A note on the effects of introducing a market for cash-settled forward contracts on electricity[*]

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Abstract

We extend the model of Allaz-Vila (1993) to a setting with uncertainty on the market demand. We study a model in which the forward market can be settled in cash, so that the market is open to risk-averse speculators. We show that the risk attitude of traders on the forward market plays a crucial role in determining the degree of competitiveness of the spot market. This because the market price of risk is positive and depends on the volatility of the spot demand. In markets with highly volatile demand the market price of risk is higher, hence the commitment effect of short-selling forwards showed by Allaz and Vila (1993) is reduced. Our model predicts then that opening a forward market in sectors with high demand uncertainty has a smaller positive impact on efficiency than in sectors with stable demand.

JEL Classification Codes: D43, L13

1 Introduction

The literature studying the effects of introducing forward markets on the degree of competitiveness of the spot markets for electricity seems overall to suggest that long-term contracts reduce the market power of producers (Powell (1993), Newbery (1995) and (1998), Green (1999)). One of the best known results in this direction is Allaz and Vila (1993), who show that the market power of producers on the final commodity market reduces when they are allowed to stipulate long-term contracts because at equilibrium they engage in forward sales in order to commit to a higher output. However, recently Mahenc and Salanie' (2004) pointed out that these results crucially depend on the form of competition present in the spot market: while quantities are strategic substitutes, prices are

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strategic complements, so that Bertrand competition is more prone to collusion. Hence, they prove that if producers compete "*a la Bertrand*", at equilibrium they buy forward contracts in order to signal they will fix a higher price on the spot market. Although the strategic insights of these models are very appealing, it is not clear to which extent these results hold in a more realistic setting in which demand is uncertain.

When demand shocks are introduced, the degree of openess of the financial forward market becomes important to determine its equilibrium. Bessembinder and Lemmon (2002) determine the forward power price in markets for forward contracts in which only wholesale producers and retailers trade, motivating this choice with the argument that only physical producers of electricity have the possibility to close their forward position at maturity. Green (2004) also considers a forward market in which only wholesalers and retailers can operate. He confirms the intuition of Newbery (2002), that high degree of competition on the spot market (or high chance to have adverse conditions there) restraints retailers to buy long term contracts, because in doing that they fix the price of electricity at a level that, ex-post, could prove to be too high.

However, it is well known that illiquidity often plagues existing forward markets. In the view of some practitioners, allowing entry of more traders could help to solve this problem: the easiest way to do this seems to allow trading of forward contracts that can be settled in cash.

In this paper, we analyze the effects of adding a market for forward contracts with cash settlement to a model of oligopolistic competition where the spot demand by retail consumers is uncertain. To do this we model a market in which a forward contract on the commodity is traded between producers and risk-averse financial speculators. In the special case of risk-neutral speculators, our model reproduces the result of Allaz and Vila (1993) showing that introducing forward markets drives the commodity price down to the competitive level. In the case of risk-averse speculators, however, we show that this strong result does not hold. Because speculators require compensation for the risk they are buying, the forward price falls below the expected spot price, leading the producers to restrain their forward sales. Our first result is that the competition-enhancing effect of forward markets decreases as spot market volatility increases due to the increase of the risk-premium, irrespectively of the form of competition on the forward market (Propositions 1 and 2). This indicates that the effectiveness of introducing forward markets for competition strongly depends on the volatility of the demand for the commodity. Hence, allowing for trading of forward contracts with cash settlement can increase efficiency substantially only in markets with stable demand (e.g. metals) while in markets subject to frequent exogenous demand shocks (e.g. energy) the opposite applies.

Moreover, this observation allows us to point out one of the main reasons why markets for cash-settled forward contracts in electricity did not develop substantially (Ong (1996)). The equilibrium price of the forward depends on the riskiness of the spot price. Since the volatility of the spot price can be influenced by the major generators (at least to some extent), the investors face a moral hazard when trading these securities. Ultimately, some screening device would

be necessary to deal with the moral hazard of generators, if market breakdown is to be avoided.

In Section 2 we present the model and our main results, comparing them to Allaz and Vila (1993). In section 3 we discuss the moral hazard problem faced by forward traders. Section 4 concludes.

