WORKING PAPER SERIES

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Working Paper No. 22/2012

OTTO VON GUERICKE
UNIVERSITÄT
MAGDEBURG

FACULTY OF ECONOMICS
AND MANAGEMENT

Impressum (§ 5 TMG)

Herausgeber:

Otto-von-Guericke-Universität Magdeburg
Fakultät für Wirtschaftswissenschaft
Der Dekan

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Bezug über den Herausgeber

ISSN 1615-4274

Herding in a Laboratory Asset Market with a Rich Action Set

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Abstract

This paper experimentally examines the efficiency of information aggregation in a simple asset market. Traders decide how to allocate an endowment of 1000 euro-cent between two assets. Only one asset will be successful and that will pay back the amount invested in it. The experiment carried out here is original in that it considered a very rich action set. We find that when the action set is sufficiently rich, traders' actions, most of the time, perfectly reveal their private information. Further, the participants in the experiment performed probability matching and took such actions, which were broadly consistent with Bayesian learning.

Keywords information cascade · information aggregation · herding · probability matching · Bayes' rule

JEL Classification G10 · D8 · C11 · C92

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1. Introduction

Efficient aggregation of dispersed private information is crucial to the effective functioning of markets. One assumption of the Efficient Market Hypothesis (Fama 1970) is that prices in competitive markets "fully reflect" available information. The theories on social learning and information cascades (Banerjee 1992, Bikhchandani, Hirshleifer, and Welch 1992 [hereafter BHW]), however, postulate that, under certain conditions, it is rational for economic agents to ignore their private information and herd on the action favored by the majority of their predecessors. The event in which the agents take identical action on all private signals is called an information cascade (in the sense of BHW). When an information cascade starts, the private information of later decision makers is never revealed and information aggregation becomes inefficient. Indeed, both market participants and economists are inclined to believe that social phenomena, such as mania and panic, are a direct consequence of information cascades. In particular, investment decisions in financial markets are often suspected of having been subjected to herd behavior.

Despite the intuitive relation between financial decisions and herd behavior, the standard theoretical models on rational herds (i.e., BHW) are subjected to at least two critiques that question the extent to which these models pertain to financial markets. In particular, the assumptions of a fixed price and a binary action set of the BHW model make its predictions inapplicable to financial markets. Avery and Zemsky (1998) show that herd behavior disappears when the BHW model is enriched with flexible prices. Drehmann, Oechssler and Roider (2005) present experimental results, which are in line with the predictions of the Avery and Zemsky's (1998) model. In this paper, the focus is on examining the role of the richness of the action set in the process of information aggregation. Lee (1993) demonstrates that, with replacement of the discrete binary decisions in the BHW model by a continuous action set, herd behavior disappears in that model. The intuition of this result is that, with a coarse action set, agents cannot fine-tune their actions to the extent they can reflect small changes in the probability of the true state. In other words, the richer the action set is, the more efficient would be the aggregation of information.

In this study, we consider a very rich action set and experimentally address the question whether social learning in a simple asset market leads to efficient information aggregation. In particular, we consider a market consisting of two assets, R and B, between which only one will be successful and pay back the amount invested in it. The probability of success of each asset is exogenously determined and equal to 50 percent. Each trader received an endowment of 1000 euro-cent which she had to fully invest, in any combination, between the two assets. In each round, there were always six traders who made investment decisions, one after the other, in an exogenously given order. The information set of each trader included the perfectly observed actions of all predecessors, a private binary signal, conditional on the true state, and knowledge of the initial distribution of the state of the world. All private signals had the same precision, with probability of receiving a signal in favor of the true state being equal to 2/3.

In situations where the action set is discrete, an information cascade eventually occurs (Banerjee 1992, BHW). The probability that an information cascade starts, however, is decreasing in the number of elements in the action set. It also holds that, with a rich action set, an information cascade can occur only if the sequence of consecutive decisions is sufficiently long. To specify the theoretical conditions that predict the occurrence of information cascade in the present experimental setting, we make the following assumptions. First, we adopt the standard in the social learning literature assumption, namely that traders calculate their posterior distributions based on Bayes' rule. Second, we assume that traders maximize logarithmic utility functions. This assumption is justified by the finding of many researchers (e.g., Fiorina 1971, Brackbill and Bravos 1962, and Arrow 1958) that people perform probability matching.

Under the parameter values and assumptions described above, theory predicts that as long as the prior probability of success of each asset is 0.9995 or lower, agents will always take actions that perfectly reveal their private signals. In other cases, an information cascade will occur and actions will no longer reveal private information. However, for shorter sequences of consecutive decisions, as the one considered in the present experiment, the prior probabilities of success of each asset will always be lower than 0.9995 and no herd behavior should be observed.

Our main finding is that in 66 percent of the cases, the subjects perfectly signaled their private information by their actions. That is, when they decided first in a sequence, they invested more than 50 percent of their endowment in the asset indicated by their signal and when they decided later in the sequence, they invested more than their direct predecessors did in the asset in favor of which they received private information. In addition, about 80 percent of the remaining decisions were not necessarily at odds with the theory but rather can be attributed to the differences in the way individual subjects performed probability matching. In fact, we included a specially designed task in the experiment in which for each subject we elicited the actions she would take under several different initial probabilities of success. The answers given on that task allow us to directly check to what extent the assumption of probability matching is justified for our experimental data. We find evidence that subjects performed probability matching, but it was not always done on a one-to one basis. Considering all these results, we conclude that in a setting with rich action set, the subjects take full advantage of their private information, most of the time, and take, in the light of the theory, correct decisions.

