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Abstract

From a large body of research studies we know that properly designed contracts can facilitate coordinated decision making of multiple actors in a supply chain (SC) so that efficiency losses for the SC as a whole can be avoided. In a newsvendor-type SC with stochastic demand it is well-known that due to the double marginalization effect a simple wholesale price contract will not achieve coordination. More complex contracts, however, do so, especially those which enable an appropriate sharing of risks between the SC actors. While the effectiveness of risk sharing contracts is well understood for SC situations with random demand and reliable supply, we do not know much about respective SC coordination problems if demand is deterministic, but supply is unreliable due to random production yield.

For a buyer-supplier SC representing the latter SC setting, it is analyzed how the distribution of risks affects the coordination of buyer's ordering and supplier's production decision. In a basic random yield, deterministic demand setting both parties are exposed to risks of over-production or under-delivery, respectively, if a simple wholesale price contract is applied. The resulting risk distribution will always be such that SC coordination cannot be achieved. It can be shown, however, that two more sophisticated contract types with penalty and reward elements for the supplier can change the risk distribution in such a way that SC coordination is possible under random yield. Additionally, it is proved that also the wholesale price contract will guarantee SC coordination if the supplier has a second (emergency) procurement source at her disposal that is more costly, but reliable. Moreover, restricting oneself to wholesale price contracts it is shown that it can be beneficial to both parties to utilize this emergency source even if it is not profitable from a SC perspective.

Key words: Supply chain coordination, contracts, random yield, risk sharing, emergency procurement

1. Introduction

Uncertainty in SCs can occur in various forms with the most prominent types being demand and supply uncertainties. Regarding the supply side, business risks primarily result from yield uncertainty. While random demand can be found in almost all industries, random yield is not as wide-spread. However, it frequently occurs in the agricultural sector or in the chemical, electronic and mechanical manufacturing industries (see Gurnani et al. (2000), Jones et al. (2001), Kazaz (2004), Nahmias (2009)). Here, random supply can appear due to different reasons such as weather conditions, production process risks or imperfect input material. As a consequence of yield uncertainty, the same production input might result in different production output quantities. In a SC context, such yield randomness obviously will affect the risk position of the actors and, thus, will have an effect on the buyer-supplier relationship in a SC. The question that arises is to what extent random yields affect the decisions of the single SC actors and the performance of the whole SC. In this study we limit ourselves to a problem setting with deterministic demand. This is to focus the risk analysis of contracting on the random yield aspect. Besides, as stated in Bassok et al. (2002), this setting is of practical relevance for production planning in some industries.

The main purpose of this paper is to explore how contracts can be used in this context in order to overcome inefficiencies arising from uncoordinated behavior. Therefore, in addition to the simple wholesale price contract, various contract types with risk sharing characteristics containing penalty or reward elements for the supplier are introduced and analyzed with respect to their coordination ability. Comparable to the newsvendor setting with stochastic demand but reliable supply, the double marginalization effect of the wholesale price contract is found in our setting. Therefore, two more contracts with alternative risk distribution are studied. One contract type which rewards the supplier and thus shifts risk to the buyer is the over-production risk sharing contract. This contract, first proposed in He and Zhang (2008), ensures that excess units from production are subsidized by the buyer. The analysis also covers a penalty contract as introduced in Gurnani and Gerchak (2007) under which the supplier is penalized for every unit of under-delivery so that risk is moved to the supplier's side. Both advanced contract types can be shown to facilitate SC coordination if contract parameters are chosen appropriately. In case a reliable but more costly second procurement option exists, it can be shown that already the wholesale price contract is sufficient to enable SC coordination as long as this option is in the hand of the supplier.

In literature there exist three major streams which are related to our research. The first one considers the context of ordering and producing under random yield. An overview of articles in this field is given by Yano and Lee (1995). Among others, the reader is referred to Gerchak et al. (1988) and Henig and Gerchak (1990) for analyzing optimal production policies in serial systems.

An extension to the approaches above provide Gerchak et al. (1994), Gurnani et al. (2000) and Pan and So (2010) who consider random yields in assembly systems.

The second body of literature concerns coordination through contracts in supply chains where risks stem from uncertain demand. A major survey article in this area which considers the coordinating properties of several types of risk sharing contracts was published by Cachon (2003). More recently, Arshinder et al. (2011) published a review article on managing uncertainty and sharing risks in a SC through appropriate coordination mechanisms including contract design.

