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Safety Stocks in Centralized and Decentralized Supply Chains under Different Types of Random Yields

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Abstract

Safety stock planning with focus on risk protection to cope with demand uncertainties is a very well researched topic in the field of supply chain management, in central as well as in local decision making systems. In contrast, there is only few knowledge about safety stock management in situations where supply risks have to be covered that are caused by uncertainties with respect to production yields. In this study, a two-stage manufacturer-retailer supply chain is considered in a single-period context that allows for an analytical study of the impact of yield randomness on safety stock determination. In order to concentrate the analysis on the effects of yield uncertainty demand will be assumed to be deterministic.

We consider three basic types of yield randomness which represent different reasons for yield losses in production processes each, namely the stochastically proportional, binomial, and interrupted geometric yield type. It will be shown that these different yield risk specifications can bring about completely different properties with regard to the way safety stocks depend on various input parameters in supply chain planning.

This holds especially for the impact of the demand size and for the influence of the level of product profitability in a supply chain. In an analytical model-based investigation it is demonstrated that these safety stock properties not only differ between the respective yield types, but also between systems of central and decentralized supply chain decision making. Thus, this study presents general insights into the importance of a correct yield type specification for an effective safety stock management and explains necessary differences in the stock distribution across supply chain stages in both centralized and decentralized settings.

Keywords

Random yield, safety stocks, supply chain decisions, yield types

1. Safety Stocks in Supply Chains

Safety stocks are held in supply chains in order to provide an economically reasonable service of delivery for end customers if various types of risks may disrupt the flow of products. These risks can result from lacking predictability of external demand, from unreliability of production and transportation processes, or from service deficiencies of outside suppliers. Safety stocks are established at different stages of a supply chain by production or procurement decisions that result in planned inflow of products at certain stocking points which differ from the expected outflow. Thus, in a periodic planning environment a safety stock at some stock point can be generally defined as expected net stock at the end of a decision period (see Silver/Pyke/Peterson, p.234) that results from respective operational decisions.

In literature there is a vast amount of contributions that deal with supply chain safety stock planning aiming to cope with demand uncertainties (see Graves and Willems 2003 and Axsäter 2003). The impact of supplier unreliability on safety stocks is also an issue that is widely addressed in scientific contributions, usually jointly with the supplier selection problem (see Minner 2003 and Burke et al. 2009). Compared to that, a fairly limited number of contributions is devoted to the problem of safety stock determination in case of production yield risks in a supply chain setting (see Li et al. 2012 and Inderfurth and Vogelgesang 2013).

Different from other risk environments, the specific problem concerning decision making under random yields is that in such a situation the procurement decision has an impact on the risk level. The basic context is illustrated in Figure 1 where in a simple supply chain context the decision variable is the input Q at a production stage which procured from an external supplier and is randomly transformed into a production output $Y(Q)$ that is used as final product to fulfill some end customer demand D. In a centralized setting the decisions on manufacturing and sales are directly connected. In the case of a decentralized supply chain manufacturer and retailer are independent decision makers, and the retailer transforms the customer demand into an upstream order that constitutes the producer's demand. Outside supplier and end customer are supply chain external actors.

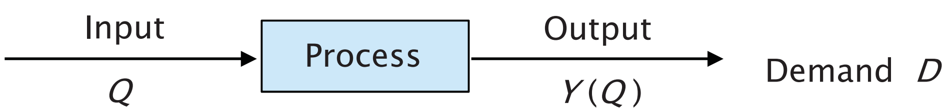

Figure 1: The basic random yield setting

Since under regular process conditions the output quantity cannot exceed the input level, the so-called yield rate $Z(Q)= Y(Q)/Q$ is always a fraction between zero and one. According to different reasons for yield uncertainty there exist different types of yield randomness which are characterized by differences in the way the stochastic yield rate depends on the production level (see Yano and Lee 1995). In this paper it will be shown that it is critically important to identify the correct yield type in practical cases because different yield characteristics will cause different structural results for production and safety stock management. This holds for both a centrally and a locally managed supply chain.

In order to concentrate this study on the pure impact of yield randomness on safety stock decisions and to facilitate analytical results two major restrictions are imposed. First, only risks from the side of production yields are considered and data concerning external customers' demand and external suppliers' delivery processes are assumed to be deterministic. Second, only a single-period context is addressed so that it is possible to model and analyze decision making also for a decentralized supply chain with independent actors in a general way. The respective problem type has already been investigated for a limited range of yield models (see e.g. Keren 2009 and Li et al. 2013). By concentrating on this type of model, we will consider a counterpart of the well-known newsvendor model with deterministic yield and random demand which is very well researched (see Khouja 1999 and Qin et al. 2011) including its extensions to decentralized supply chain settings (see Cachon 2003). From the newsvendor context under the objective of maximizing the expected profit it is well-known that the optimal procurement quantity is a critical fractile of the demand distribution. This critical ratio is high for high-margin products and low for low-margin ones. From the respective analysis it is also known that the optimal procurement level might be lower than expected demand in cases of low product profitability so that the safety stock becomes negative. Additionally, newsvendor research has revealed that in a decentralized supply chain under a simple wholesale price contract procurement level and safety stock will always be below the respective value in the centralized case where the supply chain is managed by a single decision maker. This results from the so-called double marginalization effect (see Spengler 1950) that is usually observed in a simple wholesale price contract setting.

In the sequel, it will be investigated to which extent the newsvendor results carry over to a corresponding random yield model, and which role the specific yield type will play in this context. To this end the paper is organized as follows. In Section 2, before optimization procedures for centralized and decentralized decision making are explained, the three commonly used types of yield randomness are introduced and modeled, namely the stochastically proportional, the binomial, and interrupted geometric yield process. Next, for each yield type a specific section (Sections 3 to 5) is dedicated in order to analyze safety stock determination under centralized and decentralized supply chain management and reveal specific properties for different yield situations. Finally, Section 6 concludes this study by focusing on relevant insights and addressing open research questions.

