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Corinna Krebs/Jan Fabian Ehmke

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Verantwortlich für diese Ausgabe:
Corinna Krebs and Jan Fabian Ehmke
Otto-von-Guericke-Universität Magdeburg
Fakultät für Wirtschaftswissenschaft
Postfach 4120
39016 Magdeburg
Germany

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Corinna Krebs · Jan Fabian Ehmke

Abstract Overloaded axles not only lead to increased erosion on the road surface, but also to an increased braking distance and more serious accidents due to higher impact energy. Therefore, the load on axles should be already considered during the planning phase and thus before loading the truck in order to prevent overloading. Hereby, a detailed 2D or 3D planning of the vehicle loading space is required. We model the Axle Weight Constraint for trucks with and without trailers based on the Science of Statics. We include the Axle Weight Constraint into the combined Vehicle Routing and Container Loading Problem ("2L-CVRP" and "3L-CVRP"). A hybrid approach is used where an outer Adaptive Large Neighbourhood Search tackles the routing problem and an inner Deepest-Bottom-Left-Fill algorithm solves the packing problem. Moreover, to ensure feasibility, we show that the Axle Weight Constraint must be checked after each placement of an item. The impact of the Axle Weight Constraint is also evaluated.

Keywords Vehicle Routing Problem · Container Loading · 2L-CVRP · 3L-CVRP · Axle Weights

1 Introduction

A survey by Blower and Woodrooffe (2012) of the University of Michigan analysed the status of truck safety for four countries: Australia, Brazil, China and the United States. The overloading of trucks is a major problem especially for China and Brazil. According to reports in China, 70–90% of truck accidents are related to overloaded and oversized trucks. In Brazil, 60% of the trucks

Corinna Krebs · Jan Fabian Ehmke
Department of Management Science, Otto-von-Guericke-University Magdeburg, 39106 Magdeburg, Germany
E-mail: Corinna.Krebs@ovgu.de
E-mail: Jan.Ehmke@ovgu.de

in crashes have been revealed to be overloaded and 20% of registered trucks exceeding the gross mass.
The overloading of axles can result in an overheating and failing of tyres and brakes. If the permissible axle weight of a steering axle is exceeded, the steering is more cumbersome and it is possible to loose control of the vehicle. There are also economic effects: A study by Pais et al. (2013) showed that the increased erosion of roads raises the pavement costs by more than 100%. For these reasons, it is essential to consider the load on the axles of trucks directly in the planning of routes.
In the majority of previous research on the Vehicle Routing Problem, customer demand was simply expressed as the total mass or volume. This is not sufficient to take axle weights into account, as a detailed 2D or 3D planning of the vehicle loading space is required. In this paper, we present simple, yet effective formulas for considering the axle weights suitable for all 2D and 3D Container Loading Problems. We include our approach in the combinations of Vehicle Routing and Container Loading Problems, such as the 2L-CVRP and 3L-CVRP.
This paper considers two aspects for the first time: I) We formulate an approach to consider the Axle Weight Constraint not only for trucks but also for trailers and II) we include the Axle Weight Constraint in the 3L-CVRP. Moreover, we show with a detailed real example that it is necessary to check the Axle Weight Constraint after each placement of an item, since the mass of items can act on an axle, but it can also relieve an axle. Therefore, also unloading an item can lead to an overloading of one or several axles.
We use a hybrid algorithm for tackling the Vehicle Routing and the Container Loading Problem. The routing heuristic is based on the Adaptive Large Neighbourhood Search (ALNS) by Koch et al. (2018) calling for each route a modified packing heuristic based on the Deepest-Bottom-Left-Fill (DBLF) algorithm proposed by Karabulut and İnceoğlu (2005). Both heuristics are described in detail in the following subsections. For the computational tests, we use instances from the literature, which is reviewed in Section 2. The considered problems (2L- and 3L-CVRP) are formulated in Section 3. In Section 4, the Axle Weight Constraint is described in detail. The hybrid solution approach is explained in Section 5. Section 6 presents computational results, analysing the impact of the Axle Weight Constraint on VRP solutions. Finally, conclusions are drawn in Section 7.

2 Literature Review

In this section, the literature considering the Axle Weight Constraint with their vehicle models is reviewed. To the best of our knowledge, the following overview represents all papers considering axle weights. First, the papers dealing with the Container Loading Problem or a variant of this problem are regarded since this represents a subproblem of the problems considered in this paper. Then, papers are summarized that include the axle weight constraint into the combined Container Loading and Vehicle Routing Problem.

2.1 Container Loading Problem

Although Container Loading Problems have been investigated for several decades, the axle weights of vehicles have so far only been considered in the following three papers. In the 3D Single Container Loading Problem, a number of three-dimensional boxes must be packed into one three-dimensional container while minimizing total volume utilization. In this context, the Axle Weight Constraint was first considered in a paper by Lim et al. (2013). They examine the general rules of the California Vehicle Code. They simplify the rules by formulating three semi-trailer truck models with three limits each: One limit for the front axle group, one for the rear axle group and one for the gross vehicle weight. Then, a Greedy Randomized Adaptive Search Procedure Wall-Building Algorithm packs the boxes, and the Axle Weight Constraint is checked. For this, the ideal mass center of the container is calculated. Then, the deviation between the ideal mass center and the current mass center of the container is minimized by rearranging the walls of loaded boxes.
The Multi Container Loading Problem is an extension of the Single Container Loading Problem, where a set of items needs to be packed into multiple containers while minimizing the number of used containers. For the 2D problem variant, the Axle Weight Constraint is included in mathematical models by Alonso et al. (2017). The Axle Weight Constraint is considered for trucks with two axles and is included in the model by formulating the equilibrium for the moments and forces. In Alonso et al. (2019), this model is extended by further constraints such as dynamic stability constraints.

2.2 Combined Vehicle Routing and Container Loading Problem

The Vehicle Routing Problem is one of the most studied optimization problems in logistics. This problem involves the optimal planning of routes to deliver goods to customers that are located in a depot. This is accomplished by a fleet of vehicles having a certain capacity. Since detailed 2D and 3D planning of the loading space is required to take axle weights into account, the Axle Weight Constraint has rarely been taken into account so far. In Iori et al. (2007), the combination of the 2D Container Loading Problem with the Capacitated Vehicle Routing Problem (2L-CVRP) is introduced. The Axle Weight Constraint was first considered in the 2L-CVRP by Pollaris et al. (2016). There, the objective is to minimize the total travel cost (distance). The problem is solved by a mixed integer linear programming formulation. In their model, a truck with a trailer is considered. The truck consists of two axles (steering axle and driving axle), the trailer has an axle group consisting of three axles (tridem-axle). The axle weights are calculated only for the trailer axle group and the

coupling point, which is the connection between the truck and the trailer. The separated examination of axle weights of the steering and driving axle of the truck is not considered. The axle weights are calculated by means of equilibrium of forces and moments. Pollaris et al. (2016) developed 128 instances varying in the number of customers, the number of pallets per customer and the masses of the pallets. The instances are tested with CPLEX, but for some instances, no feasible solution can be found after two hours runtime. To solve larger instances, in Pollaris et al. (2017), an Iterated Local Search approach with Sequence-Based Pallet Loading is developed. In this framework, the same assumptions are used for the axle weight calculation and the vehicle model. Additional 96 instances were created with up to 100 customers, which can be solved by the proposed metaheuristic.
The Three-Dimensional Loading Capacitated Vehicle Routing Problem (3L-CVRP) is an extension of the 2L-CVRP combining 3D Container Loading with the Capacitated Vehicle Routing Problem introduced by Gendreau et al. (2006). The 3L-CVRP has been studied intensively in the recent years so that the results for this benchmark have been improved repeatedly by researchers (e.g. Tarantilis et al. (2009), Fuellerer et al. (2010), Bortfeldt (2012) and Wei et al. (2014)). However, the Axle Weight Constraint has not been included in the 3L-CVRP yet. To the best of our knowledge, this paper considers for the first time the Axle Weight Constraint in the 3L-CVRP.

3 Problem Formulation

To model the 2L-CVRP and 3L-CVRP, we follow the convention by Bortfeldt (2012). Let $G = (N, E)$ be a complete, directed graph, where N is the set of $n+1$ nodes including the depot (node 0) and n customers to be served (node 1 to n), and E is the edge set connecting each pair of nodes. Let a distance $co_{i,j}$ ($co_{i,j} > 0$) be assigned to each edge $e_{i,j} \in E$ ($i \neq j, i, j = 0, ..., n$). The demand of customer $i \in N \setminus \{0\}$ consists of c_i items. Each item $I_{i,k}$ ($k = 1, ..., c_i$) is defined by mass $m_{i,k}$, length $l_{i,k}$, width $w_{i,k}$ and height $h_{i,k}$. The total demanded mass for a customer i is given by mc_i. The items are delivered by at most v_{max} available, homogenous vehicles. Each vehicle has a maximum load capacity D and a cuboid loading space defined by length L, width W and height H.
Let v_{used} be the number of used vehicles in a solution. A solution is a set of v_{used} pairs of routes R_v and packing plans PP_v, whereby the route R_v ($v = 1, ..., v_{used}$) is an ordered sequence of at least one customer and PP_v is a packing plan containing the position within the loading space for each item included in the route.

A solution is feasible if

(S1) All routes R_v and packing plans PP_v ($v = 1, ..., v_{used}$) are feasible (see below);

(S2) The number of used vehicles v_{used} does not exceed the number of available vehicles v_{max};
(S3) Each packing plan PP_v contains all c_i items of all customers i included in the corresponding route ($i \in R_v$).

A route R_v must meet the following routing constraints:

(R1) Each route starts and terminates at the depot and visits at least one customer;
(R2) Each customer is visited exactly once.

Each packing plan must obey the following loading constraint set P:

(C1) *Geometry*: The items must be packed within the vehicle without overlapping;
(C2) *Orthogonality*: The items can only be placed orthogonally inside a vehicle;
(C3) *Load Capacity*: The sum of masses of all included items of a vehicle does not exceed the maximum load capacity D;
(C4) *LIFO*: No item is placed above or in front of item $I_{i,k}$, which belongs to a customer served after customer i.

For the 3L-CVRP, the following loading constraints must be additionally respected:

(C5) *Rotation*: The items can be rotated 90° only on the width-length plane;
(C6) *Minimal Supporting Area*: Each item has a supporting area of at least a percentage α of its base area;
(C7) *Fragility*: No non-fragile items are placed on top of fragile items.

Moreover, the Axle Weight Constraint is considered.

(C8) *Axle Weight*: The load on the axles do not exceed the permissible axle weights.

The 2L-CVRP and 3L-CVRP consists of determining a feasible solution minimizing the number of used vehicles v_{used} and the total travel distance ttd and meeting all constraints.

4 Axle Weight Constraint

This section details our approaches for the consideration of axle weights in the problem at hand. Based on the Science of Statics, the formulas for calculating the axle weights for a box truck are derived. Then, resultant axles for replacing axle groups are introduced. Afterwards, the formulas for calculating the axle weights for trucks with trailers are shown. Then, we demonstrate that the Axle Weight Constraint must be checked after each placement of an item.

4.1 Approach for Box Trucks

The following formulas are suitable for the axle weight calculation of box trucks and box vans. These are vehicles with a cuboid-shaped cargo area without any kind of trailer. For implementation, additional specifications of the vehicle are needed. In practice, those are given by the manufacturers. Let FA_{perm} be the maximum load the vehicle's front axle can bear and RA_{perm} be the maximum load for the rear axle, respectively. Both limits are given in the unit of mass. The parameter L_f describes the distance between the front axle and the cargo area. The wheelbase WB is the distance between the front and the rear axle (see Fig. 1).