Most important to our research, a third stream of articles covers stochastic production yields in the SC interaction context which only recently has received attention in research papers. Partly, these contributions just consider different types of risk sharing contracts without analyzing their ability to facilitate SC coordination. He and Zhang (2008, 2010), Keren (2009), Wang (2009) and Xu (2010) belong to this group. In a study by Güler and Bilgiç (2009) forced compliance on the supplier's side is assumed so that the typical coordination problem under random yield is not addressed. A remaining group of contributions considers SC coordination under voluntary compliance and partly covers a broader type of random yield problems than we do. For instance, Gurnani and Gerchak (2007) and Yan et al. (2010) address contracting issues for SC coordination in a random-yield assembly system with two suppliers and a single buyer. Gurnani and Gerchak propose two types of penalty contracts, but are only able to show that they coordinate if the suppliers are left with zero profits. Insights into contract parameter determination and interaction are not given. Yan et al. extend the work of Gurnani and Gerchak by considering a salvage value for over-produced items and additionally investigate the properties of a specific type of over-production risk sharing contract where the buyer accepts the supplier's total production output, even if he has ordered less. Different from their direct statements, they observe that also this contract will not coordinate unless the supplier ends up with zero profit. Their analysis does not consider the possibility that the buyer will not order more than what is externally demanded. Our analysis will show that it is essential for constructing SC coordinating contracts in a random yield environment that this buyer-policy of ordering at demand level is taken into consideration.

Articles by Yan and Liu (2009) and He and Zhao (2011) analyze contracts for SC coordination in serial systems where they, beyond our focus, aside from random yield also include stochastic demand. While He and Zhao limit their analysis to the special case where the supplier has a second fully reliable procurement source at her disposal, Yan and Liu come quite close to our contribution as they address a situation without emergency procurement and consider both advanced contract types that we also examine. Besides considering a SC interaction where a powerful buyer can dictate all contract conditions, they also investigate the standard situation as we do where the power within the SC is more evenly distributed. For this setting they analyze the same type of over-production risk sharing contract as in Yan et al. (2010) where the buyer accepts the supplier's total production output. They show that under such a contract SC coordination which allows for the participation of the SC actors is not possible, even if this contract type is combined with a penalty contract. Coordination can only be achieved if a subsidy for over-production is combined with a buy-back arrangement for the buyer's excess stock. The contract terms, however, are very unrealistic as they form a situation where the supplier's shipment is completely independent from the buyer's order.

Our work fills research gaps from above literature and presents a comprehensive analysis of the coordination properties of different contract types with different risk sharing behavior in a serial SC with random yield and deterministic demand. It discusses basic and advanced contract types for SC settings with and without an emergency procurement. Our study not only investigates the coordination potential of these contracts and provides detailed insights into the respective contract parameter conditions, but also takes into account the role of participation constraints that must not be violated.

The rest of the paper is organized as follows. In Section 2 the SC scenarios and risk aspects considered in this research are introduced. In Section 3 the above mentioned contract designs are analyzed with respect to their SC coordination potential and general insights are presented for the case without emergency procurement. The impact of the emergency option is analyzed in Section 4. Section 5 summarizes main results and addresses aspects of further research.

2. Supply chain scenarios and risk aspects

The basic scenario considered in this paper refers to a single-period interaction within a serial SC with one buyer (indicated by subscript B) and one supplier (indicated by subscript S) as depicted in Figure 1. All cost, price, yield and demand information is assumed to be common knowledge.

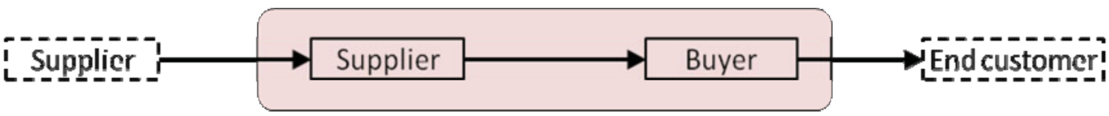

Figure 1: Serial supply chain

In order to fulfill a deterministic end customer demand D that generates a revenue p per unit, the buyer orders an amount Y from the supplier at a per unit wholesale price w. The supplier can be considered as a manufacturer who, due to production lead times, can realize only a single regular production run and, after receiving the buyer's order, has to decide upon the respective production input quantity Q associated with a per unit production cost c. However, the supplier's production process is subject to risks which lead to random output. It is assumed that the production yield is stochastically proportional, i.e. the usable output is a random fraction of the production input Q. The production output rate is denoted by z (with *pdf* $\varphi(z)$, *cdf* $\Phi(z)$ and mean μ_z) and is arbitrarily distributed between 0 and 1. Accordingly, the production output amounts to $z \cdot Q$ with $0 \le z \cdot Q \le Q$.