2. Supply Chains with Random Yields

2.1. Yield Types and Safety Stocks

There exist various reasons for randomness in the outcome of a production process. In some cases a complete production batch Q is exposed specific uncertain processing conditions (like weather conditions in agricultural production) so that there is perfect correlation of defectiveness of units within a lot. This situation is described by a so-called *stochastically proportional* (*SP*) yield model, formulated as

$$Y(Q) = Z \cdot Q \tag{1}$$

with a random yield rate Z that is characterized by a $pdf\ \varphi(.)$ and $cdf\ \Phi(.)$ with given mean μ_Z and variance σ_Z^2.

The yield situation is completely different if single defective units are generated because of independent quality problems concerning single input materials or single manufacturing operations. In this case there is no correlation of defectiveness, and the total number of non-defective units in a lot Q follows a *binomial* (*BI*) distribution. With a success probability θ for each unit, in this *BI* yield case the probabilities of yield size realizations for $Y(Q)$ are given by

$$Pr\{Y = k\} = \binom{Q}{k} \cdot \theta^k \cdot (1-\theta)^{Q-k} \ \ for\ \ k = \ 0, \dots, Q\ . \tag{2}$$

Under *BI* yield the parameters of the yield rate *Z* are expressed as

$$\mu_Z = \theta \ \text{ and } \ \sigma_Z^2 = \frac{\theta \cdot (1-\theta)}{Q} = \sigma_Z^2(Q)\ . \tag{3}$$

A third yield model applies if production is affected by a risk which results in a move of the manufacturing process from an in-control to an out-of-control state, meaning that all produced units are good before this move while they are all defective afterwards. If the probability of staying in-control is denoted by θ for any item, the yield $Y(Q)$ within a batch follows a so-called *interrupted geometric* (*IG*) distribution, characterized by

$$Pr\{Y = k\} = \begin{cases} \theta^k \cdot (1-\theta) & for\ \ k = 0,1, \dots, Q-1 \\ \quad \theta^Q & for \qquad k = Q \end{cases}$$

(4)

In this *IG* yield case the following formulas hold for the yield rate parameters (see Inderfurth and Vogelgesang (2013))

$$\mu_Z = \frac{\theta \cdot (1-\theta^Q)}{(1-\theta) \cdot Q} = \mu_Z(Q) \ \text{ and } \ \sigma_Z^2 = \frac{\theta \cdot \left(1-\theta^{1+2Q}\right) - (1-\theta) \cdot (1+2Q) \cdot \theta^{1+Q}}{(1-\theta)^2 \cdot Q^2} = \sigma_Z^2(Q)\ . \tag{5}$$

Obviously, *IG* yield is characterized by some positive level of yield correlation within a production lot.

The three types of yield randomness described above are the basic ones that are widely used to model uncertainty in the output of production processes (see Yano and Lee 1995). For decision making it is critically important to consider which yield type is relevant in a specific case. This is because these basic yield models differ in the way the yield rate parameters are affected by the production input quantity *Q*. While μ_Z and σ_Z do not depend on the level of production under *SP* yield, things are very much different for the other yield types. Under *BI* yield the yield rate variance decrease with increasing production, and under *IG* yield additionally the mean of the yield rate becomes the smaller the larger the level of production will be. These effects that are visualized in Figure 2 for a specific data set lead to qualitatively different conditions for optimal decision making concerning the size of production and safety stocks.

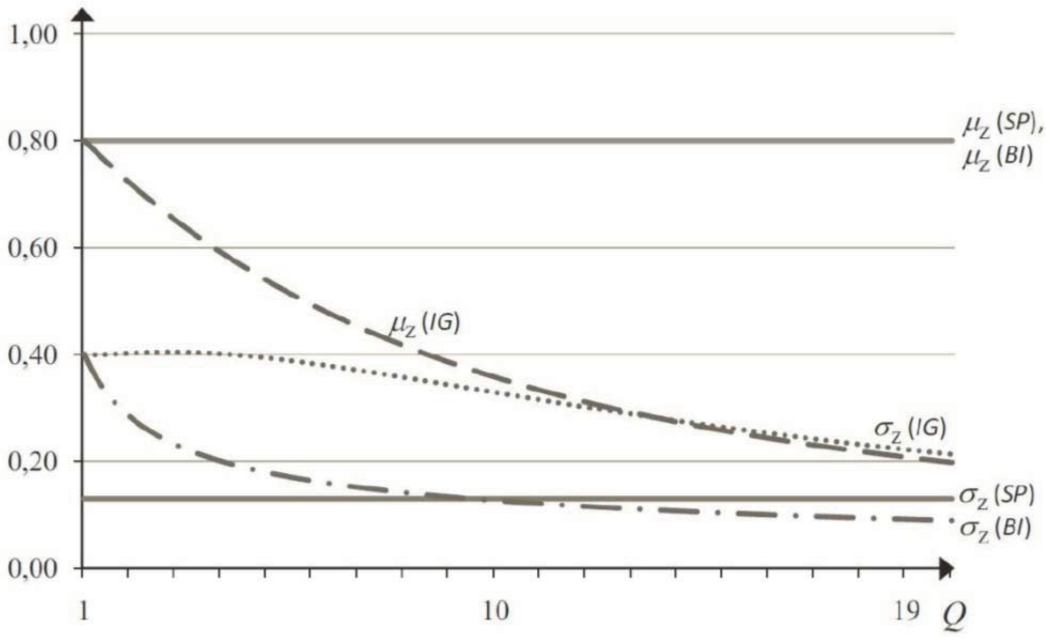

Figure 2: Yield rate parameters and batch size for $\mu_Z(SP) = \theta = 0.8$ and $\sigma_Z(SP) = 0.13$

2.2. Optimization in Centralized and Decentralized Supply Chains

According to its definition as expected net stock at the end of a decision period, in a single-period context with firm demand the safety stock is directly determined by the choice of the production level. In the current analysis risk-neutral decision makers are assumed, and the optimal decisions are defined as those which maximize the expected profit. This optimization, however, proceeds quite different for centralized and decentralized supply chains.