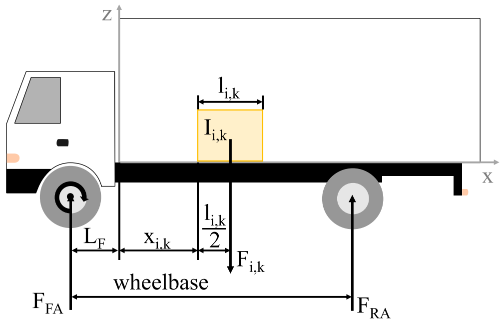


Fig. 1 Vehicle Data

An object on the surface of the earth experiences a force caused by the gravitational attraction of the earth. This force is calculated as the mass of the object times the acceleration of gravity g ($g \approx 9.81 \frac{m}{s^2}$), Therefore, each item $I_{i,k}$ acts a force $F_{i,k}$ on the vehicle, which is

$$F_{i,k} = m_{i,k} \cdot g. \tag{1}$$

The point of the force $F_{i,k}$ is located in the center of mass for each item. If the mass of an item is homogeneously distributed, the center of mass and the geometric center are at the same point. Each axle counteracts these forces, whereby F_{FA} is the force representing the front axle and F_{RA} the force for the rear axle.
Moreover, each force creates a moment, which can be determined to any point in the system. It is expedient to determine the moments to an unknown force, such as the force of the front axle F_{FA}. Each moment is calculated by the mass multiplied by the distance r from the front axle to the force. Thus, each item creates a moment $M_{i,k}$. Supposing that the mass of an item is homogeneously distributed, resulting that the point of force lays in the geometric center, the distance $r_{i,k}$ to the point of the front axle force F_{FA} is

$$r_{i,k} = L_f + x_{i,k} + l_{i,k}/2. \tag{2}$$

Thus, the moment $M_{i,k}$ created by item $I_{i,k}$ is:

$$M_{i,k} = m_{i,k} \cdot g \cdot r_{i,k} \tag{3}$$

or rather

$$M_{i,k} = m_{i,k} \cdot g \cdot (L_f + x_{i,k} + l_{i,k}/2). \tag{3b}$$

The force from the rear axle F_{RA} creates another moment, which is F_{RA} multiplied by the wheelbase WB.
In the Science of Statics, the forces and moments are in static equilibrium with their environment. Thus, the summation of forces F and of moments M are zero. Considering the direction of the forces and moments, the following formulas can be applied for a vehicle v.

Equilibrium of forces:

$$\sum_{i=1|i\in R_v}^{n} \sum_{k=1}^{c_i} F_{i,k} - F_{RA} - F_{FA} = 0, \tag{4}$$

which can be transformed to F_{FA}:

$$F_{FA} = \sum_{i=1|i\in R_v}^{n} \sum_{k=1}^{c_i} F_{i,k} - F_{RA}. \tag{4b}$$

The summation of moments must be zero and is:

$$\sum_{i=1|i\in R_v}^{n} \sum_{k=1}^{c_i} M_{i,k} - F_{RA} \cdot WB = 0, \tag{5}$$

which can be transformed to F_{RA}:

$$F_{RA} = \frac{1}{WB} \cdot (\sum_{i=1|i\in R_v}^{n} \sum_{k=1}^{c_i} M_{i,k}). \tag{5b}$$

In Equation 5b, all values are known, so that the result for F_{RA} can be calculated and inserted in Equation 4b to receive F_{FA}. The acting forces for the front and the rear axles must be below the permissible ones:

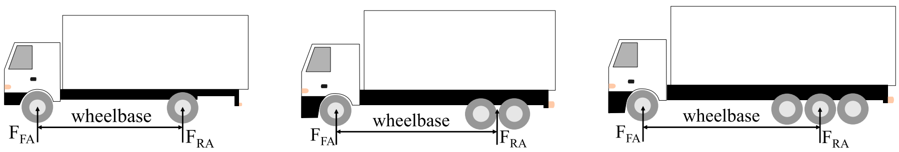
$$F_{FA} \leq FA_{perm} \cdot g \tag{6}$$

and

$$F_{RA} \leq RA_{perm} \cdot g. \tag{7}$$

4.2 Axle Group and Resultant Axle

The formulas shown in Section 4.1 are suitable for trucks with one front and rear axle each. Trucks as well as trailers can have axle groups consisting of two consecutive axles ("tandem-axle") or three consecutive axles ("tridem-axle"). For these axle groups, a so-called "resultant axle" replaces the axle group. The resultant axle is located in the center of the axle group and its value is the sum of each consecutive axle. Thus, as shown in Fig 2, for a tandem-axle, the resultant axle is between the both axles. For a tridem-axle, the resultant axle lays in the middle axle of the three axles.



(a) One Axle (b) Tandem-Axle (c) Tridem-Axle

Fig. 2 Examples for Resultant Axles replacing Axle Groups

4.3 Approach for Semi-Trailer Trucks

The calculation of the load on the axles shown in Section 4.1 is suitable for trucks without a trailer. Semi-trailer trucks, as shown in Fig. 3, have at least three axles, for which the loads must be determined: The load on the front axle and the rear axle of the tractor unit (truck) and the load on the axle group of the trailer. Their permissible load on these axles are FA_{perm} for the front axle, RA_{perm} for the rear axle and TA_{perm} for the trailer axle. Due to the three axles, the above formulas must be adapted. The Science of Statics is also applied to the following formulas.

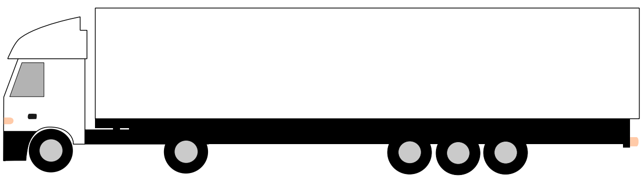
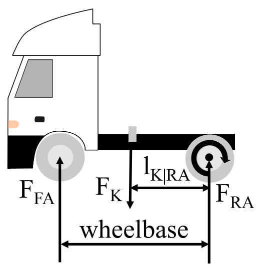
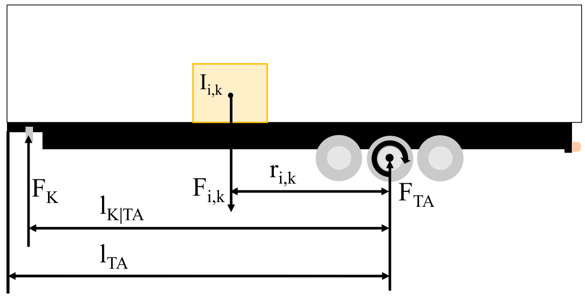


Fig. 3 Semi-Trailer Truck with Tridem Trailer Axle

Since the load on the axles of the tractor unit depends on the trailer, the forces and moments of the tractor unit and the trailer are examined separately. The kingpin on the semi-trailer connects the trailer with the tractor unit. In Fig.4 the distances, forces and moments are illustrated.



(a) Tractor Unit



(b) Trailer

Fig. 4 Forces and Moments for Semi-Trailer Trucks

In the first step, the forces and moments of the trailer are determined. As shown in Section 4.2, a resultant trailer axle can be used instead of the tridem-axle. All moments of the trailer are calculated to the center of the resultant axle. Therefore, all distances from each force to this point must be determined. Let l_{TA} be the distance between the cargo area to the resultant trailer axle. The distance between the kingpin and the resultant axle of the trailer is described by $l_{K|TA}$. The distance $r_{i,k}$ between the item $I_{i,k}$ to the resultant trailer axle is

$$r_{i,k} = l_{TA} - x_{i,k} - \frac{l_{i,k}}{2}. \tag{8}$$

The force of the kingpin and the force of the resultant trailer axle act against the item forces. As demonstrated before, the summation of forces is zero in the Science of Statics. Therefore, the following must apply for the trailer:

$$F_{TA} + F_K - \sum_{i=1|i\in R_v}^{n} \sum_{k=1}^{c_i} F_{i,k} = 0. \tag{9}$$

This can be transformed to F_{TA}:

$$F_{TA} = \sum_{i=1|i\in R_v}^{n} \sum_{k=1}^{c_i} F_{i,k} - F_K. \tag{9b}$$

Similarly, the summation of the moments must be zero, so that it is

$$F_K \cdot l_{K|TA} - \sum_{i=1|i\in R_v}^{n} \sum_{k=1}^{c_i} F_{i,k} \cdot r_{i,k} = 0, \tag{10}$$

and transformed to F_K, one gets:

$$F_K = \frac{1}{l_{K|TA}} \cdot (\sum_{i=1|i\in R_v}^{n} \sum_{k=1}^{c_i} F_{i,k} \cdot r_{i,k}). \tag{10b}$$

Since all values in Equation 10b are known, the force F_K can be calculated and inserted in Equation 9b to receive F_{TA}. In the next step, the force and moment equilibriums are described for the tractor unit. The load on the tractor unit is carried by two axles. The front axle force F_{FA} and the rear axle force F_{RA} work against the force in the kingpin F_K. The summation of forces must be zero and is

$$F_{FA} + F_{RA} - F_K = 0. \tag{11}$$

This can be transformed to F_{RA}:

$$F_{RA} = F_K - F_{FA}. \tag{11b}$$

The moments are calculated in the center of the rear axle. Let WB be the wheelbase between the front and the rear axle of the tractor unit and $l_{K|RA}$ be the distance between the rear axle and the kingpin. Then, the summation of the following moments must be zero:

$$F_{FA} \cdot WB - F_K \cdot l_{K|RA} = 0. \tag{12}$$

The force F_K was calculated before. Thus, the force F_{FA} can be determined:

$$F_{FA} = \frac{1}{WB} \cdot F_K \cdot l_{K|RA}. \tag{12b}$$

The result for F_{FA} is then inserted in Equation 11b, to receive F_{RA}. The current load on the axles must not exceed the permissible ones:

$$F_{FA} \leq FA_{perm} \cdot g, \tag{13}$$

$$F_{RA} \leq RA_{perm} \cdot g, \tag{14}$$

$$F_{TA} \leq TA_{perm} \cdot g. \tag{15}$$

4.4 Consideration of Vehicle's and Trailer's Masses

The value of the permissible axle weights may be without the consideration of vehicle mass (m_{truck}). In this case, the mass must be respected in the formulas. The mass is added as additional force in Equation 4b:

$$F_{FA} = \sum_{i=1|i \in R_v}^{n} \sum_{k=1}^{c_i} F_{i,k} + m_{truck} \cdot g - F_{RA}. \tag{4c}$$

Moreover, the mass creates a moment in the center of mass of the truck. The distance (r_{truck}) between the front axle and the center of mass of the truck must be given by the manufacturer. Thus, Equation 5b needs to be updated as follows:

$$F_{RA} = \frac{1}{WB} \cdot (\sum_{i=1|i \in R_v}^{n} \sum_{k=1}^{c_i} M_{i,k} + m_{truck} \cdot g \cdot r_{truck}). \tag{5c}$$

In case of semi-truck trailers, the masses of the tractor unit ($m_{tractor}$) and of the trailer ($m_{trailer}$) do not have to be included. In that case, the masses are added as forces in the Equations 9b and 11b, so that

$$F_{TA} = \sum_{i=1|i \in R_v}^{n} \sum_{k=1}^{c_i} F_{i,k} + m_{tractor} \cdot g - F_K. \tag{9c}$$

and

$$F_{RA} = F_K + m_{tractor} \cdot g - F_{FA}. \tag{11c}$$

The mass of the trailer and the tractor unit creates also a moment in their center of mass. The distance $r_{tractor}$ between the rear axle and the center of mass and the distance $r_{trailer}$ between the trailer axle and the center of mass must be also given by the manufacturer. The moments are added in the Equations 10b and 12b, resulting in

$$F_K = \frac{1}{l_{K|TA}} \cdot (\sum_{i=1|i \in R_v}^{n} \sum_{k=1}^{c_i} F_{i,k} \cdot r_{i,k} + m_{trailer} \cdot g \cdot r_{trailer}) \tag{10c}$$

and

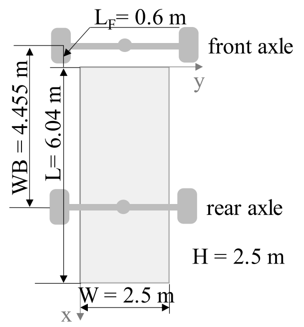
$$F_{FA} = \frac{1}{WB} \cdot (F_K \cdot l_{K|RA} + m_{tractor} \cdot g \cdot r_{tractor}). \tag{12c}$$

Other masses, such as the mass of the driver, the fuel, etc., may be added in the same way, provided that distances and masses are known.

