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A new Silver-Meal based heuristic for the single-item dynamic lot sizing problem with returns and remanufacturing

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Abstract

In a recent contribution, Teunter *et al.* [2006. Dynamic lot sizing with product returns and remanufacturing. IJPR 44 (20), 4377-4400] adapted three well-known heuristic approaches for the single-item dynamic lot sizing problem to incorporate returning products that can be remanufactured. The Silver-Meal based approach revealed in a large numerical study the best performance for the separate setup cost setting, i.e. the replenishment options remanufacturing and manufacturing are charged separately for each order. This contribution generalizes the Silver-Meal based heuristic by applying methods elaborated for the corresponding static problem and attaching two simple improvement steps. By doing this, the percentage gap to the optimal solution which has been used as a performance measure has been reduced to less than half of its initial value in almost all settings examined.

1 Introduction

Due to the increasing environmental awareness of firms and the public the research field of reverse logistics has grown steadily over the past decades. By analyzing not only the forward flow of products from a firm to its customers but also including the corresponding backward flow from the customers to the firm this research area provides valuable insights on how these flows can be managed efficiently. Among many options (see, e.g., Thierry *et al.*, 1995, for an overview on different alternatives), remanufacturing has been well established in several industries as has been reported in Kumar and Putnam (2008). When including remanufactured products in their product portfolio firms take back products from their customers, rework them to a sufficient condition in order to resell them afterwards. This saves not only a part of the value embedded in the original product but also reduces the demand for natural resources and landfill space substantially (de Brito and Dekker, 2004). In industry, the process of remanufacturing is affected by many stochastic influences as has been depicted,

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for instance, by Guide (2000) as well as Inderfurth and Langella (2006). Yet, this contribution neglects any uncertainty and presents an entirely deterministic system.

By assuming setup costs for replenishment orders and holding costs for carrying products in different conditions in stock a lot sizing problem arises. Such problems have been analyzed thoroughly for the case of static and continuous demand and return rates (see e.g. Minner and Lindner (2004) as well as Schulz and Ferretti (2008) for a brief literature review). However, the case of dynamic and discrete demands and returns has not achieved that much attention in the recent literature. Teunter *et al.* (2006) introduce a dynamic lot sizing model with returns and remanufacturing and distinguish between the case of joint and seperate setup cost for the replenishment sources remanufacturing and manufacturing. They test in a large numerical experiment three well-known heuristic approaches that were adapted from the single-item dynamic lot sizing problem without returns. In both model settings, the Silver-Meal based heuristic has been shown to be the best heuristic resulting in an average deviation of 3% from the optimal solution in the joint and 8.3% in the seperate setup cost setting. Using heuristics to handle these problems has been motivated by the fact that the authors conjecture the underlying problem of the seperate setup cost setting to be NP-hard.

Several other contributions have been made to this specific research field whereas only two shall be mentioned exemplarily. Richter and Sombrutzki (2000) discuss the dynamic lot sizing problem with returns and remanufacturing and analyze a situation in which a sufficiently large number of returned products is available. They proof that the zero-inventory property known from the dynamic lot sizing problem without returns and remanufacturing must hold in such an environment. Furthermore, they apply a Silver-Meal based algorithm to illustrate the stability of its solution. Pan *et al.* (2009) extend the analysis of Teunter *et al.* (2006) by including a disposal option for returned products and by restricting production, remanufacturing, and disposal capacities. They illustrate different problem formulations and elaborate dynamic programming algorithms to solve some of these problems to optimality.

This work proposes a generalization of the Silver-Meal based heuristic introduced by Teunter *et al.* (2006) for the separate setup cost setting by applying methods known from the corresponding static problem. Furthermore, a simple improvement heuristic is applied to the solution obtained to enhance the heuristic's performance. The remainder of this contribution is organized as follows. Section 2 presents the basic assumptions of the model analyzed in this work and describes some solution methods for the underlying problem context. Next to a mixed-integer linear program the Silver-Meal based heuristic introduced in Teunter *et al.* (2006) and our extension are depicted in this chapter. Both heuristics are tested extensively in a numerical study in the subsequent Section 3. Afterwards, Section 4 points out the improvement heuristic and tests its ideas in a numerical experiment. Finally, the last section concludes this contribution and gives a short outlook on future research opportunities.

