WORKING PAPER SERIES

Rule-based systems for leadership style selection

Kim Michelle Siegling / Thomas Spengler / Sebastian Herzog

Working Paper No. 07/2023

OTTO VON GUERICKE
UNIVERSITÄT
MAGDEBURG

FACULTY OF ECONOMICS
AND MANAGEMENT

Impressum (§ 5 TMG)
Herausgeber:
Otto-von-Guericke-Universität Magdeburg
Fakultät für Wirtschaftswissenschaft
Der Dekan

Verantwortlich für diese Ausgabe:
K. M. Siegling, T. Spengler, S. Herzog
Otto-von-Guericke-Universität Magdeburg
Fakultät für Wirtschaftswissenschaft
Postfach 4120
39016 Magdeburg
Germany

http://www.fww.ovgu.de/femm

Bezug über den Herausgeber
ISSN 1615-4274

Rule-based systems for leadership style selection

Kim Michelle Siegling, Thomas Spengler and Sebastian Herzog[1]

Abstract

In personnel economics, the choice of a leadership style is about the question of how a supervisor should lead his or her employees in such a way that operational goals are achieved. In this paper, we assume that such leadership decisions are made according to the situation. Thus, the optimal or at least a permissible leadership style has to be selected from a set of several possible leadership styles. For this choice a wide range of models has been developed in the scientific literature, from which we want to pick out and focus on the so-called normative decision model by Vroom & Yetton (Vroom/Yetton 1973). While the original model is based on univocal rules, in this paper we develop a fuzzy rule system.

JEL: A20, A22, A23, C60, M12, M21, M51

Keywords: leadership, leadership styles, rule-based systems, fuzzy logic

---

[1] Thomas Spengler, Prof. Dr., thomas.spengler@ovgu.de; Chair of Management and Organization, University of Magdeburg, P.O. Box 4120, 39016 Magdeburg (Germany), Phone: +49 391 67 58440; Kim Michelle Siegling, M. Sc., kim.siegling@ovgu.de; Chair of Management and Organization, University of Magdeburg, P.O. Box 4120, 39016 Magdeburg (Germany), Phone: +49 391 67 52626; Sebastian Herzog, M. Sc., sebastian.herzog@ovgu.de; Chair of Management and Organization, University of Magdeburg, P.O. Box 4120, 39016 Magdeburg (Germany), Phone: +49 391 67 50176.

Contents

1 Introduction

A leadership style is defined as a behavioral pattern according to which the supervisor leads his employees. There are two possible interpretations, because it is either a typical behavioral pattern of the leader in the sense of a personality constant (the supervisor always leads, across situations, following this pattern) or a leadership behavioral pattern that can be selected according to the situation (one leads depending on the current leadership situation). Leadership styles can also be differentiated by various dimensions, e.g. by participation rate (Likert 1967, Tannebaum/Schmidt 1958, Vroom 1959), the relationship orientation or the factual task orientation (Stogdill/Coons 1957). In the scientific literature, a wide range of models for leadership style selection has been presented and critically examined. These include the 3-D model of Reddin (Reddin 1970), the Managerial Grid of Blake & Mouton (Blake/Mouton 1962), the Situational Leadership Theory of Hersey & Blanchard (Hersey/Blanchard 1996), the contingency model of leadership by Fiedler (Fiedler 1967, 1978) as well as the normative decision model by Vroom & Yetton (Vroom/Yetton 1973) and the model by Vroom & Jago (Jago/Ettling/Vroom 1985, Vroom/Jago 1988) based on it. In this paper, we will focus on the Vroom & Yetton model, in which the second of the two interpretations mentioned above is used and which serves to derive recommendations for the choice of leadership style appropriate to the situation. In addition to the leadership style choice, the supervisor must make a factual decision here, and the question arises as to whether he makes this decision alone or with the help of his employees (leadership decision). Let us first (chap. 2) outline the model in a basic form formulated by Vroom & Yetton (Vroom/Yetton 1973) for such situations in which the supervisor leads not only one individual (so-called individual problems), but several employees (so-called group problems). In this basic model, the authors formulate a rule-based expert system with consistently crisp data, variables, and decision rules. We then design a new rule system based on fuzzy logic (chap. 3). Our work ends with a brief conclusion (chap. 4).

2 Vroom & Yetton's normative decision model as a system of crisp rules

2.1 Selectable leadership styles

The participation rate is the degree to which employees are involved in the decision-making process. Vroom & Yetton consider five leadership styles differentiated according to the participation rate $(I_1, I_2, \ldots, I_5)$:

I_1:= The supervisor makes the factual decision alone, based on his current level of information.

I_2:= The supervisor makes the decision on the matter alone after obtaining information from the employees.

I_3 := The supervisor makes the factual decision alone, after discussing the factual decision problem in individual meetings with the employees.

I_4:= The supervisor makes the factual decision alone, after discussing the factual decision problem with the group of employees.

I_5:= The supervisor presents the factual decision problem to the group of employees, everyone develops and evaluates alternative courses of action as a group and the group make a joint factual decision. The supervisor is an equal member of the group.

It is quickly seen that I_1 and I_2 are authoritarian leadership styles, I_3 and I_4 are consultative leadership styles and I_5 is a group-centered leadership style. Provided that one accepts the participation rate as a differentiation criterion for leadership styles - and there is nothing seriously wrong with that - this leadership style list is quite reasonable. However, one misses the complete delegation to the body (without co-decision by the superior) and the possibility of obtaining information from other persons (than one's own employees).

2.2 Determinants of the leadership situation

The leadership situation is analyzed according to a total of seven criteria in question form, whereby these are recorded dichotomously in each case and are to be answered with "yes" or "no" ($J_1, J_2, \ldots, J_7$):

J_1 := Is the quality of the decision important? (Note: Here we are asking about quality, not whether the decision itself is important).

J_2:= Does the supervisor feel sufficiently informed to make a quality factual decision?

J_3:= Does the supervisor think the factual problem is sufficiently structured?

J_4:= Is the acceptance of the factual decision on the part of the employees important for its implementation?

J_5:= Does the supervisor assume that a factual decision made in an authoritarian manner will be accepted?

J_6:= Will employees align their solution contributions with the organizational goal?

J_7 := Is it to be expected that employees will argue about the evaluation of the alternative actions?

The list of situation determinants can be accepted as reasonable, although the level of information of the employees, as provided for in another model version by Vroom & Yetton (Vroom/Yetton 1973), but also the forecast qualification of employees and supervisors could be taken into account. Also, leadership costs and revenues are at best implicitly considered in both leadership styles and leadership situations. Moreover, the dichotomization of situation determinants is based on a simplifying and complexity-reducing assumption, which is removed in later work (Vroom/Jago 1988). With seven questions, each with two possible answers, there are a total of ($2^7 =$) 128 possibilities (leadership situations) for combining the answers (variations with repetition). In the appendix, the combination possibilities (*CP*) are listed tabularly numbered.

2.3 Decision rules

For the purpose of leadership style selection, decision rules are to be applied. Vroom & Yetton propose the following seven decision rules ($DR_1, DR_2, \ldots, DR_7$) in the version presented here, where $\neg$ symbolizes negation, $\wedge$ logical and (both ... and) and $\rightarrow$ implication:

DR_1 (Information rule):[2] $J_1 \wedge \neg J_2 \rightarrow \neg I_1$

Note: This rule is undoubtedly plausible, because if decision quality is important but the supervisor is not sufficiently informed for a good factual decision, it makes no sense for him to decide based on his current level of information.

DR_2 (Trust rule): $J_1 \wedge \neg J_6 \rightarrow \neg I_5$

Note: This rule is also plausible to a certain extent, because if the quality of the decision is important but conflicts are to be expected among the employees about the factual decision to be made, they should not be allowed to participate in the decision if it is assumed that conflict resolution is not possible or at least not possible with reasonable effort. In principle, however, they can then be used in upstream stages of the decision-making process. The fact that conflicts can also have a negative impact in this process (e.g., through strategic information and consultation behavior) is apparently not considered relevant by Vroom & Yetton and therefore only I_5 is excluded here.

DR_3 (Structure rule): $J_1 \wedge \neg J_2 \wedge \neg J_3 \rightarrow \neg(I_1, I_2, I_3)$

Note: If the decision quality is important, but the supervisor is not sufficiently informed for a good factual decision and he considers the factual decision problem as unstructured, the supervisor should not decide authoritatively. This is plausible as far as it goes. Nor, according to

[2] Read: If question J_1 is answered yes and question J_2 is answered no, then do not choose leadership style I_1

Vroom & Yetton, should he or she seek advice in one-on-one meetings. That - as assumed by Vroom (1976) – I_2 and I_3 are always too cumbersome, ineffective and inefficient here is questionable and whether group discussions can make up for the deficits is at least worth discussing.

DR_4 (Acceptance rule): $J_4 \wedge \neg J_5 \rightarrow \neg(I_1, I_2)$

Note: If the acceptance of the factual decision on the part of the employees is important but it can be assumed that an authoritarian decision will not be accepted by them, then it is logical that neither of the two authoritarian leadership styles is chosen here.

DR_5 (Conflict rule): $J_4 \wedge \neg J_5 \wedge J_7 \rightarrow \neg(I_1, I_2, I_3)$:

Note: If it is important that the employees accept the factual decision, but an authoritarian factual decision is not likely to be accepted by them and conflicts over the order of preference are to be expected, then there is a case for not selecting I_1, I_2 and I_3. Whether, in this case, I_4 is actually better than I_3 is at least debatable.

DR_6 (Fairness rule): $J_4 \wedge \neg J_5 \wedge \neg J_1 \rightarrow \neg(I_1, I_2, I_3, I_4)$

Note: Vroom & Yetton consider it fair if the employees in the group (co-)decide, if the acceptance of the decision is important, an authoritarian decision is probably not accepted but the quality of the decision is irrelevant. It remains to be seen whether every employee actually considers it fair when he or she is called upon to make qualitatively irrelevant decisions.

DR_7 (Acceptance prioritization rule): $J_4 \wedge \neg J_5 \wedge J_6 \rightarrow \neg(I_1, I_2, I_3, I_4)$

Note: Here, Vroom & Yetton apparently assume that only a group decision can eliminate the presumed conflicting goals. However, this assumption is also debatable.

2.4 Decision trees

2.4.1 Original decision tree

It goes without saying that leadership style selection can be made on the basis of these seven rules - the first three of which relate to decision quality and the other four to decision acceptance - by analyzing the current leadership situation and then applying the corresponding rule(s). However, Vroom & Yetton (Vroom/Yetton 1973) also suggest the use of the following decision tree, where the complexity is reduced from 128 to 14 leadership situations, where (128-14=) 114 situations are implicitly included in this tree, although they are not explicitly listed (see figure 1):[3]

[3] In Vroom (1976), LS_1 and LS_2 are combined into one situation.

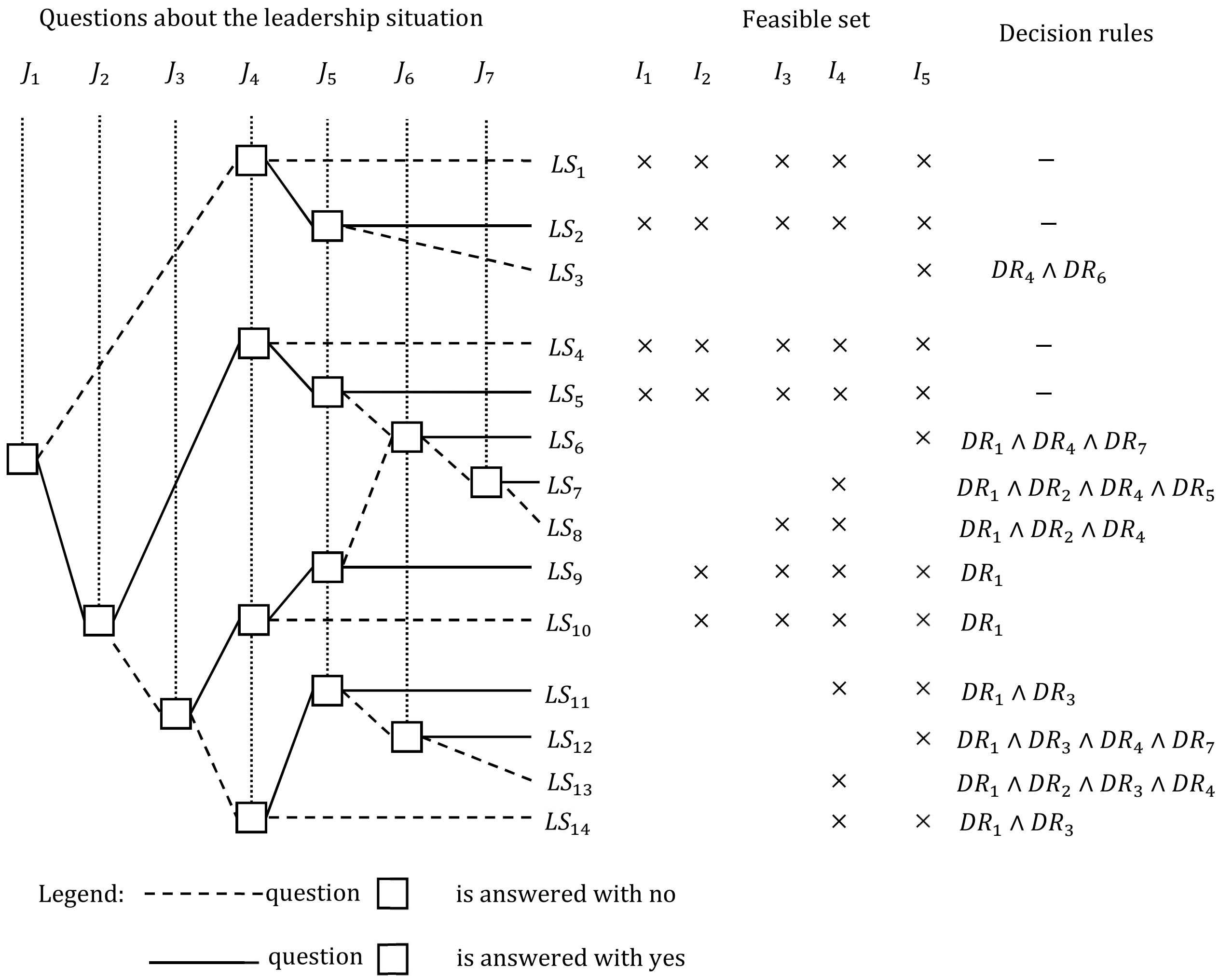

Figure 1: Decision tree for the choice of a leadership style according to Vroom & Yetton (1973)

Behind each of the 14 leadership situations, Vroom & Yetton note the feasible set of the respective selectable leadership styles and the corresponding decision rule(s). As can be seen, only in the leadership situations LS_3, LS_6, LS_{12} and LS_{13} is a single leadership style recommended. In each of the 10 other situations, there are two or more leadership styles to choose from. For these 10 situations, the authors recommend choosing either the one with the lowest participation rate (called the Time Efficient Model) or the one with the highest participation rate (called the Time Investment Model). In doing so, they assume that the decision costs and the personnel development effects increase in each case as the participation rate increases. It is then up to the supervisor to decide whether to apply the first or the second additional criterion.

Note to LS_1:

In LS_1, J_1 and J_4 are negated respectively. $\neg J_1 \wedge \neg J_4$ since five situation determinants are not explicitly requested, is contained in (2^5=) 32 possible combinations[4], namely in CP_{11}, CP_{31}, CP_{35}, CP_{39}, CP_{40}, CP_{41}, CP_{65}, CP_{69}, CP_{70}, CP_{71}, CP_{75}, CP_{76}, CP_{77}, CP_{81}, CP_{82}, CP_{83}, CP_{100}, CP_{101}, CP_{102}, CP_{106}, CP_{107}, CP_{108}, CP_{110}, CP_{111}, CP_{112}, CP_{113}, CP_{121}, CP_{122} CP_{123}, CP_{125}, CP_{126} and CP_{128}. Moreover, $\neg J_1$ occurs only in the fairness rule (DR_6) in which, however, $J_4$and not $\neg J_4$ is also required. No decision rule exists for $\neg J_4$. Thus, all five leadership styles are selectable. It is therefore not necessary to differentiate 32 leadership situations for this purpose, but these can be summarized in LS_1.