Centralized Supply Chain

In a centralized supply chain production and retailing is in one hand, and the decision is how to determine the production input Q so that the total expected supply chain profit Π_{SC} under random production yield $Y(Q)$ and deterministic customer demand D is maximized . The respective profit function can be formulated as

$$\Pi_{SC}(Q) = p \cdot E\{min(Y(Q), D)\} - c \cdot Q. \tag{6}$$

Here, p stands for the retail price and c for the (input) cost per unit. Without loss of generality, it will be assumed in the following analysis that any excess stock after production has a zero salvage value. The optimal production decision Q^* results in a safety stock level SST^C for the centralized case amounting to

$$SST^C = E\{Y(Q^*)\} - D. \tag{7}$$

For the centralized supply chain the expected physical inventory (on-hand) IOH^C at the period's end is given as

$$IOH^C = E\{Y(Q^*)|Y(Q^*) > D\} \tag{8}$$

and, thus, cannot exceed the safety stock.

Decentralized Supply Chain

Under decentralization of decision making, the producer and the retailer will maximize their local profits each. In this case the retailer first decides on his order quantity D_R released to the manufacturer, and the producer determines her production input Q_P in reaction to this supply chain internal demand. Different from a standard newsvendor situation, in the case of random yield both supply chain actors bear a risk, namely a production risk at the manufacturer's stage and a supply risk at the retailer's stage. So the retailer needs to have some information about the manufacturer's yield process and resulting delivery performance in order to form some expectation on how the manufacturer will react on his order decision. To avoid some arbitrary estimation (like 'the manufacturer always inflates an order by the reciprocal of the mean yield rate') it is assumed that for the retailer the state of information is such that he can completely retrace the producer's optimization procedure. Thus, the interplay of decisions can be formulated as a Stackelberg game, meaning that the retailer will anticipate the producer's reaction when determining his order level. The interaction within the supply chain is assumed to be characterized by a simple wholesale price contract so that the retailer has to pay a respective (internal) price w for each unit he receives from the manufacturer. Because of the production yield uncertainty the manufacturer's output might exceed the retailer's order. In this case the producer only delivers the order quantity D_R and excess production is lost. Under these circumstances the producer's profit function equals

$$\Pi_P(Q_P|D_R) = w \cdot E\{min(Y(Q_P), D_R)\} - c \cdot Q_P \ . \qquad (9)$$

Maximizing this profit leads to an optimal conditional decision $Q_P(D_R)$. Under consideration of this decision the retailer maximizes his own profit $\Pi_R(D_R|Q_P)$ which is given by

$$\Pi_R(D_R|Q_P) = p \cdot E\{min(Y(Q_P), D_R, D)\} - w \cdot E\{min(Y(Q_P), D_R)\} \qquad (10)$$

and will result in an optimal order decision $D_R(Q_P)$. After respective inserting operations the effective optimal decisions $D_R^* = D_R(Q_P(D_R))$ and $Q_P^* = Q_P(D_R^*)$ of both actors can be determined.

In a decentralized setting the global supply chain safety stock depends only on the producer's production output since under normal price conditions $w \leq p$ the retailer will never order below the (deterministic) demand level. So this safety stock SST^D is given by

$$SST^D = E\{Y(Q_P^*)\} - D. \qquad (11)$$

A split of this overall safety stock to the producer and retailer side can only be carried out arbitrarily and, thus, will be left . This, however, is different for the expected stock on-hand which can be separated into a producer's share given by

$$IOH_P^D = E\{Y(Q_P^*)|Y(Q_P^*) > D_R^*\} \qquad (12)$$

and a retailer's share which is calculated as

$$IOH_R^D = E\{Y(Q_P^*)|D_R^* \geq Y(Q_P^*) > D\}. \qquad (13)$$

Price and cost parameters must meet some economic conditions in order to guarantee that the supply chain actors are able to make profits that are positive or at least zero. So it is assumed that $c/\mu_Z \leq w \leq p$. In this context, it has to be noted that due to the specific dependency

$\mu_Z(Q)$ for *IG* yield the lower bound for w and p will depend on the choice of the production level. $1/\mu_Z$ can also be interpreted as minimum level of the product profitability defined by *w/c* for the manufacturer and *p/c* for the entire supply chain.

Yield Types and Safety Stock Properties

In order to find how different yield types affect the sign and level of safety stocks under centralized and decentralized decision making, the above optimization approaches have to be carried out for the different yield types (*SP,BI,IG*) and their respective modeling of the random variable *Y(Q)*. Before this will be investigated in detail in the forthcoming sections, some general results are presented which directly can be derived from the characteristics of the yield models. These results hold for both centralized and decentralized supply chains.

Property (1): If the production level Q does not exceed the demand size D (i.e., for $Q \leq D$), the supply chain safety stock is always negative (*SST*<0) for each yield type.

This is simply because in all yield models $E\{Y(Q)\} < D$ holds due to $Y(Q) \leq Q$.

Property (2): In the case of *IG* yield the safety stock is always negative (*SST*<0), independent of yield and price/cost parameters.

