Finance Research Letters 11 (2014) 25–35

Contents lists available at ScienceDirect

Finance Research Letters

journal homepage: www.elsevier.com/locate/frl

On the investment–uncertainty relationship: A game theoretic real option approach

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ARTICLE INFO

Article history:
Received 22 April 2013
Accepted 31 July 2013
Available online 9 August 2013

JEL classification:
G30
G13
C70
D81

Keywords:
Real option
Investment
Uncertainty
Managerial flexibility
Game theory

ABSTRACT

This paper examines the effect of uncertainty on investment timing in a game theoretic real option model. We extend the settings of the related recent literature on investment timing under uncertainty by a more general assumption, i.e. the investment is also influenced by the actions of a second player. The results show that a U-shaped investment–uncertainty relationship generally sustains even for infinite-lived investment projects and proper defined cash flows. However, timing of an investment occurs inefficiently late. Moreover, we show that the influence of uncertainty on the associated first-mover advantage becomes ambiguous, too.

1. Introduction

The timing of strategic investments is among the most common problems in corporate finance and corporate strategy, respectively. Due to inherent uncertainty an investor has to face the trade-off between early commitment to profit from first-mover advantages and late commitment due to maintenance of flexibility. The latter case is a result of the irreversible nature of most strategic investments, i.e. once an investment is made the incurred sunk costs cannot be recovered should the project be

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http://dx.doi.org/10.1016/j.frl.2013.07.006

abandoned at a later stage. Option-based valuation of investments has been proposed as an analytical tool to address these issues and the literature has provided various examples that give guidance on how to optimally time an investment under uncertainty (see e.g., Dixit and Pindyck, 1994; Trigeorgis, 1998; Kort et al., 2010). Real options express the flexibility assigned to a decision, i.e. for example the decision to delay an investment or to abandon an investment project without being obliged to.[1] Here, the simple investment opportunity represents a perpetual American Call option and the resulting investment–uncertainty relationship is traditionally considered to have negative sign, i.e. the higher the uncertainty the higher the propensity to postpone the investment.

Only recently, however, it has been shown that the investment–uncertainty relationship is not necessary monotonic and that it can be viewed through multiple lenses which may result in a differing observed sign of this relationship (see e.g. Sarkar, 2000; Lund, 2005; Wong, 2007; Gryglewicz et al., 2008). In particular, the investment–uncertainty relationship can be interpreted as

(i) the influence of uncertainty on the value of the optimal investment threshold,
(ii) the influence of uncertainty on the expected time until investment, i.e. the influence of uncertainty on the expected time to hit the threshold,
(iii) the influence of uncertainty on the probability to invest in a pre-specified time, i.e. the influence of uncertainty on the probability to hit the threshold in a pre-specified time.[2]

By correcting the usual assumption in the extent literature that the risk-adjusted return on a project is invariant to the volatility of an investment's returns Sarkar (2000) demonstrates in a numerical example that the influence of uncertainty on the probability to invest in a pre-specified time can have an ambiguous sign. However, in his example the influence of uncertainty on the value of the optimal investment threshold and on the expected time until investment is still strictly positive. His second main contribution is, though, to show that implementing the capital asset pricing model (CAPM) to account for the risk-return relationship is not a sufficient constraint in order to observe a non-monotonic investment–uncertainty relationship either by means of an optimal investment threshold or by means of an expected hitting time. Rather, the subsequent literature highlights that one of the two other constraints must hold. Firstly, it is the discounted project value of an investment that behaves according to a geometric-Brownian motion. By focusing on this assumption Wong (2007) shows that uncertainty has a U-shaped influence on the value of the investment threshold as well as on the expected time until investment. This holds in particular if the risk-adjusted rate of return on the project is positively related to uncertainty. Or secondly, the investment project life is finite. As shown by Gryglewicz et al. (2008) uncertainty has a U-shaped influence on the value of the investment threshold as well as on the expected time until investment if and only if the project life is finite and the risk-adjusted rate of return on the project is positively related to uncertainty.[3]

This paper extends the depicted stream of literature by introducing a third setting that give rise to an ambiguous uncertainty–investment relationship, i.e. the bargaining situation. In particular, we show that a U-shaped pattern of the influence of uncertainty on the value of the investment threshold and on the expected time until investment exist if the investment decision is the outcome of the sequential bargaining of two parties which both have to bear part of the investment costs and the risk-adjusted rate of return on the project is positively related to uncertainty. Such a setting may exist in any situations where two individuals jointly decide on the implementation of investment projects, such as in manager–shareholder or customer–supplier relationships. However, timing of an investment occurs inefficiently late in the sequential bargaining setting. Specifically, the socially inefficiency is the more pronounced the higher the fraction of costs the second player has to bear. While these effects are partly due to a first-mover advantage we furthermore show that the resulting first-mover advantage is also non-monotonically affected by uncertainty. We thereby add also new insights to

[1] For example Dixit and Pindyck (1994).

[2] Following Lund (2005) the investment–uncertainty relationship can also be interpreted as the influence of uncertainty on the expected aggregate investment in a pre-specified time for firms with several investment opportunities.