2 Model formulation and proposed solution methods

2.1 Basic assumptions and mixed-integer linear program

In their contribution, Teunter *et al.* (2006) introduced a dynamic lot sizing model with separate setup costs for remanufacturing and manufacturing as an extension of the well-known Wagner/Whitin model (Wagner and Whitin, 1958). The basic assumptions of this modeling approach can be outlined as follows. As depicted in Figure 1, we consider an original equipment manufacturer (OEM) that sells one product to his customers over a planning horizon of T periods. In each period $t = 1, .., T$ his customers demand a discrete and known amount of this product which will be further on denoted by D_t. The OEM provides each customer the opportunity to return her product if it is broken or when she has no further use for it. Whenever a product is returned to the OEM it is inspected whether it can be sufficiently remanufactured. All returns that pass the inspection (which will be denoted by R_t) are brought to a recoverables stock. Per unit time a recoverable product incurs holding costs of h^R and disposing it of preliminarily is assumed to be prohibitively expensive. If required, the OEM can (by paying the setup cost K^R) remanufacture x_t^R recoverable products in period t in order to bring them to an as-good-as-new condition. It shall be mentioned, that the recovery itself is always successful. After remanufacturing, the recovered products are brought to a serviceables inventory from which the customer demand is satisfied. Yet, as it is not possible to serve the entire demand from remanufacturing returned products the OEM can replenish his serviceables inventory alternatively by manufacturing x_t^M products in period t. Setting up a manufacturing lot in period t incurs fixed costs of K^M while holding a serviceable product for one period in the respective inventory costs h^M. Finally, the inventory level at the end of period t is denoted by y_t^R for the recoverables and y_t^M for the serviceables inventory.

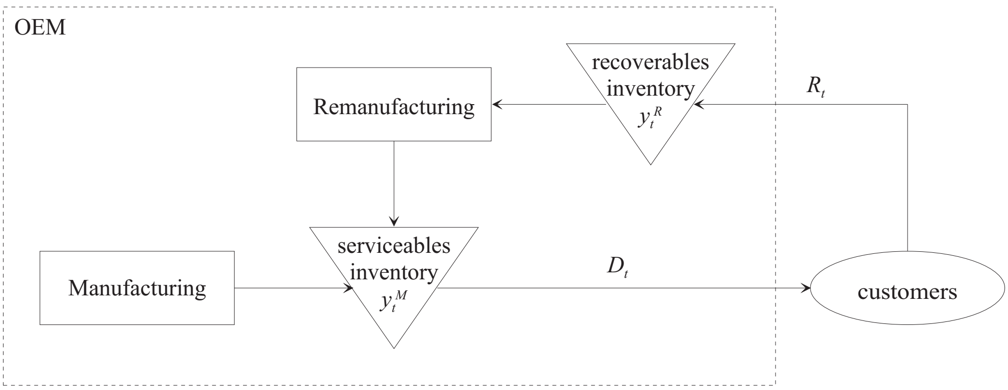

Figure 1: Dynamic lot sizing model with returns and remanufacturing

By means of mixed-integer linear programming this model can be solved to optimality. Next to the notation introduced above two more decision variables are required. If a remanufacturing lot is initiated in period t (i.e. $x_t^R > 0$) the binary decision variable γ_t^R becomes one. However, if $x_t^R = 0$ the decision variable γ_t^R remains zero. Likewise,

γ_t^M is set to one when a manufacturing lot is produced in period t and to zero if no product needs to be manufactured. The optimization model can be formulated as:

$$\min Z = \sum_{t=1}^{T} \left(K^R \cdot \gamma_t^R + K^M \cdot \gamma_t^M + h^R \cdot y_t^R + h^M \cdot y_t^M \right) \tag{1}$$

s.t.:

$$y_t^R = y_{t-1}^R + R_t - x_t^R \qquad \forall t = 1,..,T \tag{2}$$

$$y_t^M = y_{t-1}^M + x_t^R + x_t^M - D_t \qquad \forall t = 1,..,T \tag{3}$$

$$x_t^R \leq Q \cdot \gamma_t^R \qquad \forall t = 1,..,T \tag{4}$$

$$x_t^M \leq Q \cdot \gamma_t^M \qquad \forall t = 1,..,T \tag{5}$$

$$x_0^R = x_0^M = 0 \tag{6}$$

$$\gamma_t^R, \gamma_t^M \in \{0,1\} \qquad \forall i = 1,..,T$$

$$y_t^R, y_t^M, x_t^R, x_t^M \geq 0 \qquad \forall i = 1,..,T$$

The objective function (1) minimizes the sum of all relevant setup and holding costs. Constraints (2) and (3) represent inventory balance equations that describe the inventory at the end of period t as the inventory at the beginning of this period plus its inflows and minus its outflows. In order to ensure that fixed costs have to be paid whenever a lot is scheduled restrictions (4) and (5) have to be established whereas Q needs to be a sufficiently large number (e.g. the sum of all demands during the planning horizon). By imposing constraint (6) the initial inventories in both stocks are set to zero. Finally, non-negativity and binary constraints have to be defined as well to assure validity of the decisions made. Interestingly, the zero-inventory property that holds for a dynamic lot sizing model without returns and remanufacturing needs not necessarily to be valid in this model setting (as has been discussed by Teunter *et al.*, 2006), i.e. it can be optimal to schedule a (re)manufacturing lot in period t even when the serviceables inventory at the beginning of t is not depleted. This extends the results of Richter and Sombrutzki (2000) who proved the zero-inventory property to hold when there is a sufficiently large number of returned products in the recoverables stock at the beginning of the planning horizon. Moreover, Teunter *et al.* (2006) conjecture that the underlying optimization problem is NP-hard, i.e. it becomes very difficult to obtain the optimal solution for a long planning horizon. Hence, they propose several heuristic algorithms on how to handle this problem. In a large numerical study, the Silver-Meal based heuristic which will be introduced subsequently revealed the best average performance when compared to the optimal solution.

2.2 The adapted Silver-Meal heuristic by Teunter *et al.* (2006)

Unfortunately, the original Silver-Meal heuristic (Silver and Meal, 1973) cannot be applied to the model context presented above as the serviceables inventory from which all customer demands are satisfied can be replenished from two sources: manufacturing and remanufacturing. Thus, Teunter *et al.* (2006) adapted the original Silver-Meal heuristic to include both sources in form of manufacturing (option 1) as well as remanufacturing and manufacturing (option 2) in the decision-making process. The basic idea of clustering the entire planning horizon into smaller time windows (starting in period τ and ending in period z) and choosing those time windows with the smallest cost per period is kept. However, both options that will be described subsequently assume the zero-inventory property to hold.

Option 1: Manufacture only

When applying this option the entire demand in a time window is satisfied by initiating a manufacturing run in period τ. Its lot size would be

$$x_\tau^M = \sum_{i=\tau}^{z} D_i. \tag{7}$$

The associated cost per period for the entire time window (which will be denoted as $C_{\tau,z}^1$) contains the setup cost for scheduling a manufacturing lot in τ as well as the cost for carrying products in the serviceables inventory. Furthermore, the cost for holding the recoverable products in stock need to be taken into account as well. This gives for the cost per period for option 1 by using equation (2) for determining y_t^R and equation (3) for y_t^M

$$C_{\tau,z}^1 = \frac{K^M + h^M \cdot \sum_{t=\tau}^{z} y_t^M + h^R \cdot \sum_{t=\tau}^{z} y_t^R}{z - \tau + 1}. \tag{8}$$

Option 2: Remanufacture (and manufacture if necessary)

The second option introduced by Teunter *et al.* (2006) seeks to remanufacture in period τ. However, as the amount of recoverable products might not be sufficient to cover the entire demand up to period z a manufacturing lot is set up in τ if necessary. Thus, both lot sizes depend on the available number of recoverable parts which is by definition $y_{\tau-1}^R + R_\tau$. Both lot sizes are presented in the following formulae:

$$x_\tau^M = \max\left(\sum_{t=\tau}^{z} D_t - y_{\tau-1}^R - R_\tau, 0\right), \qquad x_\tau^R = \min\left(y_{\tau-1}^R + R_\tau, \sum_{t=\tau}^{z} D_t\right). \tag{9}$$

Forcing the possibly required manufacturing lot to be scheduled in τ can result in an inefficient solution when there is no immediate demand for at least one of the manufactured products. Hence, all products manufactured in τ will be held in the serviceables inventory unnecessarily until they are needed. An opportunity to overcome this deficiency will be presented later in this chapter.