4.5 Check Frequency

In the following, the check frequency for the Axle Weight Constraint is examined. According to Pollaris et al. (2016), by placing an item, the mass of the item cannot only act on an axle, but it can also relieve an axle. Therefore, when unloading an item, the load on the axles can exceed the permissible ones. Thus, it must be checked whether the constraint is fulfilled after each placement of an item. In the following, we will demonstrate this with a realistic example. The demand of four customers consists of one item each. We assume the truck is the model ML150E28FP of the manufacturer IVECO. The dimensions of the truck and of the items are shown in Fig. 5. The load capacity D is 10,100 kg and the permissible load on the axles is FA_{perm} = 5,300 kg for the front axle and RA_{perm} = 10,700 kg for the rear axle.

Vehicle



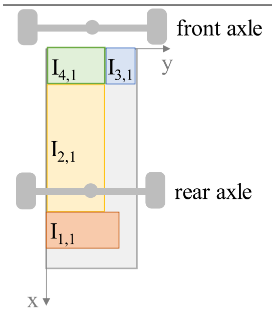
Items

Item	$m_{i,k}$ [kg]	$l_{i,k}$ [m]	$w_{i,k}$ [m]	$h_{i,k}$ [m]
$I_{1,1}$	2,000	1.00	2.00	1.60
$I_{2,1}$	1,000	3.50	1.60	1.60
$I_{3,1}$	3,000	1.00	0.80	1.60
$I_{4,1}$	4,100	1.00	1.60	1.60
	10,100			

Fig. 5 Vehicle's and Items' Dimensions

The customer visiting order for the route is $R_1 = \{1, 2, 3, 4\}$. Due to the height of the items, it is not possible to stack them. The load capacity D is complied with since the sum of the items' masses is equal to the load capacity D. Respecting the LIFO (C4) constraint, the items of the customers are unloaded in the reversed order ($order = \{I_{4,1}; I_{3,1}; I_{2,1}; I_{1,1}\}$).

To pack the items, the proposed DBLF algorithm is used (see Section 5.2). The items' positions, shown in Fig. 6, would result if the Axle Weight Constraint would be checked once after loading all items of the route R_1.



Item	$m_{i,k}$ [kg]	$l_{i,k}$ [m]	$x_{i,k}$ [m]	$s_{i,k}$ [m]	$F_{i,k}$ [N]	$M_{i,k}$ [Nm]
$I_{4,1}$	4,100	1.00	0.00	1.10	40,221	44,243
$I_{3,1}$	3,000	1.00	0.00	1.10	29,430	32,373
$I_{2,1}$	1,000	3.50	1.00	3.35	9,810	32,864
$I_{1,1}$	2,000	1.00	4.50	5.60	19,620	109,872

Fig. 6 Positions of items when checking the Axle Weight Constraint when all items have been loaded

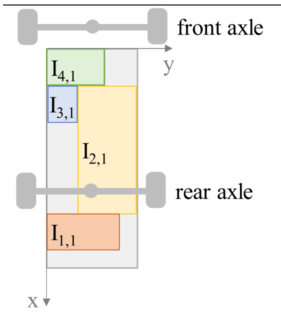
The corresponding axle weights are shown in Table 1. The load on the axles are below the permissible ones after loading all items into the vehicle's loading space. Thus, the Axle Weight Constraint seems to be fulfilled. However, when unloading the item $I_{1,1}$, the front axle gets overloaded because the item $I_{1,1}$ relieves the front axle. The front axle is even overloaded after unloading item $I_{2,1}$.

Consequently, the Axle Weight Constraint is not fulfilled for the current items' positions, shown in Fig. 6.

Loaded items	**Current Forces**		**Permissible Forces**	
	F_{FA} [N]	F_{RA} [N]	$FA_{perm} \cdot g$ [N]	$RA_{perm} \cdot g$ [N]
$I_{4,1}$; $I_{3,1}$; $I_{2,1}$; $I_{1,1}$	49,844	49,237	51,993	104,967
$I_{4,1}$; $I_{3,1}$; $I_{2,1}$	**54,886**	24,575		
$I_{4,1}$; $I_{3,1}$	**52,453**	17,198		
$I_{4,1}$	30,290	9,931		

Table 1 Axle Weights for items' positions in Fig. 6

When checking the Axle Weight Constraint after each placement of an item, the items' positions shown in Fig. 7 would result. The item $I_{4,1}$ would be still placed in the origin. The first position for item $I_{3,1}$ leads to an overload of the front axle as shown in Fig. 6. Due to the check of the Axle Weight Constraint, this position would be rejected and the next possible position according to the DBLF approach would be tested, which is in front of item $I_{4,1}$. For this position, the front axle is not overloaded and the position is feasible.



Item	$\mathbf{m_{i,k}}$ [kg]	$\mathbf{l_{i,k}}$ [m]	$\mathbf{x_{i,k}}$ [m]	$\mathbf{s_{i,k}}$ [m]	$\mathbf{F_{i,k}}$ [N]	$\mathbf{M_{i,k}}$ [Nm]
$I_{4,1}$	4,100	1.00	0.00	1.10	40,221	44,243
$I_{3,1}$	3,000	1.00	1.00	2.10	29,430	61,803
$I_{2,1}$	1,000	3.50	1.00	3.35	9,810	32,864
$I_{1,1}$	2,000	1.00	4.50	5.60	19,620	109,872

Fig. 7 Positions of items when checking the Axle Weight Constraint after each item's placement

Table 2 shows the calculated axle weights for the items' positions in Fig. 7. Since the Axle Weight Constraint is fulfilled after each item has been unloaded, the axles are not overloaded at any point in the route.
To summarize, it is necessary to check the Axle Weight Constraint after each placement of an item, although the complexity of the algorithm increases.

5 Hybrid Solution Approach

Since the 3L-CVRP can be interpreted as a combination of the Capacitated Vehicle Routing Problem (CVRP) and the 3D Container Loading Problem,

Loaded items	**Current Forces**		**Permissible Forces**	
	F_{FA} [N]	F_{RA} [N]	$FA_{perm} \cdot g$ [N]	$RA_{perm} \cdot g$ [N]
$I_{4,1}$; $I_{3,1}$; $I_{2,1}$; $I_{1,1}$	43,238	55,843	51,993	104,967
$I_{4,1}$; $I_{3,1}$; $I_{2,1}$	48,280	31,181		
$I_{4,1}$; $I_{3,1}$	45,847	23,804		
$I_{4,1}$	30,290	9,931		

Table 2 Axle Weights for items' positions in Fig. 7

we decompose the problem and use a separate algorithm for each subproblem. First, a set of routes is created which takes the routing constraints (R1, R2) into account. For each feasible route, the packing algorithm is then called, which tries to create a feasible packing plan considering the loading constraints. If no feasible packing plan can be created for a route, a new set of routes must be found. Both algorithms are described in detail in the following subsections, suitable line numbers are given in square brackets.

5.1 Routing Heuristic

The heuristic for solving the routing problem is based on the paper by Koch et al. (2018) modifying the Adaptive Large Neighborhood Search (ALNS) proposed by Ropke and Pisinger (2006). The general framework and the modifications are described below. The algorithm is shown in Alg. 1.

Algorithm 1 Adaptive Large Neighbourhood Search

Input: Instance Data, parameters
Output: best feasible solution s_{best}

1: construct initial solution s_{init}
2: $s_{best} := s_{init}$
3: $s_{curr} := s_{init}$
4: **do**
5:     select number of customers to be removed n_{rem}
6:     select removal operator *rem*
7:     select insertion operator *inst*
8:     determine next solution $s_{next} := inst(rem(s_{curr}, n_{curr}))$
9:     check acceptance of s_{next}
10:     **if** s_{next} is accepted **then**
11:         $s_{curr} := s_{next}$
12:         **if** $f(s_{curr}) < f(s_{best})$ **then**
13:             $s_{best} := s_{next}$
14:         **end if**
15:     **end if**
16:     **if** it_p reached **then**
17:         update selection probabilities for insertion and removal heuristics
18:     **end if**
19: **while** one stopping criterion is not met

5.1.1 Initial Solution

The initial solution s_{init} is constructed [1] by means of the Savings Heuristic developed by Clarke and Wright (1964), where one route is created for each customer first. Then, the savings for merging routes are calculated. Starting with the highest savings, all feasible merges are carried out while respecting the routing constraints. Based on this initial solution, the ALNS determines other feasible improved solutions.

5.1.2 Stopping Criteria

Starting with the initial solution received by the Savings Heuristic, the ALNS tries to improve the current solution as long as not one of the following stopping criteria is met [19]:

- number of total iterations $iter_{max}$;
- number of iterations without improvement $iter_{impr}$;
- computation time limit t_{max}.

5.1.3 Iteration

A new solution s_{next} is generated by choosing randomly one removal [6] and one insertion operator [7] in each iteration. The removal operator removes a number of customers n_{rem} from the current solution and the insertion operator reinserts them [8]. The number of customers to be removed n_{rem} ($n_{min} \leq n_{rem} \leq n_{max}$) is determined randomly. The generated solution is checked with respect to meeting the routing constraints. The packing procedure (see Section 5.2) is called here for each route of the solution.

5.1.4 Removal and Insertion Operators

The removal and insertion operators are described in detail in Koch et al. (2018). The following removal operators, shown in Table 3, are used in this approach. Three insertion operators are available, see Table 4.
The selection probabilities for the removal and insertion operators are adjusted according to the improvement of the solution after a defined number of iterations it_p [16-18].

5.1.5 Objective Function

A solution s is evaluated on the basis of an objective function f. It considers the number of used vehicles v_{used} as well as the total travel distance $ttd(s)$.

$$f(s) = ttd(s) + pen_v \cdot max(0, v_{used} - v_{max}) + pen_{miss} \cdot n_{miss}. \tag{16}$$

v_{max} is the number of maximal available vehicles and n_{miss} the number of customers that have not been dispatched yet. The purpose of the penalty terms pen_v and pen_{miss} is to achieve a reduction of used vehicles v_{used} and of customers that have not been dispatched yet n_{miss}.

Neighborhood Operators	Description
Shaw	Removes related customers w.r.t. distance, demand
Random	Removes random customers
Worst	Removes customers increasing the total routing costs the most
Cluster	Divides a random tour into two clusters and randomly removes one of the cluster
Neighbour graph	Removes customers increasing the average distance of a tour
Overlap	Removes customers leading to intersection of two tours
Inner Route	Removes a tour which is completely surrounded by another and splits the surrounding tour into two
Intersection	Removes customers leading to intersections within a tour
Tour Pair	Removes two intersecting tours

Table 3 Overview Removal Operators

Neighborhood Operators	Description
Greedy	Inserts customers iteratively so that an increase of routing costs is minimal
Regret-2	Inserts customers iteratively so that the maximal difference of routing costs for the best and the second best insertion in different tours is achieved
Regret-3	Inserts customers iteratively so that the sum of two differences of routing costs is maximal. The first difference is the routing cost for the best and the second best insertion in different tours, while the second difference results from the best and the third best insertion in different tours

Table 4 Overview Insertion Operators

5.1.6 Solution Acceptance

The smaller the objective function value, the better the solution. If the generated solution s_{next} is better then the current best-known s_{best} one, it is always accepted as current solution s_{curr}. A worse solution may be accepted depending on a Simulated Annealing Framework based on Kirkpatrick et al. (1983) [9]. The acceptance probability is adapted to the annealing process with a geometric cooling schedule. The best solution s_{best} is updated if its objective function value is higher than of the current solution s_{curr} [12-14].