[3] Interestingly, the influence of uncertainty on the probability to invest in a pre-specified time is analyzed in neither Wong (2007) nor Gryglewicz et al. (2008).

another stream of recent real options literature that addresses the outcome of bargaining under uncertainty (see e.g., Morellec and Zhdanov, 2005; Hackbarth and Morellec, 2008; Lambrecht, 2004; Hackbarth and Miao, 2012; Lukas and Welling, 2012).

The rest of the paper is organized as follows. Section two presents the model and characterizes the solution along two prominent measures in the real option literature, i.e. investment threshold and expected hitting time. Of special interest are the impact of bargaining and the CAPM on the investment–uncertainty relationship and the surplus distribution among the agents. Section three illustrates numerically the impact of uncertainty on timing and on the size of the first-mover advantage. Finally, section four concludes and lays out directions for future research.

2. The model

In the following, we deviate from a canonical real options model à la Dixit and Pindyck (1994). In particular, consider an investment project whose net cash flow per unit time, $x(t)$, can be expressed by the following stochastic differential equation:

$$dx(t) = \alpha x(t)dt + \sigma x(t)dW(t), \quad x(0) = x_0 \tag{1}$$

with $\sigma \in \mathbb{R}^+$ as the volatility of the cash flow stream and $\alpha \in \mathbb{R}$ as the drift rate. The investment project's life is finite of length $T \in \mathbb{R}^+$. Hence, the project's present value $V(t)$ is the given by:

$$V(t) = \mathbf{E}\left[\int_t^{T+t} x(s)e^{-\mu(s-t)}ds\right] = x_t\int_t^{T+t} e^{-(\mu-\alpha)(s-t)}ds = x_t\frac{1-e^{-(r+\lambda\rho\sigma-\alpha)T}}{r+\lambda\rho\sigma-\alpha}, \tag{2}$$

where $\mathbf{E}[\cdots]$ denotes the expectations operator, $\mu = r + \lambda\rho\sigma$ as the risk adjusted discount rate, $\lambda = (\mu_M - \mathrm{r})/\sigma_M$ denotes the market price of risk with μ_M and σ_M as the return and volatility of the market portfolio and r as the riskless rate of interest. Consequently, the dynamics of $V(t)$ are also governed by a geometric Brownian motion, i.e.:

$$dV(t) = \alpha V(t)dt + \sigma V(t)dW(t), \quad V(0) = V_0. \tag{3}$$

In contrast to the previous mentioned literature, however, we will assume that investment in the project requires the actions of two participating individuals A and B. Individual A compensates B by transferring a portion of the asset value $\psi V(x)$ to the second individual B upon investment while B times the initiation of the investment project. Moreover, investment in the project is associated with sunk costs $I \in \mathbb{R}^+$ and we will assume that A and B incur the fraction $(1-\varepsilon)I$ and εI, respectively. Here, $\varepsilon \in [0,1]$ expresses the distribution of sunk costs between the two individuals and is provided exogenously. The simplest example for such an investment setting would be the initiation of a greenfield joint venture where profits and costs are generally shared or a principal-agent setting under complete information.

We assume that time is continuous, i.e. $t \in [t_0, \infty)$ and rely on a Markovian Perfect Nash Equilibrium to determine the equilibrium strategy for both parties. In particular, as in Lukas and Welling (2012), A optimally defines ψ in stage one and conditional on the offered premium ψ B will choose a threshold value $V^*(\psi)$ or $x^*(\psi) := \left(\frac{1-e^{-(r+\lambda\rho\sigma-\alpha)T}}{r+\lambda\rho\sigma-\alpha}\right)^{-1}V^*(\psi)$, respectively, in stage two at which the offer will be accepted. Thus,

$$t^* := inf\{t \geqslant t_0 | x(t) \geqslant x^*\} = inf\{t \geqslant t_0 | V(t) \geqslant V^*\} \tag{4}$$

is the time of investing. However, B has not to decide immediately at time t_0 of the offer whether it accepts or rejects the offer. Rather, we assume that B can postpone the decision. To be more precise, while A has the action set $\psi \in (0,\infty)$, B has at every point in time the action set $\{accept, wait\}$. This degree of managerial flexibility B possesses can be interpreted as a real option. Exercising the option right refers to accepting the offer by initiating the investment project.[4]

[4] We will assume that this managerial flexibility is not limited by a fixed maturity date. Therefore the possibility to accept the offer is a perpetual real option.