Note to LS_2:
In LS_2, $\neg J_1 \wedge J_4 \wedge J_5$ is required and since four situation determinants are not explicitly requested, this requirement is contained in (2^4=) 16 possible combinations, namely CP_2, CP_9, CP_{10}, CP_{13}, CP_{14}, CP_{30}, CP_{33}, CP_{34}, CP_{37}, CP_{38}, CP_{44}, CP_{67}, CP_{68}, CP_{74}, CP_{80} and CP_{105}. Moreover, $\neg J_1$ occurs only in the fairness rule (DR_6) but in which $\neg J_4$ and not J_4 is required. Thus, for LS_2 there is no decision rule, so all five leadership styles are selectable. Thus, one does not have to differentiate 16 leadership situations for this, but can combine them in LS_2.

Note to LS_3:
In LS_3, $\neg J_1 \wedge J_4 \wedge \neg J_5$ is required, and since four situation determinants are not explicitly requested, this requirement is contained in (2^4=) 16 possible combinations, namely CP_{12}, CP_{32}, CP_{36}, CP_{42}, CP_{43}, CP_{66}, CP_{72}, CP_{73}, CP_{78}, CP_{79}, CP_{84}, CP_{103}, CP_{104}, CP_{109}, CP_{114} and CP_{124}. $\neg J_1 \wedge J_4 \wedge \neg J_5$ corresponds to the if-component of DR_6, in whose then-component I_1, I_2, I_3, and I_4 are excluded. $J_4 \wedge \neg J_5$ also occurs in DR_5 and DR_7 although these are redundant with DR_6. Also redundant is DR_4. Thus, one does not need to differentiate 16 leadership situations, but can combine them into LS_3.

Note to LS_4:
In this leadership situation $J_1 \wedge J_2 \wedge \neg J_4$ is required. Since in LS_4 again in (2^4=) 16 possible combinations it remains basically open how the remaining four situation determinants are pronounced, one has to analyze them (namely CP_5, CP_{20}, CP_{24}, CP_{25}, CP_{26}, CP_{55}, CP_{56}, CP_{57}, CP_{61}, CP_{62}, CP_{63}, CP_{95}, CP_{96}, CP_{97}, CP_{99}, CP_{120}) in more detail. While Vroom & Yetton state that for LS_4 there is no decision rule (limiting the leadership style choice), the more detailed

[4] All possible combinations can be found in the appendix.

analysis shows that for CP_{25}, CP_{56}, CP_{61}, CP_{63}, CP_{95}, CP_{97}, CP_{99}, and CP_{120} the trust rule (DR_2: $J_1 \wedge \neg J_6 \rightarrow \neg I_5$) holds, so a corresponding branch (at J_6) must be inserted in the decision tree.

Note to LS_5:

This leadership situation provides $J_1 \wedge J_2 \wedge J_4 \wedge J_5$ and is contained in (2^3=) 8 possible combinations, namely CP_1, CP_4, CP_7, CP_8, CP_{22}, CP_{23}, CP_{29} and CP_{60}. For four of these combination possibilities no decision rule exists, so that as Vroom & Yetton rightly state, the feasible set includes all five leadership styles. In CP_7, CP_{22}, CP_{29} and CP_{60} DR_2 is applicable, so similarly to LS_4 a corresponding branch (at J_6) is to be inserted in the decision tree, where I_5 is excluded.

Note to LS_6:

For LS_6 there are two paths in the decision tree. In the upper path, $J_1 \wedge J_2 \wedge J_4 \wedge \neg J_5 \wedge J_6$ holds and CP_6, CP_{21}, CP_{28}, and CP_{59}, (2^2=) 4 combination possibilities are relevant, where DR_4 and DR_7 are applicable, leaving only I_5 as selectable. In the lower path $J_1 \wedge \neg J_2 \wedge J_3 \wedge J_4 \wedge \neg J_5 \wedge J_6$ with (2^1=) 2 possible combinations (CP_{17}, CP_{53}) applies, the decision rules DR_1, DR_4, DR_5 and DR_7 and the same result as in the upper path ($\neg I_5$).

Note to LS_7:

Again, there are two paths through the tree with $J_1 \wedge J_2 \wedge J_4 \wedge \neg J_5 \wedge \neg J_6 \wedge J_7$ and with $J_1 \wedge \neg J_2 \wedge J_3 \wedge J_4 \wedge \neg J_5 \wedge \neg J_6 \wedge J_7$. In the upper path CP_{27} and CP_{58} and the decision rules DR_2, DR_4 and DR_5 apply and in the lower CP_{52} and DR_1, DR_2, DR_4 and DR_5 apply. In both paths, only I_4 remains as selectable.

Note to LS_8:

There are also two paths through the tree in LS_8 with $J_1 \wedge J_2 \wedge J_4 \wedge \neg J_5 \wedge \neg J_6 \wedge \neg J_7$ (CP_{64}, CP_{98}, DR_2, DR_4) and with $J_1 \wedge \neg J_2 \wedge J_3 \wedge J_4 \wedge \neg J_5 \wedge \neg J_6 \wedge \neg J_7$ (CP_{94}, DR_1, DR_2, DR_4) and for both with exclusion of I_1, I_2 and I_5.

Note to LS_9:

This leadership situation provides $J_1 \wedge \neg J_2 \wedge J_3 \wedge J_4 \wedge J_5$ and is contained in (2^2=) 4 possible combinations, namely CP_3, CP_{18}, CP_{19} and CP_{54}. Since there is no decision rule with J_3 as well as with J_5 the decision rule DR_1 ($\neg I_1$) applies to CP_3 and CP_{19}. However, for CP_{18} and CP_{54} DR_2 applies. This requires an additional branch (not considered by Vroom & Yetton) (at J_6) in the decision tree, where I_5 is also excluded.

Note to LS_{10}:

This leadership situation starts from $J_1 \wedge \neg J_2 \wedge J_3 \wedge \neg J_4$. This makes ($2^3$=) 8 combination possibilities (CP_{16}, CP_{49}, CP_{50}, CP_{51}, CP_{91}, CP_{92}, CP_{93}, CP_{119}) relevant and requires further branches in the tree. In all combination possibilities, I_1 must be excluded via DR_1 in each case. In CP_{50}, CP_{91}, CP_{93} and CP_{119}, DR_2 $(\neg I_5)$ additionally applies. Thus, a corresponding branch is to be inserted at J_6.

Note to LS_{11}:
In this leadership situation, $J_1 \wedge \neg J_2 \wedge \neg J_3 \wedge J_4 \wedge J_5$ is valid. And (2^2=) 4 combination possibilities (CP_{15}, CP_{47}, CP_{48}, CP_{90}) are relevant, in which the rules DR_1 $(\neg I_1)$ and DR_3 $(\neg I_1, \neg I_{2,}, \neg I_3)$ are always applicable. For CP_{47} and CP_{90}, DR_2 $(\neg I_5)$ also applies, so a branch (at J_6) in the tree becomes necessary here as well.

Note to LS_{12}:
This leadership situation provides $J_1 \wedge \neg J_2 \wedge \neg J_3 \wedge J_4 \wedge \neg J_5 \wedge J_6$, so ($2^1$=) 2 combination possibilities (CP_{46}, CP_{89}) are relevant and I_1, I_2, I_3 and I_4 are excluded respectively. The corresponding decision rules are DR_1 $(\neg I_1)$, DR_3 $(\neg I_1, \neg I_{2,}, \neg I_3)$ and DR_7 $(\neg I_1, \neg I_{2,}, \neg I_3, \neg I_4)$.
Note to LS_{13}:
In this leadership situation $J_1 \wedge \neg J_2 \wedge \neg J_3 \wedge J_4 \wedge \neg J_5 \wedge \neg J_6$ is valid. Here again (2^1=) 2 combination possibilities (CP_{88}, CP_{118}) are relevant and the leadership styles I_1, I_2, I_3 and I_5 are excluded respectively. The decision rules valid here are DR_1 $(\neg I_1)$, DR_2 $(\neg I_5)$ and DR_3 $(\neg I_1, \neg I_{2,}, \neg I_3)$.

Note to LS_{14}:
In LS_{14}, $J_1 \wedge \neg J_2 \wedge \neg J_3 \wedge \neg J_4$ holds. This makes (2^3=) 8 possible combinations (CP_{45}, CP_{85}, CP_{86}, CP_{87}, CP_{115}, CP_{116}, CP_{117}, CP_{127}) relevant and requires one more branch in the tree (at J_6). For CP_{45}, CP_{85}, CP_{87}, CP_{116} the rules DR_1 $(\neg I_1)$ and DR_3 $(\neg I_1, \neg I_{2,}, \neg I_3)$ and for the other four combinations additionally DR_2 $(\neg I_5)$ have to be applied.

2.4.2 Redesigned decision tree

Different versions of the model and various decision trees can be found in the literature (see e.g. Hill/Schmitt 1977, Lunenburg 2010, Vroom 1967, Vroom/Jago 1995, Vroom/Yetton/Jago 2015). Our illustration refers to Vroom/Yetton 1973. In our view, branching should be done in the decision tree if the corresponding feasible sets contain different leadership styles; if they include the same ones, the corresponding leadership situations can be combined. Considering the corresponding branchings and situation combinations, one arrives at the following decision tree with a total of 8 compressed and 16 differentiated leadership situations. In such a modified tree (compared to the initial tree of figure 1), we no longer symbolize the leadership situations with LS but with LS^*:

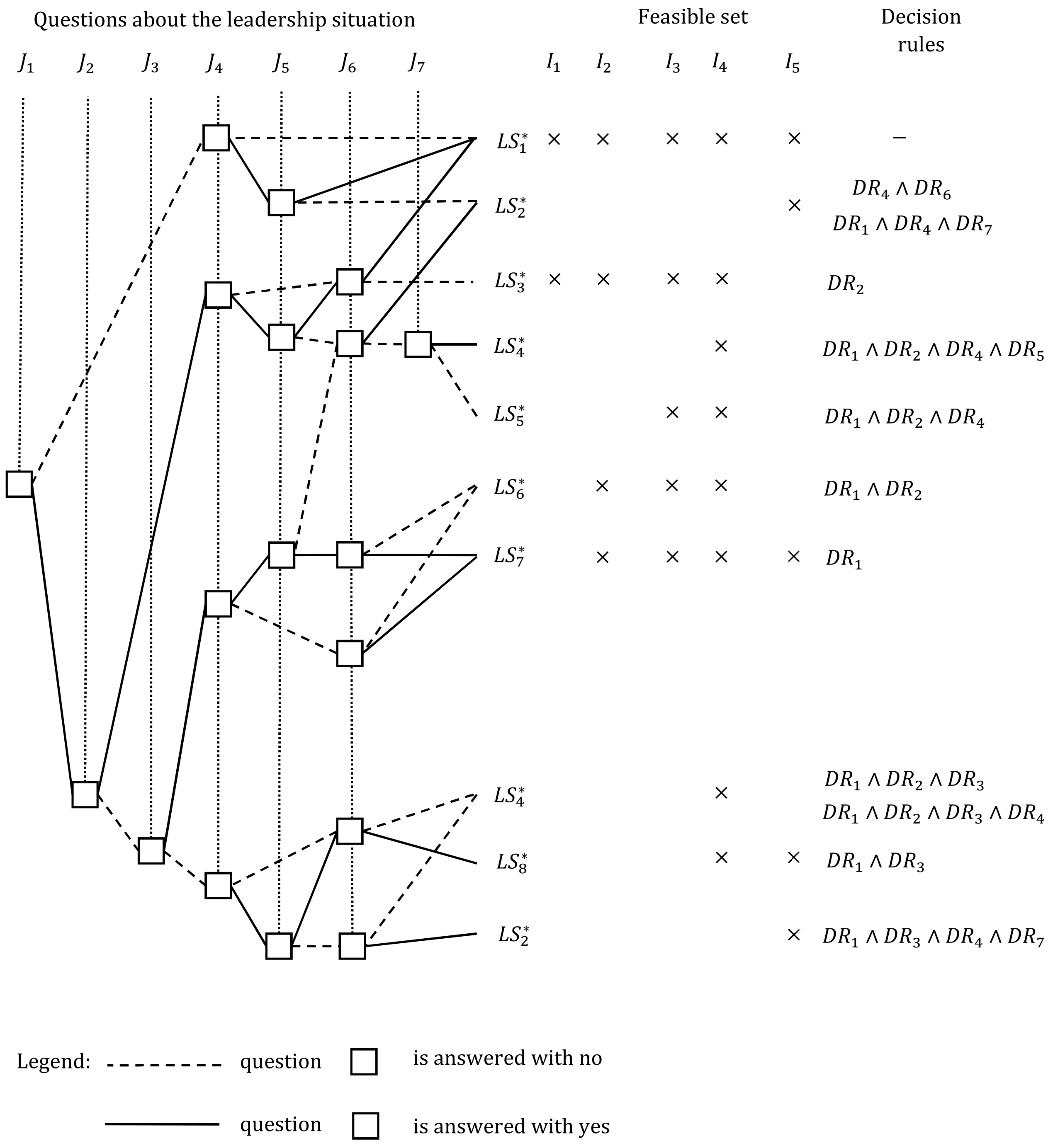

Figure 2: Adapted decision tree for choosing a leadership style

The following table 1 shows how the new leadership situations LS^* are related to the previous leadership situations LS. For example, in the line LS_1^*, we can see that it corresponds with the leadership situations LS_1, LS_2 and LS_5, since no leadership style is excluded in all these leadership situations. The leadership situations LS_2^* and LS_4^* occur twice in the modified decision tree.

If you compare the respective upper with the respective lower path through the tree here, you can see that they correspond to other initial leadership situations.

	LS_1	LS_2	LS_3	LS_4	LS_5	LS_6	LS_7	LS_8	LS_9	LS_{10}	LS_{11}	LS_{12}	LS_{13}	LS_{14}
LS_1^*	x	x	-	-	x	-	-	-	-	-	-	-	-	-
LS_2^* (u.p)	-	-	x	-	-	x	-	-	-	-	-	-	-	-
LS_2^* (l.p.)	-	-	-	-	-	-	-	-	-	-	-	x	-	-
LS_3^*	-	-	-	x	-	-	-	-	-	-	-	-	-	-
LS_4^* (u.p.)	-	-	-	-	-	-	x	-	-	-	-	-	-	-
LS_4^* (l.p.)	-	-	-	-	-	-	-	-	-	-	-	-	x	x
LS_5^*	-	-	-	-	-	-	-	x	-	-	-	-	-	-
LS_6^*	-	-	-	-	-	-	-	-	x	x	-	-	-	-
LS_7^*	-	-	-	-	-	-	-	-	x	x	-	-	-	-
LS_8^*	-	-	-	-	-	-	-	-	-	-	x	-	-	-

Legend: u.p. = upper path l.p. = lower path

Table 1: Newly determined and corresponding previous management situations

The newly constructed decision tree, as inferred, is more precise than the original tree of Vroom & Yetton (1973), since it contains branches with respect to J_6 that are not considered in the original source.

Variations of the original model were constructed later (most notably by Vroom & Jago) (Vroom/Jago 1988). In these, the dichotomy of situation determinants is abandoned and replaced by mathematical functions. This also eliminates the possibility of using decision trees (Vroom/Yetton/Jago 2015).

3 The normative decision model of Vroom & Yetton as a fuzzy rule-based system

3.1 Preliminary remarks

The initial model of Vroom & Yetton is based on Boolean (two-valued or binary) logic, which knows only two states, namely true or false, yes or no or 0 or 1. Thus an element x belongs either completely (or completely not) to a set. For the membership value of such a crisp set A

holds $\mu_A(x) \in \{0,1\}$. In the context of the so-called fuzzy logic (Buckley/Eslami 2002, Gottwald 1993, Pedrycz 1993, Piegat 2001, Zadeh 1983, Zimmermann 1987, 1996) membership values can also be graduated, such that for the membership of an element x to a fuzzy set $\tilde{A}$ $\mu_{\tilde{A}}(x) \in [0,1]$ holds (Bellmann/Zadeh 1970, Dubois/Ostasiewicz/Prade 2000, Dubois/Prade 1980a, Pedrycz 1993, Piegat 2001, Wang/Chang 2000, Zimmermann 1996). Since $\{0,1\} \subset [0,1]$ unambiguity is always a special case of fuzziness.