So, the quantity delivered to the buyer can fall below the order size and is uncertain. Production output that exceeds the buyer's order quantity is assumed to be worthless and does not generate revenue for the supplier.

From the described scenario that implicitly refers to a business relationship under a wholesale price contract, it is clear than unlike in the deterministic yield, random demand setting both actors make self-contained decisions that mutually affect the respective profits. It is also evident that both parties face specific risks. The supplier, on the one hand, is exposed to the risk of over-production if the production output exceeds the buyer's order size as any production overshoot is worthless. On the other hand, she faces the risk of under-delivery if the production output is so low that the buyer's order cannot be satisfied completely and part of her potential revenue gets lost. This under-delivery risk does also affect the buyer who additionally can be exposed to an over-ordering risk. This can occur if he orders more than externally demanded in order to compensate for a potential under-delivery by the supplier, but receives a quantity that exceeds his demand.

As a second scenario we examine the case where, after output from standard production has materialized, the supplier has an opportunity to procure extra units from a reliable source at a per unit cost c_E which is larger than the expected cost of regular production ($c_E > c / \mu_z$). This emergency source can be interpreted as an (expensive) short-term internal or external procurement option or as the possibility to rework units from the first production run. The quantity obtained from this uncapacitated source is denoted by $\hat{Q}^E$. Under these conditions, the supplier is always in a position to fulfill the buyer's order completely so that no risk of under-delivery exists. Accordingly, the buyer has no incentive to order above demand level so that for him also the over-ordering risk vanishes and a SC situation originates where only one party, namely the supplier, is bearing any risk.

The general underlying assumption in this analysis is that profitability of the business is assumed for both parties, i.e. the retail price exceeds the wholesale price which in turn exceeds the expected costs of regular production, i.e. $p > w > c / \mu_z$. The following contract analysis first refers to the scenario in which the supplier is not able to procure extra quantities after regular production. Thereafter we will address the scenario where an emergency source is available to the supplier.

3. Contract analysis without emergency procurement

Firstly, it is assumed that the supplier, once having obtained an output from regular production has no further option to retain extra products from another source so that the available quantity for filling end customer demand is random. Starting with the development of a benchmark, different contract types are studied afterwards. Finally, the results are compared to the benchmark case which will give an indication of the coordination potential of the specific contract types.

3.1. Centralized decision making

Under centralized or global decision making, i.e. all actions are conducted by one company (indicated by subscript SC), the only decision is on the production input quantity Q_{SC}. The profit Π_{SC} can then be formulated in the following way

$$\Pi_{SC}(Q_{SC}) = p \cdot E\left[\min(z \cdot Q_{SC}, D)\right] - c \cdot Q_{SC} = p \cdot \left[\int_0^{D/Q_{SC}} z \cdot Q_{SC} \cdot \varphi(z)dz + \int_{D/Q_{SC}}^{1} D \cdot \varphi(z)dz\right] - c \cdot Q_{SC} \ . \qquad (1)$$

Due to concavity of the profit function (for proof see Appendix), the optimal production input quantity is obtained by taking the first-order derivative of the profit function and setting it equal to zero. From the first-order condition (FOC) the optimal decision can be derived as

$$Q_{SC}^* \qquad \text{from} \qquad \int_0^{D/Q_{SC}} z \cdot \varphi(z)dz = \frac{c}{p} \ . \qquad (2)$$

Obviously, the SC optimal production input quantity depends on all problem data in a specific way. In detail, the structure of the optimal policy is such that the production quantity equals demand D inflated by a factor $K_{SC} > 1$ which depends on the relation of production cost and retail price as well as on the yield rate distribution, or more specifically

$$Q_{SC}^* = K_{SC}^* \cdot D \qquad \text{with} \qquad K_{SC}^* > 1 \qquad \text{and} \qquad (3)$$

$$K_{SC}^* \qquad \text{from} \qquad \int_0^{1/K_{SC}} z \cdot \varphi(z)dz = \frac{c}{p} \ . \qquad (4)$$

$K_{SC}^* > 1$ is given due to $0 < c/p < \mu_Z$. As mentioned before, the condition $p > c/\mu_Z$ always holds when profitability of the business is requested. The multiplier K_{SC}^* aims at compensating yield losses and, at the same time, accounts for the best trade-off between cost of over-production (c) and lost revenue by under-production (p) in the presence of yield randomness.