Consequently, the value of the option to invest in the project held by the reacting party B is the solution of the following maximization problem in stage two[5]:

$$F(x) = \max_{\tau} \mathbf{E}\left[\left(\frac{(1-e^{-\delta T})}{\delta}\psi x_{\tau} - \varepsilon I\right)e^{-(r+\lambda\rho\sigma)\tau}\right], \tag{5}$$

where $\delta = r + \lambda\rho\sigma - \alpha$ denotes the convenience yield (or rate of shortfall) of the investment opportunity. Hence, the value $F(x(t))$ of the investment option is solution to the differential equation

$$\frac{1}{2}\sigma^2 x(t)^2 F''(x(t)) + (\alpha - \lambda\rho\sigma)x(t)F'(x(t)) - rF(x(t)) = 0, \tag{6}$$

with the boundary condition $F(0) = 0$, the value-matching condition

$$F(x^*(\psi)) = \frac{(1-e^{-\delta T})}{\delta}\psi x^*(\psi) - \varepsilon I, \tag{7}$$

and the smooth-pasting condition

$$F'(x^*(\psi)) = \frac{(1-e^{-\delta T})}{\delta}\psi. \tag{8}$$

Solving Eq. (5) yields:

$$F(x^*(\psi)) = \begin{cases} \left(\frac{(1-e^{-\delta T})}{\delta}\psi x^*(\psi) - \varepsilon I\right)\left(\frac{x}{x^*(\psi)}\right)^{\beta} & \text{if } x < x^*(\psi) \\ \frac{(1-e^{-\delta T})}{\delta}\psi x - \varepsilon I & \text{if } x \geqslant x^*(\psi) \end{cases} \tag{9}$$

with

$$\beta = \frac{1}{2} - \frac{\alpha - \lambda\rho\sigma}{\sigma^2} + \sqrt{\left(\frac{1}{2} - \frac{\alpha - \lambda\rho\sigma}{\sigma^2}\right)^2 + \frac{2r}{\sigma^2}} > 1 \tag{10}$$

and

$$x^*(\psi) = \frac{\delta}{(1-e^{-\delta T})}\frac{\beta}{\beta - 1}\frac{\varepsilon I}{\psi}. \tag{11}$$

In contrast, the bidding firm A will choose ψ in stage one such that it maximizes

$$\begin{aligned} f(\psi) &= \max_{\psi} \mathbf{E}\left[\left(\frac{(1-e^{-\delta T})}{\delta}(1-\psi)x^*(\psi) - (1-\varepsilon)I\right)e^{-(r+\lambda\rho\sigma)t^*}\right] \\ &= \max_{\psi}\left[\left(\frac{(1-e^{-\delta T})}{\delta}(1-\psi)x^*(\psi) - (1-\varepsilon)I\right)\left(\frac{x_0}{x^*(\psi)}\right)^{\beta}\right], \end{aligned} \tag{12}$$

subject to the other party's reaction function, i.e. $x^*(\psi)$. Solving $\partial\left(\left(\frac{(1-e^{-\delta T})}{\delta}(1-\psi)x^*(\psi) - (1-\varepsilon)I\right)\left(\frac{x_0}{x^*(\psi)}\right)^{\beta}\right)\Big/\partial\psi = 0$ we get the following proposition.

Proposition 1. *The optimal offered portion ψ results to:*

$$\psi = \frac{(\beta-1)\varepsilon}{(\beta-1)+\varepsilon}. \tag{13}$$

Using this result in Eq. (11) leads to the following proposition.

[5] Throughout the analysis we assume that the current spot value is sufficient low so that investment occurs not immediately. Otherwise, immediate exercise is optimal and the results equal a 2-person ultimatum game where the first person gets the entire surplus and the remaining agents get nothing.

Proposition 2. *The optimal timing threshold x^* is given by:*

$$x^* = \frac{\delta}{(1-e^{-\delta T})}\left(1+\frac{\varepsilon}{\beta-1}\right)\frac{\beta}{\beta-1}I. \tag{14}$$

In a cooperative framework, i.e. the parties act as a central planer, the optimal timing threshold is

$$x^*_{eff} = \frac{\delta}{(1-e^{-\delta T})}\frac{\beta}{\beta-1}I. \tag{15}$$

Hence, we have

$$x^* = \left(1+\frac{\varepsilon}{\beta-1}\right)x^*_{eff}, \tag{16}$$

where x^*_{eff} denotes the investment threshold derived in Gryglewicz et al. (2008). It is obvious that our investment threshold is proportional to $x^*_{eff}(\sigma, \delta(\sigma), \beta(\delta(\sigma)))$. Thus, for the sensitivity with respect to uncertainty it follows that:

$$\frac{\partial x^*}{\partial \sigma} = \frac{\partial x^*_{eff}}{\partial \sigma} + \varepsilon\frac{\partial\left(\frac{x^*_{eff}}{\beta-1}\right)}{\partial \sigma}. \tag{17}$$

Following Gryglewicz et al. (2008), we have that the derivative of the investment threshold with respect to uncertainty is composed of at least three effects, i.e. (1) *discounting effect*, (2) *volatility effect* and (3) *a convenience yield effect*. The latter two can be combined and characterize the option effect. The *discounting effect* describes the impact of uncertainty on the discount rate of an all-equity financed firm. Here, rising uncertainty raises the discount rate of the investment and thus reduces its NPV. Consequently, a positive impact always prevails. The option effect, however, has an ambiguous sign. The direct impact of uncertainty on the option to wait is measured by the *volatility effect.* Here, a positive impact is admitted because of the insurance property of an option. In particular, an increase of uncertainty increases the upside potential while the downside payoff remains unaffected. Consequently, opportunity cost of investing rise and so does the propensity to wait. The convenience yield effect, however, is negative, i.e. higher uncertainty provokes an individual to invest earlier. This is due to the fact that due to $\delta(\sigma) = r + \lambda\rho\sigma - \alpha$ the convenience yield (or return shortfall) increases as sigma rises and thus raises the attractiveness of possessing the investment project's net cash flows, i.e. $x(t)$.