A fuzzy set $(\tilde{A})$ (Zadeh 1965) is a set of ordered second tuples $(x \in \overline{X}, \mu_{\tilde{A}}(x))$. Here $\bar{X} = \{x\}$ is a crisp (basic) set of elements x to be evaluated with respect to a fuzzy statement $\tilde{A}$. $\mu_{\tilde{A}}(x)$ is the degree to which x belongs to $\tilde{A}$. In principle, $\mu_{\tilde{A}} \to \mathbb{R}_0^+$ holds. However, by normalization one often chooses $\mu_{\tilde{A}} \to [0,1]$. The so-called supporting set of a fuzzy set $S(\tilde{A})$ is a crisp set consisting of such elements of the basic set which have a positive membership degree to $\tilde{A}$. There exist fuzzy sets based on continuous and those based on discrete basic sets $\bar{X}$. The membership functions are often continuous (see figure 3), but sometimes single discrete membership values are connected with polygonal features (see figure 4). Singletons represent a special case: These are one-element fuzzy sets for whose membership value $0 < \mu_{\tilde{A}}(x^*) \leq 1$ holds (Piegat 2001) (see figure 5).

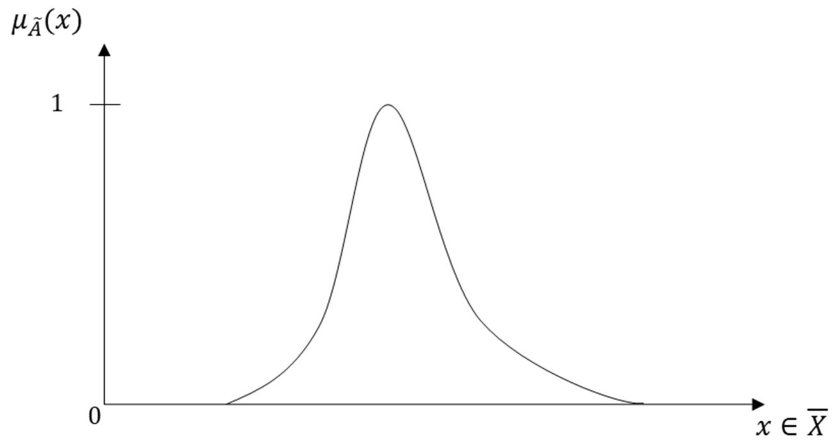

Figure 3: Example of a continuous membership function

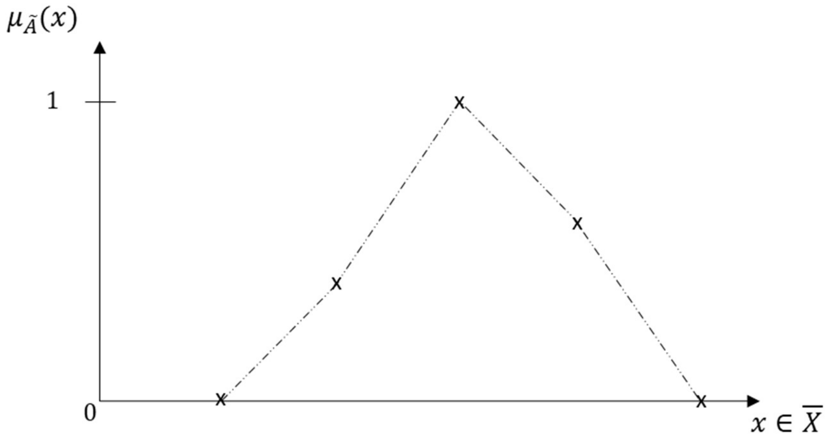

Figure 4: Example of a membership function with polygonal features

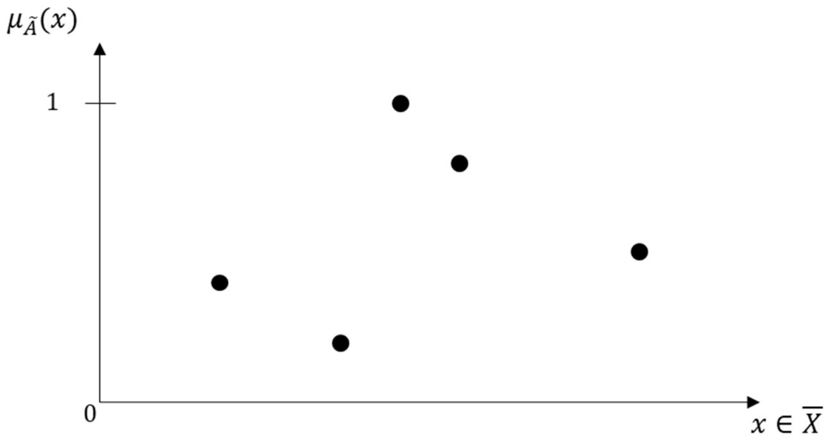

Figure 5: Examples of singletons

Crisp rule systems usually use the modus (ponendo) ponens as an inference rule (Dubois/Prade 1991, Mamdani 1981, Zimmermann 1987): it consists of (at least) two premises and one conclusion:

Premise 1: If A then C

Premise 2: A is present

Conclusion: It follows C

This inference mechanism is also used in fuzzy control systems:

Premise 1: If $\tilde{A}$ then $\tilde{C}$

Premise 2: $\tilde{A}$ is present

Conclusion: It follows $\tilde{C}$

In the context of fuzzy control (Driankov/Hellendoorn/Reinfrank 1993), linguistic variables (for $\tilde{A}$ and $\tilde{C}$) are often used. These represent quadruples (Dubois/Prade 1978, Spengler/Herzog 2023, Zadeh 1975, 1987). They consist of the name of the linguistic variable, of the set of linguistic terms, of the base set on which the linguistic variable is defined, and of a semantic rule that assigns a membership function to each linguistic term. The design of an expert system based on fuzzy rules (Hall/Kandel 1991, Zimmermann 1996) is basically carried out in three steps:

1. Step: Fuzzification of the rule input by constructing membership functions for the input variables.
2. Step: Fuzzy inference (Bouchon-Meunier 1991, Dubois/Prade 1991, Piegat 2001, Schneider/Kandel 1991, Yager 1991, Zadeh 1983) by formulating the rule base, applying the inference mechanism, and deriving the linguistic output variables (including construction of corresponding membership functions).
3. Step: Defuzzification of the fuzzy output

We will discuss these steps in more detail below. If the rule input is composed of several fuzzy premises (e.g. $\tilde{A}$ and $\tilde{B}$), which are linked by the logical AND (both ... and), and the rule inference results in a fuzzy rule output set $\tilde{C}$, then the corresponding fuzzy rule is e.g. $\tilde{A} \wedge \tilde{B} \rightarrow \tilde{C}$. The $\wedge$ -connection is formed in the fuzzy control via an operator from the group of so-called t-norms. We will restrict ourselves here to the use of the so-called minimum operator as a specific t-norm. For the definition of t-norms (Dubois/Prade 1980b, Fodor/Yager 2000, Klement/Mesiar/Pap 2004, Pap 2002, Yager 1980, Zimmermann 1996) and other corresponding operators (for example the algebraic product, the bounded difference, the drastic product, and the Yager average) see e.g. Zimmermann (1996). For example, let us imagine a case with two linguistic input variables $\tilde{A}$ and $\tilde{B}$ and with two linguistic terms each ($\tilde{A}_1, \tilde{A}_2$) and ($\tilde{B}_1, \tilde{B}_2$) and a linguistic output variable $\tilde{C}$ with three linguistic terms ($\tilde{C}_1, \tilde{C}_2, \tilde{C}_3$) then there are a total of four rules with crisp rule inputs[5] x_1 and x_2, e.g.:

Rule 1: IF $x_1 = \tilde{A}_1$ AND $x_2 = \tilde{B}_1$ THEN $y = \tilde{C}_2$

Rule 2: IF $x_1 = \tilde{A}_2$ AND $x_2 = \tilde{B}_1$ THEN $y = \tilde{C}_3$

Rule 3: IF $x_1 = \tilde{A}_1$ AND $x_2 = \tilde{B}_2$ THEN $y = \tilde{C}_1$

Rule 4: IF $x_1 = \tilde{A}_2$ AND $x_2 = \tilde{B}_2$ THEN $y = \tilde{C}_1$

As an example, let us assume that $x_1 = 0.3$ and $x_2 = 0.4$, with

$\mu_{\tilde{A}_1}(x_1) = 0.67 \qquad \mu_{\tilde{A}_2}(x_1) = 0.2 \quad \mu_{\tilde{B}_1}(x_2) = 0.6$

apply (see figure 6):

[5] Of course, fuzzy rule inputs can also be used. For reasons of simplification, however, we will refrain from doing so here.

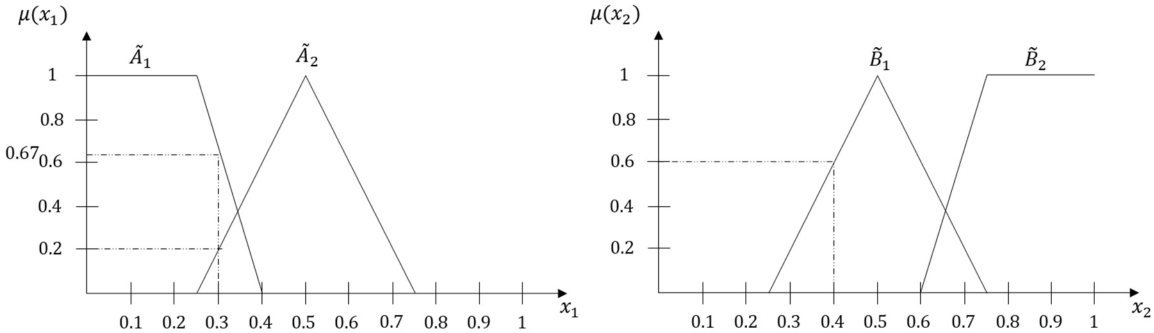

Figure 6: Exemplary representation of rule inputs

For example, the membership functions of $\tilde{C}_1$, $\tilde{C}_2$ and $\tilde{C}_3$ are as follows:

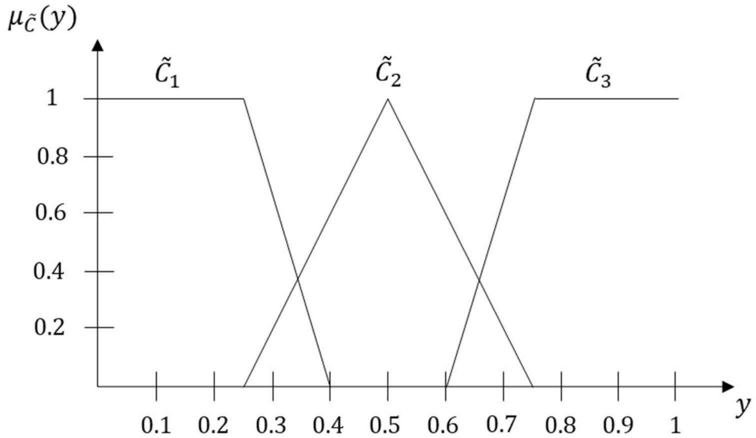

Figure 7: Exemplary representation of rule outputs

A rule is called active if all premises are true to a positive degree and their membership values are positive. Rule inputs with a truth or membership value of zero are therefore inactive. In the example, we want to model the $\wedge$- connection of the premises (as shown above) via the minimum operator as a specific *t*-norm, so that the Degree of Fulfillment (DOF) in the example results as follows:

Rule 1: $DOF = Min\left(\mu_{\tilde{A}_1}(x_1); \mu_{\tilde{B}_1}(x_2)\right) = Min(0,67; 0,6) = 0,6$

Rule 2: $DOF = Min\left(\mu_{\tilde{A}_2}(x_1); \mu_{\tilde{B}_1}(x_2)\right) = Min(0,2; 0,6) = 0,2$

Rule 3: $DOF = Min\left(\mu_{\tilde{A}_1}(x_1); \mu_{\tilde{B}_2}(x_2)\right) = Min(0,67; 0) = 0$

Rule 4: $DOF = Min\left(\mu_{\tilde{A}_2}(x_1); \mu_{\tilde{B}_2}(x_2)\right) = Min(0,2; 0) = 0$

Next, the modified membership functions of the rule outputs $\mu_{C^*}(y)$ of the individual rules have to be determined. For this purpose, we again want to use the minimum operator as a specific t-norm such that $\mu_{C^*}(y) = Min(DOF, \mu_{\tilde{C}}(y))$. The then still needed resulting membership function of the fuzzy output set $\mu_{conc}(y)$ is formed via a suitable s-norm (Klement/Mesiar/Pap 2004, Zimmermann 1996) as operator for the $\vee$-connection (either ... or ... or both). We want to use the maximum operator in the example, with $\mu_{conc}(y) = Max\left(\mu_{c_2^*}(y), \mu_{c_3^*}(y)\right)$. $\mu_{conc}(y)$ thus takes the following form (see figure 8):

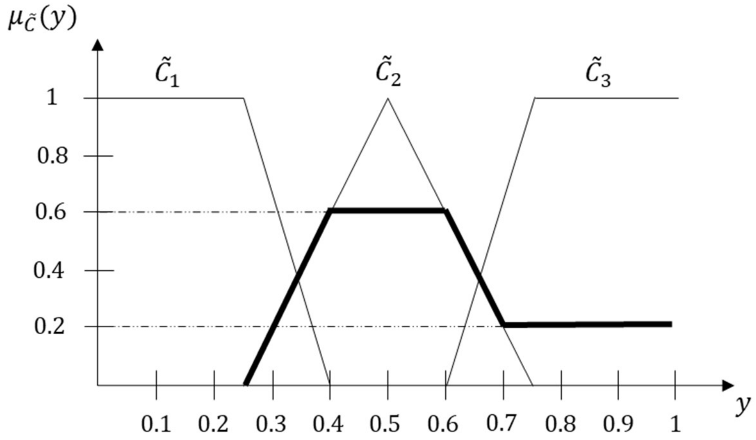

Figure 8: Fuzzy rule output with in consideration of maximum operator

3.2 Selectable leadership styles

In the original model of Vroom & Yetton (1973), the leadership styles I_1, I_2, I_3, I_4 and I_5 are discretely differentiated. Such a differentiation can also be implemented in the context of a fuzzy rule system by taking the effectiveness expressions $\tilde{E}$ of the different leadership styles as fuzzy conclusion variables of the rules in the form of singletons. E.g., for the leadership style effectiveness in the context of the information rule (DR_1: $J_1 \wedge \neg J_2 \rightarrow \neg I_1$) could apply (see figure 9):

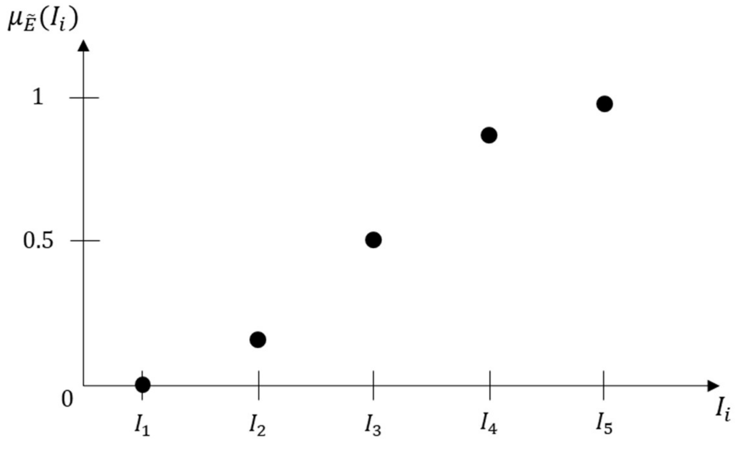

Figure 9: Leadership style effectiveness using the example of the information rule

In the present work, however, the aim is not to make a discrete but a continuous differentiation of leadership styles on the basis of a bipolar continuum of the participation rate (x_{PR}). At the poles of this continuum, $x_{PR} = 0$ (completely authoritarian leadership) and $x_{PR} = 1$ (complete delegation of factual decision-making) apply (as sketched in figure 10).