The interdependency of multiplier K_{SC}^* and the cost/price parameters becomes evident from the typical course of the integral function in (4). Figure 2 illustrates the effect which an increasing multiplier K has on the partial expectation $\int_0^{1/K} z \cdot \varphi(z)dz$. For values of K below 1, the integral is not defined as we assume the yield rate to be distributed only between 0 and 1. At a K-value of 1, the integral is exactly equal to the mean yield rate μ_Z. As the multiplier increases the integral decreases which results in an increased value for the production quantity.

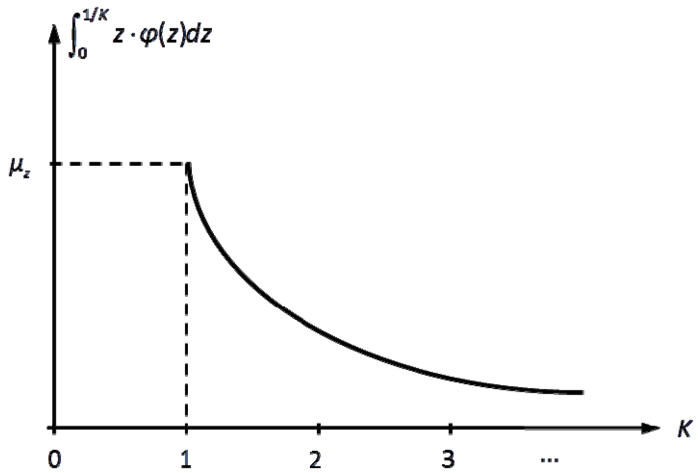

Figure 2: Effect of multiplier K on partial expectation

By inserting the optimal decision (2) into the profit function (1) and further evaluating we find the maximum profit to be

$$\Pi^*_{SC} = \Pi_{SC}(Q^*_{SC}) = p \cdot \left[1 - \Phi(1/K^*_{SC})\right] \cdot D . \tag{5}$$

It turns out that the SC profit under optimal production decision is always proportional to the demand level.

In the following, a two-member SC is considered in which two companies interact with each other and decide individually. Using the results of the previous analysis as a benchmark, the situation of decentralized decision making can be evaluated.

3.2. Decentralized decision making

Under decentralized decision making the buyer releases an order and the supplier decides on the (input) quantity for production in order to fulfill the buyer's request in a most profitable way. Depending on her overall production yield, the supplier will deliver as much as possible to satisfy the demand quantity ordered by the buyer. For all following analyses the Stackelberg game is applied as a game theoretic approach. According to the sequence of decisions the buyer is modeled as leader and the supplier as follower, i.e. the buyer anticipates the supplier's reaction to his own decision.

In this context we will investigate three different contract types the design of which results in a different way of risk sharing between the SC members. We consider the wholesale price contract as the most simple contract type first and then proceed with two more sophisticated ones which are proposed in literature for more flexible risk sharing policies.

3.2.1. Wholesale price contract

The simple wholesale price (WHP) contract states that only a constant wholesale price is charged by the supplier for each unit delivered to the buyer. No other financial transactions take place between the two parties.

Supplier's optimal decision

The supplier reacts to the buyer's order quantity Y and seeks to maximize her own profit which is given as

$$\Pi_S^{WHP}(Q_S \mid Y) = w \cdot E\left[\min(z \cdot Q_S, Y)\right] - c \cdot Q_S = w \cdot \left[\int_0^{Y/Q_S} z \cdot Q_S \cdot \varphi(z)dz + \int_{Y/Q_S}^{1} Y \cdot \varphi(z)dz\right] - c \cdot Q_S \,. \tag{6}$$

Thus, the profit function has the same structure as in (1) and, accordingly, from the FOC the optimal supplier decision can be derived as

$$Q_S^{WHP} \qquad \text{from} \qquad \int_0^{Y/Q_S} z \cdot \varphi(z)dz = \frac{c}{w} \tag{7}$$

which results in

$$Q_S^{WHP}(Y) = K_S^{WHP} \cdot Y \quad \text{with} \quad K_S^{WHP} > 1 \ \text{ for } \ w > c / \mu_z \,. \tag{8}$$

It turns out that also in the decentralized setting the optimal production decision results from inflating demand. In this case, however, the demand is not an external one but given by the buyer's order quantity. Additionally, the multiplier K_S^{WHP} is different from K_{SC}^{*} in the centralized supply chain setting. More precisely, it turns out that along with $p > w$ we find that $K_S^{WHP} < K_{SC}^{*}$ since from (7) K_S^{WHP} can be expressed as

$$K_S^{WHP} \quad \text{from} \quad \int_0^{1/K_S} z \cdot \varphi(z)dz = \frac{c}{w} \,. \tag{9}$$

Inserting the optimal decision in the supplier's profit function from (6) yields

$$\Pi_S^{WHP}(Q_S^{WHP} \mid Y) = w \cdot \left[1 - \Phi(1 / K_S^{WHP})\right] \cdot Y \,. \tag{10}$$

In case of $p \leq w$ it is obvious that $Q_S^{WHP}(Y) = 0$ or, equivalently, $K_S^{WHP} = 0$ since the buyer cannot make any profit and consequently does not order anything.