Due to sequential bargaining, however, a fourth effect occurs, i.e. the *bargaining effect* that is only present when the agents share the investment cost, i.e. $\varepsilon > 0$. To see this, expanding Eq. (17) results to:

$$\frac{\partial x^*}{\partial \sigma} = \underbrace{\frac{\partial x^*_{eff}}{\partial \delta}\frac{\partial \delta}{\partial \sigma}}_{\text{Discounting effect}} + \underbrace{\underbrace{\frac{\partial x^*_{eff}}{\partial \beta}\frac{\partial \beta}{\partial \sigma}}_{\text{Volatility effect}} + \underbrace{\frac{\partial x^*_{eff}}{\partial \beta}\frac{\partial \beta}{\partial \delta}\frac{\partial \delta}{\partial \sigma}}_{\text{Convenience yield effect}}}_{\text{Option effect}} + \underbrace{\varepsilon\frac{\partial\left(\frac{x^*_{eff}(\sigma,\delta(\sigma),\beta(\delta(\sigma),\sigma))}{\beta(\delta(\sigma),\sigma)-1}\right)}{\partial \sigma}}_{\text{Bargaining effect}}. \tag{18}$$

With respect to Eq. (18) bargaining impacts the investment trigger's sensitivity in two ways. Dismantling the bargaining effect by means of

$$\frac{\partial\left(\frac{1}{\beta-1}\right)}{\partial \beta} = \frac{\beta}{(\beta-1)}\frac{1}{x^*_{eff}}\frac{\partial x^*_{eff}}{\partial \beta}, \tag{19}$$

yields:

$$\frac{\partial x^*}{\partial \sigma} = \underbrace{\left(1+\frac{\varepsilon}{\beta-1}\right)\left(\frac{\partial x^*_{eff}}{\partial \delta}\frac{\partial \delta}{\partial \sigma}\right)}_{\text{Discounting effect}} + \left(1+\frac{\varepsilon(\beta+1)}{\beta-1}\right)\underbrace{\left(\underbrace{\frac{\partial x^*_{eff}}{\partial \beta}\frac{\partial \beta}{\partial \sigma}}_{\text{Volatility effect}} + \underbrace{\frac{\partial x^*_{eff}}{\partial \beta}\frac{\partial \beta}{\partial \delta}\frac{\partial \delta}{\partial \sigma}}_{\text{Convenience yield effect}}\right)}_{\text{Option effect of bargaining}}. \tag{20}$$

Thus, we see that the bargaining effect just amplifies or dampens the discounting effect. In particular, the first term is due to $\beta > 0$ strictly positive, i.e. $1 + \varepsilon/(\beta - 1) > 0$. Consequently, the discounting effect remains positive. Also, from Eq. (20) it is obvious that the volatility effect remains positive while the convenience yield effect remains negative stating that again a U-shape pattern might arise when uncertainty is considerable low. Consequently, the convenience yield effect dominates the other two effects and any increase in uncertainty will raise the propensity to invest. For considerable high uncertainties, however, the volatility effect dominates the convenience yield effect so that the option effect changes sign. Consequently, an increase in uncertainty will unequivocally lead to a postponement of the investment.

Eq. (20), however, also reveals that the option effect is over-proportional affected by bargaining. This is because $1 + \varepsilon/(\beta - 1) < 1 + \varepsilon(\beta + 1)/(\beta - 1)$. Hence, due to bargaining the overall impact of the three effects becomes deterred in favor of the option effect. Should the uncertainty level be very low, so that the convenience yield effect dominates we see that an increase in uncertainty will lower the additional impact of discounting due to bargaining while it will lower the additional impact of the volatility and convenience yield effect due to bargaining to a much lesser degree. Of course, the effect is further amplified the more costs are assigned to the reacting party. On the contrary, should the overall option effect become positive we observe that for an increase in uncertainty the additional discounting, volatility and convenience yield effects exert a negative impact on timing and thus leading to further postponement of the investment. Again, this effect is the more amplified the more costs are assigned to the reacting party.