Next to the holding cost for the recoverables and serviceables stock, the cost per period for the second option $C_{\tau,z}^2$ can contain both setup costs. As a manufacturing lot is only needed when the number of recoverable parts is not sufficient, the binary variable γ_τ^M represents this fact by being one if a manufacturing run is required and zero else. Therefore, the cost per period for the second option can be formulated as

$$C_{\tau,z}^2 = \frac{K^R + K^M \cdot \gamma_\tau^M + h^M \cdot \sum_{t=\tau}^{z} y_t^M + h^R \cdot \sum_{t=\tau}^{z} y_t^R}{z - \tau + 1}. \tag{10}$$

For each time window, $C^1_{\tau,z}$ is compared to $C^2_{\tau,z}$ and the smaller one is chosen. Moreover, the basic idea of the Silver-Meal heuristic is applied which means that a time window is extended as long as the smaller cost of both options does not increase. Further on, the heuristic approach introduced by Teunter *et al.* (2006) will be referred to as the SM_2 heuristic since two distinct options are evaluated. Teunter *et al.* (2006) have tested this heuristic extensively in their contribution. As a result of their numerical study a mean deviation to the optimal solution of 8.3% was observed over all instances. By generalizing the SM_2 heuristic with two additional options derived from the results of the corresponding static model we will enhance the heuristic's performance. This approach (which will be further on denoted as the SM_4 heuristic) is presented in the following.

2.3 The SM_4 heuristic

Although the dynamic lot sizing model with returns and remanufacturing has not been analyzed extensively in the literature so far, the corresponding static model (with constant demand and return rates) has received much more attention. Among many contributions, two shall be mentioned explicitly. In his work, Schrady (1967) was the first author who examined this model context. His option on how to handle this problem effectively was to create a cyclic pattern that is repeated over the entire infinite planning horizon. This cyclic pattern begins with a manufacturing lot and is always followed by a constant number of remanufacturing lots R. Teunter (2001) generalizes these findings by introducing cycles that commence with one remanufacturing lot which is always succeeded by a constant number of manufacturing lots M. He argues as well that in order to be efficient each cycle should have either one remanufacturing or one manufacturing lot. As this provides very good solutions to the static problem both cyclic patterns can be incorporated into the dynamic lot sizing model with returns and remanufacturing which will be presented subsequently. While the third option analyzes time windows with a manufacturing lot in τ that is followed by remanufacturing lots in later periods, a time window in the fourth option commences with a remanufacturing lot in τ that is succeeded by a number of manufacturing lots. Two promising effects can be observed when applying both additional options. At first, by considering more than one lot in each time window the recoverables inventory which is a critical cost factor can be controlled more accurately. Furthermore, contrary to the first two options the zero-inventory property is only presumed to hold for the first period of a time window but not within each time window any more. Hence, a (re)manufacturing lot can be scheduled although the initial serviceables inventory of the period under consideration is not zero.

Option 3: Manufacture first, remanufacture (in multiple lots) later

When applying this option, a manufacturing lot is scheduled in τ that is followed by one or more remanufacturing lots in the consecutive periods $\tau + 1$ to z. As the amount of products available in the recoverables stock needs not to be sufficient the manufacturing lot in period τ must replenish the unavailable products. The number of unavailable products in each period t ranging from $\tau + 1$ to z (which will be referred to as the net requirement NR_t) can be determined as

$$NR_t = \sum_{i=\tau}^{t} (D_i - R_i) - y_{\tau-1}^R \qquad \forall t = \tau+1, .., z. \tag{11}$$

As a manufacturing lot has to be scheduled in period τ and no remanufacturing takes place in that period, the lot size x_τ^M cannot be smaller than D_τ as the entire demand in the first period of the time window has to be met. On the other hand, this lot must be able to complement all unavailable products and corresponds therefore at least to the maximum of all net requirements. Calculating the manufacturing lot size for period τ differs from equation (9) as the timing of all returns and demands has to be taken into account for this option. Thus,

$$x_\tau^M = \max\left(D_\tau, \max_{t=\tau+1,..,z}(NR_t)\right), \qquad x_\tau^R = 0. \tag{12}$$