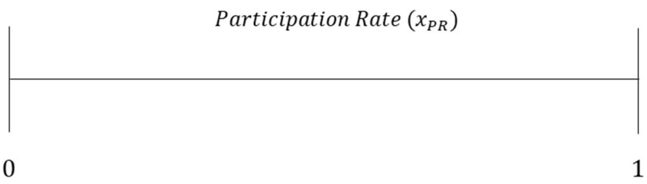

Figure 10: Continuum of leadership styles

The left pole of this continuum corresponds to I_1 in Vroom/Yetton (1973). However, there is no counterpart for the right pole in this model. The participation rate is used here as a linguistic variable with the linguistic terms *low*, *medium* and *high*, which we will consider in more detail in chapter 3.3. Of course, one can also model other linguistic terms (e.g., very low and very high) and other (e.g., piecewise continuous, bell-shaped, and trapezoidal) shapes of the membership functions.

3.3 Leadership situations

In the following we use these symbols:

$\overline{D} \coloneqq \{d | d = DQ, ILL, PS, IAAD, PAAD, GO, CE;$ d is a determinant of the leadership situation$\}$

$x_d \in [0,1]$ degree to which determinant $d \in \overline{D}$ is true

$L \coloneqq \{l | l = low, medium, high\}$ Set of linguistic terms

$\mu_d^l(x) \coloneqq$ Degree of membership of an element x to linguistic term l of determinant d

In the original model (see chapter 2), the leadership situation is analyzed according to a total of seven determinants ($J_1, J_2, \ldots, J_7$) in the form of questions, each of which is recorded dichotomously and must be answered with "yes" or "no". J_1 is about the importance of the (factual) decision quality (DQ), J_2 about the adequacy of the superior's level of information (ILL), J_3 about the structuredness of the factual problem (PS), J_4 about the importance of the acceptance of an authoritative decision ($IAAD$), J_5 about the possibility of acceptance of an authoritatively made decision ($PAAD$), J_6 about the goal orientation of the employees (GO) and J_7 about the expectation of evaluation conflicts among the employees (CE). In the fuzzy rule model to be formulated here, the evaluation of the corresponding questions or, more precisely, their truthfulness or degree of true $x_d \in [0,1]$ is not dichotomous, but in bipolar continua $x_{DQ}, x_{ILL}, x_{PS}, x_{IAAD}, x_{PAAD}, x_{GO}$ and x_{CE}. We also model these as membership functions for the linguistic terms l for criterion d as follows:

$$\mu_d^{lo}\ (x_d) = \begin{cases} 1 \ \ for\ 0 \leq x_d \leq 0.25 \\ \frac{0.4-x}{0.15} \ \ for\ 0.25 < x_d \leq 0.4 \\ 0 \ \ otherwise \end{cases} \qquad (3.1)$$

$$\mu_d^{medium}(x_d) = \begin{cases} \frac{x-0.25}{0.25} & for\ 0.25 \leq x_d \leq 0.5 \\ \frac{0.75-x}{0.25} & for\ 0.5 < x_d \leq 0.75 \\ 0 & otherwise \end{cases} \quad (3.2)$$

$$\mu_d^{high}(x_d) = \begin{cases} \frac{x-0.6}{0.15} & for\ 0.6 \leq x_d \leq 0.75 \\ 1 & for\ 0.75 < x_d \leq 1 \\ 0 & otherwise \end{cases} \quad (3.3)$$

The graphs of the membership functions are shown in figure 11:

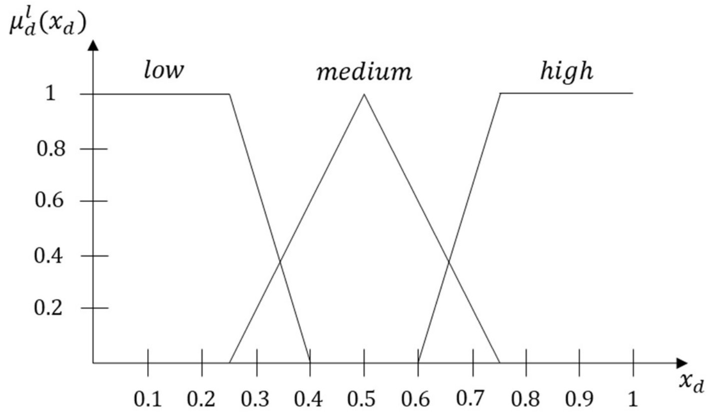

Figure 11: Graphs of the membership functions

Of course, one can again model other linguistic terms and other trajectories of membership functions.

3.4 Decision rules and rule blocks

For the purpose of leadership style selection, Vroom & Yetton bring a total of seven decision rules ($DR_1, DR_2, \ldots, DR_7$) into play in the version presented in chapter 2. However, in the fuzzy rule system to be formulated here, these are not constructed as singular rules, but as rule blocks DR_k $(k = 1, 2, \ldots, 7)$, each composed of several rules differentiated (according to the combinations of linguistic terms). In the following, we use these symbols:

$\overline{K} := \{k | k = 1, \ldots, K; k \text{ is a rule block}\}$

$\overline{M}_k := \{m = M_{k-1} + 1, M_{k-1} + 2, \ldots, M_k [m \in \overline{M}; m \text{ is a decision rule in block } k \in \overline{K}; M_0 = 0\}$

$\overline{M} := \bigcup_{k\epsilon} - \overline{M}_k$ (Set of all decision rules)

$\widetilde{PR}_k :=$ fuzzy participation rate of decision rule block k

$\widetilde{PR}_m^k :=$ fuzzy participation rate of decision rule $m \in \overline{M}_k$ in decision rule block $k \in \overline{K}$

$\widetilde{PR}_{total} :=$ total participation rate

$DOF_m :=$ Degree of fulfillment of decision rule $m\epsilon\overline{M}_k$ $\left(k \in \overline{K}\right)$

$DOF^{l,k}_{total}$ ≔ Total degree of fulfillment of linguistic term l in decision rule block $k \in \overline{K}$

Decision rule block $k = 1$:

The decision rule block $k = 1$ corresponds to the crisp information rule DR_1 from the basic model. This requires $J_1 \wedge \neg J_2 \rightarrow \neg I_1$. The fuzzy rule block $k = 1$ now demands:

$\widetilde{DQ} \wedge \widetilde{ILL} \rightarrow \widetilde{PR}_1$

Given two linguistic input variables and three linguistic terms each, there are a total of 9 rules $m \in \overline{M}_1$. These are, for example:

Rule $m \in \overline{M}_1$	$\widetilde{DQ}$	$\widetilde{ILL}$	$\widetilde{PR}^1_m$
1	low	low	medium
2	medium	low	medium
3	high	low	high
4	low	medium	medium
5	medium	medium	medium
6	high	medium	high
7	low	high	medium
8	medium	high	medium
9	high	high	low

Table 2: Rule block 1

For example, if $x_{DQ} = 0{,}7$ and $x_{ILL} = 0{,}3$, inserting in (3.1), (3.2) as well as (3.3) or from the graphs of the membership functions $\mu^l_{\widetilde{DQ}}$ and $\mu^l_{\widetilde{ILL}}$, it follows that rules 2, 3, 5 and 6 are active ($DOF > 0$) and the others are inactive ($DOF = 0$) (see the following figure 12).

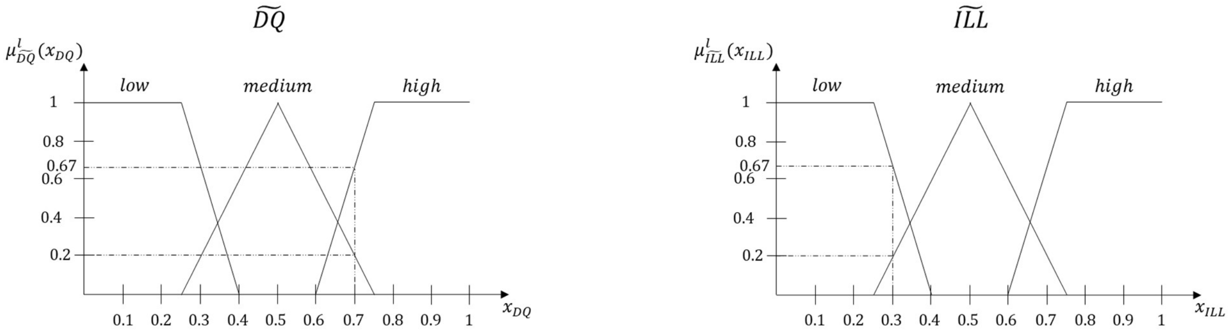

Figure 12: Membership function of $\widetilde{DQ}$ and $\widetilde{ILL}$ of decision rule block $k = 1$

Using the determination rule

$$DOF_m = Min\left(\mu^l_{\widetilde{DQ}}(x_{DQ}); \mu^l_{\widetilde{ILL}}(x_{ILL})\right) \quad \forall m \in \overline{M}_1,$$

the following active rules with the corresponding DOF_m and the expression of the participation rate $\widetilde{PR}^1_m$ are obtained for above values:

DOF_m	$\widetilde{PR}_m^1$
$DOF_2 = Min\left(\mu_{\widetilde{DQ}}^{medium}(0.7); \mu_{\widetilde{ILL}}^{low}(0.3)\right) = Min(0.2; 0.67) = 0.2$	medium
$DOF_3 = Min\left(\mu_{\widetilde{DQ}}^{high}(0.7); \mu_{\widetilde{ILL}}^{low}(0.3)\right) = Min(0.67; 0.67) = 0.67$	high
$DOF_5 = Min\left(\mu_{\widetilde{DQ}}^{medium}(0.7); \mu_{\widetilde{ILL}}^{medium}(0.3)\right) = Min(0.2; 0.2) = 0.2$	medium
$DOF_6 = Min\left(\mu_{\widetilde{DQ}}^{high}(0.7); \mu_{\widetilde{ILL}}^{medium}(0.3)\right) = Min(0.67; 0.2) = 0.2$	high

Table 3: Degrees of fulfillment

Correspondingly, the $DOF_{total}^{l,1}$ results as follows:

$DOF_{total}^{low,1} = 0$

$DOF_{total}^{medium,1} = Max(DOF_2; DOF_5) = Max(0.2; 0.2) = 0.2$

$DOF_{total}^{high,1} = Max(DOF_3; DOF_6) = Max(0.67; 0.2) = 0.67$

The participation rate of the first rule block $\widetilde{PR}_1$ and the membership function of the corresponding fuzzy output set can be read in figure 13:

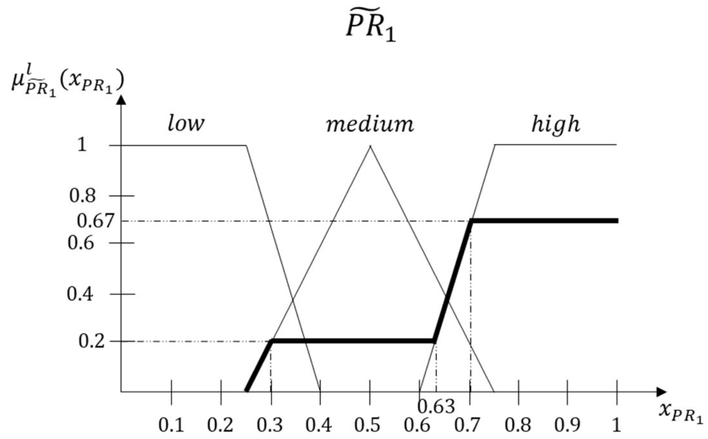

Figure 13: Membership function of $\widetilde{PR}_1$

Decision rule block $k = 2$:

The decision rule block $k = 2$ corresponds to the crisp confidence rule DR_2 from the basic model. This requires $J_1 \wedge \neg J_6 \rightarrow \neg I_5$. The fuzzy rule block $k = 2$, on the other hand, now requires:

$\widetilde{DQ} \wedge \widetilde{GO} \rightarrow \widetilde{PR}_2$

Here again, there are two linguistic input variables, each with three linguistic terms, so that nine rules $m \in \bar{M}_2$ have to be formulated. These are for example:

Rule $m \in \bar{M}_2$	$\widetilde{DQ}$	$\widetilde{GO}$	$\widetilde{PR}_m^2$
10	low	low	low
11	medium	low	low

12	high	low	low
13	low	medium	medium
14	medium	medium	medium
15	high	medium	medium
16	low	high	high
17	medium	high	high
18	high	high	high

Table 4: Rule block 2

For example, if $x_{DQ} = 0.7$ and $x_{GO} = 0.55$, inserting them into (3.1), (3.2) as well as (3.3) or from the graphs of membership functions $\mu_{\widetilde{DQ}}^{l}$ and $\mu_{\widetilde{GO}}^{l}$, it follows that rules 14 and 15 from this block are active ($DOF > 0$) and the others are inactive ($DOF = 0$) (see figure 14).

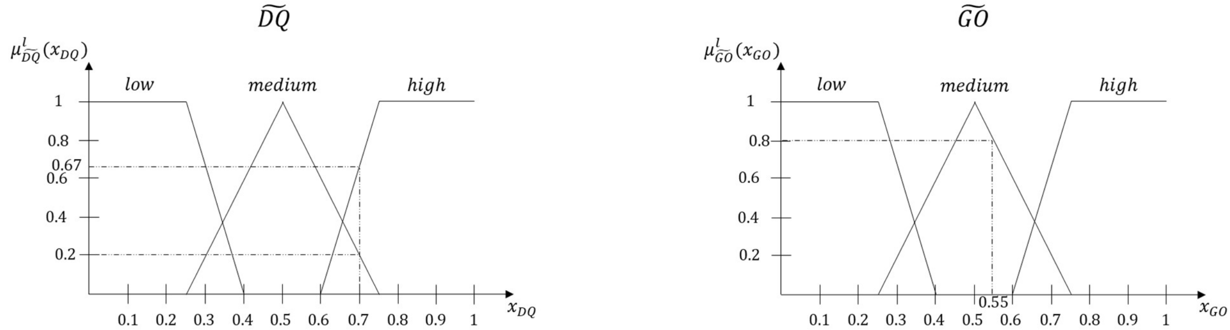

Figure 14: Membership function of $\widetilde{DQ}$ and $\widetilde{GO}$ of decision rule block $k = 2$

Using the determination rule

$$DOF_m = Min\left(\mu_{\widetilde{DQ}}^{l}(x_{DQ}); \mu_{\widetilde{GO}}^{l}(x_{GO})\right) \quad \forall m \in \overline{M}_2,$$

the following active rules with the corresponding DOF_m and the expression of the participation rate $\widetilde{PR}_m^2$ are obtained for above values:

DOF_m	$\widetilde{PR}_m^2$
$DOF_{14} = Min\left(\mu_{\widetilde{DQ}}^{medium}(0.7); \mu_{\widetilde{GO}}^{medium}(0.55)\right) = Min(0.2; 0.8) = 0.2$	medium
$DOF_{15} = Min\left(\mu_{\widetilde{DQ}}^{high}(0.7); \mu_{\widetilde{GO}}^{medium}(0.55)\right) = Min(0.67; 0.8) = 0.67$	medium

Table 5: Degrees of fulfillment

Correspondingly, the $DOF_{total}^{l,2}$ results as follows:

$DOF_{total}^{low,2} = 0$

$DOF_{total}^{medium,2} = Max(DOF_{14}; DOF_{15}) = Max(0.2; 0.67) = 0.67$

$DOF_{total}^{high,2} = 0$

For the participation rate of the second decision rule block $\widetilde{PR}_2$ and the membership function of the corresponding fuzzy output set is obtained (see figure 15):

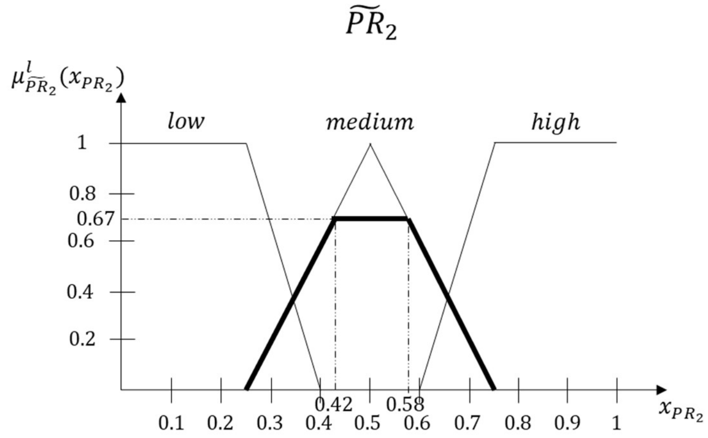

Figure 15: Membership function of $\widetilde{PR}_2$

Decision rule block $k = 3$:

The decision rule block $k = 3$ corresponds to the crisp structure rule DR_3 from the basic model. This requires $J_1 \wedge \neg J_2 \wedge \neg J_3 \rightarrow \neg(I_1, I_2, I_3)$. The fuzzy rule block $k = 3$, on the other hand, now requires:

$\widetilde{DQ} \wedge \widetilde{ILL} \wedge \widetilde{PS} \rightarrow \widetilde{PR}_3$

Here, there are now three linguistic input variables, each with three linguistic terms, so that 27 rules $m \in \bar{M}_3$ have to be formulated. These are for example:

Rule $m \in \bar{M}_3$	$\widetilde{DQ}$	$\widetilde{ILL}$	$\widetilde{PS}$	$\widetilde{PR}^3_m$
19	low	low	low	medium
20	medium	low	low	medium
21	high	low	low	high
22	low	low	medium	high
23	medium	low	medium	high
24	high	low	medium	high
25	low	low	high	high
26	medium	low	high	high
27	high	low	high	high
28	low	medium	low	medium
29	medium	medium	low	medium
30	high	medium	low	low
31	low	medium	medium	medium
32	medium	medium	medium	medium
33	high	medium	medium	high
34	low	medium	high	medium
35	medium	medium	high	high
36	high	medium	high	high
37	low	high	low	medium

38	medium	high	low	low
39	high	high	low	low
40	low	high	medium	medium
41	medium	high	medium	low
42	high	high	medium	low
43	low	high	high	high
44	medium	high	high	medium
45	high	high	high	low

Table 6: Rule block 3

For example, if $x_{DQ} = 0.7$, $x_{ILL} = 0.3$ and $x_{PS} = 0.35$, inserting them into (3.1), (3.2) as well as (3.3) or from the graphs of membership functions $\mu^{l}_{\widetilde{DQ}}$, $\mu^{l}_{\widetilde{ILL}}$ and $\mu^{l}_{\widetilde{PS}}$, it follows that rules 20, 21, 23, 24, 29, 30, 32 and 33 from this block are active $(DOF > 0)$ and the others are inactive $(DOF = 0)$ (see figure 16).