Finally, for $T \to \infty$ it is just the option effect of bargaining that determines the sign of $\partial x^*/\partial\sigma$. To see this, recall that Eq. (18) can be rewritten as:

$$\left.\frac{\partial x^*}{\partial\sigma}\right|_{T\to\infty} = \underbrace{\left(1+\frac{\varepsilon}{\beta-1}\right)\left(\frac{I\beta}{(\beta-1)^2}\frac{1}{L_1}(\sigma(\beta-1)(r+1/2(\beta-1)\lambda\rho\sigma-\mu))\right)}_{\text{Sum of all effects}} + \underbrace{\left(\frac{\varepsilon}{\beta-1}\right)\left(\frac{I\beta}{(\beta-1)^2}\frac{1}{L_1}(\sigma(\beta-1)(r+(\beta-1/2)\sigma^2)-\lambda\rho(r+(\beta-1/2)\sigma^2))\right)}_{\text{extra volatiliy and convenience yield effect due to bargaining}} \quad (21)$$

with $L_1 = (\beta - 1/2)\sigma^2 + \mu - \lambda\rho\sigma$. Consequently, the first term of the RHS disappears for low uncertainty ($\sigma \to 0$) and $\partial x^*/\partial\sigma$ becomes negative. Moreover, for considerable high values of uncertainty the discount effect and the volatility effect dominate and the overall sign becomes positive. Thus, even for infinitively long-lived investment projects a U-shaped investment–uncertainty relationship prevails should individuals bargain about investment and $\varepsilon > 0$. These results lead to the following proposition:

Proposition 3. *In a real sequential bargaining setting, i.e.* $\varepsilon > 0$, *the influence of uncertainty on the optimal timing threshold* x^* *is non-monotonic if* $\lambda\rho > 0$[6]: *For small levels of uncertainty growing uncertainty is reducing the optimal timing threshold while for high levels of uncertainty growing uncertainty is increasing the optimal timing threshold. The results also hold for infinite-lived investment projects, i.e.* $T = \infty$.

To what extent do these results hold with respect to the expected hitting time as an alternative measure for depicting the investment–uncertainty relationship? From Wong (2007) we know that as long as $\sigma \in \left[0, \sqrt{2\alpha}\right)$ we have[7]

[6] Please note, that for $\lambda\rho = 0$ the discount effect and the convenience yield effect vanish and it is only the volatility effect that determines the investment–uncertainty relationship. Thus, an increase in uncertainty always leads to a higher investment threshold. Furthermore, the result depends on the fact that the return shortfall $\delta(\sigma)$ is varying with uncertainty, while the drift rate α is constant. It is easy to show that the optimal timing threshold x^* would be monotonically increasing in uncertainty if the return shortfall would be constant while the drift rate depends on uncertainty, i.e. $\alpha(\sigma)$.

[7] If $\alpha \leqslant 0$ or $\sigma \notin \left[0, \sqrt{2a}\right)$ the probability that the investment threshold will never be reached is greater than zero and hence $\mathbf{E}[t^*] = \infty$ as long as $x_0 < x^*$.

$$\mathbf{E}[t^*] = \frac{\ln\left(\frac{x^*}{x_0}\right)}{\alpha - \frac{\sigma^2}{2}} < \infty. \tag{22}$$

and thus

$$\frac{\partial \mathbf{E}[t^*]}{\partial \sigma} = \frac{1}{(\alpha - \frac{\sigma^2}{2})^2}\left(\sigma \ln\left(\frac{x^*}{x_0}\right) + \left(\alpha - \frac{\sigma^2}{2}\right)\frac{1}{x^*}\frac{\partial x^*}{\partial \sigma}\right). \tag{23}$$

Obviously, it is $\lim_{\sigma \to \sqrt{2\alpha}} \mathbf{E}[t^*] = \infty$. Furthermore, for $\sigma = 0$ we get

$$\left.\frac{\partial \mathbf{E}[t^*]}{\partial \sigma}\right|_{\sigma=0} = \frac{1}{\alpha}\frac{1}{x^*}\frac{\partial x^*}{\partial \sigma} < 0, \tag{24}$$

whereby we use that $\frac{\partial x^*}{\partial \sigma}\big|_{\sigma=0} < 0$ if $\lambda\rho > 0$ (see e.g. Proposition 3). Thus, we can state the following proposition:

Proposition 4. *In a real sequential bargaining setting,* i.e. $\varepsilon > 0$, *the influence of uncertainty on the expected investment time is non-monotonic, if* $\lambda\rho > 0$ *and* $\alpha > 0$.[8]

One more result is deducible from Eq. (15). Because of $(1 + \frac{\varepsilon}{\beta-1}) > 1$ the parties time the investment inefficiently late after sequential bargaining which leads to the following proposition:

Proposition 5. *The investment in the project happens inefficiently late if it is bargained sequentially by the two parties and if* $\varepsilon > 0$. *Specifically, the social inefficiency is the more pronounced the higher the fraction of cost the second player has to bear.*

However, three limiting cases exist in which the sequentially bargained deal becomes social efficient: First, if the second player bears no investment costs his optimal threshold equals the social efficient threshold. As we will see in Eq. (27) the second player does not generate any surplus under that setting. Hence, this setting is equivalent to the non-game-theoretical model in Gryglewicz et al. (2008) which therefore can be seen as a special and limiting case of our game-theoretic model. Second, situations may occur where postponement of the investment would generate no extra value. In particular, a very large discount rate r implies that the individual will place a high weight on the immediate present. As a consequence, immediate investment represents the second limiting case. Likewise, in absence of uncertainty, i.e. $\sigma = 0$, a negative growth rate, i.e. $\alpha \leqslant 0$, causes $x(t)$ to remain constant or fall over time and thus it is again optimal to invest immediately if $x_0 > \frac{\delta}{(1-e^{-\delta T})} I$ and never to invest otherwise.