2　The model

There are two periods, two risk-neutral producers, i and j. At time $t = 1$ the producers and risk-averse speculators $k \in M$ buy/sell forward contracts at the forward price p_f. At time $t = 2$ a spot market for the good opens and the actual production takes place: we assume the commodity (e. g. electricity) is non-storable, so that it cannot be produced at $t = 1$. The exchange of the physical commodity occurs between the producers and consumers with an uncertain, price-sensitive demand. At the same date $t = 2$ the forward contracts are executed.

Forward market

For simplicity, we assume there is no time-discount in the model. The agents operating in the forward market can buy or sell forward contracts that call for delivery at time 2. The future realization of the spot market demand is uncertain when the forward trading takes place.

The equilibrium forward price p_f (for one unit of commodity) fixed at time $t = 1$ rules out all arbitrage possibilities and clears the forward market.

If the speculators operating in the forward market are risk-averse, they would trade the quantity h_k which maximizes the expected utility of their profits $\Pi_k = (p_f - p) h_k$, with $h_k > 0$ indicating the sells of forwards and p is the spot price; assuming they have mean-variance utility functions with risk-aversion coefficient equal to λ_k, they maximize:

$$E[U_k(\Pi_k)] = E[p_f - p]h_k - \frac{\lambda_k}{2}Var(\Pi_k) \tag{1}$$

To derive the supply of forwards we first have to solve for the spot market equilibrium.

Producers objective

We assume that the producers are risk-neutral in order to abstract from any hedging consideration and focus on the strategic aspect of forward trading. The profit of producer i at $t = 2$ is composed of two elements: what he gets selling the output at the spot price, $pq_i - c(q_i)$; and what he earns from the forwards sold at p_f, i.e. the forwards profit $p_f f_i - c(f_i)$, with $f_i > 0$ indicating a short position in the forward. We assume that for each producer the cost is $c_i(x_i) = b_i x_i$ where $x_i = q_i + f_i$ is the total amount of production taking place at $t = 2$.

Consumers demand

We assume that consumers demand at $t = 2$ is known and is linear in the spot price: $D = a - p + \theta$, where θ is the realization of the demand shock $\widetilde{\theta}$. For simplicity, we assume that the random variable $\widetilde{\theta}$ has a binomial distribution $\{\theta_L, \theta_H\}$ with probabilities $\{\pi, 1 - \pi\}$. Moreover, we normalize $E[\widetilde{\theta}] = 0$ (so that $\theta_L = -\frac{1-\pi}{\pi}\theta_H$) and $Var[\widetilde{\theta}] = \pi\theta_L^2 + (1-\pi)\theta_H^2 = \frac{1-\pi}{\pi}\theta_H^2$.

2.1 Forward market competition à la Cournot

In this section, we characterize the solution of the model in the case the producers compete in quantities on the forward market. Section 3 will analyze the case in which they compete in price.

On the spot market we always consider competition "à la Cournot", because with Bertrand competition the commitment effect of forward contracting disappears (see Green (1999) and Mahenc and Salanie' (2004)).

We characterize the equilibrium proceeding backward from stage 2.

2.1.1 Equilibrium in the spot market

Given the position in forward f_i and f_j at $t = 2$ the producers i and j choose the optimal quantity to sell on the market. The two competitors maximize the time $t = 2$ profit:

$$
\begin{aligned}
\Pi_i &= p(\theta)q_i - bx_i = (a + \theta - x_i - x_j)\, q_i - bx_i \\
\Pi_j &= p(\theta)q_j - bx_j = (a + \theta - x_i - x_j)\, q_j - bx_j
\end{aligned}
$$

where q_i, q_j are, respectively, the quantity of commodity sold on the spot market by i and j, while $x_i = q_i + f_i$ and $x_j = q_j + f_j$. Maximizing the two profit functions for a given level of production of the opponent gives the reaction functions

$$
\begin{aligned}
x_i(x_j) &= \frac{a + \theta - b + f_i - x_j}{2} \\
x_j(x_i) &= \frac{a + \theta - b + f_j - x_i}{2}
\end{aligned}
$$

Solving for the Nash equilibrium of this game, for any given realization of θ, we find the spot market equilibrium. Notice that this equilibrium is the same as in Proposition 2.1 of Allaz-Vila (1993) once we correct for the realized shock:

$$
x_i^* = x_i(\theta) = \frac{(a + \theta - b) + 2f_i - f_j}{3} \tag{2}
$$

$$
x_j^* = x_j(\theta) = \frac{(a + \theta - b) + 2f_j - f_i}{3} \tag{3}
$$

$$
p^* = p(\theta) = \frac{(a + \theta) - f_i - f_j + 2b}{3} \tag{4}
$$

The only source of uncertainty the producers and the investors are exposed is the demand realization, hence the spot price: given (4), the ex-ante $Var(p^*) = \frac{1}{9}Var[\widetilde{\theta}] = \frac{1-\pi}{9\pi}\theta_H^2$.