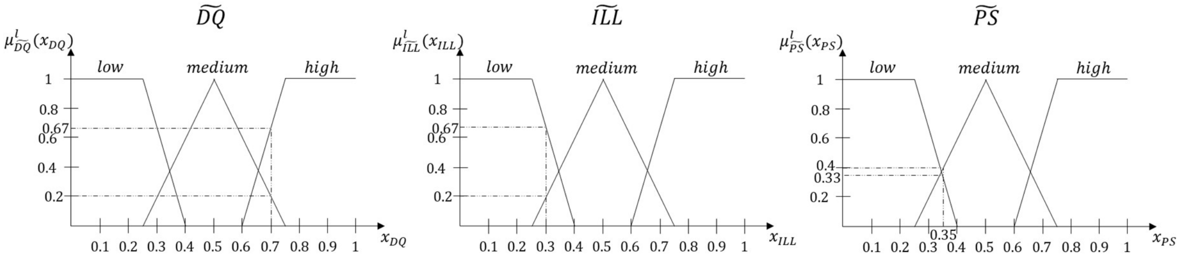

Figure 16: Membership function of $\widetilde{DQ}$, $\widetilde{ILL}$ and $\widetilde{PS}$ of decision rule block $k = 3$

Using the determination rule

$$DOF_m = Min\left(\mu^{l}_{\widetilde{DQ}}(x_{DQ}); \mu^{l}_{\widetilde{ILL}}(x_{ILL}); \mu^{l}_{\widetilde{PS}}(x_{PS})\right) \quad \forall m \in \overline{M}_3,$$

the following active rules with the corresponding DOF_m and the expression of the participation rate $\widetilde{PR}^{3}_{m}$ result for above rules:

DOF_m	$\widetilde{PR}^{3}_{m}$
$DOF_{20} = Min\left(\mu^{medium}_{\widetilde{DQ}}(0.7); \mu^{low}_{\widetilde{ILL}}(0.3); \mu^{low}_{\widetilde{PS}}(0.35)\right) = Min(0.2; 0.67; 0.33) = 0.2$	medium
$DOF_{21} = Min\left(\mu^{high}_{\widetilde{DQ}}(0.7); \mu^{low}_{\widetilde{ILL}}(0.3); \mu^{low}_{\widetilde{PS}}(0.35)\right) = Min(0.67; 0.67; 0.33) = 0.33$	high
$DOF_{23} = Min\left(\mu^{medium}_{\widetilde{DQ}}(0.7); \mu^{low}_{\widetilde{ILL}}(0.3); \mu^{medium}_{\widetilde{PS}}(0.35)\right) = Min(0.2; 0.67; 0.4) = 0.2$	high
$DOF_{24} = Min\left(\mu^{high}_{\widetilde{DQ}}(0.7); \mu^{low}_{\widetilde{ILL}}(0.3); \mu^{medium}_{\widetilde{PS}}(0.35)\right) = Min(0.67; 0.67; 0.4) = 0.4$	high
$DOF_{29} = Min\left(\mu^{medium}_{\widetilde{DQ}}(0.7); \mu^{medium}_{\widetilde{ILL}}(0.3); \mu^{low}_{\widetilde{PS}}(0.35)\right) = Min(0.2; 0.2; 0.33) = 0.2$	medium

$DOF_{30} = Min\left(\mu_{\widetilde{DQ}}^{high}(0.7); \mu_{\widetilde{ILL}}^{medium}(0.3); \mu_{\widetilde{PS}}^{low}(0.35)\right) = Min(0.67; 0.2; 0.33) = 0.2$	low
$DOF_{32} = Min\left(\mu_{\widetilde{DQ}}^{medium}(0.7); \mu_{\widetilde{ILL}}^{medium}(0.3); \mu_{\widetilde{PS}}^{medium}(0.35)\right) = Min(0.2; 0.2; 0.4) = 0.2$	medium
$DOF_{33} = Min\left(\mu_{\widetilde{DQ}}^{high}(0.7); \mu_{\widetilde{ILL}}^{medium}(0.3); \mu_{\widetilde{PS}}^{medium}(0.35)\right) = Min(0.67; 0.2; 0.4) = 0.2$	high

Table 7: Degrees of fulfillment

Correspondingly, the $DOF_{total}^{l,3}$ results as follows:

$DOF_{total}^{low,3} = 0.2$

$DOF_{total}^{medium,3} = Max(DOF_{20}; DOF_{29}; DOF_{32}) = Max(0.2; 0.2; 0.2) = 0.2$

$DOF_{total}^{high,3} = Max(DOF_{21}; DOF_{23}; DOF_{24}; DOF_{33}) = Max(0.33; 0.2; 0.4; 0.2) = 0.4$

For the participation rate of the third decision rule block $\widetilde{PR}_3$ and the membership function of the corresponding fuzzy output set is obtained (see figure 17):

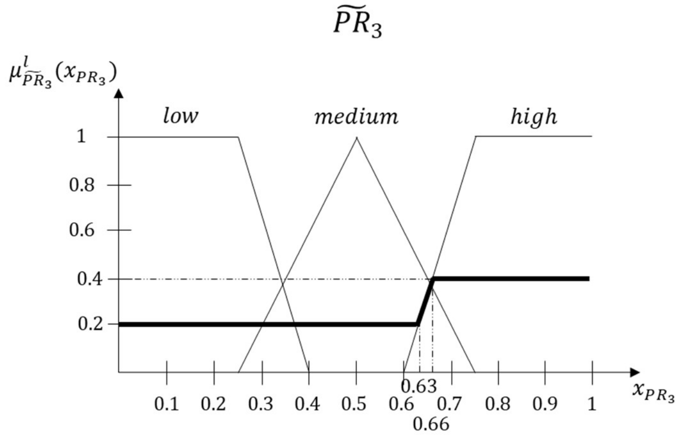

Figure 17: Membership function of $\widetilde{PR}_3$

Decision rule block $k = 4$:

The decision rule block $k = 4$ corresponds with the acceptance rule DR_4 from the basic model. This requires $J_4 \wedge \neg J_5 \rightarrow \neg(I_1, I_2)$. The fuzzy rule block $k = 4$, on the other hand, requires now:

$\widetilde{IAAD} \wedge \widetilde{PAAD} \rightarrow \widetilde{PR}_4$

Now, here again, there are two linguistic input variables, each with three linguistic terms, so that a total of nine rules $m \in \overline{M}_4$ have to be formulated. These are for example:

Rule $m \in \overline{M}_4$	$\widetilde{IAAD}$	$\widetilde{PAAD}$	$\widetilde{PR}_4$
46	low	low	medium
47	medium	low	high
48	high	low	high

49	low	medium	medium
50	medium	medium	high
51	high	medium	high
52	low	high	low
53	medium	high	medium
54	high	high	medium

Table 8: Rule block 4

For example, if $x_{IAAD} = 0.8$ and $x_{PAAD} = 0.55$, inserting them into (3.1), (3.2) as well as (3.3) or from the graphs of membership functions $\mu^l_{\widetilde{IAAD}}$, $\mu^l_{\widetilde{ILL}}$ and $\mu^l_{\widetilde{PAAD}}$, it follows that only rule 51 from this block is active ($DOF > 0$) und the others are inactive ($DOF = 0$) (see figure 18).

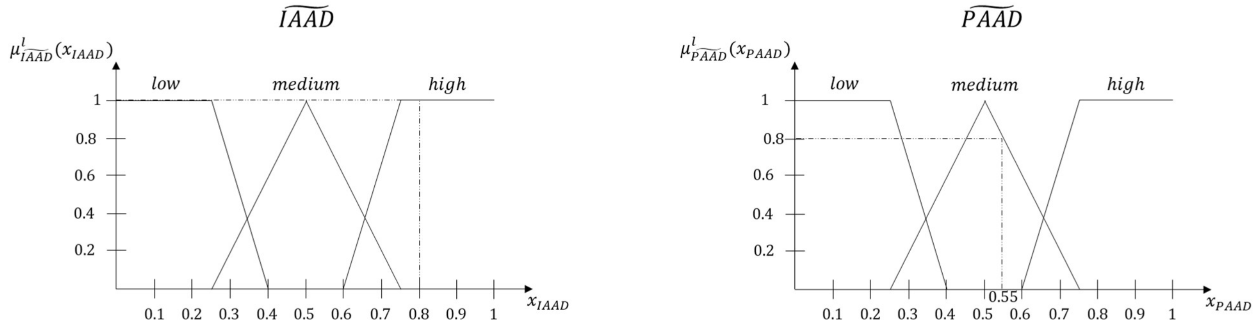

Figure 18: Membership function of $\widetilde{IAAD}$ and $\widetilde{PAAD}$ of decision rule block $k = 4$

Using the determination rule

$$DOF_m = Min\left(\mu^l_{\widetilde{IAAD}}(x_{IAAD}); \mu^l_{\widetilde{PAAD}}(x_{PAAD})\right) \quad \forall m \in \overline{M}_4,$$

the following active rule with the corresponding DOF_m and the expression of the participation rate $\widetilde{PR}^4_m$ result for above rules:

DOF_m	$\widetilde{PR}^4_m$
$DOF_{51} = Min\left(\mu^{high}_{\widetilde{IAAD}}(0.8); \mu^{medium}_{\widetilde{PAAD}}(0.55)\right) = Min(1; 0.8) = 0.8$	high

Table 9: Degree of fulfillment

Correspondingly, the $DOF^{l,4}_{total}$ results as follows:

$DOF^{low,4}_{total} = 0$

$DOF^{medium,4}_{total} = 0$

$DOF^{high,4}_{total} = 0.8$

For the participation rate of the fourth decision rule block $\widetilde{PR}_4$ and the membership function of the corresponding fuzzy output set is obtained (see figure 19):

$\widetilde{PR}_4$

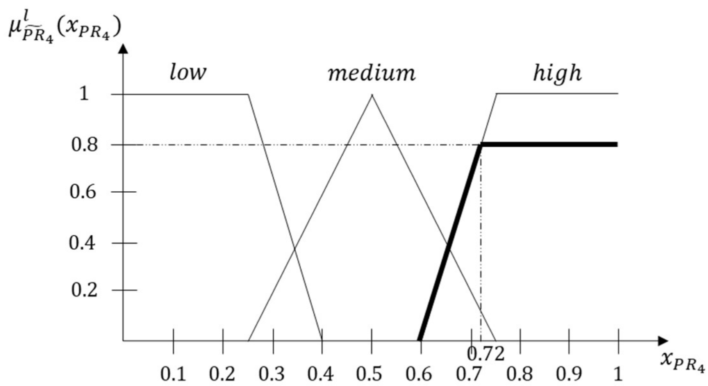

Figure 19: Membership function of $\widetilde{PR}_4$

Decision rule block $k = 5$:

The decision rule block $k = 5$ corresponds with the conflict rule DR_5 from the basic model. This requires $J_4 \wedge \neg J_5 \wedge J_7 \rightarrow \neg(I_1, I_2, I_3)$. The fuzzy rule block $k = 5$, on the other hand, requires now:

$\widetilde{IAAD} \wedge \widetilde{PAAD} \wedge \widetilde{CE} \rightarrow \widetilde{PR}_5$

Here again, there are three linguistic input variables, each with three linguistic terms, so that a total of 27 rules $m \in \bar{M}_5$ have to be formulated. These are for example:

Rule $m \in \bar{M}_5$	$\widetilde{IAAD}$	$\widetilde{PAAD}$	$\widetilde{CE}$	$\widetilde{PR}_5$
55	low	low	low	medium
56	medium	low	low	high
57	high	low	low	high
58	low	low	medium	medium
59	medium	low	medium	medium
60	high	low	medium	medium
61	low	low	high	medium
62	medium	low	high	medium
63	high	low	high	medium
64	low	medium	low	medium
65	medium	medium	low	high
66	high	medium	low	high
67	low	medium	medium	medium
68	medium	medium	medium	medium
69	high	medium	medium	high
70	low	medium	high	low
71	medium	medium	high	medium
72	high	medium	high	medium
73	low	high	low	low

74	medium	high	low	medium
75	high	high	low	medium
76	low	high	medium	medium
77	medium	high	medium	medium
78	high	high	medium	medium
79	low	high	high	low
80	medium	high	high	low
81	high	high	high	medium

Table 10: Rule block 5

For example, if $x_{IAAD} = 0.8$, $x_{PAAD} = 0.55$ and $x_{CE} = 0.7$, inserting them into (3.1), (3.2) as well as (3.3) or from the graphs of membership functions $\mu^{l}_{\widetilde{IAAD}}$, $\mu^{l}_{\widetilde{PAAD}}$ and $\mu^{l}_{\widetilde{CE}}$, it follows that the rules 69 and 72 from this block are active ($DOF > 0$) and the others are inactive ($DOF = 0$) (see figure 20).