In all consecutive periods of the time window under consideration no manufacturing lot will be set up. However, as the amount manufactured in period τ can be sufficient to satisfy the customer demand at least partly between period $\tau+1$ and z only the actually required products are remanufactured in these periods in order to avoid unnecessary holding cost for the serviceables inventory. By establishing equation (12) it is ensured that in every period between $\tau+1$ and z enough products are available in the recoverables stock to be remanufactured. The resulting lot sizes can be visualized as

$$x_t^M = 0, \qquad x_t^R = \max\left(\sum_{i=\tau}^{t} D_i - \sum_{i=\tau}^{t-1} x_i^R - x_\tau^M, 0\right) \qquad \forall t = \tau+1, .., z. \tag{13}$$

After creating a first initial solution for option 3 using formulae (12) and (13) it must be noticed that the total cost per period of this option can be very high especially when the setup cost K^R is large. Therefore, a greedy algorithm has been formulated in addition that commences in $\tau+1$ and checks two possible improvement opportunities for each remanufacturing lot. Common to both opportunities is that all products obtained in the remanufacturing lot under consideration (which has been originally scheduled in period k and contains P products) are replenished alternatively. Therefore, no remanufacturing lot is scheduled in period k in order to save the setup costs incurred. Firstly, the potential cost saving is evaluated if the manufacturing lot in τ is increased by P. Since this decision affects all remanufacturing lots between $\tau+1$ and $k-1$ formula (13) is applied to determine the corresponding lot sizes. The second opportunity comprises the option to increase the last remanufacturing lot before period k by P products as long as this amount can be found in the recoverables stock. On the other hand, if the recoverables stock does not contain enough recoverable products the difference is manufactured additionally in period τ and again all remanufacturing lots that are affected by this decision are determined using formula (13). Obviously, this option cannot be evaluated for the first remanufacturing lot in a time window. Both improvement opportunities are checked for each remanufacturing lot between $\tau+1$ and z, i.e. at most $2 \cdot (z-\tau)$ different schedules are examined. The schedule yielding the largest cost saving is chosen and the entire proceeding is repeated until no further improvement can be achieved. Finally, after the greedy local search has been applied, a number of remanufacturing lots R succeeds one manufacturing lot. The total cost per period for option 3 can thus be formulated as

$$C^3_{\tau,z} = \frac{R \cdot K^R + K^M + h^M \cdot \sum\limits_{t=\tau}^{z} y^M_t + h^R \cdot \sum\limits_{t=\tau}^{z} y^R_t}{z - \tau + 1}. \tag{14}$$

Option 4: Remanufacture first, manufacture (in multiple lots) later

This option seeks to establish a time window in which a remanufacturing run is started in period τ which is followed by at least one manufacturing lot in the consecutive periods. By assumption, the entire recoverables stock is remanufactured in the first period of the time window τ and no manufacturing lot is set up. Obviously, if the number of available recoverable products in period τ is not sufficient to meet the demand of that period D_τ, option 4 cannot be applied and option 2 provides the only solution incorporating a remanufacturing lot in τ. On the other hand, whenever at least one manufacturing lot is required to satisfy the demand up to period z and $x^R_\tau > D_\tau$ option 2 will be always dominated by option 4 because the holding cost for the serviceables inventory is smaller. This gives for period τ

$$x^M_\tau = 0, \qquad x^R_\tau = y^R_\tau + R_\tau. \tag{15}$$

In order to create an initial solution to this option the lot sizes of the remaining periods have to be determined as well. In each period from $\tau + 1$ to z all missing parts are manufactured as there are no further remanufacturing lots allowed in this time window. The respective formulae are:

$$x^M_t = \max\left(\sum_{i=\tau}^{t} D_i - \sum_{i=\tau}^{t-1} x^M_i - x^R_\tau, 0\right), \qquad x^R_t = 0 \qquad \forall t = \tau + 1, .., z. \tag{16}$$