This property is a result of the specific probability distribution of yields in (4). The probabilities of *IG* yields with a value smaller than or equal to demand D do not change if the production level is increased above D, i.e. $Pr\{Y = k\}$ is independent of Q for $k \leq D$ and *Q*>*D*. This means that the revenues in the profit functions (6) and (9) cannot be increased by increasing the production level above demand. So always $Q \leq D$ will be chosen for economic reasons, and *Property* (1) directly applies in this case.

The detailed analysis of yield type effects on safety stock characteristics is facilitated if all variables can be treated as continuous. To this end, in the sequel we assume that for *SP* yield the yield rate Z is continuous and approximate the yield *Y(Q)* in the *BI* yield case by a normal random variable with parameters from (3) (exploiting the De Moivre-Laplace theorem). Finally, under *IG* yield the respective yield expectation in (5) is treated as a continuous function in Q.

3. Safety Stocks under *SP* Yield

With *pdf* $\varphi(z)$ for the yield rate in the case of *SP* yield, the expected sales volume in (6) and (9) can be expressed as

$$E\{min(Y(Q), D)\} = \int_0^{D/Q} z \cdot Q \cdot \varphi(z) \cdot dz + \int_{D/Q}^{1} D \cdot \varphi(z) \cdot dz. \tag{14}$$

Exploiting this formulation, the profit maximization problem in the centralized and decentralized supply chain setting can be solved analytically as shown in Inderfurth and Clemens (2014). Thus, the respective production and ordering decisions can be analyzed with respect to their impact on safety stock holding, and general interrelationships can be detected.

3.1. Centralized Supply Chain

Maximizing the profit in (6) under the *SP* specific sales formula in (14) results in an optimal production quantity

$$Q^* = K^* \cdot D \quad \text{with} \quad K^* > 1 \tag{15}$$

where K^* is implicitly given by $\int_0^{1/K^*} z \cdot \varphi(z) \cdot dz = \frac{c}{p}$. (16)

From (15) it is evident that the optimal production quantity is always larger than demand with a demand inflation factor K^* that is constant. From (16) it follows that this inflation factor increases with increasing product profitability level *p/c*.

With Q^* from (15), according to (7) the safety stock in the centralized supply chain is given by

$$SST^C = \mu_Z \cdot Q^* - D = (\mu_Z \cdot K^* - 1) \cdot D\ . \tag{17}$$

Together with the K^* formulation in (16) this means that a positive safety stock will always be employed if the product profitability is sufficiently high and exceeds some critical level πc which is calculated from $\mu_Z \cdot K^* = 1$, resulting in

$$\pi c = \left[\int_0^{\mu_Z} z \cdot \varphi(z) \cdot dz\right]^{-1}. \tag{18}$$

With increasing profitability it is obvious that the supply chain safety stock will also increase. The expected on-hand inventory is given by

$$IOH^C = \int_{1/K^*}^{1} (K^* \cdot z - 1) \cdot D \cdot \varphi(z) \cdot dz \tag{19}$$

and obviously will increase with growing product margin.

3.2. Decentralized Supply Chain

The *manufacturer's optimization problem*, i.e. maximization of profit $\Pi_P(Q_P|D_R)$ in (9), equals that in the centralized problem except that external demand *D* is replaced by the retailer's order D_R and the sales price *p* by the wholesale price *w*. Accordingly, the buyer's optimal production is given by

$$Q_P(D_R) = K_P \cdot D_R \quad \text{with} \quad K_P > 1 \tag{20}$$

where K_P is defined by

$$\int_0^{1/K_P} z \cdot \varphi(z) \cdot dz = \frac{c}{w}\ . \tag{21}$$

From *w*<*p* it is obvious that the producer's inflation factor is smaller than the respective factor under centralized optimization, i.e. $K_P < K^*$.

Anticipating the producer's reaction in (20), the retailer maximizes his profit $\Pi_R(D_R|Q_P)$ in (10) by ordering an amount that is equal to or above demand *D* according to

$$D_R^* = \begin{cases} D & \text{if } K_R \le K_P \\ D \cdot \frac{K_R}{K_P} & \text{if } K_R \ge K_P \end{cases}, \tag{22}$$

where the factor K_R is determined from

$$\int_0^{1/K_R} z \cdot \varphi(z) \cdot dz = \frac{c}{p} + \frac{w}{p} \cdot \frac{1-\Phi(K_P)}{K_P} \tag{23}$$

so that also for K_R the relationship $K_R < K^*$ holds.

Given the retailer's ordering decision in (22), the supply chain interaction from (20) results in a manufacturer's production level that is characterized by

$$Q_P^* = \begin{cases} K_P \cdot D & \text{if } K_R \le K_P \\ K_R \cdot D & \text{if } K_R \ge K_P \end{cases}. \tag{24}$$

Thus, also in a decentralized supply chain the effective production input is always proportional to the demand level, but with an inflation factor which is smaller than under centralized decision making. The system-wide safety stock SST^D from (11) is calculated analogously to (17) and, due to $Q_P^* < Q^*$, must be smaller than in the centralized supply chain.

The production and safety stock level in the decentralized supply chain very much depends on the level of the wholesale price w within the feasible range $c/\mu_Z \le w \le p$. From the definition of K_P in (21) it follows that K_P is increasing in w with $K_P = 1$ for $w = c/\mu_Z$ and $K_P = K^*$ for $w = p$. The K_R definition in (23) reveals that K_R is decreasing in w with $K_R = K^*$ for $w = c/\mu_Z$ so that there exists some wholesale price w (to be determined from equalizing the right-hand sides of equations (21) and (23)) for which both inflation factors K_P and K_R are equal. Given these interrelationships, the interplay of retailer and manufacturer decision in (22) and (24) leads to a producer's decision on production and safety stock level which is equal to the centralized solution (i.e. $Q_P^* = Q^*$ and $SST^D = SST^C$) for the extreme w values $w = c/\mu_Z$ and $w = p$. In between these limits, the retailer's order is decreasing with increasing w and stays constant at demand level D after K_R reaches the K_P value, while the manufacturer's production input will first decrease and then increase again. This relationship is visualized in the following Table 1 where the results for a numerical example are presented. This example is characterized by the following data: $c = 1, p = 14, D = 100$ and Z is uniformly distributed in [0,1] so that $\mu_Z = 0.5$ holds.