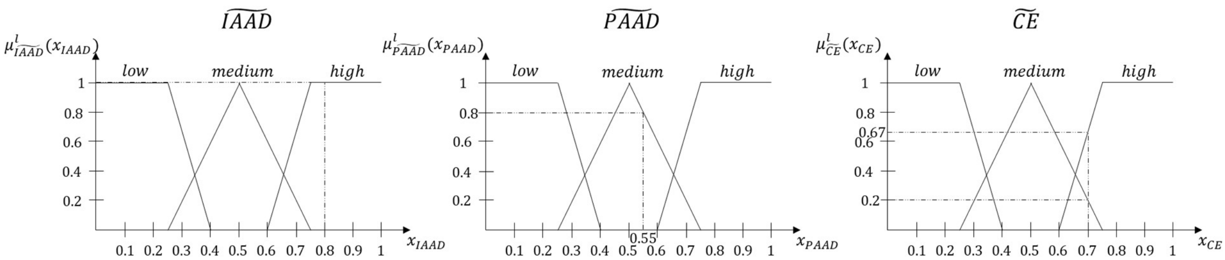

Figure 20: Membership function of $\widetilde{IAAD}$, $\widetilde{PAAD}$ and $\widetilde{CE}$ of decision rule block $k = 5$

Using the determination rule

$$DOF_m = Min\left(\mu^{l}_{\widetilde{IAAD}}(x_{IAAD}); \mu^{l}_{\widetilde{PAAD}}(x_{PAAD}); \mu^{l}_{\widetilde{CE}}(x_{CE})\right) \quad \forall m \in \overline{M}_5,$$

the following active rules with the corresponding DOF_m and the expression of the participation rate $\widetilde{PR}^{5}_{m}$ result for above rules:

DOF_m	$\widetilde{PR}^{5}_{m}$
$DOF_{69} = Min\left(\mu^{high}_{\widetilde{IAAD}}(0.8); \mu^{medium}_{\widetilde{PAAD}}(0.55); \mu^{medium}_{\widetilde{CE}}(0.7)\right) =$ $Min(1; 0.8; 0.2) = 0.2$	high
$DOF_{72} = Min\left(\mu^{high}_{\widetilde{IAAD}}(0.8); \mu^{medium}_{\widetilde{PAAD}}(0.55); \mu^{high}_{\widetilde{CE}}(0.7)\right) =$ $Min(1; 0.8; 0.67) = 0.67$	medium

Table 11: Degrees of fulfillment

Correspondingly, the $DOF^{l,5}_{total}$ results as follows:

$DOF^{low,5}_{total} = 0$

$DOF^{medium,5}_{total} = 0.67$

$DOF^{high,5}_{total} = 0.2$

For the participation rate of the fifth decision rule block $\widetilde{PR}_5$ and the membership function of the corresponding fuzzy output set is obtained (see figure 21):

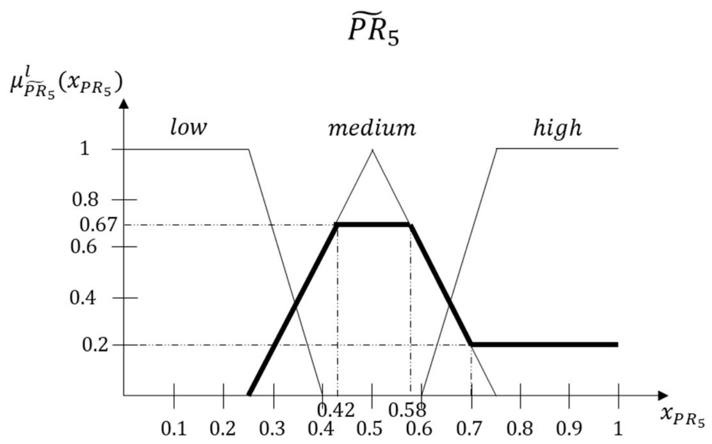

Figure 21: Membership function of $\widetilde{PR}_5$

Decision rule block $k = 6$:

The decision rule block $k = 6$ corresponds with the fairness rule DR_6 from the basic model. This requires $J_4 \wedge \neg J_5 \wedge \neg J_1 \rightarrow \neg(I_1, I_2, I_3, I_4)$. The fuzzy rule block $k = 6$, on the other hand, requires now:

$\widetilde{IAAD} \wedge \widetilde{PAAD} \wedge \widetilde{DQ} \rightarrow \widetilde{PR}_6$

Here again, there are three linguistic input variables, each with three linguistic terms, so that a total of 27 rules $m \in \bar{M}_6$ have to be formulated. These are for example:

Rule $m \in \bar{M}_6$	$\widetilde{IAAD}$	$\widetilde{PAAD}$	$\widetilde{DQ}$	$\widetilde{PR}_6$
82	low	low	low	high
83	medium	low	low	high
84	high	low	low	high
85	low	low	medium	medium
86	medium	low	medium	medium
87	high	low	medium	high
88	low	low	high	medium
89	medium	low	high	medium
90	high	low	high	high
91	low	medium	low	high
92	medium	medium	low	high
93	high	medium	low	high
94	low	medium	medium	medium
95	medium	medium	medium	high
96	high	medium	medium	high
97	low	medium	high	low
98	medium	medium	high	medium
99	high	medium	high	medium

100	low	high	low	low
101	medium	high	low	medium
102	high	high	low	high
103	low	high	medium	low
104	medium	high	medium	low
105	high	high	medium	low
106	low	high	high	low
107	medium	high	high	low
108	high	high	high	medium

Table 12: Rule block 6

For example, if $x_{IAAD} = 0.8$, $x_{PAAD} = 0.55$ and $x_{DQ} = 0.7$, inserting them into (3.1), (3.2) as well as (3.3) or from the graphs of membership functions $\mu^{l}_{\widetilde{IAAD}}$, $\mu^{l}_{\widetilde{PAAD}}$ and $\mu^{l}_{\widetilde{DQ}}$, it follows that the rules 96 und 99 from this block are active ($DOF > 0$) and the others are inactive ($DOF = 0$) (see figure 22).

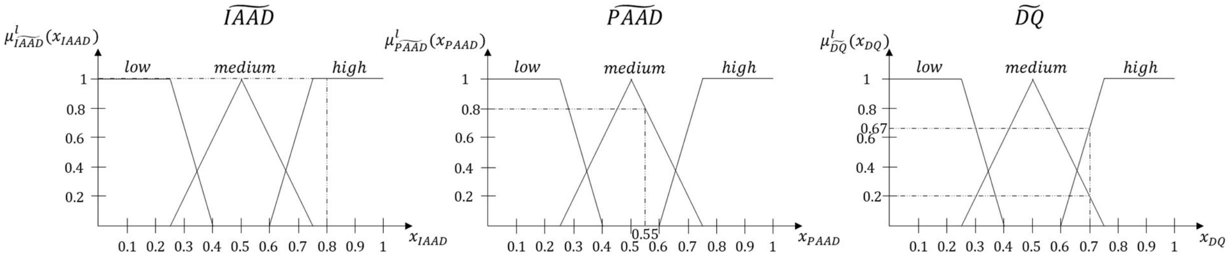

Figure 22: Membership function of $\widetilde{IAAD}$, $\widetilde{PAAD}$ and $\widetilde{DQ}$ of decision rule block $k = 6$

Using the determination rule

$$DOF_m = Min\left(\mu^{l}_{\widetilde{IAAD}}(x_{IAAD}); \mu^{l}_{\widetilde{PAAD}}(x_{PAAD}); \mu^{l}_{\widetilde{DQ}}(x_{DQ})\right) \quad \forall m \in \overline{M}_6,$$

the following active rules with the corresponding DOF_m and the expression of the participation rate $\widetilde{PR}^{6}_{m}$ result for above rules:

DOF_m	$\widetilde{PR}^{6}_{m}$
$DOF_{96} = Min\left(\mu^{high}_{\widetilde{IAAD}}(0.8); \mu^{medium}_{\widetilde{PAAD}}(0.55); \mu^{medium}_{\widetilde{DQ}}(0.7)\right) = Min(1; 0.8; 0.2) = 0.2$	high
$DOF_{99} = Min\left(\mu^{high}_{\widetilde{IAAD}}(0.8); \mu^{medium}_{\widetilde{PAAD}}(0.55); \mu^{high}_{\widetilde{DQ}}(0.7)\right) = Min(1; 0.8; 0.67) = 0.67$	medium

Table 13: Degrees of fulfillment

Correspondingly, the $DOF^{l,6}_{total}$ results as follows:

$DOF^{low,6}_{total} = 0$

$DOF^{medium,6}_{total} = 0.67$

$DOF^{high,6}_{total} = 0.2$

For the participation rate of the sixth decision rule block $\widetilde{PR}_6$ and the membership function of the corresponding fuzzy output set is obtained (see figure 23):

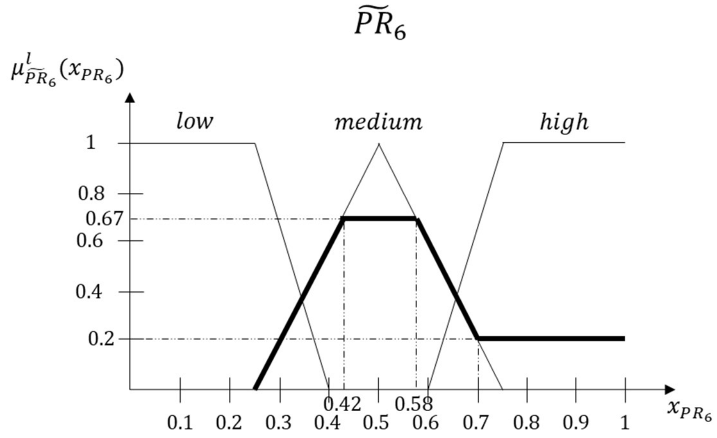

Figure 23: Membership function of $\widetilde{PR}_6$

Decision rule block $k = 7$:

The decision rule block $k = 7$ corresponds with the acceptance prioritization rule DR_7 from the basic model. This requires $J_4 \wedge \neg J_5 \wedge J_6 \rightarrow \neg(I_1, I_2, I_3, I_4)$. The fuzzy rule block $k = 7$, on the other hand, requires now:

$\widetilde{IAAD} \wedge \widetilde{PAAD} \wedge \widetilde{GO} \rightarrow \widetilde{PR}_7$

Here again, there are three linguistic input variables, each with three linguistic terms, so that 27 rules $m \in \bar{M}_7$ have to be formulated. These are for example:

Rule $m \in \bar{M}_7$	$\widetilde{IAAD}$	$\widetilde{PAAD}$	$\widetilde{GO}$	$\widetilde{PR}_7$
109	low	low	low	low
110	medium	low	low	low
111	high	low	low	low
112	low	low	medium	low
113	medium	low	medium	medium
114	high	low	medium	medium
115	low	low	high	high
116	medium	low	high	high
117	high	low	high	high
118	low	medium	low	low
119	medium	medium	low	medium
120	high	medium	low	medium
121	low	medium	medium	medium
122	medium	medium	medium	medium
123	high	medium	medium	high
124	low	medium	high	high
125	medium	medium	high	high
126	high	medium	high	high

127	low	high	low	low
128	medium	high	low	low
129	high	high	low	low
130	low	high	medium	low
131	medium	high	medium	low
132	high	high	medium	low
133	low	high	high	high
134	medium	high	high	high
135	high	high	high	high

Table 14: Rule block 7

For example, if $x_{IAAD} = 0.8$, $x_{PAAD} = 0.55$ and $x_{GO} = 0.55$, inserting them into (3.1), (3.2) as well as (3.3) or from the graphs of membership functions $\mu^{l}_{\widetilde{IAAD}}$, $\mu^{l}_{\widetilde{PAAD}}$ and $\mu^{l}_{\widetilde{GQ}}$, it follows that only the rule 123 from this block is active ($DOF > 0$) and the others are inactive ($DOF = 0$) (see figure 24).

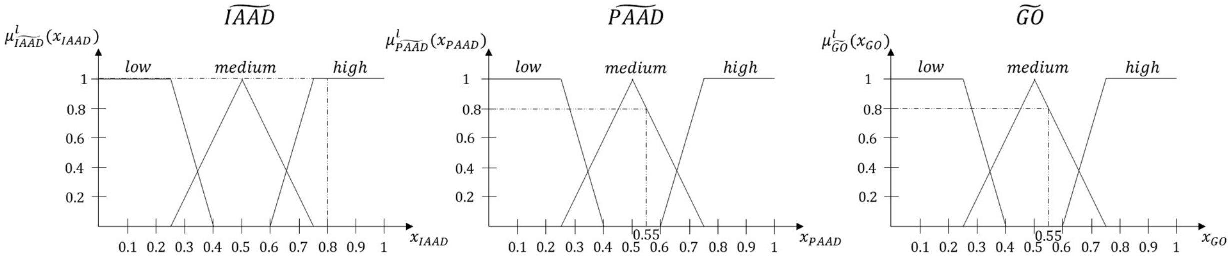

Figure 24: Membership function of $\widetilde{IAAD}$, $\widetilde{PAAD}$ and $\widetilde{GO}$ of decision rule block $k = 7$

Using the determination rule

$$DOF_m = Min\left(\mu^{l}_{\widetilde{IAAD}}(x_{IAAD}); \mu^{l}_{\widetilde{PAAD}}(x_{PAAD}); \mu^{l}_{\widetilde{GO}}(x_{GO})\right) \quad \forall m \in \overline{M}_7,$$

the following active rule with the corresponding DOF_m and the expression of the participation rate $\widetilde{PR}^7_m$ result for above rules:

DOF_m	$\widetilde{PR}^7_m$
$DOF_{123} = Min\left(\mu^{high}_{\widetilde{IAAD}}(0.8); \mu^{medium}_{\widetilde{PAAD}}(0.55); \mu^{medium}_{\widetilde{GO}}(0.55)\right) =$ $Min(1; 0.8; 0.8) = 0.8$	high

Table 15: Degree of fulfillment

Correspondingly, the $DOF^{l,7}_{total}$ results as follows:

$DOF^{low,7}_{total} = 0$

$DOF^{medium,7}_{total} = 0$

$DOF^{high,7}_{total} = 0.8$

For the participation rate of the seventh decision rule block $\widetilde{PR}_7$ and the membership function of the corresponding fuzzy output set is obtained (see figure 25):

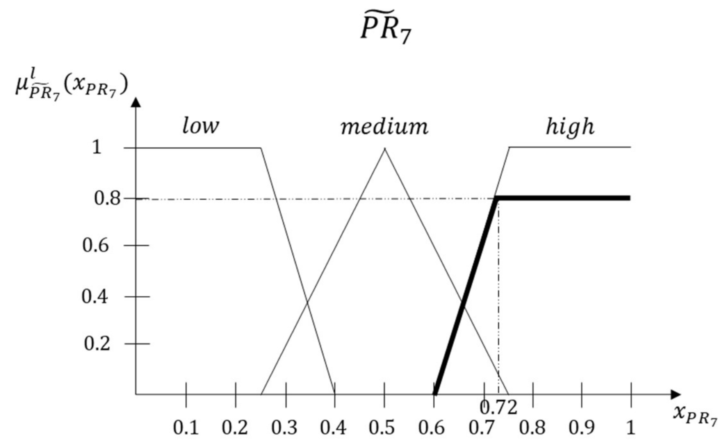

Figure 25: Membership function of $\widetilde{PR}_7$

After processing all (seven) rule blocks, the total output and the membership function of the total participation rate can be derived (see figure 26):

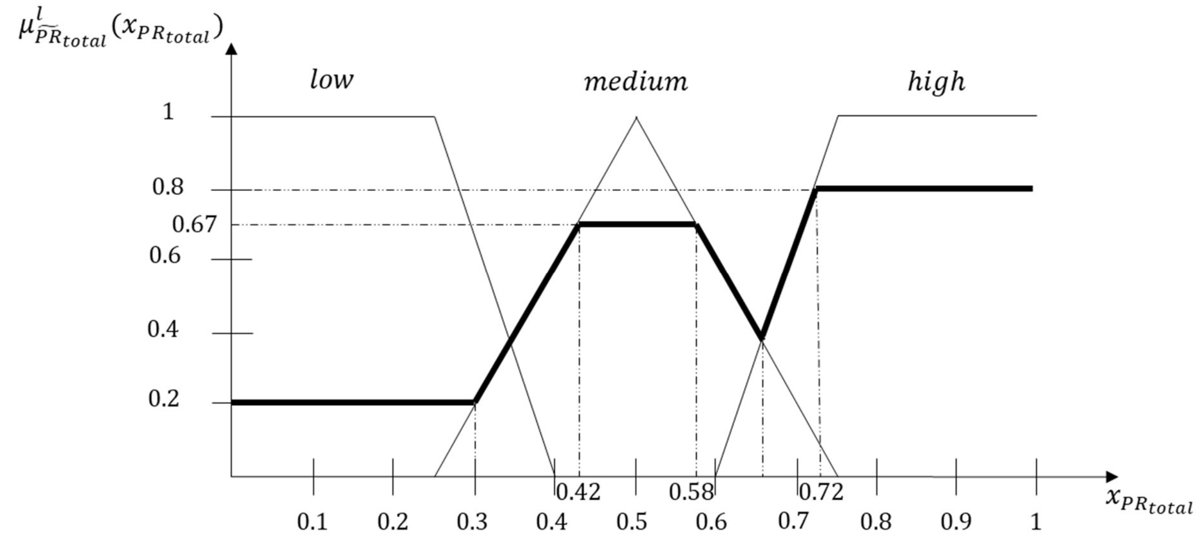

Figure 26: Membership function of $\mu^l_{PR_{total}}(x_{PR_{total}})$

3.5 Defuzzification of the fuzzy output set

To obtain a precise conclusion, it may be useful to defuzzify the fuzzy total participation rate $\widetilde{PR}_{total}$. For this purpose – if one follows the Time Investment Model (Vroom/Yetton 1973) – various maximum methods can be considered (Piegat 2001, Spengler/Herzog 2023). If the first-of-maxima-method (respectively last-of-maxima-method) is chosen, for example, $x_{PR_{total}} = 0{,}72$ (respectively 1) see figure 27. On the other hand, if one follows the Time Efficient Model (Vroom/Yetton 1973), one would choose a minimum method: With the first-of-minimum method, $x_{PR_{total}}$ would be 0 in the above example and 0.3 with last-of-minimum-method.