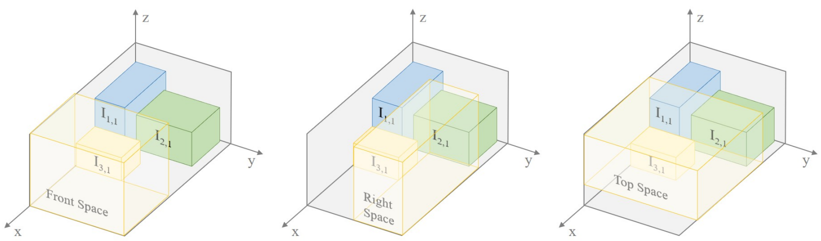
5.2 Packing Heuristic

The packing heuristic is called by the routing heuristic for each route of a solution. It is based on the DBLF algorithm proposed by Karabulut and İnceoğlu (2005). The basic concept is to place the items as far as possible to the back (first priority), to the bottom (second priority) and to the left (third priority) of the loading space. The available free spaces in the vehicle's loading space are saved in a list.

In the following, the point of origin of a Cartesian coordinate system is located in the deepest, bottom, leftmost point of the loading space. The driver's cab is located behind it accordingly. The length, width and height of the loading space are parallel to the x-, y- and z-axes. The placement of an item $I_{i,k}$ is defined by $(x_{i,k}, y_{i,k}, z_{i,k})$ of the corner, which is closest to the point of origin. In the first step of the packing heuristic, the items of each customer are stored in the set IS in a sorted order observing the following priorities:

1. fragility flag $f_{i,k}$ (non-fragile first)
2. volume (larger volume first)
3. length $l_{i,k}$ (longer first)
4. width $w_{i,k}$ (wider first).

The algorithm is shown in Alg. 2. The order of the items in the packing sequence IS are reversed to the customer's visiting order [1].
Let S be a set containing unique cuboids representing the available free spaces in the loading area after placing an item. Initially, this set consists of one space representing the total loading space [2]. Consequently, the first item of the packing sequence IS is placed in the origin. The potential spaces of the set S are always sorted based on the DBLF rule [10]. Thus, an item is placed in the deepest, bottom, leftmost point of a selected space. Each space *sp* of the set is tested as possible item position until a feasible position is found obeying all loading constraints [7]. In comparison to Karabulut and İnceoğlu (2005), the set S contains not all available spaces inside the loading space. Rather, three new spaces (Front, Right, Top), based on the feasible item placement, are created [9].



(a) Front Space (b) Right Space (c) Top Space

Fig. 8 New Spaces based on $I_{3,1}$

The *Front Space* is defined by the item's front edge (minimum x-value) and either the vehicle's door or the nearest item in front of the item (maximum x-value). Then, the minimum and maximum values for the y-axis are determined by the loading space or other items. After that, the minimum and maximum values for the z-axis are searched in the same way (see Fig. 8a).
The *Right Space* is bounded along the y-axis by the right side of the item (minimum y-value) and either by the vehicle wall or by the leftmost item

(maximum y-value). Based on these limitations, the minimum and maximum values for the x-axis are determined, then for the z-axis (see Fig. 8b).
The *Top Space* is defined by the item's top surface and either the vehicle's ceiling or an item overhanging over the current item I_p. In the next step, the minimum and maximum values for the y-axis and z-axis determined by the loading space or other items are searched (see Fig. 8c). The three new spaces (Front, Right, Top) are included in the set S considering the DBLF order.

Algorithm 2 Deepest-Bottom-Left-Fill with Spaces

Input: Instance Data
Output: Feasible placements for items

```
initialize sorted sequence of items IS
initialize set of unique available spaces S
for each item I_p ∈ IS do
    for each permitted orientation do
        for each space sp ∈ S do
            if item I_p fits in space sp then
                if placement is feasible w.r.t. loading constraints then
                    save placement for I_p
                    create new spaces
                    sort spaces based on DBLF
                    erase space sp
                    get smallest dimensions l_min and h_min of unplaced items ∈ IS
                    for each space si ∈ S do
                        update space si
                        if si too small w.r.t. l_min and h_min then
                            erase space si
                        end if
                    end for
                end if
            end if
        end for
    end for
    return false                                    ▷ No placement found
end for
return true
```

If a feasible position is found for item I_p, all remaining free spaces are checked w.r.t an intersection with item I_p. If there are one or more intersections with item I_p, then the minimum and maximum values for the x-, y- and z-axis are decreased so that no intersection with item I_p occurs any more [14]. Due to this procedure, it is guaranteed that the item does not overlap with other items or with the vehicle's walls (C1). Therefore, if an item can be placed within an available space according to the loading constraints C2-C8, an extra overlapping check for the Geometry constraint (C1) is not necessary. This is in contrast to the approach by Karabulut and İnceoğlu (2005), where an overlapping check between each item is performed.
The space sp, in which the item I_p is placed, is removed from the set [11]. Only spaces which are large and high enough for the smallest dimensions of

any item of all items of the route IS, are inserted in the set S. Therefore, the shortest length or width l_{min} and height h_{min} of any unplaced item of the route IS are searched [12]. Due to the permitted rotations, only the two measures l_{min} and h_{min} are relevant. If the length or height of any space in the set is smaller than l_{min} or h_{min}, the space is removed from the set [15-17]. If all spaces are checked and no feasible position for item I_p was found [23], the route is not feasible and is rejected. Thus, a new set of routes must be generated by the ALNS.

6 Computational Experiments

In this section, the impact of the Axle Weight Constraint on 2L-CVRP and 3L-CVRP instances is evaluated as well as the efficiency of the hybrid algorithm. This is done by testing instance sets from the literature with and without the Axle Weight Constraint (C8) and comparing the results with the benchmarks. The solution approach was coded in C++ as single-core application and compiled using the VC++ 2017 version, v14.16 compiler. The experiments were executed on one i5-7200U dual-core with 2.5 GHz and 8 GB RAM. The operating system is Windows 10.

6.1 Parameters

The ALNS for solving the routing problem and the loading constraints are parameterized as shown in Table 5. The parameters for the routing heuristic are adopted from Koch et al. (2018).

Parameter	Usage	Description	Value
$iter_{max}$	ALNS	Maximal number of iterations	25,000
$iter_{impr}$	ALNS	Maximal number of iterations without improvement	8,000
$iter_p$	ALNS	Number of iterations for updating probabilities for removal and insertion operators	100
t_{max}	ALNS	Time limit [min]	60
n_{min}	ALNS	Number of minimal customers to be removed from a route	$0.04n$
n_{max}	ALNS	Number of maximal customers to be removed from a route	$0.4n$
co_{max}	Objective Function	Maximal distance between two customers in instance	$max_{i,j \in N} co_{i,j}$
pen_v	Objective Function	Penalty term for each surplus vehicle	$10 \cdot co_{max}$
pen_{miss}	Objective Function	Penalty term for missing customers	$10 \cdot co_{max}$
α	Vertical Stability	Minimal support ratio	0.75

Table 5 Routing and Loading Parameters

6.2 Instances

We have tested our approach on three instance sets (see Table 6). The first instance set comes from Pollaris et al. (2016) and consists of 128 2L-CVRP instances, varying the number of customers in the network (10, 15, 20 and 25). The second instance set was created by Pollaris et al. (2017) and also deals with the 2L-CVRP. The set contains 96 instances with 50, 75 or 100 customers. The third instance set concerns the 3L-CVRP proposed by Gendreau et al. (2006). The number of customers is ranging between 15 and 100.

authors	**number of instances**	**problem**	**number of customers**
Pollaris et al. (2016)	128	2L-CVRP	[10, 15, 20, 25]
Pollaris et al. (2017)	96	2L-CVRP	[50, 75, 100]
Gendreau et al. (2006)	27	3L-CVRP	[15-100]

Table 6 Overview tested instances

For both 2L-CVRP instance sets, four different problem classes are created for each network by varying the number of items per customer (c_i) and the mass of demanded items (cm_i) (see Table 7). There is a low ($4 \leq c_i \leq 7$) and a high ($1 \leq c_i \leq 15$) variation in the number of items per customer i. The mass of demanded items per customer i is categorized in heavy pallets ($1,000 \leq \frac{mc_i}{c_i} \leq 1,500$) and light pallets ($100 \leq \frac{mc_i}{c_i} \leq 500$).

		composition of demand: heavy pallets	composition of demand: heavy and light pallets
demand variation:	low	class 1	class 3
	high	class 2	class 4

Table 7 Problem classes for Pollaris et al. (2016, 2017) Instances

As shown in Section 3, the 2L-CVRP considers the loading constraints Geometry (C1), Orthogonality (C2), Load Capacity (C3), LIFO (C4) and the Axle Weight Constraint (C8), whereas in the 3L-CVRP, the Rotation (C5), the Minimal Supporting Area (C6) and the Fragility (C7) are additionally taken into account.

There were no values for the axle weights assigned to the vehicles in the instances by Gendreau et al. (2006). Thus, the two-axle truck ML180E by the manufacturer IVECO was chosen. The proportion factor was calculated from the truck cargo load and the load capacity D of the instance. Then, the axle weights were proportionally scaled in order to receive a realistic proportion between vehicle load capacity and axle weights.

6.3 Results

In the following, we compare the results of our hybrid algorithm with instances from the literature. Moreover, we show the impact of the consideration of the Axle Weight Constraint on the objective values. Following the general convention of vehicle routing heuristics, for all instances, the minimization of the number of vehicles has the highest priority in our objective function and minimizing the total travel distance is of secondary importance. The consequence is that a smaller number of vehicles leads to more items per vehicle and thus, higher loads are applied to the axles. Consequently, solving the instances becomes more difficult.

6.3.1 Results for the Pollaris et al. (2016) Instances

In Pollaris et al. (2016), the 2L-CVRP with the Axle Weight Constraint is investigated. The pallets are alternately packed in two rows. Pollaris et al. (2016) use CPLEX for solving the problem with a time limit of 2 hours, within which not all instances could be solved. For the instances without a solution, our corresponding results are excluded from the difference calculation to enable fair comparison. The detailed results are in the Appendix (Tables 11-14). Table 8 shows the summed results for each network and each class.

