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Abstract: Screening contracts (or non-linear "menu of contracts") are frequently used for aligning the incentives in supply chains with private information. In this context, it is assumed that all supply chain parties are strictly (expected) profit maximizing and, therefore, sensible to even arbitrarily small pay-off differences between contract alternatives. However, previous behavioral work on contracting under asymmetric information in supply chains shows that agents (buyers) are not always strictly profit maximizing. Instead, they sometimes tend to choose contracts that have only a minor impact on their own performance but a substantially negative impact on the principal's (supplier's) and the overall supply chain's performance. Thus, these studies indicate that the buyers are in fact not strictly but only nearly profit maximizing when making their contract choices. The present work relaxes the assumption of the strictly profit maximizing buyer in a serial supply chain for a lotsizing framework with asymmetrically distributed holding cost information and deterministic end-customer demand. The study provides researchers and managers an approach on how to account for the buyer's insensitivity to arbitrarily small pay-off differences while providing a solution method for the resulting non-linear mathematical program. A numerical study compares the advantages of the "behavioral robust" contract assuming only nearly profit maximizing buyers against the classical screening contract assuming strictly profit maximizing buyers. The results highlight that supply chain performance losses can be substantially reduced under the behavioral robust contract.

Keywords: Asymmetric information · Supply chain coordination · Contracting · Behavioral modeling

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1. Introduction

A major challenge of supply chain coordination is to align the incentives of the supply chain parties such that decentralized decision making leads to an overall supply chain efficient outcome (see, e.g., Cachon, 2003). Yet, if the incentives are not sufficiently aligned, then the supply chain members might deliberately exploit the supply chain counterpart to enhance own financial ratios. In this situation, the implementation of specific concepts like, e.g., just-in-time delivery is likely to fail unless there are mechanisms avoiding the pitfalls of opportunistic behavior in advance. Very prominent tools in the supply chain management literature for avoiding opportunistic behavior are contracts that legally stipulate the business relation between the supply chain parties. These contracts strive for linking the actions of the company to individual performance measures, e.g., costs or revenues to the overall supply chain performance. In other words, contracts try to align the incentives such that optimizing the individual performance is in line with optimizing the supply chain performance. In this context, the information availability within the supply chain becomes particularly important, since opportunistic behavior might translate to deception and mistrust and, thus, to an overall deterioration of supply chain performance. The purpose of the underlying paper is to investigate how contract design under asymmetric information can contribute to supply chain coordination when relatively small incentives are insufficient for aligning the actions of the supply chain parties.

We use a stylized supply chain interaction model as presented by Inderfurth et al. (2012) and Voigt and Inderfurth (2011 a/b) in order to analyze the impact of small pay-off differences on the supply chain performance. The model captures the basic supply chain conflict that suppliers typically prefer larger delivery lot-sizes in order to exploit economies of scale, while buyers tend to choose smaller delivery lots in order to have lower average inventories. We capture this situation with a lot-sizing decision in a serial supply chain facing deterministic end-customer demand. In this context, information asymmetry arises because the supplier (principal) cannot fully assess the buyer's (agent's) advantages of lowering the average inventories, i.e., the buyer's type (e.g., low cost type or high cost type) is unknown to the supplier. If the supplier strives for increasing the buyer's order sizes he has to compensate the buyer, whereas the exact amount of the required compensation is unknown due to information asymmetry.[2]

[2] We refer to economic lotsizing models under asymmetric information that are in nature similar to the stylized strategic lotsizing model presented in the underlying work to Corbett and de Groote (1997, 2000), Corbett (2001), Burnetas et al. (2007) and Sucky (2004, 2007).

If both parties are acting fully rational and strictly profit maximizing, it can be shown that the supply chain optimal outcome cannot be achieved since the expected profit maximizing supplier offers an inefficient non-linear contract scheme. This non-linear contract scheme (so-called "Screening Contract" or "Menu-of-Contract") coordinates the supply chain to the second-best outcome, i.e., the best outcome achievable if decentralized decision makers are fully rational and strictly profit maximizing (see, e.g, Corbett and de Groote (2000), Corbett (2001), Sucky (2006), Ha (2001), Voigt and Inderfurth (2011a/b) and, for a comprehensive literature review, Voigt (2011)).

While the screening theory is pretty much developed and established in the supply chain management literature (and other related areas) for fully rational and strictly profit maximizing supply chain parties, there are only a few contributions that analyze the supply chain behavior if these critical assumptions are not met. As an example, full rationality in the present context requires that the less informed supply chain party (principal) reckons that all communication is only cheap talk, since the privately informed supply chain party (agent) will use the private information strategically and, therefore, biases the shared information in order to obtain more favorable contract terms. Since the principal anticipates this behavior, all communication will be disregarded and the supply chain ends-up in a babbling-equilibrium in which communication has no impact at all on the supply chain outcome. A relaxation of the assumption that communication has no impact in this context is presented by Voigt and Inderfurth (2011b). In contrast, the present article addresses the critical assumption of strictly profit maximizing buyers.

The strict profit maximization assumption postulates that all supply chain parties always take the (expected) profit maximizing action. The principal maximizes his expected profits by offering a menu self-selection contracts that provides incentives to reveal the private information. This revelation of private information, in turn, is also strongly linked to the profit-maximization assumption, since revelation exploits the fact that there is a unique mapping between the private information and the profit maximizing contract. Since choosing the profit maximizing contract is assumed to be in the best interest of the agent, the revealing contract choice is frequently denoted as self-selection. One direct implication of the profit maximization assumption is that the agent is always (weakly) indifferent between two contract alternatives (i.e., two contracts out of the menu-of-contracts).[3] Due to the profit maximization assumption a distinct prediction on the contract choice is made, and the principal can theoretically infer the agent's private information from his contract choice (i.e., information revelation). We denote the strictly profit-maximizing contract as the self-selection

[3] Strictly indifferent means that there is no profit difference between to contract alternatives, whereas weakly indifferent means that an arbitrarily low incentive is given for choosing the self-selection contract.

contract and the next alternative to which the buyer is (weakly) indifferent as the indifference contract.

A recent laboratory study by Inderfurth et al. (2012) shows that the strict profit-maximization assumption and, therefore, information revelation by self-selection is a critical assumption in the coordination literature under asymmetric information. In particular, their study shows that in 79% of all observations the profit maximizing contract was chosen. However, there is also a non-negligible fraction of contract choices (i.e., 21%) that cannot be explained by strict profit maximization. In 17% of the observations, the agents chose the nearly profit maximizing indifference contract. Thus, only 4% of the contract choices cannot be explained by strict or near profit maximization, whereas 2% of these 4% can be attributed to a termination of the business relationship which might be interpreted as an attempt to signal bargaining power. Inderfurth et al. (2012) are showing that contract choices that are not strictly profit maximizing have a huge negative impact on the overall supply chain performance. The present work, therefore, presents a behavioral model that explicitly accounts for the fact that some agents might be insensitive to arbitrarily small pay-off differences between a self-selection and an indifference contract and gives directions how the principal can deal with difficulties arising from this fact. Henceforth, we denote a contract that incorporates the buyer's insensitivity to arbitrarily small pay-off differences as a behavioral robust contract, since such a contract incorporates the buyers' behavior that is contrary to the standard assumption of strict profit maximization.

One approach for incorporating insensitivity to small pay-off differences into screening models was introduced by Laffont and Martimort (2002, Chapter 9.8.1), who are assuming that agents are making decision errors (so-called "trembling hand behavior") that can be described by a probability distribution. The principal accounts for these decision errors by adding a slack into the incentive constraint in order to increase the likelihood that the agent chooses the self-selection contract. The principal is then computing the optimal menu of contract by taking into consideration the probability with which a contract is chosen instead of the a-priori probability of the distribution of types, where types denotes agents having different realizations with respect to the private information (e.g., low cost type or high cost type). The underlying work incorporates a similar approach, however, in contrast to Laffont and Martimort we are extending the analysis to more than two types and allow for a different formulation of the agents decision errors. While Laffont and Martimort assume that all agents make similar decision errors that are supported by a probability distribution following a monotone hazard rate, we are assuming that the agents are either acting strictly profit maximizing or only nearly profit maximizing (as identified by Inderfurth et al., 2012). In this context, near profit maximization means that the insensitivity to pay-off differences is limited by assumption such that the buyer of a certain type will either choose the contract that was designed for his type or the indifference contract that was designed for the adjacent type. In particular, we are assuming that there is a linear dependency between the buyer's pay-off difference of the two contract alternatives (i.e., self-selection and indifference contract) and the likelihood that the profit maximizing contract is chosen. The extended analysis specifically allows investigating the impact of bunching, i.e., several types are offered the same contract (pooling equilibrium) and the consequences for optimal contract design. To this end, we are presenting a numerical study highlighting the impact of decision errors (i.e., insensitivity to small pay-off differences) on the overall supply chain performance. Such a comparison has not been performed before but is especially important for the supply chain coordination point of view, since the second best outcome serves as a benchmark of the benefits of cooperation.

Another approach accounting for the insensitivity to small pay-off differences is analyzed by Basov and Danilkina (2006) and by Basov (2009), who are analyzing the principal agent setting under the assumption that agents make probabilistic choices according to Luce (1959). In this setup, the likelihood of the contract choice depends, on the one side, on the number of contract alternatives offered and, on the other side, on the pay-off differences between those alternatives. In this approach, thus, the principal can increase the probability that the profit maximizing contract is chosen by offering several contracts that result in the same pay-off. This is done by introducing a new dimension to the product that has no impact on the agents utility associated with this product (e.g., agents have no preference for the "flavor" of a product like cereals). In contrast to this approach, we are not assuming a probabilistic choice rule in which every alternative is chosen with a positive probability. Instead, we are assuming that the actual pay-off difference between the alternatives is the only distinguishing criteria determining the probability with which contracts are chosen. Under the probabilistic choice assumption as analyzed by Basov and Danilkina (2006) and Basov (2009), the introduction of insensitivity to small pay-off differences does interestingly have no impact on the coordination deficit (i.e., gap between the sum off principals and agents pay-offs between first-best and second-best solution), while we are observing such an impact on the overall coordination deficit in our setting.