Similar to option 3, the initial solution can be quite expensive if a large setup cost K^M prevails. Therefore, a greedy algorithm can be used again to search for possible cost reductions. In contrast to the third option, this algorithm reviews all manufacturing lots. It begins by checking whether it would be less expensive to combine the second and the third manufacturing lot of the time window and proceeds in this manner (merging the third and the fourth manufacturing lot, ...) to the end of the corresponding time window. The alternative revealing the largest cost reduction is implemented and the proceeding is restarted until no further cost reductions are possible. Consequently, one remanufacturing lot is followed by a number of manufacturing lots M which can be used to determine the associated cost per period of the fourth option:

$$C^4_{\tau,z} = \frac{K^R + M \cdot K^M + h^M \cdot \sum\limits_{t=\tau}^{z} y^M_t + h^R \cdot \sum\limits_{t=\tau}^{z} y^R_t}{z - \tau + 1}. \tag{17}$$

Including options 3 and 4 into the decision-making process extends the original Silver-Meal based heuristic introduced by Teunter *et al.* (2006). We will refer to this heuristic as the SM_4 heuristic as the decision to extend the time window will be made by comparing the resulting costs per period of all four options. The following chapter tests both heuristics extensively in a numerical experiment to assess its performance.

3 Numerical experiments

In order to guarantee a fair comparison to the original heuristic of Teunter *et al.* (2006) the experimental design that has been to used to conduct the numerical study presented in this section corresponds mostly to their design. A full factorial study has been chosen in which all instances examined have a planning horizon T of twelve periods in common. Both setup cost parameters K^M and K^R can take on values of 200, 500, and 2000. While the rate of keeping a serviceable product for one period in stock (h^M) is set to one holding a recoverable product for one period (h^R) can cost 0.2, 0.5, and 0.8. All customer demands D_t have been drawn randomly from a normal distribution with a mean of 100 units per period. Likewise, the amount of returned products per period R_t has been drawn from a normal distribution with a mean of 30 (i.e. a return ratio of 30% prevails), 50, and 70. Both normal distributions were further distinguished into a small and a large variance setting. While the coefficient of variation in the small variance setting has always been set to 10% it takes on the value of 20% in the large variance setting. Contrary to the experiment conducted in Teunter *et al.* (2006), we omit the use of different demand and return patterns such as positive/negative trends and seasonal patterns. For each demand and return setting 20 instances (instead of 4 in their study) were drawn randomly. Therefore, the full factorial study considers in total $3^4 \cdot 2^2 \cdot 20 = 6480$ different examples.

For all examples both heuristic results have been calculated whereas CPLEX 11 has been used to determine the optimal solution. Both heuristics are evaluated by using the percentage gap to the optimal solution as a performance measure. The results of the numerical experiments are presented in Table 1.

By including two additional options in the decision-making process, the average performance of the SM_2 heuristic improves slightly from 7.5% to 6.1% over all instances. Comparing the performance of the SM_2 heuristic to the original numerical study in Teunter *et al.* (2006) it must be noticed that the performance in our study is slightly better which can be attributed to the differences in the experimental design. Although the SM_4 heuristic reduces the average percentage gap in almost all settings, an improvement of more than 2% can only be observed for a small setup cost for remanufacturing ($K^R = 200$) and a large holding cost for the recoverables inventory ($h^R = 0.8$). Both heuristics seem to perform well when the return ratio or the setup cost for manufacturing K^M is low and when the setup cost for remanufacturing K^R is high. Contrary, for the opposite directions the performance of both heuristics is not sufficient with average errors of more than 7%. In the next section the heuristic solutions are examined whether small modifications can be made to the initially obtained solution in order to reduce the total cost significantly.

