapt by an online retailer, but some are difficult to explain to requesting customers. For instance, offering additional short time windows based on TT and TS can be communicated quite easily, because this simply means that the delivery vehicle is expected to operate in a customer’s neighborhood anyway. However, as shown by computational experiments, TT systematically discriminates requests that are located remotely. Although simple in implementation and providing good results from a retailer’s perspective, LS is quite counterintuitive to customers. Since early booking is valuable for a retailer, early-bird customers usually expect rewards in terms of reduced prices or short time windows. However, our results show that it is better to preserve short time windows for late customers. In practice, customers could anticipate that better time windows can be achieved later and may lose their motivation to order early. They may even try to postpone their order to the last minute, although this bears the risk of not receiving the desired time window at all. To prevent this problem, we would recommend that the retailer communicates updated short time windows to all customers accepted in long time windows when the route is completed. Perhaps, the retailer could communicate updated shorter time windows, such as 20 minutes, to those who placed their orders early.

In the future, we plan to investigate the interface of offer sets and pricing for flexible time window management. Our approaches can provide valuable input for pricing decisions of delivery time windows. The considered metrics can help to estimate opportunity costs of accepting a specific customer within a specific time window better. They can also help with making pricing decisions to maintain flexibility to accept as many customers as possible. There is also potential to model customer behavior based on established
customer choice models and by considering different customer segments. Furthermore, it would be interesting to not only consider fixed given time window lengths of various types, but flexibly adapt the lengths of the time windows according to the expected level of flexibility needed at a specific time of the booking process.
References


APPENDIX

Long⇒Short (LS), Equal Demand

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