Basov and Mirrless (2009) present a study in which the agent makes random decision errors that are type dependent instead of depending on the actual pay-off differences between the alternatives. They are showing that in such a context the principal may even be better off if he is anticipating such systematic decision errors. In our approach, however, we are assuming like Basov and Danilkina (2006) and Basov (2009) that the pay-off differences resulting from the informational rents suffices for ensuring (nearly) profit maximizing contract choices, i.e., either the self-selection or the (weakly) indifference contract is chosen. This assumption is supported by the experimental study from Inderfurth et al. (2012) who are showing that strictly or nearly profit maximizing contract choices are being made in 96% (strict profit maximization: 79%, near profit maximization: 17%) of all observations. Thus, only 4% of all observations cannot be explained by (nearly) profit maximizing contract choices, which rather supports the assumption of nearly profit maximizing agents (as in Basov and Danilkina, 2006, and Basov, 2009) than agents making random decision errors independent of the pay-off structure (as in Basov and Mirrless, 2009).

Incorporating observations from experimental work (e.g., other regarding preferences, decision biases, bounded rational behavior) into theoretical operation management models in order to increase the predictive power, practical applicability and external validity of the theoretical models received an increasing attention in the recent past (we refer to Loch and Wu (2007) for a discussion on how behavioral models can enrich the operations management research). Examples of such behaviorally enriched models include, e.g., Voigt and Inderfurth (2011b) who investigate a behavioral screening model in a supply chain setting that is closest to the underlying work. They are analyzing and quantifying the pitfalls of information sharing in an asymmetric information context by introducing the assumption that a fraction of privately informed supply chain parties report their private information truthfully regardless of the underlying incentives. Supply chain models incorporating objectives different from profit maximization have been analyzed by Cui et al. (2007) and Pavlov and Katok (2011).

The contributions to the behavioral management literature and the screening literature are threefold. First, the study provides researchers and managers an approach on how to account for decision errors when offering screening contracts to agents that are insensitive to small pay-off differences in a framework where a buyer and a supplier are negotiating the terms of delivery in a just-in-time environment. It is shown that supply chain losses occurring from indifference contract choices can be substantially limited if the buyers are either actively incentivized to choose the self-selection contract by increasing the pay-off difference and/or by adjusting the order sizes in the respective screening contract.

Second, we are contributing to the operations research field of contract design by showing an efficient way for determining the contract parameters for a non-linear, non-concave optimization problem that is linearly constraint. We are showing that there are at most two critical points resulting from the evaluation of the respective Karush-Kuhn-Tucker conditions and show that there is at most one interior solution that qualifies for the global optimum. We perform the analysis for the case of more than two types, which allows identifying specific characteristics of the screening contract that would not be observable otherwise. To this end, we are introducing an algorithm that allows identifying the regions in which different buyer types will be offered the same contract (so-called "bunching").

Third, a numerical study compares the performance of the contract that accounts for insensitivity to small pay-off differences to the a-priori screening contract. It is shown, that indifference contract choices result in the majority of the cases to supply chain performance losses. However, the losses resulting from indifference contract choices can be considerably limited if the contract accounts for this phenomenon. Interestingly, though, the analysis reveals that in some specific cases the supply chain performance even improves compared to the classical a-priori screening benchmark.

The remainder is organized as follows: Section 2 outlines the stylized supply chain interaction model under full and asymmetric information. Section 3 summarizes the optimization procedure for the behavioral robust contract under asymmetric information. Section 4 introduces the measures against which the behavioral robust contract is benchmarked and Section 5 complements the numerical example with a sensitivity analysis. Section 6 concludes and gives managerial insights. A detailed derivation of the optimal behavioral robust contract is given in the Appendix, Section 8.

2. Outline of the model

A strategic lotsizing model, as introduced by Voigt and Inderfurth (2011 a, b) and Voigt (2011), is used to elaborate and quantify the impact of insensitivity to small pay-off differences in screening contracts. The model depicts a dyadic supply chain interaction in which the buyer (B) faces a deterministic and constant end-customer demand, d. The products are sold at a fixed price to the end-customer, i.e., the impact of ordering decisions on the buyer-customer interface is not considered in the underlying model.

The buyer's inventory-related costs are assumed to be linearly proportional to the stock level, which is dependent on the actual ordering decision, q. The aggregated holding cost parameter, h, is an aggregated measure for the entire buyer's disadvantages he faces if he orders in large quantities, e.g., more required warehouse space and handling equipment/workforce, more tied-up capital, etc. In turn, the supplier faces several drawbacks if he allows for low order sizes, e.g., losses of economies of scale in various logistics activities. These disadvantages are depicted by the fixed costs occurring per order, f. Hence, if the supplier agrees upon low order sizes, he will occur those fixed costs on average more frequently per period. All other cost effects are excluded in the underlying model in order to facilitate the exposition on the behavioral observations mentioned earlier. The buyer's decision variable is the order size, while lower order sizes are associated with less costs, i.e., given an order size of *q*, the buyer faces holding costs of $(h/2)\cdot q$ per period. The supplier, in turn, favors higher order sizes in order to leverage economies of scale. His fixed costs per period amount to $(f\cdot d)/q$. The profit allocation in the supply chain takes place via the transfer-price per unit, *w*. Thus, total transfers per period amount to $w\cdot d$. The buyer may choose an alternative supplier, if the costs of this supply source, R, are cheaper than the costs resulting under any contract offered from the supplier. In this case, the supplier is assumed earning $\overline{P}_S$ per unit demand (e.g., by trading with another buyer). In the present study we are assuming that the supplier will not exclude any buyer type from trade, i.e., trade is even profitable for the high cost buyer. For a discussion of so-called cut-off policies where buyer types may be excluded from trade it is referred to Ha (2001) and Cakanyildirim et al. (2012). All in all, the above situation captures the well-known supply chain conflict that supplier's tend to favor larger lot-sizes, while buyer's prefer to source in smaller lots (see, e.g., Corbett and de Groote, 2000).

We are assuming that the buyer is with a probability $\overline{\alpha}$ sensitive to even arbitrarily small pay-off differences as assumed in the classical screening literature. However, with probability $(1-\overline{\alpha})$ the buyer is insensitive to arbitrarily small pay-off differences and the magnitude of the insensitivity to pay-off difference cannot be assessed with certainty by the principal. Instead, he assumes upper and lower bounds (t_l and t_h), i.e., the pay-off difference must be at least t_l for being tangible for the buyer, and is with certainty sufficient for a pay-off difference of $t_{h.}$. Given a pay-off difference $t, t_l \leq t \leq t_h$, we are assuming that the buyer will choose the profit maximizing contract with probability $\alpha(t)$, where $0 \leq \alpha(t) \leq 1$ and $\alpha(t)$ depends linearly on t, i.e., $\partial\alpha(t)/\partial t \geq 0$ and $\partial^2\alpha(t)/\partial^2 t = 0$. Thus, the higher the pay-off difference, the higher the probability that the buyer chooses the profit-maximizing contract and vice versa. Note that we are restricting ourselves to a linear shape of $\alpha(t)$ in order to be able to prove the optimality of our resulting behavioral robust contract. In principle, though, other functional forms may be assumed and integrated into the presented framework. However, in this case the number of local maximas may be indefinite and other optimization approaches in the field of non-linear optimization or heuristic methods may be applied. Nonetheless, the general insights presented in this work will not change from a qualitative point of view, since even only locally optimal solutions (that are, for example, obtained by applying simple local search procedures) will limit the effect of out-of-equilibrium contract choices.

Under full information (FI) of all above mentioned parameters, it is in the supplier's best interest to offer a contract that maximizes his own profits, while ensuring that the buyer does not choose the alternative supply source which causes costs of *R*. The supplier's optimal contract offer consists of a combination of wholesale price and order quantity, $A^{FI} = \langle w, q \rangle$, that is determined by solving:

$$\max_{w,q,t} P_S = \alpha(t) \cdot d \cdot \left(w - f/q \right) + \left(1 - \alpha(t)\right) \cdot d \cdot \overline{P}_S \tag{2.1}$$

$s.t.$

$$\left(w + t\right) \cdot d + h/2 \cdot q \leq R \cdot d \tag{2.2}$$

$$t_l \leq t \leq t_h \tag{2.3}$$

The participation constraint (2.2) will bind in the optimal solution, and it follows directly that

$$w = R - \frac{h}{2 \cdot d} \cdot q - t \tag{2.4}$$

Note, that *t* in (2.2) can be interpreted as a fictional discount on the wholesale-price w. Since the participation constraint (2.2) must be binding in the optimal solution, the slack-variable *t* ensures that the buyer in fact has a total cost advantage when accepting the contract instead of choosing the alternative supplier of $t \cdot d = R \cdot d - h/2 \cdot q - w \cdot d$ per period between both alternatives (i.e., outside option and contract offer). Inserting (2.4) into (2.1) and deriving and solving for *q* gives the optimal order size

$$q^{FI} = \sqrt{\frac{2 \cdot f \cdot d}{h}} \tag{2.5}$$

The optimal size of the pay-off difference can be calculated from solving:

$$\frac{\partial \alpha(t)}{\partial t} \cdot \left(R - t - \frac{h}{2d} \cdot q - \frac{f}{q} - \overline{P}_S \right) - \alpha(t) = 0 \ \forall t_l \leq t \leq t_h \tag{2.6}$$

As highlighted earlier, the holding costs are assumed to be multidimensional since they are an aggregate measure for the non-value adding disadvantages of holding inventories. We model this situation by the common approach that there is a probability distribution, $p_i, i = 1,...,n$, over possible holding cost realizations $h_i, i = 1,...,n$ and $h_1 < ... < h_i < ... < h_n$. The corresponding supply chain optimal contract as if under full information is donated as $q_i^{FI}, \forall i = 1,...,n$.

This formulation is denoted as the discrete-type case, in which we distinguish between the buyer types h_i. An alternative formulation is the continuous-type case (see, e.g., Corbett, 2001), in which there is a continuum of types $\left[\underline{h},\overline{h}\right]$ with support of a density function. We refer to Kerschbamer and Maderner (1998) for a discussion and comparison of the respective model formulations.