Table 1: Performance of the SM_2 and SM_4 heuristic

	Percentage cost error to the optimal solution					
	Average		Standard deviation		Maximum	
	SM_2	SM_4	SM_2	SM_4	SM_2	SM_4
All instances	7.5%	6.1%	7.9%	7.6%	49.2%	47.3%
Demand						
Small variance	7.2%	6.0%	7.9%	7.6%	43.6%	47.3%
Large variance	7.8%	6.1%	8.0%	7.5%	49.2%	43.9%
Returns						
Small Variance	7.3%	6.1%	7.8%	7.6%	47.2%	47.3%
Large Variance	7.7%	6.1%	8.0%	7.5%	49.2%	46.3%
Return ratio						
30%	5.5%	3.7%	5.5%	4.5%	31.3%	28.5%
50%	8.5%	7.3%	9.4%	8.2%	40.1%	41.8%
70%	8.4%	7.2%	8.0%	8.7%	49.2%	47.3%
K^M						
200	4.3%	3.4%	4.5%	3.6%	20.2%	17.6%
500	5.4%	3.9%	5.2%	3.9%	25.1%	19.3%
2000	12.8%	10.9%	9.9%	10.4%	49.2%	47.3%
K^R						
200	10.9%	6.6%	9.1%	7.8%	49.2%	40.2%
500	7.9%	8.1%	6.6%	8.2%	34.7%	47.3%
2000	3.7%	3.5%	6.0%	5.7%	29.4%	25.7%
h^R						
0.2	5.9%	5.3%	8.0%	8.0%	42.9%	47.3%
0.5	7.5%	6.5%	7.7%	7.6%	49.2%	42.4%
0.8	9.1%	6.3%	7.7%	7.0%	44.4%	40.3%

4 Improvement phase

A commonly applied methodology to improve the performance of lot sizing heuristics is to use metaheuristics (see, for instance, Jans and Degraeve, 2007, for an overview). However, metaheuristics rely on an appropriate selection of parameter values which itself might be hard to determine. Therefore, this contribution omits the use of metaheuristics and tries to enhance the solutions found by the SM_2 and SM_4 heuristic by examining two possible improvement opportunities.

Improvement 1: Check whether two consecutive time windows can be combined

A first improvement to the initial solution can be found by checking whether a cost reduction can be achieved if two consecutive time windows are combined. Hence, it is examined whether one of the four (two) options introduced in chapter 2 for the SM_4 (SM_2) heuristic could improve the solution for an integrated time window that comprises both initial time windows.

Improvement 2: Check whether a remanufacturing lot can be increased

Being a myopic heuristic approach, the SM_2 and SM_4 heuristics neglect all decisions beyond the time window currently examined. Thus, some solutions revealed that recoverable products are held in stock until the end of the planning horizon although they could have been used instead of manufacturing them. Hence, the second improvement commences in the first period of the planning horizon and checks for every remanufacturing lot if more could be remanufactured to save holding costs in the recoverables inventory. However, by doing this the holding costs increase in the serviceables inventory. Since $h^M > h^R$ this procedure can only be profitable if carrying those units in the serviceables inventory is shorter than holding them in the recoverables stock. In order to be efficient, the manufacturing lot that is reduced must be scheduled either directly before or after the remanufacturing lot under consideraton, i.e. no other remanufacturing lot is scheduled in between. Whenever a remanufacturing lot suffices these prerequisites, the maximum amount that can be remanufactured in addition without changing the initial solution structure is the least recoverable inventory of the current and all subsequent periods. By remanufacturing more, either the next or the last manufacturing lot can be decreased by the same amount. Of course, the additional amount to be remanufactured in period t cannot exceed x_l^M (whereas l indicates the period of the chosen manufacturing lot) and needs to be adapted if this situation occurs. If the associated total cost can be decreased, the remanufacturing lot in period t is increased and the procedure goes on until the end of the planning horizon.

As mentioned above, both improvements can be applied to the solutions obtained by the SM_2 and SM_4 heuristic. Table 2 summarizes the results of the numerical study in which the superscript $^+$ indicates that the initial solution has been examined for both improvements.

It can be seen that the performance of the SM_4 heuristic could be enhanced substantially from 6.1% to 2.2% by applying both improvements. The larger influence on the solution improvement can be credited to improvement 1 which was able to affect the SM_4 heuristic especially. That is because by analyzing all four options introduced in Section 2 a larger flexibility in satisfying the customer demand is established in comparison to the SM_2 heuristic. Regarding the zero-inventory property, 61.4% of all heuristic solutions obtained by the SM_4 heuristic revealed at least one period in which the zero-inventory property did not hold. In contrast to the original results of the SM_2 heuristic the SM_4^+ heuristic could reduce the percentage gap to less than half of its original value in almost all settings examined. When comparing the median of all instances the improvement is even more noticeable. While the median of all instances has been 5.6% for the SM_2 heuristic the SM_4^+ heuristic could reduce it to around 1.0%. Interestingly, the SM_4^+ heuristic is able to stabilize the average performance of