Finally, we will give an answer to the question how much of wealth is distributed to the parties? In the continuation region, i.e. $x_0 < x^*$ the generated surplus equals:

$$\begin{aligned} G(V_0) = G(V(x_0)) &= \left(\frac{(1 - e^{-\delta T})}{\delta} x^* - I\right)\left(\frac{x_0}{x^*}\right)^\beta \\ &= \left(\frac{\beta}{\beta - 1} + \varepsilon\frac{\beta}{(\beta-1)^2} - 1\right) I \left(\frac{x_0}{\left(1 + \frac{\varepsilon}{\beta-1}\right)\frac{\beta}{\beta-1} I}\right)^\beta. \end{aligned} \tag{25}$$

Because A is the offering party and thus holds the bargaining power, its expected profit equals s_A-$G(V(x_0))$ with a share of the surplus of:

$$s_A = \frac{\left(\left(1 - \frac{(\beta-1)\varepsilon}{(\beta-1)+\varepsilon}\right)\frac{(1-e^{-\delta T})}{\delta} x^* - (1-\varepsilon) I\right)\left(\frac{x_0}{x^*}\right)^\beta}{\left(\frac{(1-e^{-\delta T})}{\delta} x^* - I\right)\left(\frac{x_0}{x^*}\right)^\beta} = \frac{\varepsilon + \beta - 1}{\varepsilon\beta + \beta - 1}. \tag{26}$$

[8] Like Proposition 3 this result also depends on the fact that the drift rate is seen to be constant while the return shortfall depends on uncertainty. Depending on the other parameter values switching this assumption may lead to a vanishing non-monotonicity of the uncertainty's influence on the expected time until investment.

In contrast, the reacting party receives the fraction $s_B G(V(x_0))$ with

$$s_B = 1 - s_A = \frac{\varepsilon\beta - \varepsilon}{\varepsilon\beta + \beta - 1}. \tag{27}$$

Because of $s_B = \frac{\varepsilon\beta-\varepsilon}{\varepsilon\beta+\beta-1} = \frac{(\beta-1)\varepsilon}{\varepsilon\beta+\beta-1} < \frac{(\beta-1)+\varepsilon}{\varepsilon\beta+\beta-1} = s_A$ we get the following proposition:

Proposition 6. *The expected profit for being the offering party is greater than for being the reacting party, i.e.* $s_B < s_A$.

As is shown in Wong (2007) if $\lambda\rho > 0$ we have $\frac{\partial\beta}{\partial\sigma} > 0$ for small values of uncertainty and $\frac{\partial\beta}{\partial\sigma} < 0$ for higher values of uncertainty. With $\frac{\partial s_A}{\partial\beta} = -\frac{\varepsilon^2}{(\beta\varepsilon+\beta-1)^2} < 0$ we can therefore state the following proposition:

Proposition 7. *Uncertainty has an ambivalent influence on this first-mover-advantage if* $\lambda\rho > 0$. *For small levels of uncertainty growing uncertainty is weakening the first-mover-advantage while for high levels of uncertainty growing uncertainty is strengthening the first-mover-advantage.*

This ambiguous effect is based on two opposing effects, i.e.:

$$\frac{\partial s_A}{\partial\sigma} = \frac{\partial s_A}{\partial\beta}\frac{\partial\beta}{\partial\sigma} + \frac{\partial s_A}{\partial\beta}\frac{\partial\beta}{\partial\delta}\frac{\partial\delta}{\partial\sigma}. \tag{28}$$

It is easy to show that the first term is strictly positive while the second term of the RHS is strictly negative. For significant high levels of uncertainty the impact of managerial flexibility, i.e. the first term on the RHS dominates. Obviously, the total surplus generated by the investment project is ceteris paribus the lower the more the investment is delayed relative to the situation of a central planner. Therefore, controlling the exercise of the real option can be regarded as having some bargaining power. But increasing uncertainty is diminishing this bargaining power of the second mover because a postponement of the investment is less threatening to the first-mover, i.e. to A. In particular, A also profits from a delayed investment due to a better information set and a higher flexibility value, respectively. As a consequence the first-mover advantage as proxied by the size of s_A increases. For small levels of uncertainty, however, the second term of the RHS dominates. For such initial levels of uncertainty we know that A's propensity to invest increases as uncertainty increases due to the convenience yield effect. In particular, it becomes more attractive for the first-mover to possess the investment project's net cash flows. Because B, however, controls the exercise of the real option his bargaining power increases while the first-mover's bargaining power as a consequence decreases.

3. Numerical analysis

In the following our model is analyzed numerically. Hereby we assume the following values: $r = 0.1,\ \lambda = 0.4,\ \rho = 0.7,\ I = 10,\ \alpha = 0.08,\ x_0 = 1$ and $t_0 = 0$.