	W	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**
SP	Q_P	265	220	196	180	173	187	200	212	224	235	245	255	265
Yield	D_R	265	179	138	114	100	100	100	100	100	100	100	100	100
Z~ U[0,1]	***SST***	**32**	**10**	**-2**	**-10**	**-13**	**-6**	**0**	**6**	**12**	**17**	**22**	**27**	**32**
μ_Z=0.5	IOH_P	0	4	9	12	15	20	25	30	34	39	43	47	51
	IOH_R	51	29	14	6	0	0	0	0	0	0	0	0	0

Table 1: Decisions and stocks in a decentralized supply chain for *SP* yield

In Table 1 also the values for the safety stock SST^D and the expected on-hand inventories are reported. The safety stock directly follows the manufacturer's production decision and thus reaches its highest values for the lower and the upper limiting wholesale price level where it just equals the stock under centralization. Starting with the lowest w level, with increasing wholesale price the safety stock will always decrease first and will increase again after reaching a minimum value at some intermediate price level. Naturally, the total on-hand inventory in the supply chain follows the trend of the safety stock, but at some higher level. It is interesting, however, that the IOH distribution is very different for low and for high wholesale price values. This is specifically distinct in the extreme cases. For the lowest w level the on-hand inventory is completely held by the retailer because the manufacturer's production quantity coincides with the retailer's order size. At the highest w level the complete physical inventory is on the producer's side as the retailer's order does not exceed the external demand. With increasing wholesale price the inventory on hand is continuously increasing for the producer and continuously decreasing for the retailer reaching zero at the retail stage of the supply chain as soon as the retailer's order falls to demand level.

4. Safety Stocks under *BI* Yield

As mentioned in Section 2, for the *BI* yield analysis it will be assumed that the binomial distribution of yields can be properly approximated by a normal distribution. Based on this approximation, profit maximization in the centralized and decentralized supply chain can be conducted by means of mathematical calculus. The respective analysis is performed in Clemens and Inderfurth (2014) with results that will be exploited here for safety stock analysis in the following subsections.

Under the normality assumption in the case of *BI* yield, the expected sales volume in (6) and (9) can be expressed by

$$E\{min(Y(Q), D)\} = \int_0^D y \cdot f_Q(y) \cdot dy + \int_D^Q D \cdot f_Q(y) \cdot dy \qquad (25)$$

where $f_Q(y)$ denotes the density function of a Normal distribution with parameters that – according to (3) – depend on the production level Q, i.e.

$$\mu_{Y(Q)} = \theta \cdot Q \quad \text{and} \quad \sigma_{Y(Q)} = \sqrt{\theta \cdot (1-\theta) \cdot Q} \quad . \qquad (26)$$

A comparison with the corresponding expression for *SP* yield in (14) shows that the expected sales do not depend on the *D/Q* ratio in the same simple way, because production level Q has an impact on the yield variability.

4.1. Centralized Supply Chain

When the supply chain profit in (6) is maximized under the sales formula for *BI* yield in (25) the optimal production quantity Q^* is given as implicit solution from

$$M(D, Q^*) = \frac{c}{p} \quad . \qquad (27)$$

Here, $M(D, Q)$ is defined as

$$M(D,Q) := \frac{\theta}{2} \cdot \left[2 \cdot F_S(z_{D,Q}) - \frac{\sigma_{Y(Q)}}{\mu_{Y(Q)}} \cdot f_S(z_{D,Q})\right] \tag{28}$$

where $F_S(.)$ and $f_S(.)$ stand for *cdf* and *pdf* of the standard normal distribution and $z_{D,Q}$ is the standardized variable

$$z_{D,Q} := \frac{D-\mu_{Y(Q)}}{\sigma_{Y(Q)}} \ . \tag{29}$$

From the properties of the $M(D,Q)$ function it can be shown that the optimal production quantity Q^*, and thus the respective safety stock level SST^C, will increase with increasing product profitability *p/c*. Different from the *SP* yield case, however, production level Q^* is no longer proportional to customer demand *D* and tends to the value $D/\mu_Z = D/\theta$ as the demand becomes larger and larger. This property is caused by the fact that under *BI* yield the yield rate variability is affected by the production quantity and that this variability – according to (3) – is approaching zero if the demand-triggered production volume becomes very large. This risk reduction is due to the missing correlation of defects within a production lot which, different from the *SP* situation, creates a risk-pooling effect.

With respect to the safety stock level $SST^C = \theta \cdot Q^* - D$ from (7) this means that this stock tends to zero when demand *D* moves to a very high level. In general, the safety stock might be positive or negative depending on the product profitability level *p/c*. The critical *p/c* ratio πc can be calculated from equation (27) by fixing $Q = D/\theta$ or $z_{D,Q} = 0$, respectively. This critical level amounts to

$$\pi c = \left[\theta \cdot \left(0.5 - 0.2\sqrt{(1-\theta)/D}\right)\right]^{-1} \tag{30}$$

so that, different from the *SP* yield situation in (18), the sign of the safety stock also depends on the demand level. According to (8) the expected stock on hand IOH^C will always be somewhat larger than the safety stock level.