2.1.2 No-arbitrage condition in the forward market

We now compute the forward price p_f that clears the forward market at $t = 1$, given rational expectations of the speculators. Assuming rational expectations consists in imposing that the speculators take into account the correct spot price equilibrium formula (4) to decide their optimal exposure in forward contracts. Given CARA utility functions, the optimal position for speculator k in forward contracts for a given forward price p_f and a given distribution of p^* is

$$h_k = \frac{p_f - E[p^*]}{\lambda_k Var(p^*)}$$

so that the aggregate speculators (short) position in forwards is equal to

$$H = \frac{p_f - E[p^*]}{\Lambda Var(p^*)}$$

where $\Lambda = \left(\sum_k \frac{1}{\lambda_k} \right)^{-1}$. The equilibrium forward price p_f is such that the aggregate position of speculators and producers i, j is zero:

$$f_i + f_j + H = 0$$

that gives:

$$
\begin{aligned}
\frac{p_f - E[p^*]}{\Lambda Var(p^*)} + f_i + f_j &= 0 \\
p_f - E[p^*] &= -\Lambda Var(p^*)(f_i + f_j) \\
p_f &= E[p^*] - \Lambda Var(p^*)(f_i + f_j) \\
&= E[p^*] - \Lambda \left(\tfrac{1-\pi}{9\pi} \right) \theta_H^2 (f_i + f_j) \quad (5)
\end{aligned}
$$

The forward price p_f in (5) is then the unique non-arbitrage forward price when speculators have rational expectations.

2.1.3 The choice of optimal forward exposure by the duopolists

Given the forward price p_f, the expected profit for producer i at time $t = 1$ is then

$$
\begin{aligned}
E[\Pi_i] &= E[(p^* - b)(x_i^* - f_i) + (p_f - b)f_i] \\
&= E[(p^* - b)x_i^*] + (p_f - E[p^*])f_i \\
&= E[(p^* - b)x_i^*] - \Lambda Var(p^*)(f_i + f_j)f_i
\end{aligned}
$$

where the expected profit collapses to the Allaz-Vila case if $Var(p^*) = 0$:

$$
\begin{aligned}
E[(p^* - b)x_i^*] &= E\left[\left(\tfrac{a + \tilde{\theta} - (f_i + f_j) + 2b}{3} - b \right) \tfrac{a + \tilde{\theta} - b + 2f_i - f_j}{3} \right] \\
&= \tfrac{a - b - (f_i + f_j)}{3} \tfrac{a - b + 2f_i - f_j}{3} + \tfrac{1}{9} Var(\tilde{\theta})
\end{aligned}
$$

since $E[\widetilde{\theta}] = 0$. When the producers compete in quantity on the forward market, i solves for his optimal forward position by choosing

$$\max_{f_i} E[\Pi_i] = \frac{a-b-(f_i+f_j)}{3} \frac{a-b+2f_i-f_j}{3} + \frac{1}{9}Var(\widetilde{\theta}) - \Lambda Var(p^*)(f_i+f_j)f_i \quad (6)$$

with $Var(p^*) = \frac{(1-\pi)}{9\pi}\theta_H^2$.