We address another question also: To what extent the subjects' behavior was consistent with Bayes' rule? We find that, overall, the majority of the subjects updated their beliefs correctly about the true state. On the aggregate level we find another piece of evidence that subjects acted in accordance with Bayes' rule. Namely, we show that the distribution of actual actions can be very closely replicated by combining the results of the probability matching task and Bayes' rule. This result is consistent with the findings of previous studies. Bondarenko and Bossaerts (2000), for example, report that learning in the Iowa Experimental Market is in accordance with Bayes' rule.

The rest of the paper is organized as follows. Section 2 develops the formal structure of the model and discusses the theoretical predictions. Section 3 presents the experimental design and procedure. Section 4 reports the results of the experiment. Section5 concludes.

2. The model

We base our experimental analysis on a model adapted from BHW. We consider an economy with two assets: R and B. Each trader in the economy is endowed with 1000 euro-cent for full investment in any combination of the two assets. Only one of these two assets will be successful and that will pay back the amount invested in it. The formal structure of the model can be summarized as follows:

The state of the world.—There are two states of the world $\theta = \{R, B\}$. If $\theta = R$, asset R is successful; If $\theta = B$, asset B is successful. The actual state of the world is determined by a random draw from the initial prior distribution: $\Pr(\theta = R) = \Pr(\theta = B) = 0.5$. The initial distribution is common knowledge.

The private signals.—There are two signal values $x = \{r, b\}$. All private signals are drawn from a conditionally independent and identical distribution given the actual state of the world. The precision of private signals is as follows: $\Pr(x = r / \theta = R) = \Pr(x = b / \theta = B) = 2/3$ and $\Pr(x = b / \theta = R) = \Pr(x = r / \theta = B) = 1/3$. The conditional distribution of a signal value is common knowledge.

The action set.—Each trader chooses an action $a \equiv (a_R, a_B) \in A = ([1000,0];[999,1];...;[1,999];[0,1000])$ from a finite action set, where the first number in a strategy choice stands for the amount invested in asset R and the second number for the amount invested in asset B. Traders can fine tune their investment decisions to the level of 1 euro-cent. Thus, even though the action set is discrete, it is sufficiently rich to allow for very accurate aggregation of information. Because the amount invested in both assets always sums up to 1000 euro-cents, for notational simplicity, a trader's action is sometimes denoted by a_R, instead of (a_R, a_B).

The traders.—There are countably many traders who make investment decisions in an exogenously determined order. Each trader knows the probability distribution that generates the state of the world, but none of them can observe the state that has actually been selected. All agents maximize the same utility function $U_\theta(a)$ which depends on the state of the world and the action taken. The information set of trader n includes the history of actions $h^n = (a^1, a^2, ..., a^{n-1})$ and the private signal, x^n. Traders update the prior distribution according to Bayes' rule. Following Lee (1993), and given history h^n before the private signal, the prior distribution is written as $\mu_\theta^n = \mu(\theta / h^n)$, which is updated by the private signal x^n to the posterior distribution $\pi_\theta^n = \pi(\theta / h^n, x^n)$.

Utility maximization.—Trader n solves the problem, $\max_{a \in A} E[U_\theta(a) / h^n, x^n]$. Based on findings that human beings tend to perform probability matching (Fiorina 1971, Brackbill and Bravos 1962, and Arrow 1958), we assume that traders in our model weight a monetary payoff w by a logarithmic utility function $U(w) = \ln(w)$. In the context of the social learning model under consideration, probability matching means that traders tend to allocate their endowment between the assets in proportions given by their posterior distributions. Such allocation decisions are rational to traders possessing logarithmic utility functions.

The model we present here can be seen as a hybrid between the models of BHW and Lee (1993). It differs from Lee's (1993) model in two ways. First, the objective of the traders in the model presented here is to maximize the conditional expected value of a logarithmic utility function. On the contrary, in Lee's (1993) model, the agents are concerned with the minimization of the mean squared error. Each of these two optimization problems, however, is a monotone transform of the other and warrants the same actions. The second difference is that even though the action set under consideration is very rich, it is discrete; Lee (1993), on the other hand, derives his results for a continuous action space (in his model the action space is defined as a subset of the real line). This means that, in our model, we cannot completely rule out the possibility of a cascade occurring. We use the BHW model, modified by a rich action set, to derive our theoretical predictions. The following example shows that, with an action set and parameter values as defined in this paper, the probability that a cascade occurs is strictly positive but very low for sequences including at least 13 consecutive decisions. For shorter sequences, actions should always perfectly reveal private information.

Example.—Traders can fine-tune their allocation decisions to the level of 1 euro-cent. It was already argued that if traders maximize logarithmic utility functions, they should always distribute their endowment between the two states at proportions commensurate to their posterior distributions. Consequently, as long the traders' posterior distributions are sufficiently different from their prior distributions, they can reveal their posterior distributions in a one-to-one fashion. We use Bayes' rule and the numerical probabilities defined above to find out the conditions under which the discreteness of the action set will not allow traders to reveal their private signals and how it consequently will lead to a cascade.

Table I. Numerical example

n	x^n	$\mu(R/h^n)$	$\pi(R/h^n, x^n)$	a^n
1	r	0.5	0.6667	(667, 333)
2	r	0.6667	0.8	(800,200)
.				
.				
.				
11	r	0.9990	0.9995	(1000,0)
12	r	0.9995	0.9998	(1000,0)
13	b	0.9998	0.9995	(1000,0)

In Table I, we consider a sequence of identical private signals. We assume that traders round their posteriors to the next or to the previous euro-cent using the standard rounding rules. We observe that the action of the 13th trader does not reveal her private signal. The same will hold true for all her successors. That is, after a sequence of 12 r signals, traders start a cascade in the sense of BHW, and their action reveal their private information no more. The same result holds for a sequence of 12 b signals or in instances where the absolute value of the difference between r and b signals is equal to 12. We call this difference trade imbalance and further define that the trade imbalance is positive if the number of r signals is greater than the number of b signals, and negative otherwise.[3]

When trader n solves the problem, $\max_{a \in A} E[U_\theta(a)/h^n, x^n]$, where $U(w) = \ln(w)$, $\Pr(\theta = R) = \Pr(\theta = B) = 0.5$, $\Pr(x = r/\theta = R) = \Pr(x = b/\theta = B) = 2/3$ and the action set is $A = ([1000,0];[999,1];...;[1,999];[0,1000])$, the following result holds:

Result.—An informational cascade in the sense of BHW occurs after a trade imbalance with an absolute value equal to 12. In all other cases, traders' actions perfectly reveal their private signals.