As Fig. 1 depicts the investment threshold is higher after sequential bargaining between two parties (black curves) than in the one party case (grey curves) described by Gryglewicz et al. (2008). However, independent of the fact whether the project-life is finite or infinite the influence of uncertainty on the investment threshold shows a U-shape after sequential bargaining. Specifically, an increasing fraction of the investment cost beard by the second player is shifting the curve upwards and to the right and therefore is resulting in a further postponement of the investment. As has already been stated, the model of Gryglewicz et al. (2008), Wong (2007) and Sarkar (2000) can be seen as a special case of our sequential-bargained model, indeed, if $\varepsilon = 0$ both curves will be identical. Similar results hold for the influence of uncertainty on the expected time until investment (see Fig. 2).

Now, we take a look on the third interpretation of the investment–uncertainty relationship, i.e. the probability to invest within a pre-specified time of length τ. According to Sarkar (2000) this probability can be calculated by

$$Prob(t^* \leqslant \tau) = \Phi\left(\frac{ln(\frac{x_0}{x^*}) + (\alpha - \frac{1}{2}\sigma^2)\tau}{\sigma\sqrt{\tau}}\right) + \left(\frac{x^*}{x_0}\right)^{\frac{\alpha}{\sigma^2}-1}\Phi\left(\frac{ln(\frac{x_0}{x^*}) - (\alpha - \frac{1}{2}\sigma^2)\tau}{\sigma\sqrt{\tau}}\right), \tag{29}$$

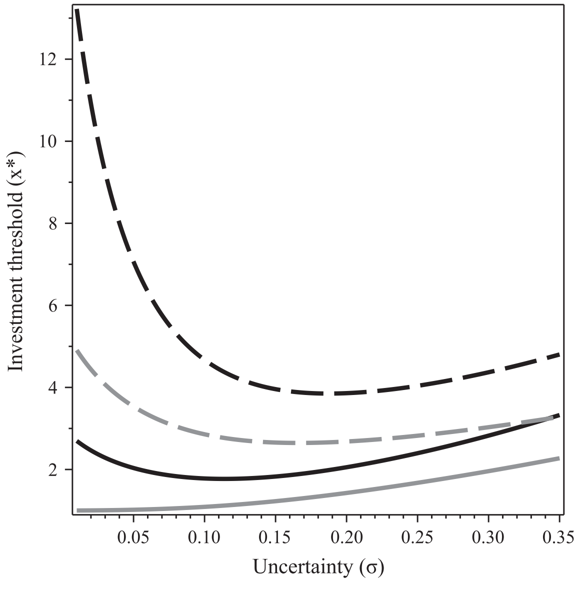

Fig. 1. Influence of uncertainty on the investment threshold x^* (black color: $\varepsilon = 0.5$; grey color: $\varepsilon = 0$; solid line: $T = \infty$; dashed line: $T = 10$).

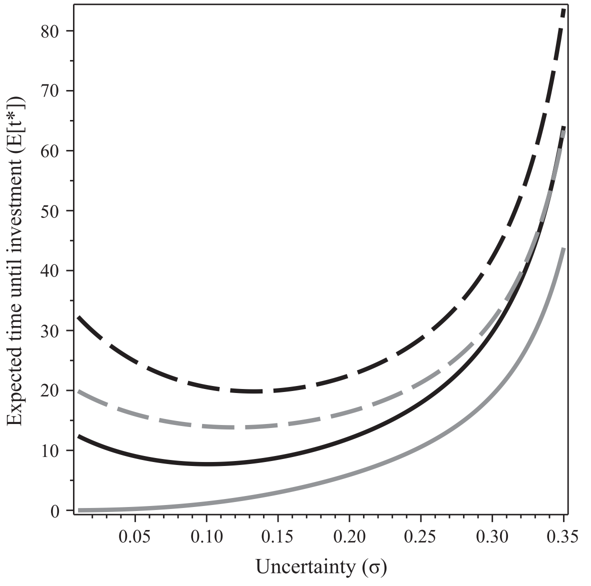

Fig. 2. Influence of uncertainty on the expected time until investment $\mathbf{E}[t^*]$ (black color: $\varepsilon = 0.5$; grey color: $\varepsilon = 0$; solid line: $T = \infty$; dashed line: $T = 10$).

whereby $\Phi(\cdots)$ is the area under the standard normal distribution. A numerical example of the bargaining case is given in Fig. 3. It can be seen that it matches with the example given by Sarkar (2000). For low values of uncertainty increasing uncertainty is enhancing the probability that the investment is carried out within the next five years while for higher values of uncertainty this probability is declining with increasing uncertainty.