Class	n	Without Axle Weight Constraint				With Axle Weight Constraint							Deviation due to		
		ALNS / DBLF				Pollaris et al. (2016)		ALNS / DBLF			diff. [%]		Axle Weight [%]		
		v_{used}	*ttd*	n viol.	time	v_{used}	*ttd*	v_{used}	*ttd*	time	v_{used}	*ttd*	v_{used}	*ttd*	time
1															
	10	24	342.42	10	110	40	354.30	26	356.01	73	-35.00	0.48	8.33	3.97	-33.61
	15	35	469.51	43	269	42	366.30	40	503.79	365	-28.57	2.03	14.29	7.30	35.45
	20	38	489.37	49	646	30	216.00	44	535.08	536	-43.33	8.44	15.79	9.34	-16.96
	25	54	674.63	68	990			63	750.44	1,502			16.67	11.24	51.75
		151	1,975.94	170	2,015	112	936.60	173	2,145.33	2,476	-34.82	2.92	14.57	8.57	22.90
2															
	10	32	396.98	25	92	40	418.90	37	417.98	84	-7.50	-0.22	15.63	5.29	-8.49
	15	42	431.63	36	263	42	394.80	46	467.63	312	-4.76	2.84	9.52	8.34	18.64
	20	68	773.29	67	473	50	485.40	80	873.12	825	0.00	9.42	17.65	12.91	74.30
	25	87	938.02	86	1,477			98	1,052.14	1,430			12.64	12.17	-3.15
		229	2,539.93	214	2,304	132	1,299.10	261	2,810.88	2,651	-3.79	4.31	13.97	10.67	15.03
3															
	10	24	330.16	0	81	40	333.10	24	330.16	23	-40.00	-0.88	0.00	0.00	-71.99
	15	33	437.97	3	271	56	441.00	33	438.68	82	-41.07	-0.53	0.00	0.16	-69.60
	20	46	554.87	4	517	70	488.00	47	557.32	188	-41.43	0.48	2.17	0.44	-63.66
	25	57	654.29	18	1,090			57	660.28	485			0.00	0.91	-55.53
		160	1,977.29	25	1,959	166	1,262.10	161	1,986.44	778	-40.96	-0.23	0.63	0.46	-60.31
4															
	10	34	410.39	14	138	40	413.70	36	422.09	41	-10.00	2.03	5.88	2.85	-70.19
	15	49	594.58	13	262	56	610.40	50	604.79	125	-10.71	-0.92	2.04	1.72	-52.41
	20	66	721.42	14	503	60	593.50	65	726.36	192	-11.67	-1.66	-1.52	0.69	-61.86
	25	86	875.39	39	1,046			89	903.68	441			3.49	3.23	-57.85
		235	2,601.78	80	1,948	156	1,617.60	240	2,656.92	798	-10.90	-0.44	2.13	2.12	-59.03
total		**775**	**9,094.94**	**489**	**8,227**	**566**	**5,115.40**	**835**	**9,599.56**	**6,703**	**-22.79**	**1.43**	**7.74**	**5.55**	**-18.52**

Table 8 Summarized Results for Pollaris et al. (2016) Instance Set

The objective values are given in columns v_{used} (number of used vehicles) and *ttd* (total travel distance). The runtime in seconds is displayed in column *time*. The column *n viol.* shows the number of customers causing an overload of the front or the rear axle. For the benchmark comparison, the results received by Pollaris et al. (2016) when obeying the Axle Weight Constraint are used. The relative differences between the benchmark results and the results received by

our hybrid algorithm including the Axle Weight Constraint are calculated in the 12th and 13th columns. Moreover, we compare the results obtained by our hybrid solution approach with and without the Axle Weight Constraint. The relative deviation of the objective values and of the runtime due to the additional Axle Weight Constraint is presented in the latter three columns.
The disregard of the axle weights leads to overloaded trucks in nearly every instance. Therefore, it is necessary to consider the Axle Weight Constraint. On average, the number of used vehicles increases by 7.74% and the total travel distance by 5.55%. The instances of class 2 are the most difficult to solve since the customers demand a large number of heavy pallets. Therefore, the consideration of the Axle Weight Constraint leads to an average increase of 13.97% of used vehicles and of 10.67% of the total travel distance. In addition, the runtime increases by 15.03% on average. The easiest class to solve is class 3. For these instances, the Axle Weight Constraint influences the results in a minor way, so that both objective values increase by less than one percent on average. However, the runtime declines rapidly by an average of 60%. The load on the axles can be calculated relatively quickly, so that infeasible positions for items are detected faster and the ALNS can terminate earlier.
The impact of the variation of the number of items on the objective values can be evaluated by comparing the results of classes 1 with 2 and of classes 3 with 4. A high variation leads to an increase of the objective values by around 2%. In addition, the runtime increases by on average of about 18%. The impact of heavy pallets on the objective values can be examined by comparing the classes 1 with 3 and classes 2 with 4. Approximately 13% more vehicles are needed and the total travel distance enlarges by around 8%. The runtime declines significantly by around 60%. Thus, the mass of items influences the Axle Weight Constraint more than the demand variation and has therefore a higher impact on the objective values.
In comparison to the results received by Pollaris et al. (2016), the number of used vehicles mostly decreases significantly and independently of the class (on average by -22.79%), while the total travel distance increases slightly (on average by 1.43%). For some instances, a reduction of both, the number of used vehicles and of the total travel distance, is achieved.
For instances with 25 customers, the approach of Pollaris et al. (2016) reaches its limits since no solution can be found within the 2 hours time limit. Moreover, two instances were not tested: For the instance with 15 customers, class 2, no. 2, we assume an error in the data, because several distances between customers are stated as zero in the distance matrix. The demanded mass per customer is not given in the instance with 20 customers, class 1, no. 2.

6.3.2 Results for the Pollaris et al. (2017) Instances

In Pollaris et al. (2017), the 2L-CVRP is solved by means of an Iterated Local Search approach with Sequence-Based Pallet Loading. In Table 9, the best results received by Pollaris et al. (2017) including the Axle Weight Constraints are compared with the results obtained by our hybrid algorithm. The detailed results are in the Appendix (Tables 15-17).

Class	n	Without Axle Weight Constraint ALNS / DBLF				With Axle Weight Constraint Pollaris et al. (2017)		ALNS / DBLF			diff. [%]		Deviation due to Axle Weight [%]		
		v_{used}	*ttd*	n viol.	time	v_{used}	*ttd*	v_{used}	*ttd*	time	v_{used}	*ttd*	v_{used}	*ttd*	time
1															
	50	109	1,185.04	184	7,307	200	1,215.40	124	1,325.22	14,075	-38.00	9.04	13.76	11.83	92.61
	75	157	1,643.92	393	22,084	296	1,684.70	184	1,849.18	28,800	-37.84	9.76	17.20	12.49	30.41
	100	211	2,156.07	575	28,800	400	2,237.80	251	2,532.63	28,800	-37.25	13.18	18.96	17.47	0.00
		477	4,985.03	1152	58,192	896	5,137.90	559	5,707.03	71,675	-37.61	11.08	17.19	14.48	23.17
2															
	50	151	1,478.26	252	7,535	200	1,513.00	176	1,695.14	11,329	-12.00	12.04	16.56	14.67	50.36
	75	231	2,332.85	457	23,414	296	2,387.50	268	2,660.31	27,720	-9.46	11.43	16.02	14.04	18.39
	100	309	2,951.03	630	28,800	400	3,002.20	362	3,450.69	28,800	-9.50	14.94	17.15	16.93	0.00
		691	6,762.14	1339	59,749	896	6,902.70	806	7,806.14	67,850	-10.04	13.09	16.64	15.44	13.56
3															
	50	109	1,172.11	40	6,945	200	1,176.00	109	1,174.46	3,495	-45.50	-0.13	0.00	0.20	-49.68
	75	160	1,663.91	155	23,846	296	1,677.90	160	1,670.58	13,930	-45.95	-0.44	0.00	0.40	-41.58
	100	213	2,101.50	278	28,800	400	2,137.40	214	2,109.16	27,458	-46.50	-1.32	0.47	0.36	-4.66
		482	4,937.52	473	59,591	896	4,991.30	483	4,954.20	44,883	-46.09	-0.74	0.21	0.34	-24.68
4															
	50	160	1,614.25	176	6,860	200	1,649.30	166	1,655.80	4,851	-17.00	0.39	3.75	2.57	-29.29
	75	247	2,426.34	288	24,536	296	2,457.40	252	2,482.09	18,625	-14.86	1.00	2.02	2.30	-24.09
	100	312	3,084.21	414	28,800	400	3,143.60	318	3,157.82	28,800	-20.50	0.45	1.92	2.39	0.00
		719	7,124.80	878	60,196	896	7,250.30	736	7,295.72	52,276	-17.86	0.63	2.36	2.40	-13.16
total		**2369**	**23,809.48**	**3842**	**237,728**	**3584**	**24,282.20**	**2584**	**25,763.09**	**236,684**	**-27.90**	**6.10**	**9.08**	**8.21**	**-0.44**

Table 9 Summarized Results for Pollaris et al. (2017) Instance Set

The number of available vehicles is not exceeded for any instance. For most instances, at least one customer leads to overloaded axles when disregarding the Axle Weight Constraint. Concerning the impact of the classes on the objective values when including the Axle Weight Constraint, findings similar to those above made for Pollaris et al. (2016) can be drawn. When considering the Axle Weight Constraint, the number of used vehicles increases on average by 9.08% and the total travel distance by 8.21%. On average, the runtime does not change significantly. For classes 1 to 3, the tendency is that the higher the number of customers, the more vehicles are needed and the longer the total travel distance. In comparison to the results received by Pollaris et al. (2017), the average number of used vehicles can be reduced by nearly 28%. However, the total travel distance increases by an average of 6%.

6.3.3 Results for the Gendreau et al. (2006) Instances

In Table 10, the results for the tested 3L-CVRP instances developed by Gendreau et al. (2006) are shown. Since this paper considers the Axle Weight Constraint for the first time in the 3L-CVRP, the comparison with the benchmark is based on the results without including this constraint.

The number of available vehicles is not exceeded for any instance. Regarding the number of customers causing an overload on either the front or the rear axle (column *n viol.*), then in 26 of 27 instances (96%), at least one of the axles is overloaded if the constraint is not included in the model. When considering the Axle Weight Constraint, the number of used vehicles remains the same for most instances (21 of 27), while the total travel distance rises for nearly every instance (25 of 27). Both objective values increase on average only slightly (around 2%). The runtime decreases or remains equal for most instances (22 of 27). On average, the runtime declines by 6.84%. For the comparison of the results without the Axle Weight Constraint, the results by Fuellerer et al. (2010) are used since their average total travel distance is the best in the literature. In this case, the number of used vehicles decreases by on average 7.17% while the total travel distance increases by on average 13.39%. The total travel distance rises for almost every instance.

7 Conclusion

In this paper, we introduced two approaches based on the Science of Statics for the consideration of axle weights of trucks with and without trailers. We showed with an example the necessity of checking the Axle Weight Constraint after each item placement. Computational experiments based on instances from the literature demonstrated that without the consideration of the Axle Weight Constraint, in almost every tour at least one customer would lead to an overload of the axles. In case of the 2L-CVRP results, when considering the Axle Weight Constraint, the number of used vehicles increased on average by 8.75% and the total travel distance by on average 7.47%. However, the runtime is reduced by on average 1%. The examination of the 2L-CVRP instances showed that the number of customers and the total travel distance increase significantly when the customers demand heavy pallets. When taking the Axle Weight Constraint into account for the 3L-CVRP, the deterioration of the objective values is small (approx. 2%). There are also positive effects on the runtime leading to a decrease of on average 6.84%. With regard to the negative consequences of an overloaded axle (increased road surface erosion and extended braking distance), the mostly small decline of the objective values, the shorter runtime and the easy implementation, we recommend the consideration of axle weights in future approaches for the Container Loading