3. Experimental design and procedure

3.1. Experimental design

The experiment involved two tasks. In each task, the subjects were asked to make several investment decisions. Each investment decision required that the subjects fully invest a total endowment of 1000 euro-cent in any combination of two assets, R and B. The subjects were informed that only one of the two assets would be successful. The success probability of asset R was given by p (with $p \in [0,1]$), and that of asset B by $(1-p)$. At the time of making their decisions, the subjects had full information about the prior probabilities of each state of the world (R or B), but not about the state of the world that had been selected. Depending on their own decisions and the true state of the world, the participants received the fraction of the

[3] Reader's attention is drawn to the difference between our definition of trade imbalance and that given in other social learning papers (see e.g. Cipriani and Guarino 2005). Many social learning studies consider a binary action set and define trade imbalance as the number of times one action exceeds the other action in the history of actions. In the prsent model, the richness of the action set allows traders, most of the time, to perfectly reveal their private signal by their actions. Therefore, as long as traders are not in cascade and rationality is assumed, the history of actions suffices to reveal the history of private signals. Once the traders start a cascade, they take identical actions for all signals and their private information can no longer be induced by successors. That is why, in the calculation of trade imbalance, only signals from traders who are not in cascades are considered; the actions of other traders are uninformative and hence not included in the calculation of the trade imbalance. The maximum possible value (in absolute terms) of the trade imbalance is, therefore, 12, after which, due to the discreteness of the action set, a cascade starts.

1000 euro-cent they invested in the successful asset. The investment in the unsuccessful asset was not reimbursed.

In Task I, the subjects were asked to simultaneously make an investment decision for each of the following pairs of probabilities of success—$(p, 1-p)$= $\{(1,0), (0.9,0.1), (0.8,0.2), (0.7,0.3), (0.6,0.4), (0.5,0.5)\}$. The subjects were informed that only one of these six investment decisions would be paid out, and that decision would be randomly determined at the end of the experiment.

In Task II, the subjects were asked to make one investment decision in each of 11(20) rounds. In each round, they made their decisions one after the other. The sequence in which the decisions were made was randomly (and exogenously) determined in each round. We call every time when a decision had to be made a period. There were six periods per round (corresponding to the investment decisions of the six participants in a given session). The probabilities of success of both assets were fixed at 0.5 for all periods and all rounds. The experimenter used these probabilities to determine the successful asset before the beginning of each round. The subjects were informed, only at the end of the experiment, which asset was successful in each round. In this way, no learning could have taken place from round to round. Before making their own decisions, the subjects were informed about the decisions of their predecessors (in case they had any). In addition, in every round, all the subjects received a binary private signal—r or b. Private signals were conditional to the true state of the world and had the following precision: $\Pr(r/R) = \Pr(b/B) = 2/3$ and $\Pr(r/B) = \Pr(b/R) = 1/3$. The subjects were told that only one round, randomly determined at the end of the experiment, would be paid out.

3.2. Experimental Procedure

The experiment was carried out at MaXLab, the experimental laboratory of the University of Magdeburg, in October 2011. Participants were recruited using ORSEE software (Greiner 2004) from a pool mostly of students from various faculties. None of the participants had any experience with probability matching and social learning. A total of 10 sessions were conducted with six subjects per session. Each subject participated in one and only one session. The experimental design was the same in all sessions, except for one parameter, namely the number of rounds in Task II; the number of rounds was over 11 in four sessions, and over 20 in the other six. Thus, in total, we collected observations from 60 participants, and over 164 rounds and 984 periods (the number of rounds and periods apply to Task II). All sessions were hand run. The experimental instructions were provided in German. The duration of the whole experiment was around 90 minutes. All subjects received a show up fee of 5 euros. Besides, they could earn, in the two tasks of the experiment a maximum of 20 euros and a minimum of 0 euros.

Upon arrival at the laboratory, the subjects were assigned a participant number (from 1 to 6) and were seated in a single-person cabin with arrangements to ensure their privacy. During the experiment, no communication was allowed among the participants. The written instructions were explained to them orally also; further they were instructed to raise their hands if they had questions, which were then answered individually.

The subjects were first given the instructions of Task I. At that moment, they knew that the experiment involved two tasks, but they had no information about the nature of Task II. In Task I, the decision played for real monetary payoff and the state of the world for that decision were determined at the end of the experiment as follows. The experimenter drew a ball from an urn containing six balls numbered 1 to 6. The number of the selected ball determined the decision which was paid out. The state of the world was determined by drawing a ball from another urn containing 10 balls, red or blue, in proportions as given by the probabilities of success of the particular decision selected to be paid out. Thus, for example, if decision 3 was chosen, the combination of success probabilities $(p, 1-p) = (0.7, 0.3)$ was replicated by placing 7 red and 3 blue balls in the second urn. If a red ball was drawn, asset R was the successful one, otherwise asset B.