Buyer's optimal decision

From the buyer's point of view, profit is random too because of an uncertain delivery quantity from the supplier which is given as $\min(z \cdot Q_S, Y)$. The buyer's expected revenue is equal to $p \cdot E\left[\min(z \cdot Q_S^{WHP}, Y, D)\right]$ and thus in general depends on both, order quantity Y and demand level D. For further evaluation we have to distinguish the two cases $Y \geq D$ and $Y \leq D$.

I. Case $Y \geq D$

In this case the buyer is ordering more than demanded by external customers in order to exaggerate a possibly insufficiently inflated supplier's production quantity. Anticipating the supplier's reaction to what he orders the buyer's profit is the following

$$\Pi_B^{WHP}(Y\,|\,Q_S^{WHP}) = p\cdot E\left[\min(z\cdot Q_S^{WHP},Y,D)\right] - w\cdot E\left[\min(z\cdot Q_S^{WHP},Y)\right]$$
$$= p\cdot\left[\int_0^{D/Q_S^{WHP}} z\cdot Q_S^{WHP}\cdot\varphi(z)dz + \int_{D/Q_S^{WHP}}^{1} D\cdot\varphi(z)dz\right] - w\cdot\left[\int_0^{Y/Q_S^{WHP}} z\cdot Q_S^{WHP}\cdot\varphi(z)dz + \int_{Y/Q_S^{WHP}}^{1} Y\cdot\varphi(z)dz\right].$$

As the buyer is anticipating the decision of the supplier which, in detail, is $Q_S^{WHP} = K_S^{WHP}\cdot Y$, this information can be used as an input to decision making and the profit function transforms to

$$\Pi_B^{WHP}(Y\,|\,Q_S^{WHP}) = p\cdot\left[\int_0^{D/(K_S^{WHP}\cdot Y)} (z\cdot K_S^{WHP}\cdot Y - D)\cdot\varphi(z)dz + D\right] - w\cdot\left[\int_0^{1/K_S^{WHP}} (z\cdot K_S^{WHP}\cdot Y - Y)\cdot\varphi(z)dz + Y\right]. \tag{11}$$

Exploiting the concavity of the above profit function (for proof see Appendix), from the FOC the buyer's optimal order quantity is

$$Y^{WHP} \quad \text{from} \quad \int_0^{D/(K_S^{WHP}\cdot Y)} z\cdot\varphi(z)dz = \frac{c}{p} + \frac{w}{p}\cdot\frac{1-\Phi(1/K_S^{WHP})}{K_S^{WHP}} \tag{12}$$

which results in

$$Y^{WHP} = K_B^{WHP}\cdot\frac{D}{K_S^{WHP}} \quad \text{with a multiplier} \quad 1 < K_B^{WHP} < K_{SC}^{*}. \tag{13}$$

This result means that it is optimal for the buyer to inflate the external demand by an own multiplier after deflating it with the supplier's multiplier. The buyer's multiplier K_B^{WHP} is given from (12) as

$$K_B^{WHP} \quad \text{from} \quad \int_0^{1/K_B} z\cdot\varphi(z)dz = \frac{c}{p} + \frac{w}{p}\cdot\frac{1-\Phi(1/K_S^{WHP})}{K_S^{WHP}}. \tag{14}$$

The result from (14) only holds as long as $K_B^{WHP} > K_S^{WHP}$ is valid. Otherwise, according to (13) the buyer's order does not exceed his demand and the second case must be analyzed.