However, in fuzzy control, the center-of-gravity-method is also frequently used. The center of gravity (*COG*) of an area can be understood as its center point. The *COG* of a membership function is the center of mass of the membership values. In order to compute centroids, one must determine first of all the contents of the area. As is well known, integral calculus is used for this purpose, especially for (at least partially) curved function graphs. For the exact procedure in detail, see e.g. Spengler/Herzog (2023).

The membership function of $\widetilde{PR}_{total}$ has the following shape in above example:

$$\mu_{\widetilde{PR}_{total}}\left(x_{PR_{total}}\right) = \begin{cases} 0.2 \quad for\ 0 \leq x_{PR_{total}} \leq 0.3 \\ \dfrac{x-0.25}{0.25} \quad for\ 0.3 < x_{PR_{total}} \leq 0.42 \\ \dfrac{2}{3} \quad for\ 0.42 < x_{PR_{total}} \leq 0.58 \\ \dfrac{0.75-x}{0.25} \quad for\ 0.58 < x_{PR_{total}} \leq 0.66 \\ \dfrac{x-0.6}{0.15} \quad for\ 0.66 < x_{PR_{total}} \leq 0.72 \\ 1 \quad for\ 0.72 < x_{PR_{total}} \leq 1 \end{cases}$$

With the center-of-gravity-method, the following characteristics can be determined for the abscissa coordinate x_{cog} respectively for the ordinate coordinate $\mu(x_{cog})$ of the centroid of the area (see figure 27):

$$x_{cog} = 0{,}62$$

$$\mu\left(x_{cog}\right) = 0{,}31$$

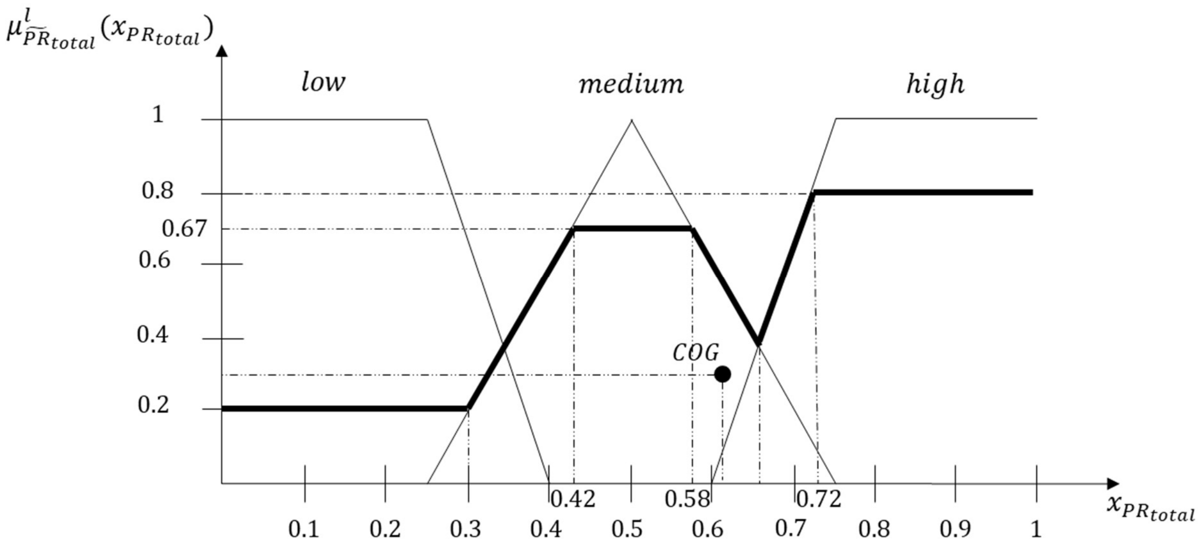

Figure 27: Representation of the fuzzy output set and corresponding center of gravity

$x_{cog} = 0{,}62$ can then be interpreted as the mean participation rate in the above example.

4 Conclusion

The normative decision model of Vroom & Yetton represents a predominantly plausibly designed expert system based on crisp rules.[6] It has been widely tested, evaluated, and criticized (Auer-Rizzi/Reber 2013, Duncan/France/Ginter 2003, Erffmeyer 1983, Field 1979, 1982, Field/Andrews 1998, Field/House 1990, Horgan/Simeon 1990, Gomolka/Mackin 1984, Margerison/Glube 1979, Pate 1987, Samosudova 2017, Vignesh 2020, Vroom 2003, Vroom/Jago 2007, Wedley/Field 1984, Zimmer 1978).

Most considerations of the Vroom/Yetton model refer to the crisp case, although a few (and little differentiated) considerations of the fuzzy case have also been made in the scientific literature (Sosnin/Molotov 2017). In addition, connections have already been made to the AHP model (Mls/Otĉenáŝková 2013), the locus of control concept (Selart 2005) and framing theory (Moser 1990).

Furthermore, practical applications of the model have also been studied (e.g., by Kağnıcıoğlu/Ateşb 2014, Pasewark 1990).

The Vroom/Yetton model is an instrument for the determination of a situationally appropriate leadership decision, whereby the univocal assessment of the characteristics with regard to existing situational determinants is assumed. However, the crisp assessment of the situation determinants is hardly possible in practice, which is why the fuzzification of this model makes sense from our point of view and enables an effective application.

In considering the fuzzy case, we deliberately focus in this paper on fuzzy rule systems rather than fuzzy decision trees, although such have already been constructed in the scientific literature (Baldwin/Xie 2005, Cintra/Monard/ Camargo 2012, Hall/Lande 1998, Janikow 1998, Lertworaprachaya/Yang/John 2010, Olaru/Wehenkel 2003, Yuan/Shaw 1995). However, we consider our approach with continous input and output sets to be more promising. The rule system presented here comprises 135 fuzzy decision rules grouped into seven rule blocks. These blocks are modeled on the univocal rule system of Vroom & Yetton (1973). When applying the ambiguous rule system, one can leave the resulting output set in the fuzzy state or defuzzify it. We conclude the third chapter by showing suitable defuzzification possibilities.

---

[6] In chapter 2.4 of this paper, we formulate an improved decision tree compared to the original.

Appendix

Case 1: None of the questions is answered with "no". There is exactly 1 possibility here.

CP	J_1	J_2	J_3	J_4	J_5	J_6	J_7
1	**yes**	**yes**	**yes**	**yes**	**yes**	**yes**	**yes**

Case 2: One of the questions is answered with "no". Here the binomial coefficient has to be determined and there are exactly $\binom{7}{1} = \frac{7!}{1! \cdot (7-1)!} = 7$ possibilities (combinations without repetition) to answer the questions with "yes" and "no". The general rule for the binomial coefficient is: $\binom{n}{k} = \frac{n!}{k! \cdot (n-k)!}$.

CP	J_1	J_2	J_3	J_4	J_5	J_6	J_7
2	**no**	**yes**	**yes**	**yes**	**yes**	**yes**	**yes**
3	**yes**	**no**	**yes**	**yes**	**yes**	**yes**	**yes**
4	**yes**	**yes**	**no**	**yes**	**yes**	**yes**	**yes**
5	**yes**	**yes**	**yes**	**no**	**yes**	**yes**	**yes**
6	**yes**	**yes**	**yes**	**yes**	**no**	**yes**	**yes**
7	**yes**	**yes**	**yes**	**yes**	**yes**	**no**	**yes**
8	**yes**	**yes**	**yes**	**yes**	**yes**	**yes**	**no**

Case 3: Two of the questions are answered with "no". Here there are exactly $\binom{7}{2} = \frac{7!}{2! \cdot (7-2)!} =$ 21 possibilities to answer the questions with "yes" and "no".

CP	J_1	J_2	J_3	J_4	J_5	J_6	J_7
9	**no**	**no**	**yes**	**yes**	**yes**	**yes**	**yes**
10	**no**	**yes**	**no**	**yes**	**yes**	**yes**	**yes**
11	**no**	**yes**	**yes**	**no**	**yes**	**yes**	**yes**
12	**no**	**yes**	**yes**	**yes**	**no**	**yes**	**yes**
13	**no**	**yes**	**yes**	**yes**	**yes**	**no**	**yes**
14	**no**	**yes**	**yes**	**yes**	**yes**	**yes**	**no**
15	**yes**	**no**	**no**	**yes**	**yes**	**yes**	**yes**
16	**yes**	**no**	**yes**	**no**	**yes**	**yes**	**yes**
17	**yes**	**no**	**yes**	**yes**	**no**	**yes**	**yes**
18	**yes**	**no**	**yes**	**yes**	**yes**	**no**	**yes**

19	yes	no	yes	yes	yes	yes	no
20	yes	yes	no	no	yes	yes	yes
21	yes	yes	no	yes	no	yes	yes
22	yes	yes	no	yes	yes	no	yes
23	yes	yes	no	yes	yes	yes	no
24	yes	yes	yes	no	no	yes	yes
25	yes	yes	yes	no	yes	no	yes
26	yes	yes	yes	no	yes	yes	no
27	yes	yes	yes	yes	no	no	yes
28	yes	yes	yes	yes	no	yes	no
29	yes	yes	yes	yes	yes	no	no

Case 4: Three of the questions are answered with "no". Here there are exactly $\binom{7}{3} = \frac{7!}{3!\cdot(7-3)!} =$ 35 possibilities to answer the questions with "yes" and "no".

CP	J_1	J_2	J_3	J_4	J_5	J_6	J_7
30	no	no	no	yes	yes	yes	yes
31	no	no	yes	no	yes	yes	yes
32	no	no	yes	yes	no	yes	yes
33	no	no	yes	yes	yes	no	yes
34	no	no	yes	yes	yes	yes	no
35	no	yes	no	no	yes	yes	yes
36	no	yes	no	yes	no	yes	yes
37	no	yes	no	yes	yes	no	yes
38	no	yes	no	yes	yes	yes	no
39	no	yes	yes	no	no	yes	yes
40	no	yes	yes	no	yes	no	yes
41	no	yes	yes	no	yes	yes	no
42	no	yes	yes	yes	no	no	yes
43	no	yes	yes	yes	no	yes	no
44	no	yes	yes	yes	yes	no	no
45	yes	no	no	no	yes	yes	yes
46	yes	no	no	yes	no	yes	yes
47	yes	no	no	yes	yes	no	yes

48	yes	no	no	yes	yes	yes	no
49	yes	no	yes	no	no	yes	yes
50	yes	no	yes	no	yes	no	yes
51	yes	no	yes	no	yes	yes	no
52	yes	no	yes	yes	no	no	yes
53	yes	no	yes	yes	no	yes	no
54	yes	no	yes	yes	yes	no	no
55	yes	yes	no	no	no	yes	yes
56	yes	yes	no	no	yes	no	yes
57	yes	yes	no	no	yes	yes	no
58	yes	yes	no	yes	no	no	yes
59	yes	yes	no	yes	no	yes	no
60	yes	yes	no	yes	yes	no	no
61	yes	yes	yes	no	no	no	yes
62	yes	yes	yes	no	no	yes	no
63	yes	yes	yes	no	yes	no	no
64	yes	yes	yes	yes	no	no	no

Case 5: Four of the questions are answered with "no". Here there are exactly $\binom{7}{4} = \frac{7!}{4!\cdot(7-4)!} =$ 35 possibilities to answer the questions with "yes" and "no".

CP	J_1	J_2	J_3	J_4	J_5	J_6	J_7
65	no	no	no	no	yes	yes	yes
66	no	no	no	yes	no	yes	yes
67	no	no	no	yes	yes	no	yes
68	no	no	no	yes	yes	yes	no
69	no	no	yes	no	no	yes	yes
70	no	no	yes	no	yes	no	yes
71	no	no	yes	no	yes	yes	no
72	no	no	yes	yes	no	no	yes
73	no	no	yes	yes	no	yes	no
74	no	no	yes	yes	yes	no	no
75	no	yes	no	no	no	yes	yes
76	no	yes	no	no	yes	no	yes

77	**no**	**yes**	**no**	**no**	**yes**	**yes**	**no**
78	**no**	**yes**	**no**	**yes**	**no**	**no**	**yes**
79	**no**	**yes**	**no**	**yes**	**no**	**yes**	**no**
80	**no**	**yes**	**no**	**yes**	**yes**	**no**	**no**
81	**no**	**yes**	**yes**	**no**	**no**	**no**	**yes**
82	**no**	**yes**	**yes**	**no**	**no**	**yes**	**no**
83	**no**	**yes**	**yes**	**no**	**yes**	**no**	**no**
84	**no**	**yes**	**yes**	**yes**	**no**	**no**	**no**
85	**yes**	**no**	**no**	**no**	**no**	**yes**	**yes**
86	**yes**	**no**	**no**	**no**	**yes**	**no**	**yes**
87	**yes**	**no**	**no**	**no**	**yes**	**yes**	**no**
88	**yes**	**no**	**no**	**yes**	**no**	**no**	**yes**
89	**yes**	**no**	**no**	**yes**	**no**	**yes**	**no**
90	**yes**	**no**	**no**	**yes**	**yes**	**no**	**no**
91	**yes**	**no**	**yes**	**no**	**no**	**no**	**yes**
92	**yes**	**no**	**yes**	**no**	**no**	**yes**	**no**
93	**yes**	**no**	**yes**	**no**	**yes**	**no**	**no**
94	**yes**	**no**	**yes**	**yes**	**no**	**no**	**no**
95	**yes**	**yes**	**no**	**no**	**no**	**no**	**yes**
96	**yes**	**yes**	**no**	**no**	**no**	**yes**	**no**
97	**yes**	**yes**	**no**	**no**	**yes**	**no**	**no**
98	**yes**	**yes**	**no**	**yes**	**no**	**no**	**no**
99	**yes**	**yes**	**yes**	**no**	**no**	**no**	**no**

<u>Case 6:</u> Five of the questions are answered with "no". Here there are exactly $\binom{7}{5} = \frac{7!}{5!\cdot(7-5)!} =$ 21 possibilities to answer the questions with "yes" and "no".

CP	J_1	J_2	J_3	J_4	J_5	J_6	J_7
100	**no**	**no**	**no**	**no**	**no**	**yes**	**yes**
101	**no**	**no**	**no**	**no**	**yes**	**no**	**yes**
102	**no**	**no**	**no**	**no**	**yes**	**yes**	**no**
103	**no**	**no**	**no**	**yes**	**no**	**no**	**yes**
104	**no**	**no**	**no**	**yes**	**no**	**yes**	**no**
105	**no**	**no**	**no**	**yes**	**yes**	**no**	**no**

106	no	no	yes	no	no	no	yes
107	no	no	yes	no	no	yes	no
108	no	no	yes	no	yes	no	no
109	no	no	yes	yes	no	no	no
110	no	yes	no	no	no	no	yes
111	no	yes	no	no	no	yes	no
112	no	yes	no	no	yes	no	no
113	no	yes	yes	no	no	no	no
114	no	yes	no	yes	no	no	no
115	yes	no	no	no	no	no	yes
116	yes	no	no	no	no	yes	no
117	yes	no	no	no	yes	no	no
118	yes	no	no	yes	no	no	no
119	yes	no	yes	no	no	no	no
120	yes	yes	no	no	no	no	no

<u>Case 7:</u> Six of the questions are answered with "no". Here there are exactly $\binom{7}{6} = \frac{7!}{6!\cdot(7-6)!} = 7$ possibilities to answer the questions with "yes" and "no".