4.2. Decentralized Supply Chain

Like in the analysis for *SP* yield the *producer's optimal decision* corresponds to the optimal production decision in the centralized system given that demand and price are represented by the local data D_R and *w*. Thus the manufacturer's response function $Q_P(D_R)$ is implicitly defined from

$$M(D_R, Q_P) = \frac{c}{w} \tag{31}$$

with the same properties as described for centralized decision making.

The retailer's reaction when maximizing his profit $\Pi_R(D_R|Q_P)$ from (10) is gained from the solution of the following equation

$$-w \cdot \left[1 - F_S(z_{D_R,Q_P})\right] + \left[p \cdot M(Q_P, D) - w \cdot M(Q_P, D_R)\right] \cdot \frac{dQ_P(D_R)}{dD_R} = 0 \tag{32}$$

as long as the respective D_R value is larger than demand *D*. Otherwise $D_R^* = D$ is optimal. When exploiting the retailer-producer interaction in (31) the producer's optimal decision Q_P^*, unfortunately, cannot be expressed in a closed-form manner. From $M(D,Q^*) < M(D_R^*, Q_P^*)$

for $w < p$, however, it follows that $Q_P^* < Q^*$ so that also the supply chain safety stock SST^D in a decentralized supply chain will not exceed the respective stock level in a centralized system. It also can be shown that for the lower and upper wholesale price bound within the feasible range of w the manufacturer's optimal production level is equal to the optimal quantity in the centralized supply chain, i.e. $Q_P^* = Q^*$. For $w = p$ this results from (31) because the retailer's response equals external demand ($D_R^* = D$) in this case. For $w = c/\mu_Z = 1/\theta$ it can be derived from (32) that the retailer will order $D_R^* = Q^*$ and the manufacturer will choose her production level according to this order. The course of orders and production quantities for changing w values within the feasible range resembles that in the *SP* yield case. With increasing wholesale price w the retailer's order D_R^* is decreasing and reaches the size of external demand at some critical price, while the production quantity Q_P^* is first decreasing, but increasing again after it reaches some minimum level. This is also illustrated by a numerical example in Table 2 where the same data are used as in the *SP* yield example in Table 1 except for the yield description. For *BI* yield here a success probability of $\theta = 0.5$ is chosen which equals the μ_Z value in the *SP* yield case.

	w	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**
	Q_P	215	211	207	205	205	207	209	210	211	212	213	214	215
BI	*D_R*	215	109	104	101	100	100	100	100	100	100	100	100	100
Yield	***SST***	**8**	**6**	**4**	**3**	**3**	**4**	**5**	**5**	**6**	**6**	**7**	**7**	**8**
Θ=0.5	*IOH_P*	0	2	3	4	4	5	6	6	6	7	7	8	8
	IOH_R	8	4	2	0	0	0	0	0	0	0	0	0	0

Table 2: Decisions and stocks in a decentralized supply chain for *BI* yield

The values of safety stock SST^D and stock on hand *IOH* held by the producer and retailer that are reported in Table 2 follow directly from the production and order decisions. A comparison with Table 1 shows that the dependency of these stock values on the wholesale price *w* has the same structure as under *SP* yield. While the *BI* levels are considerably lower their course is much smoother. This effect stems from the property of *BI* yield that, different from *SP* yield, the risk from the random yield rate (in terms of its variance) is decreasing with increasing order and production level.

From the previous analysis it is apparent that both yield types, *SP* and *BI*, have an impact on safety stock management in centralized and decentralized supply chains which is different in terms of stock levels, but results in the same qualitative structure concerning the influence of prices on stocks.

5. Safety Stocks under *IG* Yield

From *Property 2* we know that in case if *IG* yield the production level will never exceed the respective demand (i.e. $Q \leq D$) so that the expected sales quantity in (6) and (9) will reduce to

$$E\{min(Y(Q), D)\} = E\{Y(Q)\} = \frac{\theta \cdot (1-\theta^Q)}{(1-\theta)} \quad . \tag{33}$$

From this sales function it follows that the respective profit functions which have to be optimized do not depend on the demand level, except for the condition that $Q \leq D$ holds. As mentioned in subsection 2.2., under *IG* yield the lower bound $c/\mu_Z(Q)$ for w and p depends on the batch size *Q*. Since $\mu_Z(Q)$ in (5) is montonously decreasing in *Q*, the minimum value of this bound is given for *Q*=1 and amounts to c/θ.

5.1. Centralized Supply Chain

Using the result in (33), the profit function from (6) can be expressed as

$$\Pi_{SC}(Q) = p \cdot \frac{\theta}{1-\theta} \cdot (1-\theta^Q) - c \cdot Q \quad . \tag{34}$$

It is easy to show that $\Pi_{SC}(Q)$ is a concave function so that the optimal production level can by determined by exploiting the first-order optimality condition

$$\frac{d\Pi_{SC}(Q)}{dQ} = -p \cdot \frac{\theta}{1-\theta} \cdot \ln\theta \cdot \theta^Q - c = 0 \; .$$

This results in the following solution

$$Q^+ = \frac{1}{\ln\theta} \cdot \ln\left[-\frac{1-\theta}{\theta \cdot \ln\theta} \cdot \frac{c}{p}\right] \; . \tag{35}$$

Thus, together with the restriction $Q \leq D$ the optimal production level in a centrally managed supply chain is given by

$$Q^* = \begin{cases} Q^+ & \text{if } \; Q^+ \leq D \\ D & \text{if } \; Q^+ \geq D \end{cases} \; . \tag{36}$$

From (35) it follows that Q^+ is steadily increasing with increasing product profitability *p/c* so that there exists some critical profitability level π_D for which Q^+ equals *D*. This level can be directly determined from (35) and is given by

$$\pi_D = \frac{1-\theta}{\theta^{D+1} \cdot \ln\theta} \quad . \tag{37}$$

Thus, if the profitability is sufficiently high, i.e. if $p/c \geq \pi_D$, the optimal production level will always be equal to external demand. In this context it has to be mentioned that under *IG* yield the minimal profitability level which guarantees non-negative profits is equal to $1/\theta$.