In the standard game-theoretic equilibrium, the supplier offers a menu of contracts $A_i = \langle w_i, q_i \rangle, i = 1,...,n$, in which the contract A_i minimizes the buyers costs for holding cost h_i (incentive constraint). Again, we assume that a buyer type h_i with a pay-off difference of t_i will choose the profit maximizing contract offer with probability $\alpha(t_i)$. For notational convenience we introduce $\alpha(t_i) = \alpha_i$.

$$\max_{w_i,q_i,t_i} E(P_S) = \sum_{i=1}^{n} p_i \cdot d \cdot \left[\alpha_i \cdot \left(w_i - \frac{f}{q_i} \right) + (1-\alpha_i)\left(w_{i+1} - \frac{f}{q_{i+1}} \right) \right] + (1-\alpha_n) \cdot p_n \cdot \overline{P}_S \cdot d \tag{2.7}$$

$s.t.$

$$(w_i + t_i) \cdot d + \frac{h_i}{2} q_i \le w_{i+1} \cdot d + \frac{h_i}{2} q_{i+1}, \ \forall i = 1,...,n-1 \tag{2.8}$$

$$(w_n + t_n) \cdot d + \frac{h_n}{2} q_n \le R \cdot d \tag{2.9}$$

$$q_{i+1} \le q_i \ \forall i = 1,...,n-1 \tag{2.10}$$

$$t_l \le t_i \le t_h \ \forall i = 1,...,n \tag{2.11}$$

The above formulated approach is already a reduced form of the classical optimization problem which already takes into account some properties that must hold in the optimal menu of contracts. Therefore, some constraints that will not bind in the optimal solution can be eliminated (see Sappington, 1983). These properties are: (1) $q_{i-1}^{AI} \ge q_i^{AI}$ and $w_{i-1}^{AI} \le w_i^{AI} \ \forall i = 1,...,n$, (2) the participation constraint for the buyer facing holding costs h_n binds (i.e., the buyer is weakly indifferent between choosing contract A_n or choosing the alternative supplier), and (3) the buyer facing holding costs h_i $(i = 1,...,n-1)$ is (weakly) indifferent between the contracts A_i and A_{i+1}, $\forall i = 1,...,n-1$.

Property (1) states that the wholesale price decreases with increasing order sizes. Hence, the menu of contracts can be interpreted as a quantity discount. Properties (2) and (3) point out that a buyer is always indifferent between at least two alternatives. However, in our approach we introduce the slack variable $t_i, t_l \le t_i \le t_h$ ensuring in the optimization program formulation that the indifference property holds. However, t_i will not be included in the resulting contract offers $A_i = \langle w_i, q_i \rangle$, but will only be a fictional number controlling for the pay-off difference that will result between the contract offers. As mentioned earlier, we are assuming that the buyer will choose with probability α_i the profit maximizing contract A_i and with probability $(1-\alpha_i)$ the next contract alternative A_{i+1} resulting in a loss of t_i. As in Laffont and Martimort (2002), Basov and Danilkina (2006), and Basov (2009), we are applying the concept of nearly rational profit-maximizing buyers (agents). In this context, nearly profit maximizing means that the upper bound t_h is sufficiently low for ensuring that the buyer will only either choose the self-selection or the indifference contract, i.e. (A_i, A_{i+1}), and not any other contract, $A_j, j \notin (i, i+1)$, out of the menu of contracts.

3. *Optimal contract parameters under asymmetric information*

In the classical formulation of this problem, i.e., $\alpha_i = 1$ and $t_i = 0 \ \forall i = 1,...,n$, the objective function is concave and there is therefore only one critical point qualifying for an interior optimal solution. However, by introducing α_i the objective function loses its concavity condition. Yet, we can show that there are at most one local minimum and one local maximum and the optimal contract parameters can therefore be efficiently obtained by determining the local maximum out of the two critical points resulting from evaluating the KKT conditions.
Setting up the Lagrange function gives:

$$\begin{aligned}
\max \ \ell = &\sum\nolimits_{i=1}^{n} \alpha_i \cdot p_i \cdot \left(w_i - \frac{f}{q_i} \right) \cdot d + \sum\nolimits_{i=1}^{n-1} (1-\alpha_i) \cdot p_i \cdot \left(w_{i+1} - \frac{f}{q_{i+1}} \right) \cdot d \\
&+ (1-\alpha_n) \cdot p_n \cdot \overline{P}_S \cdot d - \sum\nolimits_{i=1}^{n-1} \lambda_{i,i+1} \left((w_i + t_i) \cdot d + \frac{h_i}{2} q_i - w_{i+1} \cdot d - \frac{h_i}{2} q_{i+1} \right) \\
&- \mu \cdot \left((w_n + t_n) \cdot d + \frac{h_n}{2} q_n - R \cdot d \right) - \sum\nolimits_{i=1}^{n} \left(\gamma_i \cdot (t_l - t_i) + \omega_i (t_i - t_h) \right) \\
&- \sum\nolimits_{i=2}^{n-1} \sum\nolimits_{j>i} \beta_{ij} \left(q_j - q_i \right)
\end{aligned} \tag{3.1}$$

The Lagrange parameter β_{ij} in (3.1) is related to the implementability condition (2.10). For certain parameter combinations, the solution to the relaxed problem which neglects the implementability condition (2.10) gives $q_i < q_{i+1}$. However, such a solution cannot be optimal, since it does not satisfy all incentive constraints (see for a discussion, e.g., Sappington, 1983). In the majority of the contributions regarding screening contracts in the literature, this case is ruled out by imposing restrictions on the agent's utility function (here: buyer's cost function) and/or the a-priori probability distribution $p_i, i=1,...,n$.[4] If such conditions are satisfied, then the relaxed problem without the implementability condition will always yield the supply chain optimum in the classical screening model. However, in the present model formulation, bunching occurs even in situations that would not be expected in the classical screening model. Hence, imposing restrictions on the a-priori distribution and the buyer's cost function alone cannot rule out bunching, because the actual frequency of contract choices is not only determined by the a-priori distribution but also by the buyer's insensitivity to pay-off differences. Hence, we explicitly consider this situation and provide an algorithm that may be used to determine the optimal contract parameters for those situations in which the implementability condition (2.10) binds. We refer to the Appendix, Section 8 for a detailed derivation of the optimal menu of contracts.

The optimal contracts A_i=<q_i,w_i> in the menu of contracts result from (see):

$$q_i = q_{zj} \;\; for\; i \in \left(j \geq z \,|\, z, z+1,...., j \right) \; and \; q_{zj} \in M^+ \;\; \forall i = 1,...,n \tag{3.2}$$

where q_{zj} and M^+ are determined according to the algorithm mentioned below.

The optimal wholesale prices are:

$$w_i = w_{i+1} - t_i + \frac{h_i}{2d}\left(q_{i+1} - q_i\right) \; \forall i = 1,...,n-1 \tag{3.3}$$

$$w_n = R - t_n - \frac{h_n}{2d} \cdot q_n. \tag{3.4}$$

The optimal pay-off difference, t_i, results from solving the following non-linear equation system.

$$\frac{\partial \alpha_i}{\partial t_i} \cdot p_i \cdot \left(w_i - \frac{f}{q_i} - w_{i+1} + \frac{f}{q_{i+1}} \right) - \left(\sum_{k=1}^{i-1} p_k + \alpha_i \cdot p_i \right) = 0 \; \forall i = 1,...,n \tag{3.5}$$

Note, the function (3.5) is uni-modular for a linear shape of α_i, i.e., there is at most one local minimum (see Appendix (8.29) - (8.31)). Thus, there are at most two solutions satisfying the KKT condition, whereby one yields a global minimum and the other the global maximum (as long as the maximum is an inner solution). Thus, for solving the non-linear equation system well established methods (e.g., Newton–Raphson method) can be used for determining the roots to (3.5).

[4] In the continuous formulation of the principal-agent problem, e.g., bunching is ruled out by assuming a probability distribution that follows a monotone hazard rate. In contrast, in the discrete type case, bunching will never occur in the two-type case, or if the a-priori distribution and utility function satisfies certain conditions (see, Kerschbamer and Maderner, 1998).

The key for determining the optimal contract parameters (3.2) - (3.5) is to determine the order sizes that will be identical for different buyer types. Let q_{zj} denote the order sizes for which $q_z = q_{z+1} = \ldots = q_j \ \forall j \geq z$ hold, i.e. for $z < j$ some buyer types will be offered an identical order size. Once we know the order sizes q_{zj}, the optimal order sizes q_i follow from (3.2). The optimal order sizes can be determined via the following algorithm.[5] The general idea of the algorithm is to solve (3.2) to (3.5) without the implementability condition (2.10), and then to successively adding those constraints $q_z \geq q_{z+1} \geq \ldots \geq q_j$ that are not satisfied in the relaxed solution.

From evaluating the KKT-conditions (see appendix, (8.23)) we get:

$$q_{zj} = \sqrt{\frac{2fd}{h_j + \frac{\sum_{k=1}^{z-2} p_k + \alpha_{z-1} \cdot p_{z-1} \cdot \left(h_j - h_{z-1}\right)}{\sum_{k=i}^{j-1} p_k + \left(\left(1-\alpha_{z-1}\right) p_{z-1} + \alpha_j \cdot p_j\right)}}} \quad \forall i = 1, \ldots, n \text{ and } j \geq i \tag{3.6}$$

Furthermore we define

$$x_{zj} = \begin{Bmatrix} 1, \text{ if } q_{zj} \in M^+ \\ 0, \text{ else} \end{Bmatrix} \forall z, j = 1, \ldots, n \text{ and } j \geq z \,. \tag{3.7}$$

Start:

$x_{ii} = 1 \ \forall i = 1, \ldots, n$

$x_{ij} = 0 \ \forall i \neq j \text{ and } j > i$

***Solve* (3.2) to (3.5)** $\rightarrow q_i \ \forall i = 1, \ldots, n$

Do while Not $q_i > q_{i+1} \ \forall i = 1, \ldots, n-1$

$y = \arg\min \left\{ i \mid q_i < q_{i+1} \right\}$

$z = y + 1$

Do while not $q_y > q_z$

$x_{y,z-1} = x_{z,z} = 0$

$x_{yz} = 1$

***Solve* (3.2) to (3.5)** $\rightarrow q_i \ \forall i = 1, \ldots, n$

$z = z + 1$

Loop

Loop

End

[5] Methods for optimal bunching have been developed for the continuous type case (see, e.g., Nöldeke and Samuelson, 2007), however, to the best of our knowledge there have been no systematic approach being reported for the discrete type case.