The necessary and sufficient conditions for a maximum of the program (6) are:

$$\frac{\partial \left(\frac{a-b-(f_i+f_j)}{3} \frac{a-b+2f_i-f_j}{3} - \Lambda \frac{(1-\pi)}{9\pi}\theta_H^2 (f_i+f_j)f_i \right)}{\partial f_i} = 0$$

$$\Rightarrow f_i = -\frac{1}{2}\frac{a\pi - b\pi - \Lambda\theta_H^2 f_j - f_j\pi + \Lambda\theta_H^2 f_j\pi}{-2\pi - \Lambda\theta_H^2 + \Lambda\theta_H^2\pi} = \frac{a-b-f_j - \frac{\Lambda\theta_H^2 f_j}{\pi} + \Lambda\theta_H^2 f_j}{4 + \frac{2\Lambda\theta_H^2}{\pi} - 2\Lambda\theta_H^2}$$

$$\frac{\partial^2 \left(\frac{a-b-(f_i+f_j)}{3} \frac{a-b+2f_i-f_j}{3} - \Lambda \frac{(1-\pi)}{9\pi}\theta_H^2 (f_i+f_j)f_i \right)}{\partial f_i^2} = \frac{2(\Lambda\theta_H^2(\pi-1)-2\pi)}{9} < 0$$

so that the f.o.c. characterizes the global maximum.

The symmetric condition $f_j = \frac{a-b-f_i - \frac{\Lambda\theta_H^2 f_i}{\pi} + \Lambda\theta_H^2 f_i}{4 + \frac{2\Lambda\theta_H^2}{\pi} - 2\Lambda\theta_H^2}$ together with $f_i = \frac{a-b-f_j - \frac{\Lambda\theta_H^2 f_j}{\pi} + \Lambda\theta_H^2 f_j}{4 + \frac{2\Lambda\theta_H^2}{\pi} - 2\Lambda\theta_H^2}$ gives us:

$$f_i = f_j = f = \frac{(a-b)\pi}{5\pi + 3\Lambda\theta_H^2(1-\pi)} = \frac{(a-b)}{5 + 3\Lambda\theta_H^2 \frac{1-\pi}{\pi}} = \frac{(a-b)}{5 + 3\Lambda Var(\theta)}$$

which is always lower than the forward position in the solution of Allaz-Vila (1993) (see their Proposition 2.3). This proves the following result.

Proposition 1 *If the speculators $k \in M$ operating in the forward market are strictly risk-averse, then the optimal forward position for producers is lower than the one with certain demand schedule.*

The interpretation of proposition 1 is simple. Risk-averse speculators buy forward contracts at a price which is lower than the expected spot price, since they ask a premium in order to bear the price risk. The discount is proportional to the variance of the spot price. The rationale of their behavior is different from Green (2004), where electricity retailers (who act as buyers of forward contracts) reduce their buy because they are afraid that adverse market conditions will let them face a very low spot price at $t = 2$; here we consider financial speculators, who do not operate on the physical spot market, but who care about the risk they are buying. In equilibrium the producers will optimally reduce their short forward position because the cost of selling forwards increases with the risk premium. The lower the short position of each producer, the lower the degree

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of competitiveness on the spot market at $t = 2$. When demand shocks are highly unpredictable the cost of selling forwards is so high that the introduction of a forward market has little impact on the overall efficiency.

2.2 Forward market competition à la Bertrand

Green (1999) argues that a less competitive market in long-term contracts has less impact on the spot market allocation. He proves that adding a forward market in which producers compete "à la" Cournot does not change the spot market allocation. On the other hand, he argues that a very competitive forward market, with competition "à la" Bertrand should produce larger effects on the spot market.

We verify this intuition extending the result of Proposition 1 to the case in which the producers compete in price in the forward market.

Proposition 2 *If the speculators $k \in M$ operating in the forward market are strictly risk-averse, and if the producers compete in prices on the forward market, then the optimal forward position for producers is lower than the one with certain demand schedule when*

$$Var(\theta) > \frac{9}{2}\Lambda^{-1}$$

Proof: Rewriting the f.o.c. for the profit maximization for producer i we obtain:

$$\frac{d\left(\frac{a-b-(f_i+f_j)}{3}\frac{a-b+2f_i-f_j}{3} - \Lambda\frac{(1-\pi)}{9\pi}\theta_H^2(f_i+f_j)f_i\right)}{df_i} =$$

$$\frac{1}{9}\frac{a\pi-b\pi-4f_i\pi-f_j\pi-2\Lambda\theta_H^2 f_i+2\Lambda\theta_H^2 f_i\pi-\Lambda\theta_H^2 f_j+\Lambda\theta_H^2 f_j\pi}{\pi} + \frac{1}{9}\frac{-2a\pi+2b\pi-f_i\pi+2f_j\pi-\Lambda\theta_H^2 f_i+\Lambda\theta_H^2 f_i\pi}{\pi}\frac{df_j}{df_i} =$$