The safety stock SST^C can be determined according to (7), resulting in the following closed-form expression

$$SST^C = \begin{cases} \frac{\theta}{1-\theta} \cdot \left(1-\theta^{Q^+}\right) - D & \text{if } \; Q^+ \leq D \\ \frac{\theta}{1-\theta} \cdot (1-\theta^D) - D & \text{if } \; Q^+ \geq D \end{cases} \; . \tag{38}$$

This confirms the finding in *Property 2* that under *IG* yield the safety stock always must be negative, i.e. $SST^C < 0$. From (35) it is easy to see that the optimal production and, thus, the safety stock level is increasing with increasing product profitability *p/c* as long as $Q^* < D$. Like for the production quantity, the safety stock will be constant for each profitability level which exceeds the critical value π_D.

Since production always undershoots demand there is no stock on hand SOH^C that is held in the supply chain. A next consequence is that the fill rate *fr* as a service measure can simply be expressed by

$$fr(Q,D) = \frac{E\{Y(Q)\}}{D} = \frac{\theta\cdot(1-\theta^Q)}{(1-\theta)\cdot D} \quad . \tag{39}$$

The highest possible service level is reached for all instances where Q^* equals D or, equivalently, when profitability *p/c* is larger than the critical level π_D. This means that a fill rate level $fr(D,D)$ cannot be exceeded even if the product profitability is arbitrarily high. This maximum fill rate is increasing with increasing success parameter θ and decreasing with increasing demand level *D*. As consequence, under *IG* yield the optimal service level can become extremely low if the customer demand reaches a very high level.

5.2. Decentralized Supply Chain

Like in the other yield situations, in a decentralized setting with local optimization the producer's profit function $\Pi_P(Q_P|D_R)$ has the same structure as the global one. The only difference to the profit in (34) is that the external price *p* is replaced by the wholesale price *w* and that the condition $Q_P \leq D_R$ has to be taken into account. Thus, as optimal production level for a given retailer's order we get

$$Q_P(D_R) = \begin{cases} Q_P^+ & \text{if } Q_P^+ \leq D_R \\ D_R & \text{if } Q_P^+ \geq D_R \end{cases} \tag{40}$$

with

$$Q_P^+ = \frac{1}{\ln\theta}\cdot\ln\left[-\frac{1-\theta}{\theta\cdot\ln\theta}\cdot\frac{c}{w}\right] \quad . \tag{41}$$

The retailer knows that the manufacturer will never produce more than his own order D_R and that a production level above customer demand *D* will not affect his probabilities of receiving *D* or less units. Thus, he has no incentive to order more than *D* units (i.e., $D_R \leq D$) so that the profit function in (10) reduces to

$$\Pi_R(D_R|Q_P) = (p-w)\cdot E\{Y(Q_P)\} \qquad \text{with} \qquad Q_P \leq D_R \quad . \tag{42}$$

Due to the regular price relationship $w \leq p$ the retailer's optimal response is to order as much as possible under the restriction $D_R \leq D$ what results in an optimal order quantity of

$$D_R^* = D \quad . \tag{43}$$

Thus, from the producer-retailer interaction in (40) we finally find as optimal production level of the manufacturer

$$Q_P^* = \begin{cases} Q_P^+ & \text{if } Q_P^+ \leq D \\ D & \text{if } Q_P^+ \geq D \end{cases} \tag{44}$$

Since a comparison of (35) and (41) reveals that $Q_P^+ \leq Q^+$, it is obvious from the production levels in (36) and (44) that always $Q_P^* \leq Q^*$ holds. This means that also under *IG* yield the production level and hence the supply chain safety stock is smaller in a decentralized setting than under central decision making, except for $w = p$ where the results are identical. As consequence, also in a decentralized supply chain the safety stock is negative (i.e. $SST^D < 0$), and because of $Q_P^* \leq D_R^* = D$ neither the producer nor the retailer will hold any physical inventory ($IOH_P^D = IOH_R^D = 0$).

The properties concerning the impact of product profitability on production level and safety stock that were found for the centralized supply chain carry over to the decentralized setting if *w/c* is interpreted as profitability measure instead of *p/c*. Here, the lower bound on *w/c* is determined from the respective zero profit condition like in the centralized case at a production level of Q_P^+ instead of Q^+. Thus, different from the situation under *SP* and *BI* yield, with increasing wholesale price w in its feasible range $c/\theta \leq w \leq p$ the course of production and safety stock does not have a U-shape, but is characterized by a steady increase in the *IG* yield case until a maximum is reached when *w/c* exceeds the critical profitability π_D in (37). Furthermore, also under decentralization of decision making the fill rate *fr* can be calculated like in (39) with Q_P^* as production level *Q*. So the minimum service level is given for minimum product profitability $1/\theta$, and the maximum level holds for all cases with $w/c \geq \pi_D$.