The algorithm is initialized with the relaxed problem (2.7) without implementability condition (2.10), i.e., the optimal contract parameters q_i can be determined by setting $z = j$ which translates to the case that there is no bunching. If the implementability condition (2.10) is satisfied, then the optimal solution is found (since introducing additional constraints can only reduce the objective's value). However, if the relaxed solution violates the implementability condition, then a so-called "bunching" occurs and some contracts out of the menu of contracts will be assigned the same order quantity. Therefore, it has to be determined which order sizes needs to be bunched. In every iteration, the problem needs to be solved according to (3.2) to (3.5), however, in each iteration the set of bunched contracts, M^+, will be updated (expressed by changes of (3.7)). This is done within the two loops. The first loop ensures that all regions in which bunching might occur are identified, while the second loop identifies within each region how many order sizes exactly needs to be bunched.

Example: Assume there are seven buyer types, $h_1,...,h_7$, and in the optimal the contracts $A_2 = A_3 = A_4$ and $A_6 = A_7$ will be bunched. The initialization yields $q_1 > q_2 < q_3 < q_4 > q_5 > q_6 < q_7$. In the outer loop, the first bunching region (starting from A_2) is identified, while the the inner loop determines that three contracts, namely $A_2 = A_3 = A_4$ need to be bunched. Afterwards, the outer loop identifies the second bunching region (starting from A_6) while the inner loop identifies that the contracts $A_6 = A_7$ are being bunched. It thus follows $M^+ = \{q_{11}, q_{24}, q_{55}, q_{67}\}$ which translates to the variables $x_{11} = x_{24} = x_{55} = x_{67} = 1 \; and \; x_{ij} = 0 \; \forall ij \notin \{11, 24, 55, 67\}$.

4. Performance benchmarks and numerical example

In order to assess the impact of the buyer's insensitivity to small pay-off differences, we compare the behavioral robust contract (***br***) to the a-priori screening contract with and without self-selection. The benchmark of the a-priori contract with self-selection (***as***) depicts the classical situation in which all buyers are strictly profit maximizing, while the a-priori contract with insensitivity to small pay-off differences (***ai***) depicts the situation in which the buyer is in fact not strictly profit maximizing but the supplier nonetheless offers the a-priori screening contract. Table 1 summarizes the assumptions underlying the respective benchmarks. The supply chain performance is measured as the performance gap (coordination deficit, *CD*) between the expected supply chain costs in the respective benchmarks, i.e., $E(C_{SC}{}^{i}), \forall i \in \{as, ai, br\}$ and the expected supply chain costs in a centralized setting (supply chain optimum), $E(C_{SC}{}^{SC})$:

$$CD^{i} = E(C_{SC}{}^{i}) - E(C_{SC}{}^{SC}) \; \forall i \in \{as, ai, br\} \tag{4.1}$$

Table 1: Performance benchmarks

	Screening contract based on	
	a-priori probabilities	expected frequency of contract choices
Strict profit maximization: $\overline{\alpha} = 1$	as	
Near profit maximization: $0 < \overline{\alpha} < 1$	ai	br

Supply Chain Performance: An upper bound for the expected coordination deficit is CD^{ai}, i.e., the coordination deficit that results if the a-priori screening contract is offered, but not all buyers are strictly profit maximizing, i.e., there is a certain probability $(1-\overline{\alpha})$ that the supplier is interacting with a buyer who is insensitive to small pay-off differences. Obviously, accounting for the insensitivity to small pay-off differences can only improve the supply chain performance, since it is anticipated that indifferences contract might occur. This, in turn, shifts the probability mass towards higher holding costs levels which also increases order sizes. Interestingly, though, the expected coordination deficit CD^{as} is not a lower bound for the coordination deficit. The reason is that the shift of probability mass from lower types h_i to higher types h_j, j > i, tends to increase the order sizes q_j, which has a positive effect on the supply chain performance for the types h_j. If this positive effect is not offset by the buyer choosing with probability $(1-\alpha_i)$ the order size q_j instead of q_i $(q_j \le q_i)$, then the supply chain performance can even improve compared to the classical case. An example in which supply chain performance improves compared to the classical screening model with self-selection is shown in Section 5, Figure 4b.

Supplier's performance: The supplier's expected profits are obviously largest in the classical benchmark with self-selection and lowest in the classical benchmark with indifference contract choices.[6] However, the anticipation of indifference contract choices and the respective offering of behaviorally optimized contracts dampen the impact of indifference contract choices. Hence, the following relation for the supplier's expected profits holds: $E\left(P_S^{ai}\right) \le E\left(P_S^{br}\right) \le E\left(P_S^{as}\right)$.

[6] If the supplier could increase self-selection without any costs – as in the classical screening benchmark – then his profits would increase.

Buyer's costs: The buyer's costs can only decrease due to indifference contract choices compared to the classical screening benchmark. Obviously, the indifference contract choice has only an arbitrarily small impact on the buyer's performance. However, since the supplier tends to adjust the order sizes of higher types upwards, the informational rents of the lower types increase. Overall, thus, the expected costs decrease since the effect of increased informational rents is only marginally mitigated by arbitrarily small costs caused by indifference contract choices.

The following numerical example depicts a situation with three buyer types (h_1, h_2, h_3).[7] It is assumed that the buyer's insensitivity to small pay-off differences can be described by the following function, α , that is linear in $t_i, 0 \le t_i \le t_h$ given that the buyer acts with probability $\overline{\alpha}$ strictly profit maximizing: $\alpha(t_i) = 1 + \left(1 - \overline{\alpha}\right) \cdot \frac{t_i - t_h}{t_h}$ (see Figure 1).

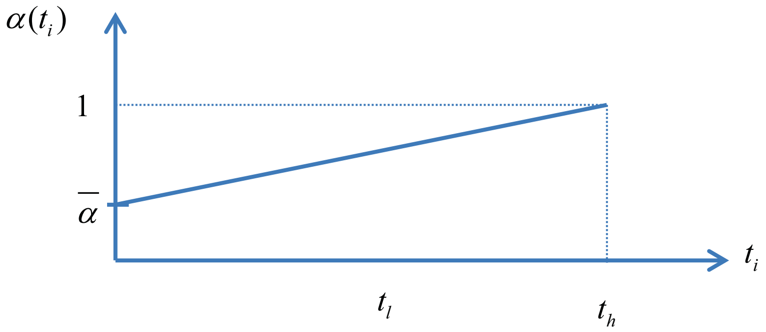

Figure 1: Pay-off insensitivity of buyers

Table 1 presents the parameters for the baseline example without bunching. Note, that the costs of the outside option, *R,* are set such that the supplier always yields higher profits than compared to the outside option. Trade with all buyers is always profitable if trade is profitable with the highest type, i.e.,

$$\left(w_n - \frac{f}{q_n}\right) \cdot d \ge \overline{P}_S . \tag{4.2}$$

Replacing w_n with its optimal value (3.4) in (4.2) and replacing q_n with the highest value that can result, i.e., the supply chain optimal quantity q_n^{FI} , and the highest possible additional incentive, t_h ,

[7] In order to investigate the propensity of bunching that is caused by the insensitivity to small pay-off differences, the numerical example is based on three buyer types, since bunching will never occur in the two-type case. The following presents a numerical baseline example for the case with and without bunching. For a discussion that two distinctive types are not sufficient for analyzing all effects in screening models see Kerschbamer and Maderner, 1998).

gives a value for *R* allowing the supplier to make profits even when trading with the high cost buyer: $R = \overline{P}_S + f / q_n^{SC} + t_h + h_n / (2d) \cdot q_n^{SC} + \mathrm{c}$, where *c* defines the supplier's level of profits. Note, the difference between *c* and $\overline{P}_S$ is that higher $\overline{P}_S$ might exclude buyers from trade (see, e.g., Ha (2001) and Cakanyildirim et al. (2012)), while a higher level of *c* translates to the buyer having higher costs of sourcing from an alternative supplier. Thus, the higher the costs of the buyer's outside option, the higher the wholesale-price the supplier may claim.

Moreover, it may be difficult to assess in a practical situation the buyers insensitivity to small pay off differences, i.e., to assess the form of α_i. As an approximation we assume that the highest required additional incentive can be obtained by the cost difference between the contract choices A_1/A_2 and A_3 in the classical a-priori screening contract, since the experimental results from Inderfurth et al. (2012) indicate that this profit difference is large enough for incentivizing the respective (nearly) profit maximizing contract choice.

Table 2: Parameter values in numerical example for the case without bunching

Parameter	
$(f, d) = (800, 100)$	$(t_h, \overline{\alpha}) = (0.6, 0.5)$ [1]
$(p_1, p_2, p_3) = (0.3, 0.4, 0.3)$	$(\overline{P}_S, R, c) = (0, 15, 5)$ [2]
$(h_1, h_2, h_3) = (1, 3, 5)$	1) $t_h = (h_2 - h_1) / 2d \left(q_2^{as}(\overline{\alpha}) - q_3^{as}(\overline{\alpha})\right)$ 2) $R = \overline{P}_S + f / q_3^{SC} + t_h + h_3 / (2d) \cdot q_3^{SC} + \mathrm{c}$

For the given parameter values, Table 3 summarizes the optimal contract parameters/decision variables and Table 4 summarizes the cost effects for the supplier/buyer/supply chain given the respective contract choices A_i and types h_i, i=1,2,3.

The coordination deficits are $CD^{as} = 20.42$ (i.e., 3.07 % of optimal SC costs), $CD^{br} = 28.45$ (i.e., 4.28 % of optimal SC costs), and $CD^{ai} = 59.75$ (i.e. 8.98 % of optimal SC costs). Hence, if the classical screening approach is taken without considering the insensitivity to small pay-off differences, then the performance gap is approximately three times as large as predicted by the classical screening theory (a large impact on supply chain performance was also observed by Inderfurth et al., 2012). However, the performance losses resulting from indifference contract choices can be substantially limited with a behavioral robust contract where the cost increase is only 40% compared to the classical screening benchmark without indifference contract choices.