Class	n	Without Axle Weight Constraint								With Axle Weight C.			Deviation due to Axle Weight [%]		
		Fuellerer et al. (2010)		ALNS / DBLF		diff. [%]		n		ALNS / DBLF					
		v_{used}	ttd	v_{used}	ttd	v_{used}	ttd	viol.	time	v_{used}	ttd	time	v_{used}	ttd	time
1	15	4	297.65	4	302.02	0.00	1.47	2	38.04	4	320.42	14.62	0.00	6.09	-61.57
2	15	5	334.96	5	334.96	0.00	0.00	2	1.51	5	336.52	2.13	0.00	0.46	41.45
3	20	4	362.27	4	400.76	0.00	10.63	6	57.68	4	429.98	48.85	0.00	7.29	-15.30
4	20	6	430.89	6	440.68	0.00	2.27	6	7.97	6	462.47	7.65	0.00	4.94	-4.04
5	21	6	406.50	5	437.41	-16.67	7.60	6	94.23	6	454.14	37.24	20.00	3.82	-60.48
6	21	6	495.85	6	498.32	0.00	0.50	6	15.18	6	512.64	8.46	0.00	2.87	-44.28
7	22	6	732.52	5	780.03	-16.67	6.49	4	140.17	5	781.42	167.33	0.00	0.18	19.38
8	22	6	735.14	6	845.34	0.00	14.99	0	154.51	6	845.34	105.95	0.00	0.00	-31.43
9	25	8	630.13	8	658.43	0.00	4.49	6	25.79	8	706.33	24.75	0.00	7.28	-4.05
10	29	8	711.45	7	828.43	-12.50	16.44	1	349.52	7	844.54	351.10	0.00	1.94	0.45
11	29	8	718.25	7	777.34	-12.50	8.23	1	836.43	7	818.54	250.22	0.00	5.30	-70.08
12	30	9	612.63	9	619.38	0.00	1.10	13	40.93	9	646.71	40.36	0.00	4.41	-1.40
13	32	8	2,391.77	7	2,721.25	-12.50	13.78	6	612.51	7	2,737.69	315.67	0.00	0.60	-48.46
14	32	9	1,222.17	7	1,419.70	-22.22	16.16	1	939.70	8	1,449.88	572.65	14.29	2.13	-39.06
15	32	9	1,182.86	7	1,366.39	-22.22	15.52	2	810.96	7	1,383.12	473.04	0.00	1.22	-41.67
16	35	11	698.61	11	708.65	0.00	1.44	9	24.59	11	719.24	31.05	0.00	1.49	26.26
17	40	14	862.18	14	866.40	0.00	0.49	21	39.26	14	887.34	34.87	0.00	2.42	-11.19
18	44	11	1,112.18	10	1,227.85	-9.09	10.40	2	1,895.53	10	1,258.17	2,263.30	0.00	2.47	19.40
19	50	12	671.60	10	753.45	-16.67	12.19	3	2,419.47	11	784.93	1,696.30	10.00	4.18	-29.89
20	71	18	515.39	15	594.84	-16.67	15.42	2	3,600.00	16	592.47	3,600.00	6.67	-0.40	0.00
21	75	17	951.87	16	1,127.35	-5.88	18.44	1	3,600.00	16	1,137.66	3,600.00	0.00	0.91	0.00
22	75	18	1,030.12	16	1,188.75	-11.11	15.40	2	3,600.00	17	1,208.05	3,600.00	6.25	1.62	0.00
23	75	17	971.05	16	1,158.27	-5.88	19.28	8	3,600.00	16	1,165.79	3,600.00	0.00	0.65	0.00
24	75	16	1,057.39	16	1,163.52	0.00	10.04	18	2,980.04	16	1,239.87	2,530.06	0.00	6.56	-15.10
25	100	22	1,207.97	21	1,551.05	-4.55	28.40	2	3,600.00	21	1,592.22	3,600.00	0.00	2.65	0.00
26	100	26	1,453.39	24	1,734.79	-7.69	19.36	1	3,600.00	25	1,797.88	3,600.00	4.17	3.64	0.00
27	100	23	1,333.16	23	1,722.47	0.00	29.20	10	3,600.00	23	1,734.40	3,600.00	0.00	0.69	0.00
total		**307**	**23,129.95**	**285**	**26,227.84**	**-7.17**	**13.39**	**141**	**36,684.02**	**291**	**26,847.76**	**34,175.59**	**2.11**	**2.36**	**-6.84**

Table 10 Results for Gendreau et al. (2006) Instances

Problems. For future work, we plan to integrate the Axle Weight Constraint directly in the packing algorithm to improve the selection process for the items' positions. In addition, items with high densities will be considered.

Conflict of interest

The authors declare that they have no conflict of interest.

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8 Appendix

Class	n	Without Axle Weight Constraint				With Axle Weight Constraint							Deviation due to		
		ALNS / DBLF				Pollaris et al. (2016)		ALNS / DBLF			diff. [%]		Axle Weight [%]		
		v_{used}	*ttd*	n viol.	time	v_{used}	*ttd*	v_{used}	*ttd*	time	v_{used}	*ttd*	v_{used}	*ttd*	time
1															
	1	3	38.43	1	2.72	5	45.20	3	40.07	8.34	-40.00	-11.36	0.00	4.26	206.66
	2	3	38.49	1	8.87	5	38.50	3	41.17	11.08	-40.00	6.94	0.00	6.97	24.97
	3	3	39.35	1	11.32	5	39.50	3	41.19	10.42	-40.00	4.27	0.00	4.67	-7.98
	4	3	41.85	3	15.41	5	45.70	4	44.53	7.10	-20.00	-2.56	33.33	6.40	-53.91
	5	3	51.71	2	7.35	5	51.70	3	54.00	9.99	-40.00	4.45	0.00	4.44	35.85
	6	3	43.42	0	14.92	5	44.20	3	43.42	8.69	-40.00	-1.76	0.00	0.00	-41.76
	7	3	45.22	1	35.04	5	45.20	4	45.90	8.71	-20.00	1.54	33.33	1.50	-75.13
	8	3	43.96	1	14.20	5	44.30	3	45.74	8.58	-40.00	3.26	0.00	4.06	-39.57
2															
	1	3	41.19	2	8.20	5	44.30	3	42.09	13.34	-40.00	-4.99	0.00	2.18	62.71
	2	4	44.72	5	14.25	5	51.30	5	48.07	8.76	0.00	-6.30	25.00	7.49	-38.52
	3	3	56.32	3	6.31	5	56.80	4	58.30	9.13	-20.00	2.63	33.33	3.51	44.74
	4	4	50.26	4	9.69	5	50.70	5	53.98	7.33	0.00	6.47	25.00	7.41	-24.30
	5	4	49.93	2	13.72	5	53.80	4	52.52	8.43	-20.00	-2.37	0.00	5.20	-38.56
	6	4	49.52	5	10.74	5	53.30	5	53.51	11.53	0.00	0.40	25.00	8.07	7.32
	7	7	64.57	2	20.35	5	68.20	7	64.57	16.10	40.00	-5.32	0.00	0.00	-20.88
	8	3	40.48	4	8.44	5	40.50	4	44.94	9.28	-20.00	10.96	33.33	11.02	10.01
3															
	1	3	37.45	0	8.81	5	37.40	3	37.45	2.01	-40.00	0.13	0.00	0.00	-77.23
	2	3	37.38	0	4.43	5	38.30	3	37.38	1.65	-40.00	-2.39	0.00	0.00	-62.76
	3	3	40.96	0	11.51	5	41.00	3	40.96	2.30	-40.00	-0.09	0.00	0.00	-80.02
	4	3	43.37	0	12.22	5	43.40	3	43.37	3.24	-40.00	-0.08	0.00	0.00	-73.52
	5	3	38.79	0	2.91	5	40.80	3	38.79	1.46	-40.00	-4.92	0.00	0.00	-49.86
	6	3	41.33	0	19.94	5	41.30	3	41.33	6.77	-40.00	0.08	0.00	0.00	-66.04
	7	3	44.38	0	12.93	5	44.40	3	44.38	2.83	-40.00	-0.04	0.00	0.00	-78.12
	8	3	46.48	0	8.56	5	46.50	3	46.48	2.53	-40.00	-0.03	0.00	0.00	-70.48
4															
	1	4	57.27	4	8.13	5	57.30	5	58.64	3.99	0.00	2.34	25.00	2.40	-50.96
	2	5	47.26	1	18.56	5	49.30	5	47.40	5.11	0.00	-3.86	0.00	0.28	-72.49
	3	3	46.90	0	16.38	5	46.90	3	46.90	3.74	-40.00	0.00	0.00	0.00	-77.19
	4	4	53.34	3	14.22	5	53.30	5	57.18	4.37	0.00	7.28	25.00	7.20	-69.30
	5	4	44.70	3	19.41	5	44.70	4	45.54	5.11	-20.00	1.88	0.00	1.87	-73.65
	6	3	50.21	0	21.55	5	52.20	3	50.21	4.59	-40.00	-3.81	0.00	0.00	-78.72
	7	6	57.15	0	24.85	5	59.90	6	57.15	7.83	20.00	-4.59	0.00	0.00	-68.49
	8	5	53.56	3	14.44	5	50.10	5	59.07	6.28	0.00	17.90	0.00	10.28	-56.55
total		**114**	**1,479.95**	**49**	**420.37**	**160**	**1,520.00**	**123**	**1,526.24**	**220.60**	**-23.13**	**0.41**	**7.89**	**3.13**	**-47.52**

Table 11 Pollaris et al. (2016) Instances with 10 customers

Class	n	Without Axle Weight Constraint ALNS / DBLF				With Axle Weight Constraint Pollaris et al. (2016)		With Axle Weight Constraint ALNS / DBLF			diff. [%]		Deviation due to Axle Weight [%]		
		v_{used}	*ttd*	n viol.	time	v_{used}	*ttd*	v_{used}	*ttd*	time	v_{used}	*ttd*	v_{used}	*ttd*	time
1															
	1	4	59.25	5	53.16			5	63.65	37.56			25.00	7.42	-29.34
	2	4	54.57	8	25.97			5	66.42	62.34			25.00	21.72	140.03
	3	5	62.15	6	40.35	7	62.10	5	67.18	50.73	-28.57	8.18	0.00	8.10	25.75
	4	4	54.11	6	33.38	7	55.10	5	58.35	60.30	-28.57	5.90	25.00	7.84	80.61
	5	4	56.63	6	12.81	7	58.40	5	62.43	47.85	-28.57	6.91	25.00	10.25	273.58
	6	5	64.08	6	40.42	7	68.50	5	65.34	40.90	-28.57	-4.62	0.00	1.97	1.19
	7	5	62.54	0	33.70	7	63.60	5	62.54	23.18	-28.57	-1.67	0.00	0.00	-31.22
	8	4	56.20	6	29.45	7	58.60	5	57.89	41.83	-28.57	-1.22	25.00	3.00	42.02
2															
	1	6	56.57	5	48.86	7	59.70	7	61.89	33.33	0.00	3.67	16.67	9.40	-31.78
	3	6	60.32	4	37.66	7	64.00	7	64.63	42.64	0.00	0.98	16.67	7.14	13.24
	4	5	58.70	9	43.26	7	58.70	6	68.79	31.24	-14.29	17.18	20.00	17.18	-27.78
	5	7	62.02	7	32.46	7	75.40	8	72.24	110.07	14.29	-4.19	14.29	16.48	239.06
	6	6	59.45	5	24.04			6	61.63	29.19			0.00	3.66	21.43
	7	6	68.77	2	46.87	7	68.80	6	72.12	31.85	-14.29	4.82	0.00	4.87	-32.04
	8	6	65.80	4	29.62	7	68.20	6	66.34	33.41	-14.29	-2.72	0.00	0.83	12.81
3															
	1	4	56.20	0	32.91	7	56.20	4	56.20	11.12	-42.86	0.00	0.00	0.00	-66.20
	2	4	51.52	0	16.54	7	51.50	4	51.52	7.98	-42.86	0.05	0.00	0.00	-51.73
	3	4	53.31	0	20.10	7	54.70	4	53.31	5.70	-42.86	-2.54	0.00	0.00	-71.65
	4	4	57.83	0	56.70	7	57.80	4	57.83	16.10	-42.86	0.05	0.00	0.00	-71.61
	5	4	55.62	3	44.46	7	56.70	4	56.33	15.86	-42.86	-0.65	0.00	1.28	-64.32
	6	5	58.00	0	34.31	7	58.00	5	58.00	7.66	-28.57	0.00	0.00	0.00	-77.67
	7	4	59.04	0	27.05	7	59.00	4	59.04	6.70	-42.86	0.06	0.00	0.00	-75.24
	8	4	46.44	0	39.13	7	47.10	4	46.44	11.32	-42.86	-1.40	0.00	0.00	-71.08
4															
	1	6	81.68	4	27.23	7	93.20	6	85.53	9.62	-14.29	-8.23	0.00	4.72	-64.69
	2	6	79.70	4	51.12	7	83.40	6	80.90	14.88	-14.29	-3.00	0.00	1.49	-70.90
	3	7	77.86	0	38.07	7	78.40	7	77.86	10.62	0.00	-0.69	0.00	0.00	-72.10
	4	7	87.99	5	36.33	7	88.00	8	93.15	45.73	14.29	5.85	14.29	5.86	25.89
	5	6	65.36	0	32.18	7	65.40	6	65.36	14.04	-14.29	-0.07	0.00	0.00	-56.37
	6	6	74.71	0	27.42	7	74.70	6	74.71	11.78	-14.29	0.01	0.00	0.00	-57.03
	7	6	63.94	0	25.20	7	64.00	6	63.94	9.90	-14.29	-0.09	0.00	0.00	-60.71
	8	5	63.34	0	24.37	7	63.30	5	63.34	8.07	-28.57	0.06	0.00	0.00	-66.89
total		**159**	**1,933.70**	**95**	**1,065.12**			**169**	**2,014.89**	**883.51**	**-21.94**	**0.59**	**6.29**	**4.20**	**-17.05**