The subjects proceeded with Task II immediately after all answer sheets from Task I were collected. After all the subjects read the experimental instructions, the experimenter determined the state of the world for the first round by drawing a ball from an urn containing 10 balls—5 red and 5 blue. If a red (blue) ball was drawn, asset R (B) was the successful one. Once the actual state of the world was determined and recorded (but not announced to the participants), the experimenter privately approached all the subjects and asked them to draw a ball from an urn containing three balls—2 red and 1 blue in case asset R was the successful one, otherwise 1 red and 2 blue. The color of the ball, which each subject drew, stood for her private signal—red for $x^n = r$ and blue for $x^n = b$. As a next step, the experimenter determined the order in which the subjects were going to make their decisions by consecutively drawing all balls from an urn containing 6 numbered balls (1 to 6). Finally, the experimenter approached the first subject in the sequence and asked her to make and record her investment decision. The experimenter then announced that decision loudly and the rest of the participants recorded it on their decision sheets. After this, the experimenter approached the second subject in the sequence and repeated the same procedure. When the sixth subject in the sequence made her decision and that was recorded by everybody else, the first round ended and the second round started. The same procedure was followed in all the rounds. The round, played for real money, was determined at the end of the experiment by drawing a ball from an urn containing 11 (or 20) consecutively numbered balls (1 to 11 [20]).

4. Results

A. Probability Matching

One of the key assumptions of the present model is that the traders possess logarithmic utility functions. The first part of the experiment measured to what extent this assumption was justified by the experimental data. If subjects' utility function can well be approximated by a logarithmic function, we should find evidence of probability matching.

In Task I, all the subjects were asked to make an investment decision for each of the six pairs of success probabilities—$(p,1-p)$= $\{(1,0), (0.9,0.1), (0.8,0.2), (0.7,0.3), (0.6,0.4), (0.5,0.5)\}$, where the fist number in a probability string stood for the probability of success of asset R and the second number for the probability of success of asset B. It was observed that even though the subjects could choose actions from a very rich action set, the majority of them exclusively made allocations which were multiples of 100. Those allocations which were odd multiples of 50 were selected only infrequently, while those which were multiples of other less prominent numbers were practically never selected. Figure I depicts the relative frequency with which each of the six most prominent asset allocations ([500, 500], [600, 400], [700, 300]… [1000, 0]) was made for each pair of success probabilities. The first number in an action string denotes the amount invested in asset R and the second number the amount invested in asset B. Allocations, other than the six most prominent allocations, were only a few; therefore, they were all pooled together in Figure I and depicted under the common label “other”.

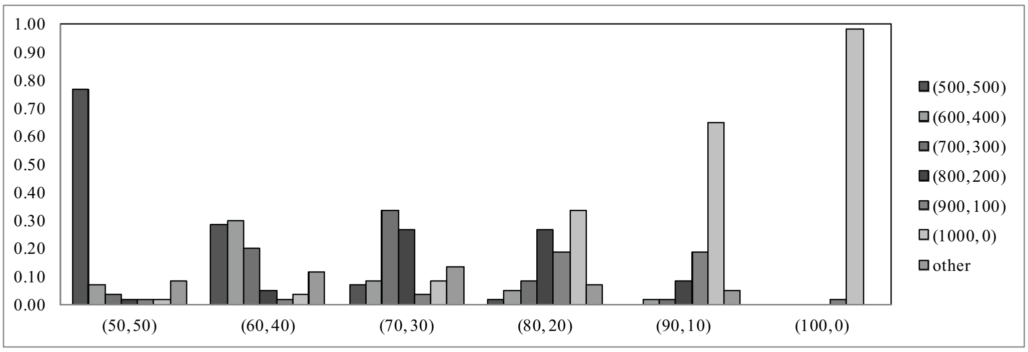

FIGURE I. Relative frequencies of allocation decisions (in Task I)

For all but one pair of success probabilities, the frequency distributions of asset allocations are peaked at the allocation that maps in a one-to-one fashion the given probability of success. In the case of $(p, 1-p) = (0.8, 0.2)$, (1000, 0) was the allocation most often preferred by the subjects, followed by (800, 200). This shows that the participants indeed performed probability matching in the present experiment. However, not all their allocations matched the success probabilities perfectly. For example, for the probability pair $(p, 1-p) = (0.6, 0.4)$, the allocations (500, 500), (600, 400), (700, 300) were selected with approximately the same frequency. We observe a similar pattern for other pairs of probabilities of success also. Thus, the present data show that the subjects did perform probability matching, but the mapping of success probabilities into asset allocation was not always on a one-to-one fashion.

B. *Rationality*

We now address the question to what extent subjects behaved rationally in Task II, i.e., consistent with the theoretical predictions. For that purpose, the subjects' overall behavior in the second part of the experiment needs to be discussed first.

In Task II, we observe allocation decisions similar to those in Part I. The subjects took actions, which were, almost exclusively multiples of 100; they seldom selected actions which were odd multiples of 50 or other less prominent numbers. For the sessions in which the subjects played over 20 rounds, no pronounced difference could be identified in the behavior between earlier and later rounds, indicating that the number of rounds played by the participants in the game did not influence their allocation choices. Furthermore, the subjects behaved very similarly in all but one session. Participants of the session, in which the allocation choices differed from those made in the other sessions, almost exclusively selected the action $a = (500, 500)$, which guaranteed them a sure payoff of 5 euros, but it did not convey their private information. That session could not have contributed to any learning and was hence excluded from subsequent analysis. Also, to account for the fact that the subjects probably needed some time to learn how to play the game, the decisions from the first three rounds of the remaining nine sessions were excluded from analysis. Thus, in total, we study the behavior of 54 subjects, over 126 rounds and 756 periods.