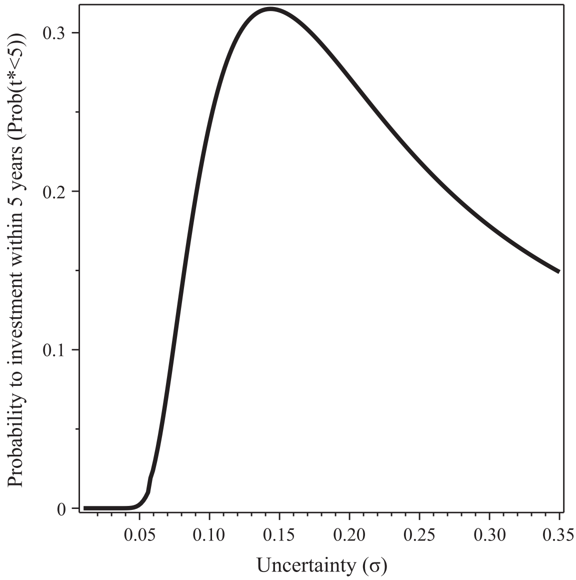

Fig. 3. Influence of uncertainty on the probability to invest within 5 years. ($\varepsilon = 0.5$; $T = \infty$).

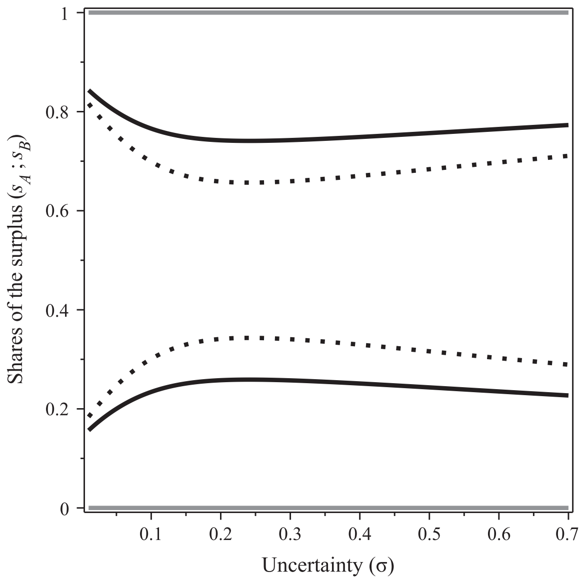

Fig. 4. The shares of the surplus of the offering party (s_A) and of the reacting party (s_B) depending on the amount of uncertainty and the size of incurred costs the parties face, $\varepsilon = 0.95$ (black, dots), $\varepsilon = 0.5$ (black, solid), and $\varepsilon = 0$ (grey).

According to Proposition 7, the influence of uncertainty on the shares of the surplus of the two players is ambiguous. As can be seen in Fig. 4 this influence is following a U-shape, too. Starting with

$$\lim_{\sigma \to 0} s_A = \frac{\frac{r}{\alpha} - 1 + \varepsilon}{\frac{r}{\alpha}\varepsilon + \frac{r}{\alpha} - 1} > \frac{1}{2}, \tag{30}$$

the share of the first player is first decreasing with increasing uncertainty. For higher levels of uncertainty, however, the share of the first-player is increasing with increasing uncertainty. Under infinite uncertainty the first-player would get the whole surplus, $\lim_{\sigma\to\infty} s_A = 1$.

From Fig. 4 it is also apparent that the ambiguity is controlled by the distribution of the investment costs, i.e. $(1-\varepsilon)I$ and εI, respectively. In particular, should the first player bear no or moderate costs, i.e. $0 \leqslant \varepsilon < 0.5$, the ambiguity is well pronounced. This is due to the fact, that the second player makes his timing decision contingent not only on the size of uncertainty but on the degree of irreversibility as represented by the size of sunk costs, too. Consequently, the first mover profits from *B*'s managerial flexibility the less cost he incurs. In contrast, the higher the fraction of overall costs devoted to *A* becomes the weaker becomes the ambiguous effect of uncertainty on the shares of the surplus. Hence, should he incur the full cost, the sharing of surplus does no longer depend on the size of uncertainty. In this limiting case, the result resembles the one of a simple ultimatum game.

4. Conclusion

While the standard real option optimization framework and the deduced uncertainty–investment relationship, respectively, are characterized by a game against nature a proper treatment of games between individuals is missing. As an approach to fill this gap we have set up a model that builds on the assumptions of Gryglewicz et al. (2008), Wong (2007) and Sarkar (2000) but treats investment timing as an outcome of a sequential game with two individuals. The results show that a U-shaped investment–uncertainty relationship generally sustains. However, timing of an investment occurs inefficiently late. This inefficiency is increasing with the fraction of investment cost the second player has to bear. Furthermore the results show that in the sequential bargaining game a first-mover-advantage always prevails, i.e. the first player gets a higher fraction of the combined surplus than the second player. The amount of this first-mover-advantage is influenced by uncertainty in a U-shaped pattern, too. While for small values of uncertainty the first-mover-advantage is decreasing with increasing uncertainty for higher values of uncertainty it is increasing with increasing uncertainty. Future directions in which this work could be extended is to consider information asymmetries or moral hazard and to investigate how these risks affect timing and optimal contracting (see e.g. Hackbarth and Miao, 2012).

Acknowledgements

The authors are particularly grateful to the Editor-in-Chief Ramazan Gencay, the handling Co-Editor and two anonymous referees for their insightful comments and valuable suggestions. Any remaining errors are the sole responsibility of the authors.

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