These general results are illustrated by a numerical example where except for the yield parameter the same data are chosen like for the *SP* and *BI* yield examples. In order to report reasonable numerical results a high success probability of $\theta = 0.98$ is chosen for the *IG* example. The respective results (including fill rate data) for this example are presented in Table 3 and confirm that the safety stocks always stay in the minus region resulting in fill rates that range between 24 % for w=2 and 43 % as highest level that is reached for $w/c > \pi_D = 7.7$. Even for the highest feasible wholesale price value this upper level will not be exceeded. The minimum wholesale price level that guarantees non-negative profits is somewhat lower than *w*=2 and can be calculated to be *w*=1/0.98=1.02.

	w	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**
IG	Q_P	34	54	68	79	88	96	100	100	100	100	100	100	100
Yield	D_R	100	100	100	100	100	100	100	100	100	100	100	100	100
Θ=0.98	***SST***	**-76**	**-67**	**-63**	**-61**	**-59**	**-58**	**-57**	**-57**	**-57**	**-57**	**-57**	**-57**	**-57**
	fr (in%)	**24**	**33**	**37**	**39**	**41**	**42**	**43**	**43**	**43**	**43**	**43**	**43**	**43**

Table 3: Decisions and stocks in a decentralized supply chain for *IG* yield

The safety stock analysis for situations with *IG* yield raises some critical questions concerning the relevance of the modelled decision problem. The general negativity of safety stock values and especially the continuously decreasing service level for increasing demand, even for extremely high product profitability, make it doubtful that one would face such a decision context widely in practice. In the above example with a demand of *D*=100 it needs a success probability of $\theta = 0.999$ to guarantee a fill rate of 95%. So, under both centralized and decentralized supply chain conditions the solution of the modelled optimization problem will only lead to acceptable solutions if a very high process quality is existing and/or if demand has only a fairly low level. Under other planning conditions in case of *IG* yield the production process should be organized in such a way that more than a single production run per period can be carried out to satisfy the period's demand. This facilitates the execution of smaller production batches with lower risk of large yield losses and helps to guarantee arbitrarily high service levels in cases of high product profitability. An overview of approaches that optimize the number and batch size of production runs for a given fixed cost per run is found in Grosfeld-Nir and Gerchak (2004).

6. Insights and Future Research

There is a bunch of findings that emerge from the above analysis of yield randomness and its impact on safety stock holding in supply chains under both central and local planning conditions. A first major insight is that it is not only the degree of yield risk but, even more importantly, the type of yield randomness that matters.

In general, in a centralized setting the safety stock size increases steadily with an increasing profit margin of the product. For *IG* yield, however, this increase is strictly limited because here production never exceeds demand so that the safety stock will never take on a positive value. Thus, under *IG* yield the service level might remain at a very low size even if the product profitability is extremely high. With *SP* and *BI* yield negative safety stock will only occur if the profit margin is relatively low. The safety inventory will always be positive if a critical profitability level is exceeded. Furthermore, the impact of demand is even more diverse for the different yield types. With increasing demand level the safety stock in continuously increasing or, in case of negative value, decreasing under *SP* yield while this stock tends to approach zero under *BI* yield. In the case of *IG* yield the safety stock will only change with increasing demand if the product profitability is sufficiently high.

In a decentralized setting the supply chain safety stock is directly determined by the manufacturer's production decision. Under all yield types this stock is lower than the respective safety level under central decision making as long as price and cost parameters are such that both supply chain members make positive profits. This is a consequence of the double marginalization effect that is also well-known from other supply chain problem areas if a simple wholesale price contract is applied. Although the manufacturer's production problem has the same structure as the decision problem of the central supply chain planner, the general safety stock properties from the centralized setting do not simply carry over. This is because the manufacturer's response to the retailer's order is anticipated and causes a retailer's reaction that results in an order size that might exceed the external supply chain demand. One faces such a situation if from retailer's view the product profitability is relatively high. This effect is never found under *IG* yield, but it exists in case of *SP* and *BI* yield so that under these yield types an increase of the manufacturer's product margin (in terms of the ratio of wholesale price and unit production cost) will not necessarily lead to an increase of production and safety stock level, but can even result in a decrease. Interestingly, the distribution of safety stock dependent stock on hand between the supply chain members differs considerably according to the relative product profitability which depends on the wholesale price level. While the stock on hand concentrates on the retailer side for a low manufacturer's and high retailer's margin the opposite holds if the margin relationship is the other way around.

Further research is necessary in order to reveal to which extent the above insights carry over to more complex supply chain structures like those with several stages and multiple producers and retailers. The same holds for the investigation of situations where yield risks come along with demand uncertainty. Up to now, studies that address these cases like those by Gurnani and Gerchak (2007) and by Güler and Bilgic (2009), for example, only refer to problems with *SP* yield and need to be extended to the other yield types. An extension of the above research to multi-period problems can be based on already existing studies for centralized supply chains (see Inderfurth and Vogelgesang 2013), but needs the solution of highly complex game-theoretic problems in the case of local supply chain decision making. A very valuable extension would lie in the consideration of multiple production runs, particularly in situations with *IG* yield where the safety stock analysis in this paper has only limited practical relevance for cases with low process quality and high product margin. For this problem type solutions only exist for centrally coordinated supply chains (see Grosfeld-Nir and Gerchak 2004) while it is a major challenge to solve these problems in a decentralized setting with producer-retailer interaction. Finally, an additional field for future research is given if the current research is extended to manufacturer-retailer interactions that base upon more complex contract types than the simple wholesale price contract. For many problem areas a large variety of contracts has been analyzed which aim to coordinate local supply chain decisions to the optimal central solution (see Cachon 2003). From recent research contributions addressing the above random yield problem (see Inderfurth and Clemens 2014 and Clemens and Inderfurth 2015) it is known that two contract types, namely the so-called overproduction risk sharing and the penalty contract, achieve coordination if their parameters are chosen such that the retailer always is incentivized to place orders according to the external demand. Under these conditions also under decentralized decision making the safety stock is equal to that of the centralized solution in case of *SP* and *BI* yield. It is not clear, however, if this result also holds for other contract types that enable coordination like, for instance, the pay-back-revenue-sharing contract proposed in Tang and Kouvelis (2014). Furthermore, it is completely unknown which types of contracts will support coordination if *IG* yield is considered and how they affect safety stock management in the supply chain.

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