Table 3: Optimal contract parameters/decision variables for the case without bunching

Contract offer	Order size	Wholesale Price	Additional incentive	Supply Chain Optimal order size
$A_1 = <q_1, w_1>$	q_1= 400.00	w_1= 8.72	t_1= 0.35	q_1^{SC}=400.00
$A_2 = <q_2, w_2>$	q_2= 182.58	w_2= 10.15	t_2= 0	q_2^{SC} =230.94
$A_3 = <q_3, w_3>$	q_3= 151.19	w_3= 10.62	t_3= 0.60	q_3^{SC} =178.89

Table 4: Cost/profit effect in dependence of contract choice and holding cost realization for the case without bunching

		A_1	A_2	A_3	Alternative option	Expected profits/cost
Supplier's profit		671.51	576.78	532.89	0	577.27
Buyer's costs	h_1	1071.51	1106.23	1137.63	1500.00	1271.17
	h_2	1471.51	1288.81	1288.81	1500.00	
	h_3	1871.51	1471.40	1440.00	1500.00	
Supply chain costs	h_1	400.00	529,45	604.74	907.11	693.91
	h_2	800.00	712.03	755.93	907.11	
	h_3	1200.00	894.61	907.11	907.11	

Bunching: An example in which there is no bunching in the a-priori screening contract, but bunching in the behavioral robust contract can be easily constructed from the example by setting $\bar{\alpha} = 0.1$. Obviously, a change in $\bar{\alpha}$ does not impact the classical a-priori screening contract since such a parameter does not exist in this model. However, by changing $\bar{\alpha}$ the marginal benefits of increasing t_i increase, since a larger fraction of buyers is incentivized choosing the self-selection contract. If all other parameter values from Table 2 remain the same then the following optimal decision variables (Table 5) and profits/costs (Table 6) result:

Table 5: Optimal contract parameters/decision variables for the case with bunching

Contract offer	Order size	Wholesale Price	Additional incentive	
$A_1 = <q_1, w_1>$	q_1= 400.00	w_1= 8.71	t_1= 0.6	
$A_2 = <q_2, w_2>$	q_2= 154.37	w_2= 10.54	t_2= 0	Bunching of contracts A_2 and A_3
$A_3 = <q_3, w_3>$	q_3= 154.37	w_3= 10.54	t_3= 0.60	

Table 6: Cost/profit effect in dependence of contract choice and holding cost realization for the case with bunching

		A_1	A_2=A_3	Alternative option	Expected profits/cost
Supplier's profit		671,26	535,84	0	576,47
Buyer's costs	h_1	1071,26	1131,26	1500	1267,63
	h_2	1471,26	1285,63	1500	
	h_3	1871,26	1440,00	1500	
Supply chain costs	h_1	400,00	595,42	904,16	691,17
	h_2	800,00	749,79	904,16	
	h_3	1200,00	904,16	904,16	

The coordination deficits for the classical screening approach remains at the same level, i.e., $CD^{as} = 20.42$ (i.e., 4.28 % of optimal SC costs), however, the adjusted coordination gap for the behavioral robust contract reduces to $CD^{br} = 25.71$ (i.e., 3.86 % of optimal SC costs), caused by an upward adjustment of the order size q_3. In turn, the coordination deficit for the classical screening model with indifference contract choices increases substantially to $CD^{ai} = 91.2$ (i.e. 13.71 % of optimal SC costs). This significant increase is obviously driven by the fact that the frequency of indifference contract choices increases which is not anticipated in the benchmark ***ai***, i.e., buyer type h_i chooses with the probability $(1-\alpha_i)$ the even more downwards distorted order size q_{i+1}.

The following Section 5 presents a sensitivity analysis, in order to gain some deeper insight on the impact of the specific parameter values on the coordination deficit.

5. Sensitivity analysis

In the following we perform a sensitivity analysis with respect to the above baseline example. We concentrate on the coordination deficits in percent of the optimal supply chain costs, since they indicate under which situations the supply chain performance is especially vulnerable if buyers are insensitive to small pay-off differences.

The coordination deficit in percent of optimal supply chain costs is relatively insensitive to parameter changes of the fixed costs, *f*, and the demand rate, *d*. For low values of *f* and *d*, the relative attractiveness of the supplier's outside option, $\overline{P}_S$, decreases and, thus, the supplier may increase the additional incentive and, therefore, $\alpha(t_3)$. However, for all changes of *f* and *d*, $\alpha(t_1)$ and $\alpha(t_2)$ remain constant.

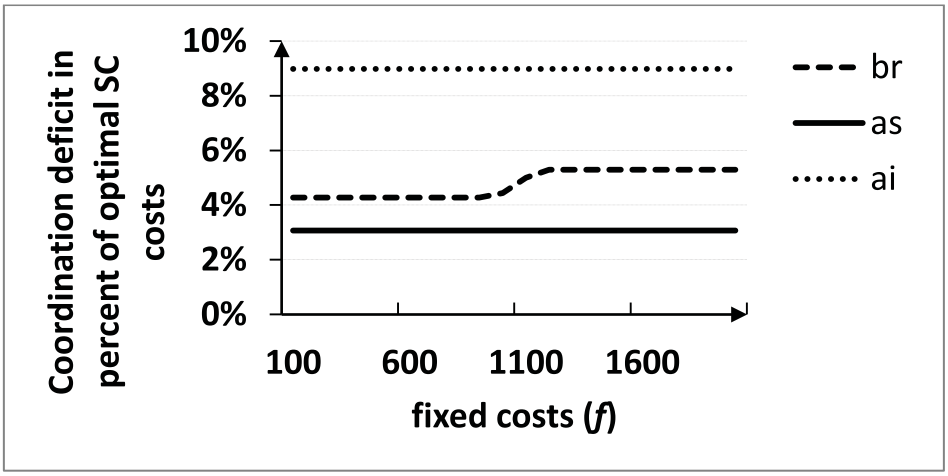

Figure 2: Coordination deficit in % of SC optimal costs in dependence of fixed costs *f*

When changing the a-priori probability distribution the impact of bunching on supply chain performance becomes obvious. The following Figure 3 depicts the benchmark in dependence of p_2, where $p_1 = p_3 = (1 - p_2)/2$.

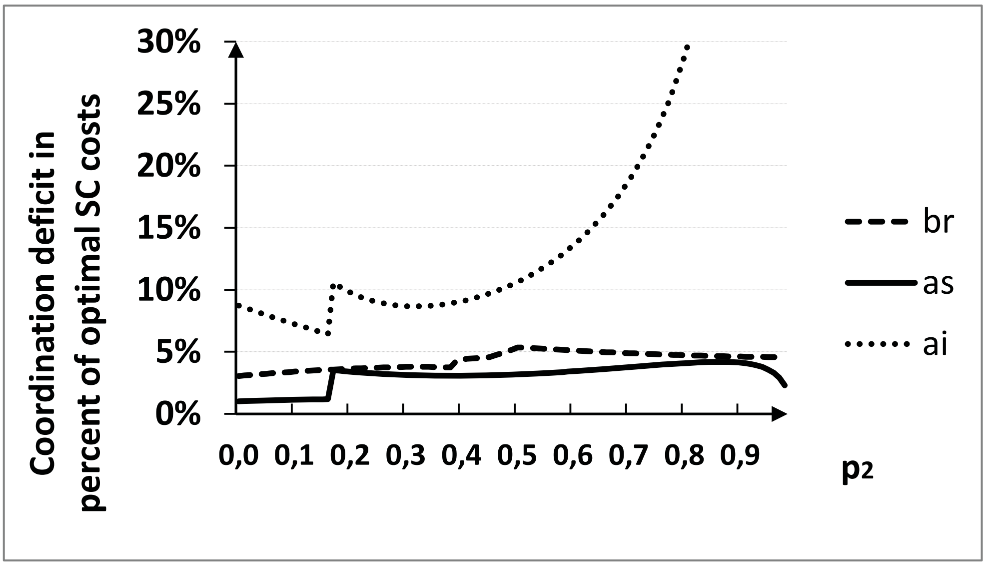

Figure 3: Coordination deficit in % of SC optimal costs in dependence of a-priori probability p_2

For $p_2 < 0.33$, we observe bunching of the order sizes q_2 and q_3 for the behavioral robust contract, since it is not optimal for the supplier to design a distinctive contract for buyer type h_2, because the informational rent induced for buyer type h_1 would be too high. In turn, bunching occurs for the a-priori contracts only for $p_2 < 0.18$, highlighting that considering bunching in contract design becomes much more important when there is not a one to one mapping between a-priori distribution and frequency of contract choices. Interestingly, the impact on the coordination deficit of bunching is much higher in the classical setting. A closer look on Figure 3 reveals that in the a-priori contract settings, the coordination deficit is on a lower level when bunching occurs. However, such a significant effect is not observable in the behavioral contract setting, because in those parameter regions where the contracts A_2 and A_3 are very similar (i.e., there is almost bunching), it is not optimal for the supplier to additionally incentivize the insensitive buyers by increasing t_2. Thus, with probability $(1-\overline{\alpha})$ the contract A_3 will still be chosen and the effect of bunching on the coordination deficit is therefore smoothened.

The degree of uncertainty is also impacted by the distance between the holding cost realizations. Let a define the distance between the holding cost realizations, i.e., $h_1 = 1$, $h_2=h_1+a$, and $h_3=h_2+a$. The larger the distance between the holding cost realizations, the higher the informational rents the buyer is receiving and, therefore, the higher efficiency losses caused by asymmetrically distributed information. Figure 4a Figure 4b depict the coordination deficits in dependence of *a*. The non-monotonicity for values around $a = 2.3$ is due to the supplier reducing the additional incentive for not choosing the alternative supplier, i.e., t_3. Obviously, the higher h_3, the lower the profits the supplier may extract from the buyer and, thus, the higher the willingness to let him choose the alternative supplier. Interestingly, for $a \le 0.2$ the coordination deficit CD^{br} is even lower than in the a-priori screening contract under self-selection assumption, i.e., CD^{as}, highlighting that CD^{as} cannot be generally regarded as a lower bound for the expected supply chain deficit (see Figure 4b). As mentioned before, this is due to the fact that the shift of probability mass increases order sizes for higher cost types and this positive effect is not offset by choosing the indifference contract.