In Section 2, it was argued that, as long as the trade imbalance is less than 12 (in absolute terms), the subjects' actions should always perfectly reveal their private information. That is, if $x^n = r$, trader n should always invest a bit more than what her direct predecessor did in asset R; otherwise, she should invest a bit more than what her direct predecessor did in asset B. When the trade imbalance is equal to 12 (in absolute terms), the subjects should take identical actions ([1000, 0] or [0, 1000] depending on whether the trade imbalance is 12 or -12, respectively) that would no longer reveal their signals. The present experimental design allowed the testing of only the first prediction because we studied social learning for sequences consisting of exactly six traders. The second prediction is particularly difficult to test in laboratory experiments, because even by considering longer sequences of consecutive investment decisions, a trade imbalance of 12 (in absolute terms) will occur very infrequently and it will be hardly feasible to collect so many observations as would be required for a meaningful statistical inference.

We report the following results. In 499 (66 percent) periods, the subjects perfectly signaled their private signals by their actions. That is, when they decided first in a sequence, they invested more than 50 percent of their endowment in the asset indicated by their signal and when they decided later in the sequence, they invested more than what their direct predecessors did in the asset in favor of which they received private information. However, because the subjects almost exclusively selected actions which were multiples of 100, they often overbid their predecessors by more than what would have been predicted by the theory. In 144 (19.5 percent) periods, the subjects took the same action as that of their predecessors. In 56 (7.4 percent) periods, the subjects invested more than 50 percent (and in a few cases exactly 50 percent) of their endowment in the asset for which they had a favorable signal, but the amount invested in that asset was lower than the amount invested (in the same asset) by their direct predecessors. In 31 (4,1 percent) periods, the subjects invested more than 50 percent of their endowment in the asset against which they had private signal and the amount invested in that asset was higher than the amount invested (in the same asset) by their direct predecessors. Finally, the remaining 26 (3.4 percent) periods, reflect the decisions of the subjects who had no predecessors and invested an equal portion of their endowment in both assets; or, they reflect the decisions of the subjects who, exactly as their predecessors, invested an equal portion of their endowment in both assets.

These results show that in two thirds of the periods, the subjects behaved rationally and in a little more than half of the periods in which subjects behaved irrationally they did so because they decided not to take advantage of their private information in further fine-tuning of their actions and, instead, they simply copied the behavior of their direct predecessors. This can be explained as due to their unwillingness to rely on their own judgment about the likelihood of each asset being successful or due to overreliance on the judgments of their direct predecessors. Alternatively, different ways of mapping the probabilities into actions might also explain the high percentage of periods in which the subjects copied the behavior of their direct predecessors. This explanation seems plausible in light of Task I results.

By a similar argument, one can explain even the behavior in the 56 periods in which the subjects invested more than 50 percent (and in a few cases exactly 50 percent) of their endowment in the asset for which they had a favorable signal but the amount invested in that asset was lower than the amount invested (in the same asset) by their direct predecessors. Overall, we are confident to conclude that the actual behavior in Task II is in line with the theoretical predictions.

C. Bayes' rule

Another key assumption of the theoretical model presented in Section 2 is that traders apply Bayes' rule to update the probability of the state of the world. In this sub-section, we address the question to what extent the decisions taken by the subjects of this experiment were consistent with Bayes' rule. To address this question, we first try to find out the posteriors on which the subjects based their decisions. We call these posteriors induced posteriors. Next, we calculate the "true" under Bayes' rule posteriors. We call these, Bayes' posteriors. Finally, we compute the difference and the absolute difference between the induced posteriors and the Bayes' posteriors. These differences are interpreted as a measure of the "error" and the "absolute error" committed by the subjects in updating their probabilities.

Neither the induced posteriors nor the Bayes' posteriors can be calculated straight away. To carry out these calculations, we make some additional assumptions. To find out the induced posteriors, we need to take the inverse of the function which the subjects used to translate their estimates of the probability of the true state into actions. In Task I, the subjects' actions were elicited for different probabilities of success. Assuming that, in both tasks of the experiment, the participants mapped probabilities into actions in the same way, we use the answers given in Task I to transform the subjects' actions into induced posteriors. For this, we make three additional assumptions. First, because the subjects' actions were elicited only for the case when asset R was exactly as likely or more likely than asset B to be successful, we assume that the subjects' mapped probabilities into actions in a symmetric way around the pair of success probabilities $(p,1-p)=(0.5,0.5)$. That is, if for example, in Task I, trader n was willing to invest 700 in asset R and 300 in asset B for $(p,1-p)=(0.7,0.3)$, we assume that she would be willing to invest 300 in asset R and 700 in asset B for $(p,1-p)=(0.3,0.7)$. Second, in the cases where the subjects made the same allocation choices for different pairs of probabilities of success (e.g., they invested 900 in R and 100 in B for $(p,1-p)=\{(0.7,0.3),(0.8,0.2),(0.9,0.3)\}$) while converting their actions into probabilities, we take the average of the probabilities that warranted the same action (that is, following the foregoing example, we induce that $(p,1-p)=(0.8,0.2)$). Third, as the subjects took actions in Task II, which sometimes did not exactly match any action taken in Task I, we find out between which actions, taken in Task I, that particular action lay and transform it into a probability by assuming that all actions between the two boundary actions in Task I are uniformly distributed. For example, if trader n took action $a=(700,300)$ in Task II and in Task I she indicated that she would invest 600 in R and 400 in B for $(p,1-p)=(0.6,0.4)$, and 800 in R and 200 in B for $(p,1-p)=(0.7,0.3)$, we calculate the probability corresponding to the action $a=(700,300)$ as $(p,1-p)=(0.65,0.35)$.

To calculate Bayes' posteriors, we need to know the sequence of signals, which the subjects induced from the actions of their predecessors. Considering the results reported in the previous sub-section, we identify the following heuristic by means of which 621 (82.1%) of the 756 private signals could be correctly induced.

