



Sharing and Anti-Sharing in Teams.

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Abstract

Compared to budget-balanced Sharing contracts, Anti-Sharing may improve the efficiency of teams. The Anti-Sharer collects a fixed payment from all team members; he receives the actual output and pays out its value to them. If a team member becomes Anti-Sharer, he will be unproductive in equilibrium. Hence, internal Anti-Sharing fails to yield the first-best outcome. Anti-Sharing is more likely to yield a higher team profit than Sharing, the larger the team, the curvature of the production function, or the marginal effort cost. Sharing is more likely to be better, the greater the marginal product, the cross-partial of the production function, or the curvature of the effort cost.

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1 Introduction

It is well established that budget-balanced sharing contracts do not motivate risk-neutral team members to choose efficient effort.¹ For risk-averse teams, Rasmusen (1987) has shown that sharing contracts may implement first-best effort. Strausz (1999) has proposed an efficient sharing rule for sequential teams. However, an optimal contract for teams of risk-neutral agents who choose their non-verifiable effort simultaneously has not yet been derived.

“Anti-Sharing” is an attempt to solve the sharing problem. Under an Anti-Sharing contract, the team members have to make a fixed payment to the Anti-Sharer who, in turn, is obliged to pay the full team output to each member. Hence, all team members except for the Anti-Sharer are residual claimants and

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¹See the general proof in Holmstrom (1982). The inefficiency of relative performance evaluation in teams was demonstrated by Choi (1993).

have stronger incentives to spend effort than under a sharing contract. The role of the Anti-Sharer may be played by an outsider or by one of the team-members. External Anti-Sharing is closely related to “bonding”, as it was mentioned by Holmstrom (1982, 328), and can induce first-best efforts.²

This paper focuses on internal Anti-Sharing, which leads to a theory of the firm in the spirit of Alchian/Demsetz (1972). In their paper, one team member becomes residual claimant and (perfectly) monitors the others to solve the sharing problem. Anti-Sharing neither presupposes monitoring abilities, nor is the resulting hierarchy based on authority. We set up a simple model with homogeneous agents and compare symmetric Sharing with internal Anti-Sharing. As the internal Anti-Sharer remains unproductive in equilibrium, first-best efforts will not be reached. However, the team profit can be higher than under a sharing contract.

2 The model

2.1 Inefficiency of the Sharing Contract

Consider n risk-neutral agents who spend effort $e_i \geq 0, i = 1..n$ to produce an output $Y(e)$, where $e = (e_1..e_n)$ represents the effort vector of all n players.³ The production function $Y(e)$ is twice differentiable, continuous, with positive but diminishing marginal returns, and with non-negative cross-partials. Individual efforts are assumed to be non-verifiable. We denote the effort disutility of agent i as $C(e_i)$ and assume $C_1 > 0 < C_{11}$ as well as $C(0) = 0$ and $C_1(0) = 0$.⁴ Players’ utility functions are separable in wealth and effort cost. To keep the model simple, we assume the agents to be homogeneous, i.e., they are identical with regard to effort costs, utility functions, and marginal productivity. Moreover, for all agents the second derivatives and the cross-partials of the production function are assumed to be identical.

The socially optimal effort vector e^* maximizes the team profit $T(e) = Y(e) - \sum_{i=1}^n C(e_i)$ and, thus, satisfies the first order conditions⁵ $Y_i(e_{(-i)}^*, e_i) - C_1(e_i) = 0, i = 1..n$. As Holmstrom (1982) has demonstrated, a budget-balanced “Sharing” contract does not induce the players to choose e_i^* . Sharing is budget balanced if the players’ shares, denoted as $s_i \geq 0$, add up to 1. Under such a contract, at least one player receives a share smaller than one. The incentives are, thus, insufficient at least for some players even if all other players choose

²However, external Anti-Sharing differs from outside enforcement of non-balanced sharing.

³Let $e_{(-i)}$ denote the effort vector of all n players except player i , i.e., $(e_1..e_{i-1}, e_{i+1}..e_n)$. Consequently, $e_{(-i,-j)}$ is the effort vector without the contributions of players i and j . For convenience, we write $e = (e_{(-i)}, e_i) = (e_{(-i,-j)}, e_i, e_j)$.