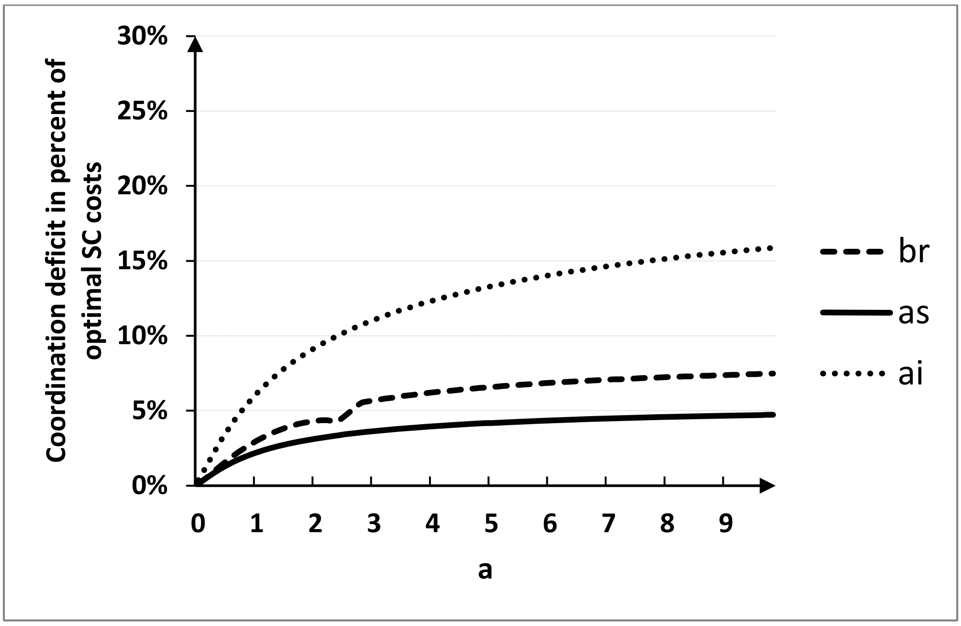

Figure 4a: Coordination deficit in % of SC optimal costs in dependence of distance between holding cost realizations

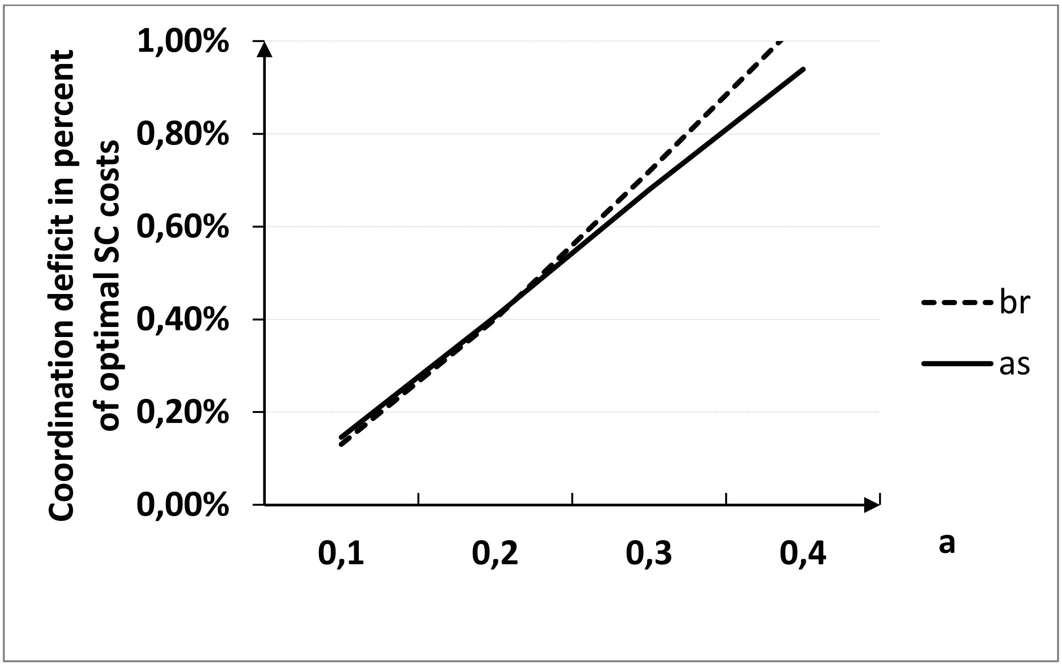

Figure 4b: Coordination deficit in % of SC optimal costs in dependence of distance between holding cost realizations

Finally, we show the impact of the fraction of buyers who are sensitive to small pay-off differences, i.e., $\bar{\alpha}$ (see Figure 5). For low $\bar{\alpha}$, the benefits of increasing the pay-off difference t_i becomes relatively larger, since $\frac{\partial \alpha}{\bar{\alpha}} = -\frac{t_i - t_h}{t_h - t_l} < 0$, i.e., the larger $\bar{\alpha}$ the flatter the slope of $\alpha(t_i)$ and vice versa.

Thus, for small values of $\bar{\alpha}$ one might expect that it is profitable setting $t_i = t_h \forall i$,i.e., the classical assumption of all buyers being profit maximizing is resembled. However, the numerical highlights, that the classical benchmark under the profit-maximization assumption will not be reached, since bunching occurs for $\bar{\alpha} \leq 0.4$ resulting in a coordination deficit compared to the classical a-priori screening contract with self-selection (*as*). Hence, even though incentivizing self-selection is relatively cheap, the classical benchmark will not be achieved, because the additional incentives required for self-selection increase the buyer type's h_1 informational rent and, thus, the benefits of bunching increase which is an additional source of supply chain inefficiency. Yet, once bunching is just not anymore optimal, CD^{br} is decreases which holds for $0.4 \leq \bar{\alpha} \leq 0.46$.

However, since the marginal cost for incentivizing the buyer's self-selection are increasing with $\bar{\alpha}$, the coordination deficit is increasing in our example for $0.46 < \bar{\alpha} \leq 0.68$. In these cases, the supplier reduces the incentive for self-selection, t_i, which is not offset by the probability that the buyer is profit maximizing. Thus, in this region the probability of self-selection $\alpha(\bar{\alpha}, t_i)$ tends to decrease which is caused by lower values t_i.

Yet, for $\bar{\alpha} > 0.68$ the coordination deficit is decreasing, since the lower willingness to incentivize self-selection is offset by the higher probability $\bar{\alpha}$ that is independent from t_i, i.e. $\alpha(\bar{\alpha}, t_i)$ is increasing. Note, that $\bar{\alpha} = 1$ resembles the classical assumption within screening theory and, therefore, all benchmarks fall together for this specific case.

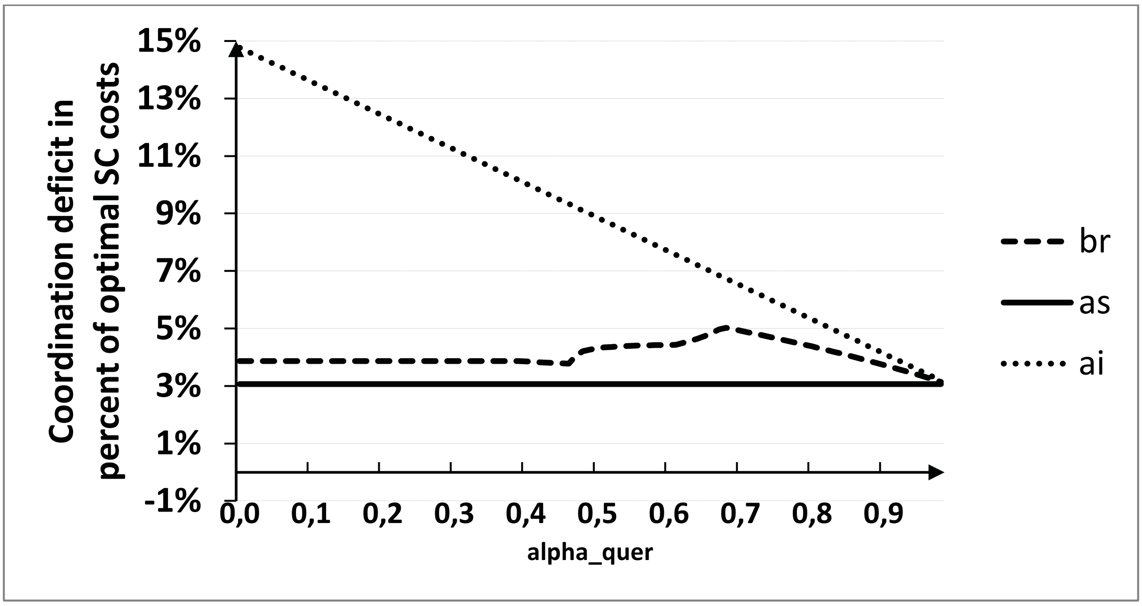

Figure 5: Coordination deficit in % of SC optimal costs in dependence of buyers sensitive to small pay-off differences

6. Conclusion and Managerial Insights

There is a growing body of work analyzing the inefficiencies of supply chain interactions under asymmetric information in common principal-agent settings. Traditionally, it is assumed that the agent (here: buyer) is strictly profit-maximizing. However, recent experimental work in the behavioral operations management area highlights that the strict profit maximization assumption is critical since buyers tend to be insensitive to small pay-off differences and, therefore, choose between strictly profit maximizing and only nearly profit maximizing contracts. Yet, such only nearly profit maximizing contracts have a substantially negative impact on the supplier's and overall supply chain's pay-offs, while the impact on the buyer's pay-offs is obviously negligible.

The present work relaxes the critical profit maximization assumption by assuming that the buyer in a serial supplier-buyer supply chain is only nearly profit maximizing, i.e., for sufficiently low pay-off differences between two contract alternatives the buyer will choose with a certain probability the profit maximizing contract and with the complementary probability the only nearly profit maximizing contract. We show that the supplier can account for such behavior by increasing the pay-off differences between the contract alternatives and/or by adjusting the order sizes in the respective menu of contracts. Therefore, we introduce a slack variable into the buyer's binding incentive constraints that allows the supplier to control for the pay-off differences between the contracts alternatives and, therefore, to influence the probabilities with which the respective contracts are chosen (see also Laffont and Martimort, 2002).

First, the analysis reveals that the second best benchmark that is predicted by the classical screening theory substantially overestimates the actual supply chain performance, since the nearly profit maximizing buyer will not choose the already downwards distorted profit maximizing order size, but sometimes the even more downwards distorted order size that is only nearly profit maximizing. However, if the supplier accounts for this behavior by designing a behavioral robust contract that accounts for insensitivity to small pay-off differences, then the performance losses can be substantially limited. Since the performance losses are mainly born by the supplier (note, the buyer is at least nearly profit maximizing), the supplier obviously has an incentive to anticipate such behavior and adjust the contract parameters accordingly. Nonetheless, we conclude that the inefficiencies arising from asymmetrically distributed information are underestimated in the coordination literature and along with this the benefits of cooperation (truthful information sharing and trusting information processing).