0

and with Bertrand competition on the forward market: $\frac{df_j}{df_i} = -1$ then

$\frac{1}{9}\frac{3a\pi-3b\pi-3f_i\pi-3f_j\pi-\Lambda\theta_H^2 f_i+\Lambda\theta_H^2 f_i\pi-\Lambda\theta_H^2 f_j+\Lambda\theta_H^2 f_j\pi}{\pi} = 0$

The optimal forward position is given by the solution:

$$f_i = -\frac{3a\pi - 3b\pi - \Lambda\theta_H^2 f_j - 3f_j\pi + \Lambda\theta_H^2 f_j\pi}{-\Lambda\theta_H^2 + \Lambda\theta_H^2\pi - 3\pi}$$

and imposing symmetry: $f_i = f_j = f$ and solving for f :

$$f^B = -\frac{3}{2}\pi\frac{a-b}{-\Lambda\theta_H^2+\Lambda\theta_H^2\pi-3\pi} = \frac{a-b}{2+\frac{2}{3}\Lambda Var(\theta)}$$

Comparing the result in presence of Bertrand competition f^B with the result with Cournot competition, $f^C = \frac{a-b}{5+3\Lambda Var(\theta)}$, we can conclude that $f^B > f^C$ and $f^B < f^{Allaz-Vila} \Leftrightarrow 2 + \frac{2}{3}\Lambda Var(\theta) > 5 \Leftrightarrow Var(\theta) > \frac{9}{2}\Lambda^{-1}$. ∎

If the shock $\widetilde{\theta}$ has high variance, the forward market does not achieve the perfectly competitive allocation as in Allaz and Vila (1993) even if it is very competitive.

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3   Moral hazard in the forward market

The expected profits for the producers reduce with higher variance of the spot price (see (6)). This because a high volatility of p^* increases the risk premium asked by the speculator to bear the price risk. However, given their market power, the producers could in principle stabilize the price at $t = 2$, adjusting their production in order to accomodate the demand shock.

In this section we investigate whether this strategy is feasible (and profitable).

Proposition 3 *The producers cannot credibly commit to adjust their production so that p is known ex-ante, even if the productions are contractible.*

Proof: Suppose productions x_i and x_j are observable and verifiable, as well as θ: speculators can then include in the forward contract a clause that invalidates the contract whenever $x_i(\theta) + x_j(\theta) \neq a + \theta - k$, where k is any prespecified (constant) spot price. However, then each producer at $t = 1$ has an interest to sell forward all its capacity, violating the constraint above. Considering the ex-ante profit function $E[(p^* - b) x_i^*] - \Lambda Var(p^*)(f_i + f_j) f_i$ with $Var(p) = 0$ and $p^* = k$, one gets immediately that both i and j have interest to produce at the maximum of their capacity when $k > b$, or to choose $x = 0$ when $k < b$. In any case, their choice is not consistent with $Var(p) = 0$.

If the productions are not contractible, the promise made at $t = 1$ to stabilize the spot price at $t = 2$ is not believed by forward buyers unless it is credible. They correctly anticipate i and j will behave as Cournot duopolists at $t = 2$; however, in order to stabilize the price, i and j would then need to sell quantities dependent on the shock realization, i.e. $f_i(\theta)$, $f_j(\theta)$; this is not possible since the realization of the demand shock is not know at $t = 1$. $\blacksquare$

Proposition 3 illustrates quite clearly what problems a financial regulator encounters when trying to create a liquid market for forward contracts based on cash settlement for a perishable commodity like electricity: the sellers of the insurance (i.e. the speculators) are exposed to moral hazard by the producers, because the latter have market power. We believe that this is the reason why these markets have not developped in greater extent.

4   Conclusion

In this paper we show that introducing forward markets in a duopoly does not always enhance the perfectly competitive allocation on the spot markets, as in Allaz and Vila (1993). Their result relies crucially on the absence of any element of uncertainty over the spot demand of the commodity and on the risk-neutrality of forward buyers, like financial intermediaries. The efficiency-enhancing effect of forward contracts is lower when the demand of the good is very uncertain. Moreover, the market power of producers allows them to endogenously control up to some extent the spot price variability: this has a negative impact on the participation of financial speculators into forward markets, even if cash-settlement is allowed.

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