Heuristic 1: Let $a_R^n \neq 500$ *and, if applicable,* $a_R^{n-1} = \ldots = a_R^1 = 500$.

i) If $a_R^n > 500$, *then* $x_{induced}^n = r$. *If* $a_R^n < 500$, *then* $x_{induced}^n = b$. *In addition,* $x_{induced}^{n-1},\ldots,x_{induced}^1$ *were undetermined and* $a_R^{n-1},\ldots,a_R^1$ *ignored when successors calculate their posteriors.*

ii) If $a_R^n > 500$ *and* $a_R^{n+1} = a_R^n$ *or* $a_R^{n+1} > a_R^n$, *then* $x_{induced}^{n+1} = r$. *If* $a_R^n > 500$ *and* $a_R^{n+1} < a_R^n$, *then* $x_{induced}^{n+1} = b$. *If* $a_R^n < 500$ *and* $a_R^{n+1} = a_R^n$ *or* $a_R^{n+1} < a_R^n$, *then* $x_{induced}^{n+1} = b$. *If* $a_R^n < 500$ *and* $a_R^{n+1} > a_R^n$, *then* $x_{induced}^n = r$. *Follow iteratively the same procedure to find* $x_{induced}^{n+2},\ldots,x_{induced}^6$.

iii) If there is a trader υ *such that* $\upsilon > n$ *and* $a_R^v = 500$, *then if* $a_R^{v+1} > 500$, $x_{induced}^{v+1} = r$ *; if* $a_R^{v+1} < 500$, *then* $x_{induced}^{v+1} = b$*; if* $a_R^{v+1} = 500$, *then* $x_{induced}^{v+1} = x_{induced}^{v}$.

We calculate Bayes' posteriors by applying Bayes' rule on the signals induced by means of Heuristic 1. Figure II depicts the average absolute error and the average error for each of the 54 participants in our sample. Observations between the vertical grid lines were always taken from subjects who played in one session.

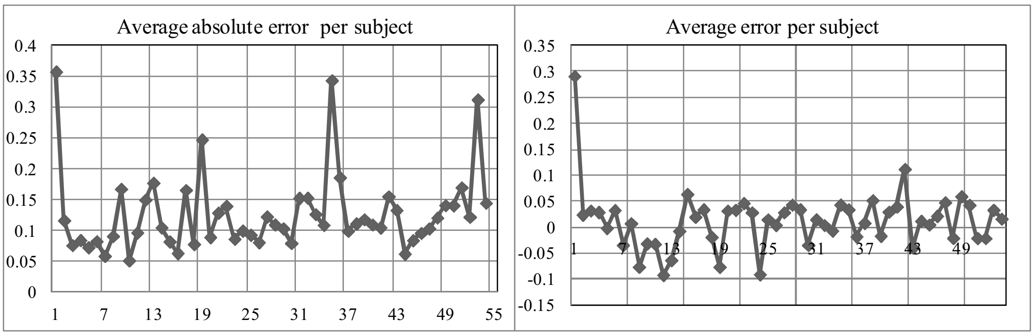

FIGURE II. Average absolute errors and average errors

In the left panel of Figure II, it can be seen that some subjects made very small absolute errors, implying that the assumption of Bayes' learning provided them good approximation of actual behavior. There are, however, also subjects who deviated from Bayes' posteriors, on average, by as much as 35 percent. For the subjects as a whole, the average absolute error is 12.7 percent. The right panel of Figure II, depicts the average error per subject. Participants sometimes overestimated and sometimes underestimated the Bayes' probabilities; the average errors indicate to what extent these positive and negative errors canceled out. If the average error per subject is very close to zero, one can conclude that when subjects play over many periods, their behavior can well be described by Bayes' learning. From Figure II, it can be observed that all but two subjects made an average error of less than 10 percent, while the majority of them made an average error of less than 5 percent. For the subjects as a whole, the average error is 1.26 percent which, by excluding the outlier who made an average error of 29 percent, decreases to 0.73 percent. We interpret these results as evidence that when the subjects make decisions over several periods, their behavior is consistent with Bayesian learning.

D. *Actual actions and theoretical predictions on the aggregate level*

In this sub-section, we look at the data at an aggregate level. We derive the theoretical distributions of actions for each of the six periods (over all 126 rounds) and juxtapose them next to the distribution of actual actions. Depending on whether the sequence of signals used was derived by means of Heuristic 1 or the actual sequence of signal realizations, we obtain two different theoretical distributions. These distributions were calculated as follows. The Bayes' posteriors, obtained as explained in the previous sub-section, were converted into actions by using the results from Task I, averaged over all participants and depicted in Figure I. Furthermore, we made similar calculations by using actual signal realizations. The graphs in Figure III depict the distribution of actual actions for each period, as well as the two theoretical distributions of actions.

With the help of Heuristic 1, 82.1 percent of the signals could be identified. It can be observed from Figure III that the distributions of actions derived from the true signals are almost identical to the one derived from Heuristic 1. What is even more interesting is that these two distributions resemble very closely the distribution generated by actual actions. The absolute difference between the relative frequencies of actual actions and those calculated based on the true signals is lower than 10 percent in 86 percent of the cases, and 5 percent in 60 percent of the time. The absolute difference between the relative frequencies of actual actions and those calculated based on signals induced by means of Heuristic 1 is lower than 10 percent in 90 percent of the cases, and 5 percent in 70 percent of the time. The smallness of the distances between the actual distribution of actions and the two theoretical distributions provides another piece of evidence that actual behavior in Task II can well be approximated by Bayes' rule.