CP	J_1	J_2	J_3	J_4	J_5	J_6	J_7
121	no	no	no	no	no	no	yes
122	no	no	no	no	no	yes	no
123	no	no	no	no	yes	no	no
124	no	no	no	yes	no	no	no
125	no	no	yes	no	no	no	no
126	no	yes	no	no	no	no	no
127	yes	no	no	no	no	no	no

<u>Case 8:</u> All questions are answered with "no". There is exactly 1 possibility here.

CP	J_1	J_2	J_3	J_4	J_5	J_6	J_7
128	no	no	no	no	no	no	no

Check: $1 + \binom{7}{1} + \binom{7}{2} + \binom{7}{3} + \binom{7}{4} + \binom{7}{5} + \binom{7}{6} + 1 = 128$

References

Auer-Rizzi, W./Reber, G. (2013): Leadership styles: Inertia and changes in the Czech Republic, in: Journal for East European Management Studies, Vol. 18, No. 1, pp. 9-35.

Baldwin, J. F./Xie, D. (2005): Simple fuzzy logic rules based on fuzzy decision tree for classification and prediction problem, in: International Conference on Intelligent Information Processing (IIP2004), October 21–23, 2004, Springer US: China pp. 175-184.

Bellmann R. E./Zadeh, L. A. (1970): Decision-Making in a Fuzzy Environment, in: Management Science, Vol. 17, No. 4, pp. 141-164.

Blake, R. R./Mouton, J. S./Bidwell, A. C. (1962): Managerial grid, in: Advanced Management-Office Executive, Vol. 1, No. 9, pp. 12-15.

Bouchon-Meunier, B. (1991): Inferences With Imprecisions and Uncertainities in Expert Systems, in: Kandel, A. (Ed.) Fuzzy Expert Systems, pp. 43-54, CRC Press: London et al.

Buckley, J. J./Eslami, E. (2002): An Introduction to Fuzzy Logic and Fuzzy Sets, Springer: Heidelberg/New York.

Cintra, M. E./Monard, M. C./Camargo, H. A. (2012): FuzzyDT—a fuzzy decision tree algorithm based on C 4.5, in: Proceedings of the Brazilian Congress on Fuzzy Systems, June 10-15, 2012, IEEE: Australia, pp. 199-211.

Driankov, D./Hellendoorn, H./Reinfrank, M. (1993): An Introduction to Fuzzy Control, Springer: Berlin/Heidelberg.

Dubois, D./Ostasiewicz, W./Prade, H. (2000): Fuzzy Sets: History and Basic Notions, in: Dubois, D./Prade, H. (Ed.): Fundamentals of Fuzzy Sets, pp. 21-124., Springer: Boston et al.

Dubois, D./Prade, H. (1978): Operations on Fuzzy Numbers, in: International Journal of Systems and Science, Vol. 9, No. 6, pp. 613-626.

Dubois, D./Prade, H. (1980a): Fuzzy Sets and Systems: Theory and Applications, Academic Press: New York et al.

Dubois, D./Prade, H. (1980b): New results about properties and semantics of fuzzy set-theoretic operators, in: Wang, P. P./ Chang, S. K. (Ed.): Fuzzy Sets, pp. 59-75, Springer: Boston.

Dubois, D./Prade, H. (1991): Fuzzy sets in approximate reasoning, Part 1: Inference with possibility distributions, in: Fuzzy Sets and Systems, Vol. 40, pp. 143-202.

Duncan, W. J./LaFrance, K. G./Ginter, P. M. (2003): Leadership and decision making: A retrospective application and assessment, in: Journal of Leadership & Organizational Studies, Vol. 9, No. 4, pp. 1-20.

Erffmeyer, E. S. (1983): The Vroom-Yetton Model of Decision Making: an Empirical Evaluation in Applied Settings, LSU Digital Commons: Louisiana (Doctoral dissertation, University of Louisiana).

Fiedler, F. E. (1967): A Theory of Leadership Effectiveness, McGraw-Hill Series in Management: New York.

Fiedler, F. E. (1978): The contingency model and the dynamics of the leadership process, in: Advances in experimental social psychology, Vol. 11, pp. 59-112.

Field, R. G. (1979): A critique of the Vroom-Yetton contingency model of leadership behavior, in: Academy of Management Review, Vol. 4, No. 2, pp. 249-257.

Field, R. G. (1982): A test of the Vroom-Yetton normative model of leadership, in: Journal of Applied Psychology, Vol. 67, No. 5, pp. 523-532.

Field, R. H./Andrews, J. P. (1998): Testing the incremental validity of the Vroom–Jago versus Vroom–Yetton models of participation in decision making, in: Journal of Behavioral Decision Making, Vol. 11, No. 4, pp. 251-261.

Field, R. H./House, R. J. (1990): A test of the Vroom-Yetton model using manager and subordinate reports, in: Journal of applied psychology, Vol. 75, No. 3, pp. 362-366.

Fodor, J./Yager, R. R. (2000): Fuzzy-Set-Theoretic Operators and Quantifiers, in: Dubois, D./ Prade, H. (Ed.): Fundamentals of Fuzzy Sets, pp. 125-193, Springer: Boston et al.

Gomolka, E. G./Mackin, J. L. (1984): Individual vs. Group Grade: An Exercise in Decision Making, in: Developments in Business Simulation and Experiential Learning, Proceedings of the Annual ABSEL conference, Hawaii, Vol. 11, pp. 128-132.

Gottwald, S. (1993): Fuzzy Sets and Fuzzy Logic. Foundations of Application from a Mathematical Point of View, Springer: Wiesbaden.

Hall, L. O./Kandel, A. (1991): The Evolution From Expert Systems to Fuzzy Expert Systems, in: Kandel, A. (Ed.) Fuzzy Expert Systems, pp. 3-21, CRC Press: London et al.

Hall, L. O./Lande, P. (1998): Generation of Fuzzy Rules from Decision Trees, in: Journal of Advanced Computational Intelligence and Intelligent Informatics, Vol. 2, No. 4, pp. 128-133.

Hersey, P./Blanchard, K. H./ Johnson, D. E (1996): Management of Organizational Behavior, 7th edition, USA: Prentice Hall.

Hill, T. E./Schmitt, N. (1977): Individual differences in leadership decision making, in: Organizational Behavior and Human Performance, Vol. 19, No. 2, pp. 353-367.

Horgan, D. D./Simeon, R. J. (1990): Mentoring and participation: An application of the Vroom-Yetton model, in: Journal of Business and Psychology, Vol. 5, No. 1, pp. 63-84.

Jago, A. G./Ettling, J. T./ Vroom, V. H. (1985): Validating a Revision to the Vroom/Yetton Model: First Evidence, in: Academy of Management Proceedings, Briarcliff Manor (NY 10510), pp. 220-223.

Janikow, C. Z. (1998): Fuzzy decision trees: issues and methods, in: IEEE Transactions on Systems, Man, and Cybernetics - Part B (Cybernetics), Vol. 28, No. 1, pp. 1-14.

Kağnıcıoğlu, C. H./Ateşb, S. S. (2014): Vroom and Yetton's Decision Tree for the Selection of Leadership Style: An Implementation in Ataturk Airport, in: Global Interdisciplinary Business-Economics Advancement Conference, Conference Proceedings, May 15-18, GIBA: Florida, pp. 415-421.

Klement, E. P./Mesiar, R./Pap, E. (2004): Triangular norms. Position paper I: basic analytical and algebraic properties, in: Fuzzy Sets and Systems, Vol. 143, pp. 5-26.

Lertworaprachaya, Y./Yang, Y./John, R. (2010): Interval-valued fuzzy decision trees, in: International Conference on Fuzzy Systems, July 18-23, Spain: IEEE, pp. 1-7.

Likert, R. (1967): The human organization: its management and values, McGraw-Hill Book Company: New York.

Lunenburg, F. C. (2010): Models of decision making, in: Focus on Colleges, Universities, And Schools, Vol. 4, No. 1, pp. 1-9.

Mamdani, E.H. (1981): Advances in the Linguistic Synthesis of Fuzzy Controllers, in: Mamdani, E.H./Gaines, B.R. (eds.): Fuzzy Reasoning and its Applications, Academic Press: London, pp. 311-323.

Margerison, C./Glube, R. (1979): Leadership decision-making: An empirical test of the vroom and yetton model, in: Journal of Management Studies, Vol. 16, No. 1, pp. 45-55.

Mls, K./Otĉenáŝková, T. (2013): Analysis of complex decisional situations in companies with the support of AHP extension of Vroom-Yetton contingency model, Februrary, in: IFAC Proceedings Volumes, Austria, Vol. 46, No. 9, pp. 549-554.

Moser, S. B. (1990): The influence of framing on the decision over how much participation to allow from subordinates in organizational decision-making: an exploration of the Vroom and Jago model (Doctoral dissertation, University of Cincinnati).

Olaru, C./Wehenkel, L. (2003): A complete fuzzy decision tree technique, in: Fuzzy sets and systems, Vol. 138, No. 2, pp. 221-254.

Pap, E. (2002): Aggregation Operators in Engineering Design, in: Calvo, T./ Mayor, G./ Mesiar, R. (Ed.): Aggregation Operators. New Trends and Applications, pp. 195-223, Physica: Heidelberg, New York.

Pasewark, W. R./Welker, R. B. (1990): A Vroom-Yetton Evaluation of Subordinate Participation in Budgetary Decision Making, in: Journal of Management Accounting Research, Vol. 2, No. 1, pp. 113-126.

Pashiardis, P. (1993): Group decision making: The role of the principal, in: International Journal of Educational Management, Vol. 7, No. 2, pp. 8-11.

Pate, L. B./Heiman, D. C. (1987): A Test of the Vroom-Yetton Decision Model in Seven Field Settings, in: Personnel Review, Vol. 16, No. 2, pp. 22-26.

Pedrycz, W. (1993): Fuzzy Control and Fuzzy System, 2. Edition, Wiley: New York et al.

Piegat, A. (2001): Fuzzy Modeling and Control, Physica: Heidelberg, New York.

Reddin, W. J. (1970): Managerial effectiveness, New York: ERIC Institute of Educational Science.

Samosudova, N. V. (2017): Modern leadership and management methods for development organizations, MATEC Web of Conferences, October 28 – November 01, 2016, in: EDP Sciences, Taiwan, pp. 1-10.

Schneider, M./Kandel, A. (1991): General Purpose Fuzzy Expert Systems, in: Kandel, A. (Ed.) Fuzzy Expert Systems, pp. 23-41, CRC Press: London et al.

Selart, M. (2005): Understanding the role of locus of control in consultative decision-making: a case study, in: Management Decision, Vol. 43, No. 3, pp. 397-412.

Skrynnyk, O. (2020): Surrogate leadership model for digital organizational systems, in: Business Ethics and Leadership, Vol. 4, No. 4, pp. 140-146.

Sosnin, P./Molotov, R. (2017): Fuzzy logic in VYJ Decision Model, INTERACTIVE SYSTEMS: Problems of Human-Computer Interaction – Collection of scientific papers, in: USTU, Ulyanovsk, pp. 224-229.

Spengler, T./Herzog, S. (2023): Defuzzification in Scenario Management – A theoretical and practical Guide. In: FEMM Working Paper Series, Working Paper No. 05/2023, Magdeburg.

Stogdill, R. M./Coons, A. E. (1957): Leader behavior: Its description and measurement, OSU Bureau of Business Research, College of Commerce a Columbus: Ohio.

Tannenbaum, R./Schmidt, W. H. (1973): How to choose a leadership pattern, In: Harvard Business Review Press, Vol. 1958, No. 2, pp. 95-101.

Vignesh, M. (2020): Decision Making using Vroom-Yetton-Jago model with a practical application, in: International Journal for Research in Applied science & Engineering Technology, Vol. 8, No. 1, pp. 330-337.

Vroom, V. H. (1959): Some personality determinants of the effects of participation (Doctoral dissertation, University Göttingen).

Vroom, V. H. (1964): Work and motivation, John Wiley Sons: New York, London and Sydney.

Vroom, V. H. (1967): Leadership, in: Handbook of Industrial and Organizational Psychology, Dunnette, M. D. (Ed.), Rand McNally College Publishing Company: Chicago, pp. 1527-1551.

Vroom, V. H. (1981): Führungsentscheidungen in Organisationen, in: Die Betriebswirtschaft, Vol. 41, No. 1, pp. 183-193.

Vroom, V. H. (2003): Educating managers for decision making and leadership, in: Management decision, Vol. 41, No. 10, pp. 968-978.

Vroom, V. H./Jago, A. G. (1988): Managing participation: A critical dimension of leadership, in: Journal of Management Development, Vol. 7, No. 5, pp. 32-42.

Vroom, V. H./Jago, A. G. (1991): Flexible Führungsentscheidungen: Management der Partizipation in Organisationen, C. E. Poeschel Verlag: Stuttgart.

Vroom, V. H./Jago, A. G. (1995): Situation effects and levels of analysis in the study of leader participation, in: The Leadership Quarterly, Vol. 6, No. 2, pp. 169-181.

Vroom, V. H./Jago, A. G. (2007): The role of the situation in leadership, in: American psychologist, Vol. 62, No. 1, pp. 17-24.

Vroom, V. H./Yetton, P. W. (1973): Leadership and decision-making, University of Pittsburgh Press: USA.

Vroom, V./Yetton, P. W./Jago, A. (2015): Normative decision process theory, in: Organizational Behavior 1. Essential theories of Motivation and Leadership (Miner, J. B.), Routledge: London and New York, pp. 207-232.

Wang, H.-F./Chang, W.-Y. (2000): Fuzzy Scenario Analysis in Strategic Planning, in: Int. J. General Systems, Vol. 30, No. 2, pp. 193-207.

Wedley, W. C./Field, R. H. (1984): A predecision support system, in: Academy of Management Review, Vol. 9, No. 4, pp. 696-703.

Yager, R. R. (1980): On a General Class of Fuzzy Connectives, in: Fuzzy Sets and Systems, Vol. 3, pp. 235-242.

Yager, R. R. (1991): On the Representation of Relational Production Rules in Expert Systems, in: Kandel, A. (Ed.) Fuzzy Expert Systems, pp. 55-67, CRC Press: London et al.

Yuan, Y./Shaw, M. J. (1995): Induction of fuzzy decision trees, in: Fuzzy Sets and systems, Vol. 69, No. 2, pp. 125-139.

Zadeh, L. A. (1965): Fuzzy Sets, in: Information and Control, Vol. 8, pp. 338-353.

Zadeh, L. A: (1975): The Concept of a Linguistic Variable and its Application to Approximate Reasoning – Part I, in: Information Sciences, Vol. 8, pp. 199-249.

Zadeh, L. A. (1978): Fuzzy Sets as a Basis for a Theory of Possibility, in: Fuzzy sets and systems, Vol. 1, pp. 3-28.

Zadeh, L. A. (1983): The role of fuzzy logic in the management of uncertainty in expert systems, in: Fuzzy sets and systems, Vol. 11, No. 1-3, pp. 199-227.

Zadeh, L. A. (1987): The Concept of a Linguistic Variable and its Application to Approximate Reasoning – Part II, in: Yager, R. R./ Ovchinnikov, S./ Ton, R. M./ Nguyen, H. T. (Ed.): Fuzzy Sets and Applications: Selected Papers by L. A. Zadeh, pp. 271-327, Wiley: New York et al.

Zimmer, R. J. (1978): Validating the Vroom-Yetton normative model of leader behavior in field sales force management and measuring the training effect of Telos on the leader behavior of district managers (Doctoral dissertation, Virginia Tech University).

Zimmermann, H.-J. (1987): Fuzzy Sets, Decision Making, and Expert Systems, Kluwer Academic: Boston et al.

Zimmermann, H.-J. (1996): Fuzzy Set Theory – and Its Applications, Third Printing, Kluwer-Nijhoff: Boston et al.

Otto von Guericke University Magdeburg
Faculty of Economics and Management
P.O. Box 4120 | 39016 Magdeburg | Germany

Tel.: +49 (0) 3 91 / 67-1 85 84
Fax: +49 (0) 3 91 / 67-1 21 20

www.fww.ovgu.de/femm

ISSN 1615-4274