Second, the present work shows how to deal from an Operations Research point of view with the difficulties arising from incorporating insensitivity to small pay-off differences into a stylized lotsizing model that depicts the well-known conflict that suppliers typically favor large order sizes while buyers favor to order in smaller lots (e.g., just in time mode). It is shown that the resulting optimization problem loses its favorable concavity property. However, it is proven for the case of a linear dependency between the additional incentive and the probability of self-selection that there are at most two critical points, whereas one critical point qualifies for an optimal solution. Thus, standard approaches for solving non-linear equation systems can be used for determining the critical points which allows for example practitioners to solve the evolving problems with standard software.

Third, closely related to the issue of determining the optimal contract parameters, it is shown that under the behavioral robust contract it is in more instances optimal offering the same contract to several buyer types (i.e., bunching) than in the classical screening theory which is an additional source of inefficiencies in supply chain management. Yet, in most contributions regarding asymmetric information the optimality of bunching is ruled out by limiting the number of types and/or restricting the a-priori distribution of types. Yet, such restrictions that are generally accepted within the screening literature are not applicable in the present context, since there is not a one-to-one mapping of the a-priori distribution to the frequency of contract choices. Hence, we are presenting an algorithm that allows identifying the relevant bunching areas for an unlimited number of discretely distributed types.

From a supply chain manager's perspective, our findings imply that relying on indifference-based equilibrium models (e.g., screening contracts) can seriously harm the supply chain performance, since managers cannot assume that their business partners will generally take the profit-maximizing action, when the pay-off differences to the next best alternatives are very small. Thus, for the manager it seems worthwhile to investigate how a varying the size of the incentives in screening contracts affects the frequency of buyer's deviation from the equilibrium choice.

Finally, the present study is subject to some limitations and may be extended in several ways. First, we are restricting ourselves to nearly profit maximizing buyers and a linear dependency between the additional incentive and the probability of self-selection. However, if a certain sensitivity to pay-off differences is not satisfied, then not even the indifference contract may be chosen. Possible ways to model such situations are presented e.g. by Basov (2009) and Basov and Danilkina (2006), who are assuming that the agents are following a probabilistic choice rule in which every type chooses each contract with positive probability. Second, we are assuming that it is never profitable for the buyer to exclude buyers from trade (so-called cut-off policy). However, we conjecture that the main insights remain valid in such settings, since excluding buyers will typically only be a local phenomenon (i.e., if all buyers are excluded from trade, then the problem becomes trivial). Third, we motivate that near profit maximization results from the buyers making small decision errors, e.g., when calculating their profits. However, it may also be worthwhile investigating how fairness or reciprocity concerns (see, e.g., Fehr and Schmidt (1999), Bolton and Ockenfels (2000)) impact the buyers contract choices. In such cases, the buyers tendency towards contracts with small pay-off differences might be driven by the intention to punish the supplier for perceived unfair behavior, e.g., leaving only the reservation profit for the high cost type.

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8. Appendix

$$\max \ell = \sum_{i=1}^{n} \alpha_i \cdot p_i \cdot \left(w_i - \frac{f}{q_i} \right) \cdot d + \sum_{i=1}^{n-1} \left(1-\alpha_i\right) \cdot p_i \cdot \left(w_{i+1} - \frac{f}{q_{i+1}} \right) \cdot d + \left(1-\alpha_n\right) \cdot p_n \cdot \overline{P}_S \cdot d -$$
$$\sum_{i=1}^{n-1} \lambda_{i,i+1} \left(\left(w_i + t_i\right) \cdot d + \frac{h_i}{2} q_i - w_{i+1} \cdot d - \frac{h_i}{2} q_{i+1} \right) - \mu \cdot \left(\left(w_n + t_n\right) \cdot d + \frac{h_n}{2} q_n - R \cdot d \right) -$$
$$\sum_{i=1}^{n} \left(\gamma_i \cdot \left(t_l - t_i\right) + \omega_i (t_i - t_h) \right) + \sum_{i=2}^{n-1} \sum_{j>i} \beta_{ij} \left(q_j - q_i \right) \tag{8.1}$$

$$\frac{\partial L}{\partial q_i} = \frac{\alpha_i \cdot p_i \cdot f \cdot d}{q_i^{\,2}} + \frac{(1-\alpha_{i-1}) \cdot p_{i-1} \cdot f \cdot d}{q_i^{\,2}} - \lambda_{i,i+1} \cdot \frac{h_i}{2} + \lambda_{i-1,i} \cdot \frac{h_{i-1}}{2} + \sum_{j>i} \beta_{ij} \leq 0 \quad , \forall i = 1, \ldots, n-1 \tag{8.2}$$

$$\frac{\partial L}{\partial q_n} = \frac{\alpha_n \cdot p_n \cdot f \cdot d}{q_n^{\,2}} + \frac{(1-\alpha_{n-1}) \cdot p_{n-1} \cdot f \cdot d}{q_n^{\,2}} + \lambda_{n-1,n} \cdot \frac{h_{n-1}}{2} - \mu \cdot \frac{h_n}{2} + \sum_{i=1}^{n-1} \beta_{in} \leq 0 \tag{8.3}$$

$$\frac{\partial L}{\partial w_i} = \alpha_i \cdot p_i \cdot d + \left(1-\alpha_{i-1}\right) \cdot p_{i-1} \cdot d - \lambda_{i,i+1} \cdot d + \lambda_{i-1,i} \cdot d \leq 0 \, , \forall i = 1, \ldots., n-1 \tag{8.4}$$

$$\frac{\partial L}{\partial w_n} = \alpha_n \cdot p_n \cdot d + \left(1-\alpha_{n-1}\right) \cdot p_{n-1} \cdot d + \lambda_{n-1,n} \cdot d - \mu \cdot d \leq 0 \tag{8.5}$$

$$\frac{\partial L}{\partial t_i} = \frac{\partial \alpha_i}{\partial t_i} \cdot p_i \cdot \left(w_i - \frac{f}{q_i} \right) \cdot d - \frac{\partial \alpha_i}{\partial t_i} \cdot p_i \cdot \left(w_{i+1} - \frac{f}{q_{i+1}} \right) \cdot d - \lambda_{i,i+1} \cdot d - \gamma_i + \omega_i \leq 0 \quad , \forall i = 1, \ldots, n-1 \tag{8.6}$$

$$\frac{\partial L}{\partial t_n} = \frac{\partial \alpha_n}{\partial t_n} \cdot p_n \cdot \left(w_n - \frac{f}{q_n} \right) \cdot d - \frac{\partial \alpha_n}{\partial t_n} \cdot p_n \cdot \overline{P}_S \cdot d - \mu \cdot d - \gamma_n + \omega_n \leq 0 \tag{8.7}$$

$$\frac{\partial L}{\partial \lambda_{i,i+1}} = -\left(\left(w_i + t_i\right) \cdot d + \frac{h_i}{2} q_i - w_{i+1} \cdot d - \frac{h_i}{2} q_{i+1} \right) \leq 0 \quad , \forall i = 1, \ldots, n-1 \tag{8.8}$$

$$\frac{\partial L}{\partial \mu} = -\left(\left(w_n + t_n\right) \cdot d + \frac{h_n}{2} q_n - R \cdot d \right) \leq 0 \tag{8.9}$$

$$\frac{\partial L}{\partial \gamma_i} = -t_l + t_i \geq 0 \quad , \forall i = 1, \ldots, n \tag{8.10}$$

$$\frac{\partial L}{\partial \omega_i} = -t_i + t_h \geq 0 \quad , \forall i = 1, \ldots, n \tag{8.11}$$

$$\frac{\partial L}{\partial \beta_{ij}} = q_j - q_i \leq 0 \tag{8.12}$$

$$\frac{\partial L}{\partial \beta_{ij}}\beta_{ij}, \frac{\partial L}{\partial q_i}q_i = 0, \frac{\partial L}{\partial w_i}w_i = 0, \frac{\partial L}{\partial t_i}t_i = 0, \frac{\partial L}{\partial \lambda_{i,i+1}}\lambda_{i.i+1} = 0,$$
$$\frac{\partial L}{\partial \mu}\mu = 0, \frac{\partial L}{\partial \gamma_i}\gamma_i = 0, \frac{\partial L}{\partial \omega_i}\omega_i = 0, \frac{\partial L}{\partial \beta_{ij}}\beta_{ij} \quad (8.13)$$

Let $\hat{p}_i$ denote the probability that contract A_i is chosen, i.e.

$$\hat{p}_i = \alpha(t_i)p_i + (1-\alpha_{i-1})p_{i-1}, \forall i = 2,...,n$$
$$\hat{p}_1 = \alpha(t_i)p_i$$

From (8.5) follows

$$\hat{p}_n \cdot d + \lambda_{n-1,n} \cdot d - \mu \cdot d = 0 \quad (8.14)$$

And it follows directly that

$$\mu = \hat{p}_n + \lambda_{n-1,n} \quad (8.15)$$

From (8.4) follows

$$\lambda_{i,i+1} = \hat{p}_i + \lambda_{i-1,i} \quad (8.16)$$

Inserting (8.16) into (8.15) for all $i = 1,...,n-1$ gives

$$\mu = \hat{p}_n + \hat{p}_{n-1} + ... + \hat{p}_1 = p_1 + p_2 + ... + \alpha_n \cdot p_n = 1-(1-\alpha(t_n))p_n \quad (8.17)$$

Inserting (8.17) into (8.15) gives

$$\lambda_{n-1,n} = 1 - p_n - (1-\alpha_{n-1}) \cdot p_{n-1} \quad (8.18)$$

And it follows from (8.16) that

$$\lambda_{n-2,n-1} = \lambda_{n-1,n} - \hat{p}_{n-1} = 1-(1-\alpha_n)p_n - \hat{p}_n - \hat{p}_{n-1}$$
$$....$$
$$\lambda_{i-1,i} = \sum_{k=1}^{i-2} p_k + \alpha(t_{i-1}) \cdot p_{i-1} \quad (8.19)$$
$$\lambda_{i,i+1} = 1-(1-\alpha_n)p_n - \hat{p}_n - ... - \hat{p}_{i+1} = \sum_{k=1}^{i-1} p_k + \alpha_i \cdot p_i = \sum_{k=1}^{i} \hat{p}_k$$
$$\lambda_{1,2} = \alpha_1 \cdot p_1$$