II. Case $Y \le D$

In this case the buyer anticipates that the supplier is inflating his order to such a high extent (above demand *D*) that he is better off to order at or below demand level. Under this condition his expected revenue does no longer depend on the demand so that his profit function is different from the former case $Y \ge D$ and can be written as

$$\Pi_B^{WHP}(Y\,|\,Q_S^{WHP}) = p\cdot E\left[\min(z\cdot Q_S^{WHP},Y)\right] - w\cdot E\left[\min(z\cdot Q_S^{WHP},Y)\right].$$

After inserting $Q_S^{WHP} = K_S^{WHP}\cdot Y$, the resulting profit function is

$$\Pi_B^{WHP}(Y\,|\,Q_S^{WHP}) = (p-w)\cdot\left[\int_0^{1/K_S^{WHP}} (z\cdot K_S^{WHP}\cdot Y - Y)\cdot\varphi(z)dz + Y\right], \tag{15}$$

where, different from the situation in (11), the profit is a linear function of the order quantity *Y*. Since the first-order derivative which is

$$\frac{\partial}{\partial Y}\Pi_B^{WHP}(Y\,|\,Q_S^{WHP}) = (p-w)\cdot\left[\frac{c}{w}\cdot K_S^{WHP} + 1 - \Phi(1/K_S^{WHP})\right],$$

is positive due to $p > w$, the buyer's optimal decision is to increase his order to the upper level *D*. So we find that in this case (as long as the profitability condition holds) the buyer's order will be equal to the external demand *D*.

Summarizing cases I and II, we find that the buyer will order a quantity equal to

$$Y^{WHP} = \begin{cases} \dfrac{K_B^{WHP}}{K_S^{WHP}} \cdot D & \text{if} \quad K_B^{WHP} \geq K_S^{WHP} \\ D & \text{if} \quad K_B^{WHP} \leq K_S^{WHP} \end{cases}, \tag{16}$$

with multipliers K_S^{WHP} and K_B^{WHP} given from (9) and (14). With these order decisions the buyer's maximal profit in (17) can be derived as follows

$$\Pi_B^{WHP}(Y^{WHP} \mid Q_S^{WHP}) = \begin{cases} p \cdot \left[1 - \Phi(1/K_B^{WHP})\right] \cdot D & \text{if} \quad K_B^{WHP} \geq K_S^{WHP} \\ (p-w) \cdot \left[\dfrac{c}{w} \cdot K_S^{WHP} + 1 - \Phi(1/K_S^{WHP})\right] \cdot D & \text{if} \quad K_B^{WHP} \leq K_S^{WHP} \end{cases}. \tag{17}$$

Interaction of buyer and supplier decisions

Along with the supplier's decision function from (8) and the buyer's decision in (16), the following production decision within the decentralized SC setting under WHP contract turns out to be made

$$Q_S^{WHP}(Y^{WHP}) = \begin{cases} K_B^{WHP} \cdot D & \text{if} \quad K_B^{WHP} \geq K_S^{WHP} \\ \mathrm{K}_S^{\mathrm{WHP}} \cdot D & \text{if} \quad K_B^{WHP} \leq K_S^{WHP} \end{cases}. \tag{18}$$

As it is shown above that $K_B^{WHP} < K_{SC}^*$ and $K_S^{WHP} < K_{SC}^*$ it is obvious that $Q_S^{WHP}(Y^{WHP}) < Q_{SC}^*$ will always hold under the general price/cost condition $p > w > c/\mu_z$. It follows that SC coordination is not enabled when only a wholesale price is fixed in the parties' contract because this type of contract will always lead to an under-production decision. This results from the so-called double marginalization effect which states that when both parties aim for positive profits, each SC stage charges a mark-up on the cost it incurs when selling to successive stages. As a result the supplier inflates the buyer's order too low while the buyer does not compensate this effect by raising his order sufficiently above the demand level.

From our analysis, there exist two limiting cases where decentralized and centralized decision making result in the same production decision (i.e., $Q_S^{WHP}(Y^{WHP}) = Q_{SC}^*$). First, when the wholesale price equals the expected production cost (i.e., $w = c/\mu_Z$), we find from (9) and (14) that $K_S^{WHP} = 1$ and $K_B^{WHP} = K_{SC}^*$. This scenario, however, violates the business profitability condition for the supplier resulting in a supplier's unwillingness to participate in the interaction. Second, when the wholesale price equals the retail price (i.e., $w = p$), analogously we find that $K_S^{WHP} = K_{SC}^*$ and $K_B^{WHP} < K_S^{WHP}$ so that according to (18) also in this case $Q_S^{WHP}(Y^{WHP}) = Q_{SC}^*$ holds. However, this second scenario also violates the participation constraints for SC interaction since due to zero contribution margin this time the buyer will not make any profit.