⁴With regard to functions (in capital letters), the index i denotes the first derivative with respect to the i^{th} argument, while an index ij denotes a second derivative with respect to the i^{th} and j^{th} argument. With regard to variables (lower case letters), an index i denotes the i^{th} player. Subsequently, we use the superscript S to indicate the sharing contract, while AS refers to internal Anti-Sharing.

⁵We assume the second-order conditions for a finite maximum to be satisfied, hence the Hessian to be negative definite, which requires the cross partials to be not too great. This implies the second-order conditions for the individual players’ maximization problems under Sharing and Anti-Sharing to hold as well.

efficiently. Let e_i^S denote player i 's equilibrium effort under the symmetric sharing contract with equal shares, i.e., $s_i = 1/n$. Each player $i = 1..n$ maximizes his individual payoff $Y(e_{(-i)}^S, e_i)/n - C(e_i)$, and the first-order condition for an internal solution is $Y_i(e_{(-i)}^S, e_i) = nC_1(e_i), i = 1..n$. The individual payoff in equilibrium amounts to $Y(e^S)/n - C(e_i^S)$. Hence, for $n > 1$ the equilibrium efforts are suboptimal. Sharing not only induces each of the agents to choose lower than efficient effort, this also reduces the marginal productivity of each agent if cross-partials of the production function are positive, which further reduces the agents' motivation.

2.2 Internal Anti-Sharing

Without loss of generality, we assign the role of the internal Anti-Sharer to the 1st team member. Each of the other agents $i = 2..n$ promises to pay an amount $p_i \geq 0$ to him. All players choose their effort, denoted as e_i^{AS} . The actual output is produced $Y(e^{AS})$ and transferred to the Anti-Sharer. He pays out its value to each of the other team members, net of p_i . The Nash equilibrium analysis starts with player 1.

Proposition 1: *In equilibrium the internal Anti-Sharer chooses zero effort.*

Proof: The Anti-Sharer receives the lump sum payments p_i from the other players as well as the actual output; he has to pay out $(n - 1)$ times the value of the actual output; and he bears his own effort costs. Thus, he chooses $e_1^{AS} = \arg \max \left[\sum_{i=2}^n p_i + Y(e_1, e_{(-1)}^{AS}) - (n - 1)Y(e_1, e_{(-1)}^{AS}) - C(e_1) \right]$. The first derivative with respect to e_1 is $(2 - n) \cdot Y_1(e_1, e_{(-1)}^{AS}) - C_1(e_1)$. As $n \geq 2$, this expression is negative, hence $e_1^{AS} = 0$. \square

The intuition behind this result is the distortion of the Anti-Sharer's incentives to spend effort. He receives the actual output once, but has to pay its value $(n - 1)$ times to the other team members. For $n \geq 2$, his individual marginal profit from spending effort is negative.

We denote as $\hat{e}_{(-1)}$ the "constrained efficient" efforts of players $i = 2..n$ which maximize the team profit under the constraint $e_1 = 0$, i.e., $\hat{e}_{-1} = \arg \max \left[Y(0, e_{(-1)}) - \sum_{i=2}^n C(e_i) \right]$, and derive our second result.

Proposition 2: *Under an internal Anti-Sharing contract, a Nash equilibrium exists in which players $i = 2..n$ choose constrained efficient efforts \hat{e}_i .*

Proof: Under the condition $e_1 = 0$, the first-order conditions for constrained efficient efforts are $Y_i(0, e_{(-1,-i)}, e_i) = C_1(e_i), i = 2..n$. Anticipating $e_1 = 0$, each of the team members $i = 2..n$ chooses $e_i^{AS} = \arg \max \left[Y(0, e_{(-1,-i)}^{AS}, e_i) - C(e_i) - p_i \right]$. As p_i is independent of e_i , the first-order conditions are identical to those for constrained efficient effort. \square

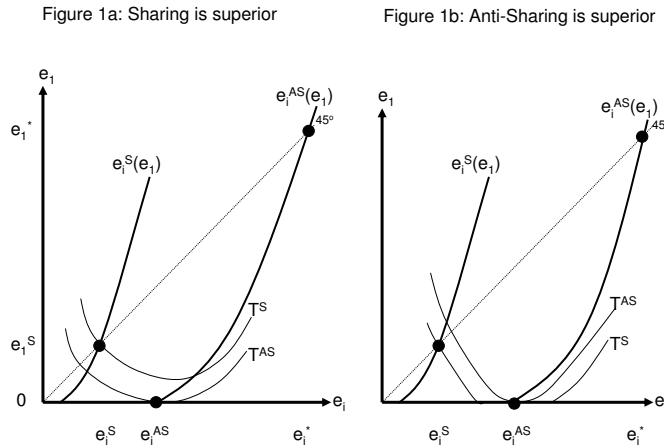
An arbitrage argument allows us to derive the side payments p_i endogenously. If, in the constrained efficient equilibrium, the Anti-Sharer's payoff is greater, each team member wants to become Anti-Sharer; if it is lower, then no one wants to assume this role. Hence $(2 - n)Y(\hat{e}) + (n - 1)p_i = Y(\hat{e}) - p_i - C(\hat{e})$.