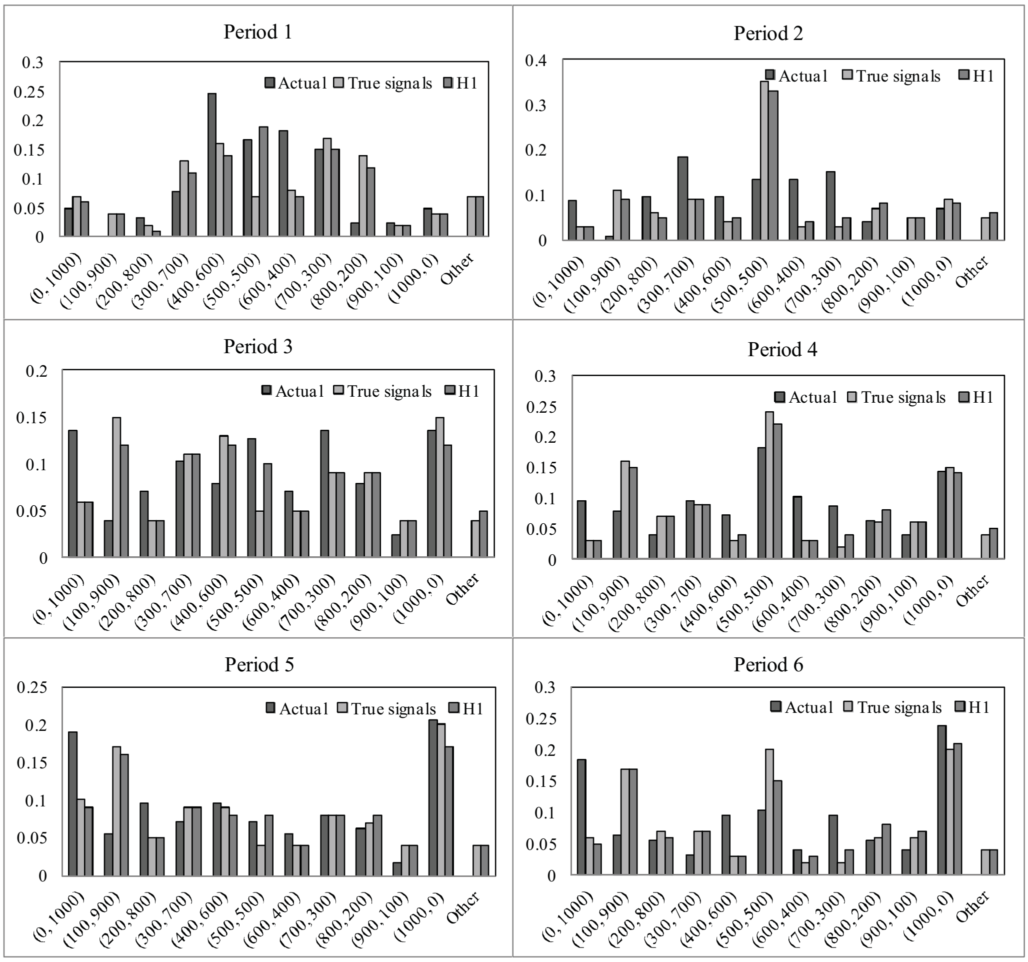

FIGURE III. Actual and theoretical distributions of actions

5. Conclusion

In this paper, we experimentally test the theoretical predications of a model based on the theory of BHW. Our experimental design is original in that we could study social learning in a setting where agents can take actions from a very rich action set, in contrast to the models of many previous studies whose action sets have only two elements. Consistent with the argument of Lee (1993) that the richer the action set, the more efficient would be the aggregation of information, we find that the majority of decision choices made by the subjects in the present experiment perfectly reveal private information.

This paper also addresses the questions whether the subjects perform probability matching and whether they take actions in line with Bayes' law. Consistent with previous findings, we find that participants in our experiment performed probability matching. The mapping of probabilities into actions, however, was not always done in a one-to-one fashion. The present results also indicate, that, both on the individual and aggregate levels, Bayes' law seems to provide good approximation of the rules that subjects use to update their beliefs about the true state.

Even though the asset market we consider in this paper is very simple, our results have interesting implications for the study of information aggregations in different markets, such as betting markets, prediction markets, and financial markets. Our results imply that herding should not be an issue in any of these markets. In fact, empirical studies from financial markets find only little evidence of herding among institutional investors (e.g., Lakonishok, Shleifer, and Vishny 1992, Grinblatt, Titman, and Wermers 1995, Wermers 1999). Some researchers also consider that market prices seem to perform well as information aggregators (see e.g. Forsythe, Nelson, Neumann, and Wright 1992, Berg, Forsythe, Nelson, and Rietz 2005, Berg, Nelson, and Rietz 2003).

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Appendix Written Experimental Instructions

Welcome to our today's experiment! Below you will find the description of the experiment and then you will be asked to make a series of decisions. Please read the following information very carefully. If you have any questions, please ask before the experiment starts. Please note that during the whole experiment, communication with the other participants is not allowed. Thank you!

The Experiment

This is a hand-run experiment consisting of two parts. You get separate instructions for each part of the experiment. At the beginning of the experiment, you have been assigned a participant number. Please write that number in the upper right blank on all of your answer sheets.

At the beginning of each part of the experiment, you get the printed instructions for this part. Please read the complete instructions at first and ask any questions you may have. After that, please make your decisions.

You receive a show-up fee of 5 euros. During the experiment you are given the opportunity to earn more money. Your earrings depend partly on your decisions and partly on chance. At the end of the experiment you will be informed about your total earnings and you will be privately paid.

Part I

Instructions

In this part of the experiment, you are asked to make several investment decisions. You receive an endowment of 1000 euro-cent. Your task is to decide how to divide that endowment between two investments—Investment R and Investment B. You can divide the endowment between the two investments in any way you like.