Summarizing, under a WHP contract SC coordination cannot be achieved in the described random yield context. From (10) and (17) the total SC profit Π_{SC}^{WHP} in the case of decentralized decision making will sum up to

$$\Pi_{SC}^{WHP} = \begin{cases} \left[w \cdot \dfrac{K_B^{WHP}}{K_S^{WHP}} \cdot \left[1 - \Phi(1/K_S^{WHP}) \right] + p \cdot \left[1 - \Phi(1/K_{\mathbf{B}}^{WHP}) \right] \right] \cdot D & \text{if } K_B^{WHP} \geq K_S^{WHP} \\ \left[(p-w) \cdot \dfrac{c}{w} \cdot K_S^{WHP} + p \cdot \left[1 - \Phi(1/K_S^{WHP}) \right] \right] \cdot D & \text{if } K_B^{WHP} \leq K_S^{WHP} \end{cases}$$

which for $p > w > c/\mu_z$ is always smaller than the SC optimal profit Π_{SC}^{*} in (5).

As described, the form of risk sharing inherent in the WHP contract does not result in a sufficiently high buyer's order and/or supplier's production volume to enable SC coordination. Next, it will be investigated if two other contract types from literature with different risk sharing properties are able to induce SC coordinating decisions.

3.2.2. Over-production risk sharing contract

The over-production risk sharing (ORS) contract ensures that in case of random production yields the risk of producing too many units (compared to the quantity ordered) will be shared among the two parties so that the supplier bears less risk and is motivated to respond to the buyer's order with a higher production quantity. Under this contract, the buyer commits to pay for all units produced by the supplier. While he pays the wholesale price w per unit for deliveries up to his actual order volume, quantities that exceed this amount are compensated at a lower price w_O. Thus, this contract type is characterized by two contract parameters w and w_O. In order to exclude situations where the supplier can generate unlimited profits from over-production the following parameter restrictions are set: $w_O < c/\mu_z < w$. As the supplier can salvage all units she has an incentive to produce a larger lot compared to the situation under the simple WHP contract. This increase might provide the potential to align the supplier's production decision with the SC optimal one.

In this context two contract variants have to be distinguished depending on the way a possible over-production is handled by the parties. A first variant is characterized by the fact that the buyer just financially compensates the supplier for over-production without accepting deliveries that exceed his order size. This Pull-ORS contract leaves him in a different risk position as when the parties agree that the supplier will deliver the whole production output irrespective of the buyer's order so that some kind of Push-ORS contract is given.

Supplier's optimal decision

The profit to optimize by the supplier is identical for both contract variants. It now also includes the rewards from over-production and is given by

$$\Pi_S^{ORS}(Q_S\,|\,Y) = w\cdot E\big[\min(z\cdot Q_S, Y)\big] + w_O\cdot E\big[\max(z\cdot Q_S - Y, 0)\big] - c\cdot Q_S$$
$$= w\cdot\left[\int_0^{Y/Q_S} z\cdot Q_S\cdot\varphi(z)dz + \int_{Y/Q_S}^{1} Y\cdot\varphi(z)dz\right] + w_O\cdot\int_{Y/Q_S}^{1}(z\cdot Q_S - Y)\cdot\varphi(z)dz - c\cdot Q_S\ . \tag{19}$$

The first-order derivative of this profit turns out to be

$$\frac{\partial}{\partial Q_S}\Pi_S^{ORS}(Q_S\,|\,Y) = (w - w_O)\cdot\int_0^{Y/Q_S} z\cdot\varphi(z)dz + w_O\cdot\mu_z - c\ .$$

Given the above parameter restrictions, from the FOC the optimal production quantity can be obtained as

$$Q_S^{ORS} \qquad \text{from} \qquad \int_0^{Y/Q_S} z\cdot\varphi(z)dz = \frac{c - w_O\cdot\mu_z}{w - w_O} \tag{20}$$

which results in

$$Q_S^{ORS} = K_S^{ORS}\cdot Y \qquad \text{with} \qquad K_S^{ORS} > 1 \quad \text{and} \tag{21}$$

$$K_S^{ORS} \qquad \text{from} \qquad \int_0^{1/K_S} z\cdot\varphi(z)dz = \frac{c - w_O\cdot\mu_z}{w - w_O}\ . \tag{22}$$

Given the production policy in (21) with the multiplier in (22), the supplier's profit in (19) simplifies to

$$\Pi_S^{ORS} = (w - w_0)\cdot\big[1 - \Phi(1/K_S^{ORS})\big]\cdot Y\ . \tag{23}$$

Buyer's optimal decision

The buyer's profit function depends on the specific variant of ORS contract that is applied. Under a Pull-ORS type (exclusion of over-delivery) the buyer maximizes a profit which compared to the WHP contract is reduced by the supplier's reward for over-produced items

$$\Pi_B^{ORS}(Y\,|\,Q_S^{ORS}) = p\cdot E\big[\min(z\cdot Q_S^{ORS}, Y, D)\big] - w\cdot E\big[\min(z\cdot Q_S^{ORS}, Y)\big] - w_O\cdot E\big[\max(z\cdot Q_S^{ORS} - Y, 0)\big]\ . \tag{24}$$

For evaluating this profit function we have to distinguish the cases $Y \geq D$ and $Y \leq D$.