This implies $p_i = (n - 1)[Y(\hat{e}) - C(\hat{e}_i)]/n$. In this arbitrage-free equilibrium, each team member $i = 1..n$ earns $[Y(\hat{e}) - (n - 1)c(\hat{e}_n)]/n$.

2.3 Comparison of Sharing and Anti-Sharing

Anti-Sharing Pareto-dominates Sharing if it yields a greater team profit $T(e) = Y(e) - \sum c(e_i)$, because side-payments are allowed. We will use the properties of best-response functions and iso-team-profit curves to derive criteria for this comparison.

If the cross-partials of the production function $Y(e)$ are positive then the individual payoff functions of the n team members also exhibit positive cross-partials (the only exception is the internal Anti-Sharer). The team members $i = 2..n$ play a symmetric supermodular game, which is also supermodular in the effort of the Anti-Sharer $i = 1$. We can therefore limit the following analysis to the relation between the effort choices of one representative player $i = 2..n$ (which we subsequently label as i) and the special player $i = 1$.



The slope of a player's best response function is positive if the cross-partials of this player's individual profit function with the other player's effort is positive,⁶ hence if $Y_{ij} > 0$, because $Y_{ii} - C_{11} < 0$. Figures 1a and 1b show the best-response function $e_i(e_1)$ under the Sharing (respective left curve) and the Anti-Sharing contract (respective right curve).

Under Sharing, player i 's individual best reply to 1 is never greater than under Anti-Sharing: $e_i^{AS}(e_1) \geq e_i^S(e_1)$. The Nash equilibrium under Sharing is represented by $e_i^S(e_i)$, i.e., the bold dot on the 45-degree-line. The slope of $e_i^S(e_1)$ is decreasing in n .⁷ Hence, the Nash equilibrium under Sharing is shifted towards

⁶See Vives (2005).

⁷As figures 1a, 1b show e_1 on the vertical axis, a lower slope would mean that $e_i^S(e_1)$ becomes steeper.

the origin if n is increased. The best-response function of the Anti-Sharer, is $e_1(e_i) = 0$. The Nash equilibrium under Anti-Sharing is represented by the bold dot at $e_i^{AS}(0)$, while the fixed point $e_i^{AS}(e_1) = e_1$ represents the first-best outcome.

Figures 1a and 1b also show iso-team-profit curves. Figure 1b depicts the case in which Anti-Sharing yields a higher team profit than Sharing, while figure 1a displays an example in which Sharing is superior. Our next proposition derives those properties of iso-team-profit curves which are relevant for the comparison between Sharing and Anti-Sharing.

Lemma: *Any iso-profit curve in an $e_i - e_1$ -diagram (where e_i represents the symmetric effort choice of players $i = 2..n$), when crossing the 45-degree line, is strictly decreasing with slope $-(n-1)$ and convex. It has a minimum (i.e. is still convex) when crossing the best-response function $e_i^{AS}(e_1)$.*

Proof: Any iso-profit curve can be expressed as an implicit function $T[e_1(e_i), e_i] = t$, where e_i represents identical effort choices of players $i = 2..n$, while t denotes the profit level under scrutiny. The slope of the iso-profit curve is derived from the total differential $(n-1)T_i de_i + T_1 de_1 = 0$, which is equivalent to

$$\frac{de_1(e_i)}{de_i} = -\frac{(n-1)T_i}{T_1}. \quad (1)$$

which proves the first claim. Further differentiation of the total differential with respect to e_i yields

$$\frac{d^2 e_1}{de_i^2} = (n-1)[(n-2)T_{ij} + T_{ii}] + T_{i1} \frac{de_1}{de_i} + \frac{d^2 e_1}{de_i^2} T_1 + \frac{de_1}{de_i} \left[(n-1)T_{i1} + T_{11} \frac{de_1}{de_i} \right]$$