Table 12 Pollaris et al. (2016) Instances with 15 customers

Class	n	Without Axle Weight Constraint				With Axle Weight Constraint							Deviation due to		
		ALNS / DBLF				Pollaris et al. (2016)		ALNS / DBLF			diff. [%]		Axle Weight [%]		
		v_{used}	*ttd*	n viol.	time	v_{used}	*ttd*	v_{used}	*ttd*	time	v_{used}	*ttd*	v_{used}	*ttd*	time
1															
	1	6	73.99	8	59.03	10	74.20	6	83.70	60.62	-40.00	12.80	0.00	13.12	2.68
	3	5	72.57	8	124.19			6	79.17	118.92			20.00	9.09	-4.25
	4	5	72.01	0	63.93	10	72.00	5	72.01	18.75	-50.00	0.01	0.00	0.00	-70.67
	5	6	70.76	7	262.85			7	72.89	53.66			16.67	3.02	-79.58
	6	5	61.75	10	40.57			7	72.14	109.28			40.00	16.82	169.40
	7	6	69.80	8	54.85	10	69.80	6	78.54	58.04	-40.00	12.52	0.00	12.51	5.82
	8	5	68.50	8	40.27			7	76.64	116.88			40.00	11.89	190.23
2															
	1	8	91.67	9	49.05			9	101.18	85.26			12.50	10.38	73.84
	2	8	87.77	8	50.40	10	87.80	9	94.14	65.23	-10.00	7.22	12.50	7.25	29.42
	3	9	113.04	7	63.33			11	133.23	183.57			22.22	17.87	189.88
	4	10	99.62	9	54.43	10	101.40	12	108.25	190.50	20.00	6.76	20.00	8.66	250.02
	5	9	94.44	7	43.39			10	107.60	66.37			11.11	13.93	52.97
	6	8	104.60	10	56.44	10	111.30	10	117.94	81.33	0.00	5.97	25.00	12.76	44.10
	7	9	90.80	9	62.34	10	93.50	10	106.10	99.88	0.00	13.47	11.11	16.84	60.21
	8	7	91.35	8	93.94	10	91.40	9	104.68	52.82	-10.00	14.53	28.57	14.60	-43.77
3															
	1	6	67.30	0	94.52	10	67.30	6	67.30	21.97	-40.00	0.01	0.00	0.00	-76.75
	2	5	68.52	4	35.54	10	68.50	6	70.97	27.29	-40.00	3.61	20.00	3.58	-23.22
	3	6	78.72	0	86.30	10	78.80	6	78.72	23.88	-40.00	-0.10	0.00	0.00	-72.33
	4	6	63.05	0	42.81	10	63.10	6	63.05	21.57	-40.00	-0.08	0.00	0.00	-49.61
	5	6	68.28	0	53.22	10	68.30	6	68.28	16.93	-40.00	-0.03	0.00	0.00	-68.20
	6	6	78.42	0	68.41	10	78.40	6	78.42	24.99	-40.00	0.02	0.00	0.00	-63.47
	7	5	63.62	0	57.02	10	63.60	5	63.62	20.49	-50.00	0.02	0.00	0.00	-64.07
	8	6	66.97	0	79.15			6	66.97	30.77			0.00	0.00	-61.12
4															
	1	7	80.88	7	71.20			6	82.28	35.29			-14.29	1.72	-50.44
	2	6	59.53	2	92.55			6	60.45	17.50			0.00	1.54	-81.09
	3	7	88.92	0	78.04	10	88.90	7	88.92	20.19	-30.00	0.02	0.00	0.00	-74.13
	4	10	86.61	5	72.39	10	89.30	10	89.24	30.21	0.00	-0.07	0.00	3.04	-58.27
	5	10	100.34	0	62.28	10	100.30	10	100.34	26.94	0.00	0.04	0.00	0.00	-56.74
	6	10	122.65	0	47.04	10	127.60	10	122.65	21.42	0.00	-3.88	0.00	0.00	-54.48
	7	7	92.65	0	31.46	10	97.50	7	92.65	15.66	-30.00	-4.97	0.00	0.00	-50.21
	8	9	89.84	0	47.92	10	89.90	9	89.84	24.59	-10.00	-0.06	0.00	0.00	-48.69
total		**218**	**2,538.95**	**134**	**2,138.84**			**236**	**2,691.89**	**1,740.79**	**-23.33**	**3.17**	**8.26**	**6.02**	**-18.61**

Table 13 Pollaris et al. (2016) Instances with 20 customers

Class	n	Without Axle Weight Constraint ALNS / DBLF				With Axle Weight Constraint ALNS / DBLF			Deviation due to Axle Weight [%]		
		v_{used}	*ttd*	n viol.	time	v_{used}	*ttd*	time	v_{used}	*ttd*	time
1											
	1	7	84.55	5	99.62	8	93.83	162.10	14.29	10.98	62.73
	2	7	78.23	10	82.37	8	86.63	232.68	14.29	10.75	182.49
	3	6	77.98	10	62.36	7	85.36	114.08	16.67	9.46	82.94
	4	7	90.03	9	84.17	8	102.75	247.34	14.29	14.13	193.85
	5	7	85.80	9	370.68	8	93.70	242.00	14.29	9.20	-34.72
	6	7	93.42	8	121.80	8	102.79	116.25	14.29	10.03	-4.55
	7	7	83.05	9	86.19	8	90.09	210.95	14.29	8.47	144.75
	8	6	81.57	8	82.91	8	95.29	177.03	33.33	16.83	113.52
2											
	1	12	122.55	14	295.81	13	142.48	296.91	8.33	16.26	0.37
	2	11	111.38	12	111.69	12	124.82	211.62	9.09	12.07	89.46
	3	9	90.00	4	313.79	10	98.22	104.77	11.11	9.14	-66.61
	4	14	168.54	14	286.34	15	174.82	210.49	7.14	3.72	-26.49
	5	10	114.44	11	102.73	12	123.14	113.16	20.00	7.60	10.16
	6	9	101.66	10	133.11	12	111.36	155.69	33.33	9.54	16.97
	7	12	120.18	10	104.90	12	131.54	116.29	0.00	9.45	10.86
	8	10	109.27	11	128.20	12	145.75	221.17	20.00	33.38	72.52
3											
	1	8	90.22	5	166.04	8	91.05	66.14	0.00	0.92	-60.17
	2	7	73.45	0	158.07	7	73.45	52.59	0.00	0.00	-66.73
	3	7	79.42	0	59.07	7	79.42	24.67	0.00	0.00	-58.24
	4	7	79.92	8	240.28	7	83.26	157.14	0.00	4.17	-34.60
	5	7	82.47	0	137.06	7	82.47	43.01	0.00	0.00	-68.62
	6	8	83.65	5	123.64	8	85.47	40.16	0.00	2.18	-67.52
	7	6	81.68	0	105.94	6	81.68	52.88	0.00	0.00	-50.08
	8	7	83.48	0	99.78	7	83.48	48.06	0.00	0.00	-51.84
4											
	1	11	125.07	4	113.61	12	130.37	69.52	9.09	4.24	-38.81
	2	8	85.68	0	159.38	8	85.68	75.52	0.00	0.00	-52.62
	3	11	120.76	7	85.78	12	130.54	44.69	9.09	8.10	-47.90
	4	11	115.85	7	119.36	11	119.34	38.68	0.00	3.02	-67.60
	5	10	104.86	3	97.37	11	106.09	33.47	10.00	1.18	-65.63
	6	11	99.64	4	120.40	11	101.02	58.19	0.00	1.38	-51.67
	7	12	120.78	5	265.60	12	125.00	68.54	0.00	3.49	-74.20
	8	12	102.74	9	84.34	12	105.63	52.19	0.00	2.81	-38.12
total		**284**	**3,142.33**	**211**	**4,602.36**	**307**	**3,366.54**	**3,857.98**	**8.10**	**7.13**	**-16.17**

Table 14 Pollaris et al. (2016) Instances with 25 customers

Class	n	Without Axle Weight Constraint ALNS / DBLF				With Axle Weight Constraint Pollaris et al. (2017)		ALNS / DBLF			diff. [%]		Deviation due to Axle Weight [%]		
		v_{used}	*ttd*	n viol.	time	v_{used}	*ttd*	v_{used}	*ttd*	time	v_{used}	*ttd*	v_{used}	*ttd*	time
1															
	1	14	152.22	26	900.03	25	154.10	16	169.18	1,284.42	-36.00	9.79	14.29	11.15	42.71
	2	13	133.82	20	1,423.20	25	143.30	15	148.30	2,436.57	-40.00	3.49	15.38	10.82	71.20
	3	14	145.42	21	803.63	25	147.70	15	159.53	1,196.99	-40.00	8.01	7.14	9.70	48.95
	4	13	150.79	23	1,329.26	25	152.10	15	168.41	1,321.98	-40.00	10.72	15.38	11.69	-0.55
	5	14	166.07	25	534.00	25	168.20	16	187.57	1,160.97	-36.00	11.51	14.29	12.95	117.41
	6	14	153.23	23	512.52	25	156.20	16	171.86	920.65	-36.00	10.02	14.29	12.16	79.63
	7	13	141.19	19	951.02	25	146.30	15	159.09	3,076.28	-40.00	8.74	15.38	12.68	223.47
	8	14	142.31	27	853.82	25	147.50	16	161.29	2,676.94	-36.00	9.35	14.29	13.34	213.52
2															
	1	17	170.32	30	758.90	25	177.50	21	197.63	712.29	-16.00	11.34	23.53	16.03	-6.14
	2	19	200.52	36	711.18	25	203.80	22	228.80	2,700.47	-12.00	12.27	15.79	14.10	279.72
	3	17	188.44	28	1,335.51	25	192.80	21	218.31	1,497.53	-16.00	13.23	23.53	15.85	12.13
	4	22	183.02	31	1,250.24	25	187.00	25	201.43	917.26	0.00	7.72	13.64	10.06	-26.63
	5	21	190.86	35	838.87	25	194.50	24	216.77	2,098.57	-4.00	11.45	14.29	13.57	150.17
	6	17	168.37	30	696.19	25	168.40	19	190.46	970.96	-24.00	13.10	11.76	13.12	39.47
	7	19	184.23	30	868.79	25	189.40	22	217.43	990.73	-12.00	14.80	15.79	18.02	14.04
	8	19	192.48	32	1,075.03	25	199.60	22	224.31	1,441.62	-12.00	12.38	15.79	16.54	34.10
3															
	1	14	143.91	8	786.31	25	144.30	14	144.20	338.59	-44.00	-0.07	0.00	0.20	-56.94
	2	14	156.92	0	857.83	25	157.30	14	156.92	326.53	-44.00	-0.24	0.00	0.00	-61.94
	3	13	143.84	7	702.37	25	144.30	13	145.07	376.57	-48.00	0.54	0.00	0.86	-46.39
	4	13	149.13	9	1,106.10	25	149.10	13	149.04	511.67	-48.00	-0.04	0.00	-0.06	-53.74
	5	14	158.35	0	1,026.38	25	159.80	14	158.35	587.57	-44.00	-0.91	0.00	0.00	-42.75
	6	13	143.30	16	630.19	25	144.30	13	144.21	283.03	-48.00	-0.06	0.00	0.64	-55.09
	7	14	146.71	0	725.26	25	146.60	14	146.71	565.68	-44.00	0.07	0.00	0.00	-22.00
	8	14	129.96	0	1,110.73	25	130.30	14	129.96	505.34	-44.00	-0.26	0.00	0.00	-54.50
4															
	1	18	181.36	17	754.39	25	184.60	19	187.38	775.66	-24.00	1.51	5.56	3.32	2.82
	2	19	209.98	26	728.62	25	210.70	21	217.78	370.82	-16.00	3.36	10.53	3.71	-49.11
	3	19	197.32	23	702.10	25	199.00	19	203.64	1,128.27	-24.00	2.33	0.00	3.20	60.70
	4	20	193.66	16	866.07	25	199.50	21	195.90	381.44	-16.00	-1.81	5.00	1.15	-55.96
	5	21	198.10	19	745.34	25	205.50	22	203.35	550.30	-12.00	-1.05	4.76	2.65	-26.17
	6	19	212.45	22	1,315.05	25	212.80	19	214.56	619.88	-24.00	0.83	0.00	0.99	-52.86
	7	20	199.41	23	670.95	25	202.90	21	202.57	611.00	-16.00	-0.16	5.00	1.58	-8.93
	8	24	221.97	30	1,077.68	25	234.30	24	230.63	413.26	-4.00	-1.57	0.00	3.90	-61.65
total		**529**	**5,449.66**	**652**	**28,647.57**	**800**	**5,553.70**	**575**	**5,850.62**	**33,749.85**	**-28.13**	**5.35**	**8.70**	**7.36**	**17.81**