Only one of the two investments will be successful. Investment R will be successful with probability p and Investment B will be successful with probability $(1-p)$. If an investment is successful, you receive the money invested in it. Otherwise, you receive nothing.

Example

p	1-p	Investment R	Investment B	Total investment
50%	50%	600	400	1000 cent

In this example, Investment R will be successful with probability $p = 50\%$ and Investment B will be successful with probability $(1-p) = 50\%$. If you invest 600 cent in Investment R and 400 cent in Investment B, depending on which investment turns out to be successful, your earnings will be determined as follows:

i) Investment R is successful: earnings = 600 cent.

ii) Investment B is successful: earnings = 400 cent.

The successful investment will be determined by a random draw from an urn containing 10 balls. The balls are in two different colors—red and blue. If a red ball is drawn, then Investment R will be the successful investment. If a blue ball is drawn, then Investment B will be the successful investment. The proportions of the red and blue balls in the urn are determined based on the probabilities of success of the two investments (i.e., 1 ball corresponds to 10 percent, 2 balls to 20 percent etc.).

Payoff mechanism

You will be asked to make one investment decision for each of the following combinations of success probabilities:

	p	1-p	Investment R (p)	Investment B (1-p)	Totao investment
1	100%	0%			1000 cent
2	90%	10%			1000 cent
3	80%	20%			1000 cent
4	70%	30%			1000 cent
5	60%	40%			1000 cent
6	50%	50%			1000 cent

Only 1 of the 6 decisions will be paid out. The decisions which will be played for real money will be determined at the end of the experiment. For this purpose, one ball will be randomly drawn from an urn containing six, consecutively numbered from 1 to 6 balls. The number of the ball which has been drawn determines the decision which will be paid out.

Depending on your investment decision you will receive:

The amount of the 1000 euro-cent which you have invested in the successful investment.

Participant number:

Decision sheet *(for Player 1)*

Please use the table below to record your investment decisions.

Nr.:	p	1-p	Invesment R	Investment B	Total investment
1	100%	0%			1000 cent
2	90%	10%			1000 cent
3	80%	20%			1000 cent
4	70%	30%			1000 cent
5	60%	40%			1000 cent
6	50%	50%			1000 cent

p: The probability with which Investment R will be successful

$1-p$: The probability with which Investment B will be successful

Part II

Instructions

This part of the experiment consists of 20 rounds. In each round, you will be asked to make an investment decision. In each round, you receive an endowment of 1000 euro-cent. Your task is to decide how to divide that endowment between two investments—Investment R and Investment B. You can divide the endowment between the two investments in any way you like.

In each round, only one of the two investments will be successful. Investment R will be successful with probability $p = 50\%$ and Investment B will be successful with probability $(1-p) = 50\%$. If an investment is successful, you receive the money invested in it. Otherwise, you receive nothing.

At the beginning of each round, the successful investment will be determined by the experimenter by a random draw from an urn containing 10 balls—5 red and 5 blue. If a red ball is drawn, then Investment R will be the successful investment. If a blue ball is drawn, then Investment B will be the successful investment. You will not be able to observe the color of the ball which has been drawn. At the end of the experiment, you will be informed about the color of the balls which have been drawn in each round.

You and the rest of the participants will receive a private signal about the successful investment. Each of you will be able to observe only your own private signal. The receipt of the private signal will take place in the following way. The experimenter will come around each of you and you will draw a ball from an urn containing 3 balls. The balls are in two different colors—red and blue. In the rounds in which Investment R is successful, the urn will contain 1 blue and 2 red balls. In the rounds in which Investment B is successful, the urn will contain 1 red and 2 blue balls. This means that the probability of drawing a red ball is 2/3 when Investment R is successful and 1/3 when Investment B is successful. Similarly, the probability of drawing a blue ball is 1/3 when Investment R is successful and 2/3 when Investment B is successful.

After all of you receive a private signal, you will be asked to make an investment decision. All investment decisions will be made in a sequence. The experimenter will approach you in the order in which the investments decisions are going to be made. When the first person in the sequence makes her/his decision, the experimenter will loudly announce that decision to the rest of the participants. After that, the second person in the sequence will be approached and asked to make her/his decision which will also be loudly announced to the rest of the participants. This process will be repeated until the last, sixth, person in the sequence has made her/his decision. After that a new round begins. The order in which the decisions are made is randomly determined at the beginning of each round.

You are asked to complete your investment decisions in the decision table that follows. The results for each round are recorded on a separate row. The round numbers are listed on the left side of each row. Next to the round number is a blank which should be used to record your private draw. The columns numbered (1) through (6) should be used to record the investment decisions in the sequence they are announced. Write your decision in the column, (1) through (6), that corresponds to the order in which you are approached and circle your decision to distinguish it from others' decisions. The last two column of the table will be filled out by the experimenter at the end of the experiment.

The payoff mechanism

Only 1 of the 20 rounds will be paid out. The round played for real money will be randomly determined at the end of the experiment. For this purpose, a ball will be randomly drawn from an urn containing 20 consecutively numbered from 1 to 20 balls. The number of the ball which has been drawn determines the number of the round which will be paid out.

Depending on your investment decision you will receive:

The amount of the 1000 euro-cent which you have invested in the successful investment.

Participant number:

The decision table

Round	Private signal red/blue	Invest-ment	1	2	3	4	5	6	Successful investment R or B	Earnings
1		R								
		B								
2		R								
		B								
3		R								
		B								
4		R								
		B								
5		R								
		B								
6		R								
		B								
7		R								
		B								
8		R								
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18		R								
		B								
19		R								
		B								
20		R								
		B								

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ISSN 1615-4274