Inserting (8.17) and (8.19) into (8.3) and (8.2) gives

$$q_i(t_i, t_{i-1}) = \sqrt{\frac{2 \cdot f \cdot d}{h_i + \frac{\left(\sum_{k=1}^{i-2} p_k + \alpha_{i-1} \cdot p_{i-1}\right) \cdot (h_i - h_{i-1}) + \sum_{j>i} \beta_{ij}}{\left((1-\alpha_{i-1}) p_{i-1} + \alpha_i p_i\right)}}} \tag{8.20}$$

If the implementability condition is not binding in the optimal menu of contracts, then $\beta_{ij} = 0$ and the optimal order sizes result from

$$q_i = \sqrt{\frac{2 \cdot f \cdot d}{h_i + \frac{\left(\sum_{k=1}^{i-2} p_k + \alpha_{i-1} \cdot p_{i-1}\right) \cdot (h_i - h_{i-1})}{\left((1-\alpha_{i-1}) p_{i-1} + \alpha_i p_i\right)}}} \quad \forall i = 1, \ldots, n \tag{8.21}$$

Yet, if bunching occurs in the optimal menu of contracts, then at least one $\beta_{i,j} > 0$. We define q_{ij} as the order sizes that are bunched, i.e., $q_{ij} = q_i = q_{i+1} = \ldots = q_j \; \forall j \geq i$. In this case, the necessary KKT condition (8.2) changes to:

$$\frac{\partial L}{\partial q_{ij}} = \sum_{z=i}^{j} \left(\frac{\alpha_z \cdot p_z \cdot f \cdot d}{q_{ij}^{\;2}} + \frac{(1-\alpha_{z-1}) \cdot p_{z-1} \cdot f \cdot d}{q_{ij}^{\;2}} - \lambda_{z,z+1} \cdot \frac{h_z}{2} + \lambda_{z-1,z} \cdot \frac{h_{z-1}}{2} \right) \leq 0 \;\; , \forall i = 1, \ldots, n-1 \tag{8.22}$$

Inserting (8.19) and rearranging gives

$$q_{ij} = \sqrt{\frac{2fd}{h_j + \frac{\left(\sum_{k=1}^{i-2} p_k + \alpha_{i-1} \cdot p_{i-1}\right) \cdot (h_j - h_{i-1})}{\sum_{k=i}^{j-1} p_k + \left((1-\alpha_{i-1}) p_{i-1} + \alpha_j \cdot p_j\right)}}} \tag{8.23}$$

Given the optimal contract parameters that can be determined via the algorithm stated in Section 3,

the optimal wholesale prices result from (8.9)

$$w_n^{AI} = R - t_n - \frac{h_n}{2d} \cdot q_n^{AI}(t_n) \tag{8.24}$$

and (8.8)

$$w_i = w_{i+1} - t_i + \frac{h_i}{2d}(q_{i+1} - q_i) \tag{8.25}$$

As long as the additional incentive is an interior solution (i.e., as long as $\gamma_i, \omega_i = 0$), the optimal value for t_i can be obtained by solving the equation system resulting from inserting (8.19) into (8.6), i.e.,

$$\frac{\partial \alpha_i}{\partial t_i} \cdot p_i \cdot \left(\left(w_i - w_{i+1} \right) + \left(\frac{f}{q_{i+1}} - \frac{f}{q_i} \right) \right) - \left(\sum_{k=1}^{i-1} p_k + \alpha_i \cdot p_i \right) = 0 \; \forall i = 1, \ldots, n-1 \tag{8.26}$$

And for inserting (8.24) and (8.25) gives l_i which denotes the marginal revenues/costs of increasing t_i.

$$l_i = \frac{\partial \alpha_i}{\partial t_i} \cdot \frac{p_i}{d} \cdot \left(-t_i d - \frac{h_i}{2} \left(q_i - q_{i+1} \right) - fdq_i^{-1} + fdq_{i+1}^{-1} \right) - \left(\alpha_i \cdot p_i + \sum_{k=1}^{i-1} p_k \right) = 0 \; \forall i = 1, \ldots, n-1 \tag{8.27}$$

Respectively, t_n can be obtained by inserting (8.17) into (8.7)

$$\frac{\partial \alpha_n}{\partial t_n} \cdot p_n \left(R - t_n - \frac{h_n}{2d} \cdot q_n - \frac{f}{q_n} - \overline{P}_S \right) - \alpha_n p_n = 1 - p_n \tag{8.28}$$

Note, that the equations (8.26)/(8.27) are uni-modular and have therefore at most two roots.

Proof:

The first derivative w.r.t. t_i while considering $\frac{\partial^2 \alpha_i}{\partial^2 t_i} = 0$ gives:

$$\frac{\partial l}{\partial t_i} = -\frac{\partial \alpha_i}{\partial t_i} \cdot \frac{p_i}{d} \cdot d - \frac{\partial \alpha_i}{\partial t_i} \cdot p_i - fdq_{i+1}^{-2} \cdot \frac{\partial q_{i+1}}{\partial t_i} + \frac{h_i}{2} \frac{\partial q_{i+1}}{\partial t_i} - \left(\frac{h_i}{2} \frac{\partial q_i}{\partial t_i} - fdq_i^{-2} \cdot \frac{\partial q_i}{\partial t_i} \right) \tag{8.29}$$

Let the first term,

$$x_1 = -\frac{\partial \alpha_i}{\partial t_i} \cdot \frac{p_i}{d} \cdot d - \frac{\partial \alpha_i}{\partial t_i} \cdot p_i = -2 \cdot \frac{\partial \alpha_i}{\partial t_i} \cdot p_i \leq 0 \tag{8.30}$$

denote the effect of changing t_i if the order sizes are not adjusted (e.g., the order sizes are calculated with $\alpha = \overline{\alpha}$), where $l_i = x_1 + x_2$. (8.30) is constant and negative, i.e., (8.27) would be strictly monotonically decreasing if the order sizes were not adjusted. However, in the optimal solution, the order sizes q_i and q_{i+1} will only be adjusted, if the marginal revenues of increasing the order size are larger than the marginal cost. Thus, it follows that adjusting the order sizes can only have a positive contribution in (8.27) and it follows that there can be at most one maximum.

Let x_2 denote the effect of the order size adjustment, i.e.,

$$x_2 = -fdq_{i+1}^{-2} \cdot \frac{\partial q_{i+1}}{\partial t_i} + \frac{h_i}{2}\frac{\partial q_{i+1}}{\partial t_i} - \left(\frac{h_i}{2}\frac{\partial q_i}{\partial t_i} - fdq_i^{-2} \cdot \frac{\partial q_i}{\partial t_i}\right) \geq 0 \quad (8.31)$$

The effect of the order size adjustment is positive (which follows directly from

$\left(\frac{fd}{q_{i+1}} + \frac{h_i}{2}q_{i+1}\right) - \left(\frac{fd}{q_i} + \frac{h_i}{2}q_i\right) \geq 0$) and this positive effect becomes larger with increasing t_i which follows directly from (8.31). Thus, it follows that (8.27) is unimodular (a monotonically increasing positive term, (8.31), is added to a constant negative term, (8.30), and thus there are at most two roots for each equation in (8.29) satisfying the necessary KKT conditions. If there are two roots, then the first necessary condition is a local maximum (the marginal revenues of increasing t_i are larger than the marginal costs), and the second root is necessarily a local minimum. The following

Figure 6 depicts the case with two roots (y_1 and y_2). For ti < y1 the marginal revenues (i.e., incentivizing choosing a larger order size) are larger than the marginal costs (i.e., the informational rent to the buyer). Thus, for a local maximum the supplier should increase t_i to y_1. For $y_1<t_i<y_2$, the marginal costs are higher than the marginal benefits, and increasing t_i to y_2 will yield a local maximium. Increasing t_i further contributes positively to the objective function. Thus, if the additional benefits gained by increasing t_i from y_2 to t_h offset the cost occurred from increasing t_i from y_1 to y_2, then the global optimum will be t_h, otherwise it will be y_1. □

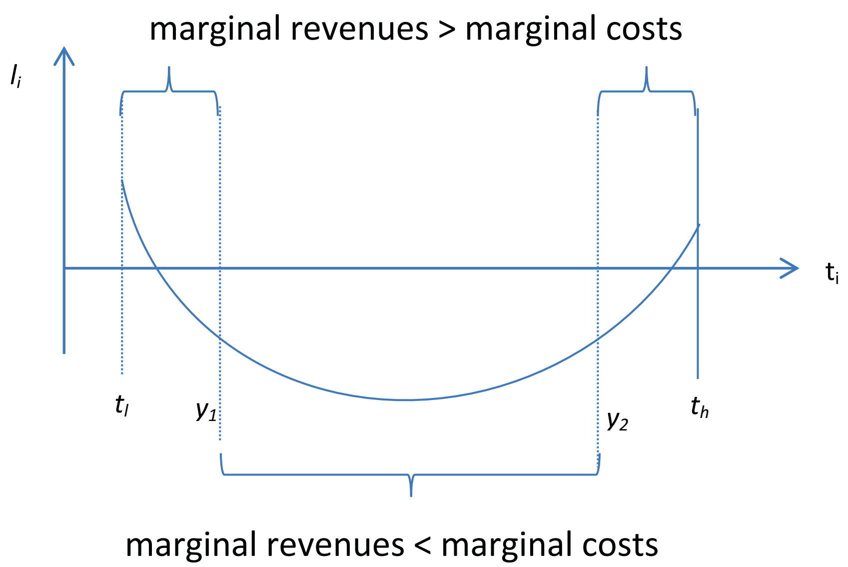

Figure 6: Critical points resulting from the KKT conditions

Let y_1 denote a root for which

$$l_i(y_1 - \varepsilon) > 0 \text{ and } l_i(y_2 - \varepsilon) < 0 \tag{8.32}$$

then:

- y_1 is the global maximum if y_2 does not exist
- y_1 **or** t_h are the global maximum if y_2 exist
- t_l **or** t_h are the global maximum if only y_2 exist

Thus, for solving the non-linear equation system, a simple local search procedure can be applied, and the above conditions needs to be tested for identifying the optimal solution.

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