Table 15 Pollaris et al. (2017) Instances with 50 customers

Class	n	Without Axle Weight Constraint ALNS / DBLF				With Axle Weight Constraint Pollaris et al. (2017)		ALNS / DBLF			diff. [%]		Deviation due to Axle Weight [%]		
		v_{used}	*ttd*	n viol.	time	v_{used}	*ttd*	v_{used}	*ttd*	time	v_{used}	*ttd*	v_{used}	*ttd*	time
1															
	1	19	201.08	47	2,659.39	37	203.60	22	224.22	3,600.00	-40.54	10.13	15.79	11.51	35.37
	2	19	206.18	52	2,305.14	37	213.60	23	234.22	3,600.00	-37.84	9.65	21.05	13.60	56.17
	3	20	207.82	48	1,895.56	37	213.30	23	235.11	3,600.00	-37.84	10.23	15.00	13.13	89.92
	4	20	211.69	51	3,362.08	37	217.50	23	233.39	3,600.00	-37.84	7.30	15.00	10.25	7.08
	5	20	198.89	47	2,472.58	37	201.80	23	223.49	3,600.00	-37.84	10.75	15.00	12.37	45.60
	6	20	198.68	49	3,068.29	37	202.80	24	221.20	3,600.00	-35.14	9.07	20.00	11.34	17.33
	7	20	213.19	48	3,600.00	37	219.70	23	241.66	3,600.00	-37.84	10.00	15.00	13.36	0.00
	8	19	206.40	51	2,721.14	37	212.40	23	235.88	3,600.00	-37.84	11.06	21.05	14.28	32.30
2															
	1	27	273.75	62	1,937.59	37	283.50	32	314.95	3,600.00	-13.51	11.09	18.52	15.05	85.80
	2	28	260.82	54	2,224.36	37	262.80	32	294.61	3,600.00	-13.51	12.11	14.29	12.96	61.84
	3	27	297.58	56	3,600.00	37	308.20	32	345.84	3,600.00	-13.51	12.21	18.52	16.22	0.00
	4	28	274.50	58	3,600.00	37	282.50	33	315.68	2,520.27	-10.81	11.75	17.86	15.00	-29.99
	5	33	323.72	56	2,008.49	37	329.20	36	356.23	3,600.00	-2.70	8.21	9.09	10.04	79.24
	6	24	261.90	52	3,600.00	37	264.30	29	298.28	3,600.00	-21.62	12.86	20.83	13.89	0.00
	7	31	314.82	58	3,600.00	37	323.70	37	367.31	3,600.00	0.00	13.47	19.35	16.67	0.00
	8	33	325.75	61	2,843.68	37	333.30	37	367.40	3,600.00	0.00	10.23	12.12	12.79	26.60
3															
	1	20	220.62	21	3,600.00	37	223.70	20	222.34	1,707.66	-45.95	-0.61	0.00	0.78	-52.57
	2	20	194.50	24	2,808.36	37	196.00	20	194.95	1,804.43	-45.95	-0.53	0.00	0.23	-35.75
	3	20	201.41	20	2,834.70	37	203.30	20	201.78	1,483.98	-45.95	-0.75	0.00	0.18	-47.65
	4	20	203.65	19	2,664.04	37	204.80	20	204.79	1,822.16	-45.95	0.00	0.00	0.56	-31.60
	5	20	204.28	27	3,197.15	37	205.60	20	205.46	2,631.51	-45.95	-0.07	0.00	0.58	-17.69
	6	20	228.44	0	2,985.64	37	230.00	20	228.44	1,371.91	-45.95	-0.68	0.00	0.00	-54.05
	7	20	201.57	20	2,694.81	37	202.00	20	202.13	1,962.38	-45.95	0.06	0.00	0.27	-27.18
	8	20	209.44	24	3,061.13	37	212.50	20	210.69	1,146.38	-45.95	-0.85	0.00	0.59	-62.55
4															
	1	33	337.19	42	2,905.66	37	340.60	34	339.63	1,614.66	-8.11	-0.28	3.03	0.72	-44.43
	2	33	326.67	38	3,184.82	37	331.90	33	334.17	1,514.44	-10.81	0.68	0.00	2.30	-52.45
	3	35	327.00	41	2,056.90	37	332.60	36	342.22	2,929.20	-2.70	2.89	2.86	4.65	42.41
	4	29	322.13	43	3,600.00	37	326.80	30	335.52	3,600.00	-18.92	2.67	3.45	4.16	0.00
	5	27	270.52	26	3,600.00	37	271.70	28	274.33	1,864.23	-24.32	0.97	3.70	1.41	-48.22
	6	30	286.42	34	3,172.32	37	287.80	30	288.19	2,528.07	-18.92	0.14	0.00	0.62	-20.31
	7	32	287.81	37	2,416.22	37	295.70	32	294.27	1,242.38	-13.51	-0.48	0.00	2.24	-48.58
	8	28	268.60	27	3,600.00	37	270.30	29	273.76	3,332.32	-21.62	1.28	3.57	1.92	-7.44
total		**795**	**8,067.02**	**1293**	**93,880.05**	**1,184**	**8,207.50**	**864**	**8,662.16**	**89,075.98**	**-27.03**	**5.54**	**8.68**	**7.38**	**-5.12**

Table 16 Pollaris et al. (2017) Instances with 75 customers

Class	n	Without Axle Weight Constraint ALNS / DBLF				With Axle Weight Constraint Pollaris et al. (2017)		ALNS / DBLF			diff. [%]		Deviation due to Axle Weight [%]		
		v_{used}	*ttd*	n viol.	time	v_{used}	*ttd*	v_{used}	*ttd*	time	v_{used}	*ttd*	v_{used}	*ttd*	time
1															
	1	27	271.07	69	3600,00	50	278.00	31	316.70	3600,00	-38.00	13.92	14.81	16.83	0.00
	2	27	279.35	75	3600,00	50	289.10	30	328.67	3600,00	-40.00	13.69	11.11	17.66	0.00
	3	25	253.79	69	3600,00	50	265.00	31	302.12	3600,00	-38.00	14.01	24.00	19.04	0.00
	4	27	273.13	74	3600,00	50	285.80	33	319.86	3600,00	-34.00	11.92	22.22	17.11	0.00
	5	26	259.66	64	3600,00	50	266.40	31	301.61	3600,00	-38.00	13.22	19.23	16.16	0.00
	6	26	273.67	74	3600,00	50	285.40	32	320.48	3600,00	-36.00	12.29	23.08	17.11	0.00
	7	26	274.16	77	3600,00	50	287.60	31	330.04	3600,00	-38.00	14.76	19.23	20.38	0.00
	8	27	271.25	73	3600,00	50	280.50	32	313.16	3600,00	-36.00	11.64	18.52	15.45	0.00
2															
	1	40	387.77	87	3600,00	50	397.30	48	454.29	3600,00	-4.00	14.35	20.00	17.16	0.00
	2	37	361.77	74	3600,00	50	365.70	43	427.52	3600,00	-14.00	16.91	16.22	18.17	0.00
	3	39	355.38	79	3600,00	50	363.90	44	410.13	3600,00	-12.00	12.70	12.82	15.41	0.00
	4	40	382.35	79	3600,00	50	388.90	48	453.43	3600,00	-4.00	16.59	20.00	18.59	0.00
	5	39	364.70	79	3600,00	50	370.80	46	416.47	3600,00	-8.00	12.32	17.95	14.20	0.00
	6	38	331.54	77	3600,00	50	337.70	45	389.21	3600,00	-10.00	15.25	18.42	17.39	0.00
	7	38	382.83	75	3600,00	50	390.30	45	447.41	3600,00	-10.00	14.63	18.42	16.87	0.00
	8	38	384.70	80	3600,00	50	387.60	43	452.21	3600,00	-14.00	16.67	13.16	17.55	0.00
3															
	1	26	246.67	36	3600,00	50	253.50	26	249.12	2,860.27	-48.00	-1.73	0.00	0.99	-20.55
	2	27	265.77	34	3600,00	50	270.30	27	266.24	2,997.81	-46.00	-1.50	0.00	0.17	-16.73
	3	27	272.10	38	3600,00	50	277.20	28	271.59	3600,00	-44.00	-2.02	3.70	-0.19	0.00
	4	26	251.54	39	3600,00	50	259.20	26	252.71	3600,00	-48.00	-2.50	0.00	0.47	0.00
	5	28	264.60	41	3600,00	50	266.70	28	266.66	3600,00	-44.00	-0.01	0.00	0.78	0.00
	6	27	276.92	43	3600,00	50	281.10	27	277.47	3600,00	-46.00	-1.29	0.00	0.20	0.00
	7	26	258.56	47	3600,00	50	263.10	26	260.05	3600,00	-48.00	-1.16	0.00	0.58	0.00
	8	26	265.34	0	3600,00	50	266.30	26	265.34	3600,00	-48.00	-0.36	0.00	0.00	0.00
4															
	1	38	393.15	54	3600,00	50	398.30	39	409.93	3600,00	-22.00	2.92	2.63	4.27	0.00
	2	40	414.18	48	3600,00	50	421.50	41	416.37	3600,00	-18.00	-1.22	2.50	0.53	0.00
	3	44	429.40	56	3600,00	50	437.00	45	442.23	3600,00	-10.00	1.20	2.27	2.99	0.00
	4	37	358.42	48	3600,00	50	363.80	38	358.82	3600,00	-24.00	-1.37	2.70	0.11	0.00
	5	38	354.13	55	3600,00	50	361.70	38	362.57	3600,00	-24.00	0.24	0.00	2.39	0.00
	6	38	365.60	58	3600,00	50	375.40	38	376.70	3600,00	-24.00	0.35	0.00	3.03	0.00
	7	39	393.40	48	3600,00	50	401.20	40	408.53	3600,00	-20.00	1.83	2.56	3.85	0.00
	8	38	375.94	47	3600,00	50	384.70	39	382.67	3600,00	-22.00	-0.53	2.63	1.79	0.00
total		**1,045**	**10,292.80**	**1,897**	**115,200.00**	**1,600**	**10,521.00**	**1,145**	**11,250.31**	**113,858.08**	**-28.44**	**6.93**	**9.57**	**9.30**	**-1.16**

Table 17 Pollaris et al. (2017) Instances with 100 customers

Otto von Guericke University Magdeburg
Faculty of Economics and Management
P.O. Box 4120 | 39016 Magdeburg | Germany

Tel.: +49 (0) 3 91 / 67-1 85 84
Fax: +49 (0) 3 91 / 67-1 21 20

www.fww.ovgu.de/femm

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