Table 2: Performance of the SM_2^+ and SM_4^+ heuristic

	Percentage cost error to the optimal solution					
	Average		Standard deviation		Maximum	
	SM_2^+	SM_4^+	SM_2^+	SM_4^+	SM_2^+	SM_4^+
All instances	6.9%	2.2%	7.9%	2.9%	49.2%	24.3%
Demand						
Small variance	6.6%	2.1%	7.9%	2.8%	43.5%	18.9%
Large variance	7.2%	2.4%	8.0%	3.0%	49.2%	24.3%
Returns						
Small Variance	6.8%	2.2%	7.8%	2.9%	47.2%	21.1%
Large Variance	7.1%	2.3%	8.0%	2.9%	49.2%	24.3%
Return ratio						
30%	4.9%	1.2%	5.4%	1.8%	31.3%	12.1%
50%	8.0%	2.3%	9.3%	2.7%	39.8%	16.2%
70%	8.0%	3.3%	8.0%	3.5%	49.2%	24.3%
K^M						
200	3.5%	2.3%	4.0%	2.6%	20.2%	13.5%
500	4.8%	2.1%	4.9%	2.5%	23.7%	12.8%
2000	12.6%	2.3%	9.9%	3.4%	49.2%	24.3%
K^R						
200	10.0%	1.9%	9.4%	2.1%	49.2%	11.8%
500	7.3%	3.4%	6.6%	3.2%	34.7%	19.1%
2000	3.6%	1.4%	5.9%	2.9%	29.4%	24.3%
h^R						
0.2	5.8%	1.7%	8.0%	2.5%	42.9%	21.1%
0.5	7.0%	2.3%	7.7%	3.0%	49.2%	24.3%
0.8	8.1%	2.8%	7.8%	3.0%	44.4%	20.6%

all settings to lie between 1.2% and 3.4%. Although the SM_4^+ heuristic has reduced the maximum deviation from the optimal solution considerably as can be observed in the right hand side of both Tables 1 and 2 there are still instances which perform poorly. Nevertheless, the SM_4^+ heuristic was able to achieve that in only 2% of all instances the percentage gap was larger than 10%. On the contrary, 18% of all instances exhibited a percentage gap of more than 10% when using the SM_2 heuristic.

5 Conclusion and Outlook

This contribution extends the seminal work of Teunter *et al.* (2006) in the area of simple heuristics for the dynamic lot sizing problem with returns and remanufacturing. In their work the authors introduced a Silver-Meal based heuristic that analyzes two options to meet the customer demand. This work included two more options to be analyzed that are well-known from the corresponding static lot sizing problem. By doing this, the percentage gap to the optimal solution which has been used as a performance measure could be reduced slightly from 7.5% to 6.1% (mean over all instances). Afterwards, two simple procedures were applied to the initial solutions found by the SM_2 and SM_4 heuristic to improve the results they created. The average percentage gap to the optimal solution could be reduced over all instances to 2.2% when using the SM_4 heuristic's solution as initial one. Comparing this result to the heuristic introduced by Teunter *et al.* (2006), the average percentage gap has thus been reduced to less than half of its original value.

Future research efforts can be directed to a more detailed modelling of the remanufacturing process. While in this contribution all remanufacturing operations have been subsumed to a single stage, in industry the process of remanufacturing contains next to the disassembly of returned products also the cleaning and rework of the parts obtained, and finally the re-assembly into as-good-as-new products. Furthermore, including the option to dispose of recoverable parts when they are not required and variable unit cost for remanufacturing and manufacturing alter the decision-making process. Another promising research opportunity would be to test the heuristics in a rolling planning horizon environment. As has been shown by Blackburn and Millen (1980) the heuristic might outperform even the optimal solution because of its schedule stability. Another interesting aspect of rolling planning horizon environments that can be analyzed in this context is the uncertainty of demand and return realizations at the end of each planning roll which becomes more accurate as closer one gets to this period.

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