Setting this equal to zero and rearranging, making use of (1), leads to

$$\frac{d^2 e_1}{de_i^2} = \frac{n-1}{T_i} \left[\frac{nT_i T_{i1}}{T_1} - (n-2)T_{ij} - T_{ii} - \frac{(n-1)T_i^2 T_{11}}{T_1^2} \right] \quad (2)$$

With $T_{ij} > 0$, the sign of this expression is ambiguous. However, on the 45-degree line (with $e_i = e_1$) the assumption of symmetric agents implies $T_i = T_1 > 0$, $T_{ii} = T_{11} < 0$, and $T_{ij} = T_{i1} \geq 0$. In this case, the right hand side of (2) can be simplified to

$$\frac{n-1}{T_i} [nT_{ij} - T_{ii} - (n-2)T_{ij} - (n-1)T_{ii}] = \frac{n-1}{T_i} [2T_{ij} - nT_{ii}] > 0. \quad (3)$$

This proves the convexity. To prove the last statement, recall that the slope of the iso-team-profit curve equals the slope of an iso-profit curve of players $i = 2..n$ under the Anti-Sharing contract. Hence, the best-response function $e_i^{AS}(e_1)$ consists of *unique* minima of the iso-team-profit curve. Thus, the latter curves are strictly convex between the 45-degree-line and $e_i^{AS}(e_1)$. \square

Now we have completed the toolbox needed to compare the team outcomes under the Sharing and Anti-Sharing. Sharing yields the higher team profit if the iso-team-profit-curve has its minimum (on the best response function of player i) at $e_1 > 0$ (see figure 1a). Anti-Sharing yields the higher team payoff if

the Sharing iso-profit line crosses the vertical axis to the left of $e_i^{AS}(0)$, as it is displayed in figure 1b. As all the iso-team-profit curves have slope $-(n-1)$ on the 45-degree line, it only depends on their curvature which of these two cases is true. The smaller the curvature, i.e., the smaller T_{ij} or T_i or the greater T_{ii} , the more likely it is that the Anti-Sharing contract is the better one. Recall that $T_i = Y_i - C_1$, $T_{ii} = Y_{ii} - C_{11}$, and $T_{ij} = Y_{ij}$. These insights from the Lemma establish our final result.

Proposition 3: *Anti-Sharing is more likely to be the superior contract, the greater n , Y_{ii} , or C_1 , and the smaller Y_1 , Y_{ij} , or C_{11} .*

The results regarding n and T_{ij} highlight the main intuition behind the comparison of the two team contracts. The inefficiency of the Sharing contract is caused by the fact that all members face inefficiently low incentives. The Sharing problem increases with a greater n . The inefficiency of Anti-Sharing is due to the lack of player 1's contribution, but this problem becomes less relevant the greater the team. On the other hand, a higher cross-partial of the production function may increase the inefficiency of Anti-Sharing. Look at the extreme case of a Cobb-Douglas or Leontief production function: the best reply of all players i to $e_1 = 0$ would be $e_i^{AS}(0) = 0$. Hence, complete substitution of the inputs is a necessary condition (albeit not sufficient) for Anti-Sharing to yield a higher team profit than Sharing.

3 Discussion

We have demonstrated that internal Anti-Sharing can make a team better off, compared to the symmetric sharing contract. However, as the internal Anti-Sharer has no incentive to contribute positive effort, this contract also fails to achieve the first-best solution; it can even be worse than Sharing. The convexity of iso-team-profit curves as well as the team size are the decisive factors for the question which contract makes the team better off. In an asymmetric case, the least productive partner should become Anti-Sharer. The productivity of partner i , however, is not only to be measured by his own marginal productivity, but also by his impact on the other players via the cross-partials.

It is worth discussing a modification of the Anti-Sharing contract: let the productive agent pay a higher fixed payment to the Anti-Sharer who, in turn, pays more than the actual output to the productive team members. This would induce the latter to spend even more effort, which might make up for the lack of effort on the Anti-Sharer's side. However, we have shown that the Anti-Sharer contract as described above is indeed "constrained efficient", i.e., it maximizes the team profit under the condition $e_1 = 0$. In this paper, we have only compared Sharing and Anti-Sharing, and disregarded other contract structures which may perform better than both of these. Hence, a general theory of second-best team contracts still needs to be developed.

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