I. Case $Y \geq D$

Here the buyer's profit function can be written as

$$\Pi_B^{ORS}(Y\,|\,Q_S^{ORS}) = p\cdot\left[\int_0^{D/(K_S^{ORS}\cdot Y)} (z\cdot K_S^{ORS}\cdot Y - D)\cdot\varphi(z)dz + D\right] - w\cdot\left[\int_0^{1/K_S^{ORS}} (z\cdot K_S^{ORS}\cdot Y - Y)\cdot\varphi(z)dz + Y\right]$$
$$-w_O\cdot\int_{1/K_S^{ORS}}^{1} (z\cdot K_S^{ORS}\cdot Y - Y)\cdot\varphi(z)dz\ . \tag{25}$$

The buyer's optimal order decision can be derived from the FOC using the derivative

$$\frac{\partial}{\partial Y}\Pi_B^{ORS}(Y\,|\,Q_S^{ORS}) = p\cdot K_S^{ORS}\cdot\int_0^{D/(K_S^{ORS}\cdot Y)} z\cdot\varphi(z)dz - (w - w_O)\cdot\left[(1 - \Phi(1/K_S^{ORS})) + K_S^{ORS}\cdot\int_0^{1/K_S^{ORS}} z\cdot\varphi(z)dz\right] - w_O\cdot\mu_z\cdot K_S^{ORS}$$

resulting in a policy where the buyer's order quantity is proportional to his external demand like for the WHP contract

$$Y^{ORS} = K_B^{ORS} \cdot \frac{D}{K_S^{ORS}} \quad \text{with a multiplier} \quad 1 < K_B^{ORS} < K_{SC}^{*} \quad \text{and with} \tag{26}$$

$$K_B^{ORS} \text{ from } \int_0^{1/K_B} z \cdot \varphi(z) dz = \frac{c}{p} + \frac{(w - w_O)}{p} \cdot \frac{\left(1 - \Phi(1/K_S^{ORS})\right)}{K_S^{ORS}} . \tag{27}$$

Note that for $w_O = 0$ the optimal decisions from the WHP contract emerges as $K_S^{ORS} = K_S^{WHP}$ and thus, $K_B^{ORS} = K_B^{WHP}$.

This result only holds as long as $K_B^{ORS} > K_S^{ORS}$ is valid. Otherwise, the second case must be analyzed.

II. Case $Y \leq D$

In this case, the demand level *D* is irrelevant and the buyer's profit function from (24) transforms to

$$\Pi_B^{ORS}(Y \mid Q_S^{ORS}) = (p - w) \cdot \left[\int_0^{1/K_S^{ORS}} (z \cdot K_S^{ORS} \cdot Y - Y) \cdot \varphi(z) dz + Y \right] - w_O \cdot \int_{1/K_S^{ORS}}^{1} (z \cdot K_S^{ORS} \cdot Y - Y) \cdot \varphi(z) dz , \tag{28}$$

which is linear in *Y* with a first-order derivative that after inserting the integral expression from (22) results in

$$\frac{\partial}{\partial Y} \Pi_B^{ORS}(Y \mid Q_S^{ORS}) = \left[\frac{p \cdot (c - w_0 \cdot \mu_Z)}{w - w_0} - c \right] \cdot K_S^{ORS} + (p - w + w_O) \cdot \left[1 - \Phi(1/K_S^{ORS}) \right] . \tag{29}$$

Thus, the buyer's optimal decision is to increase his order to the upper level *D* if the derivative in (29) is positive, and to order nothing if not. So, given our parameter restrictions, we find that

$$Y^{ORS} = D \quad \text{if} \qquad p \cdot (c - w_0 \cdot \mu_Z) \geq c \cdot (w - w_0) . \tag{30}$$

Interaction of buyer and supplier decisions

The investigation if the supplier-buyer interaction can result in SC coordination will be carried out separately for the two cases $Y \geq D$ and $Y \leq D$.

In **case I** ($Y \geq D$) the supplier's production quantity will be $Q_S^{ORS}(Y^{ORS}) = K_S^{ORS} \cdot Y^{ORS} = K_B^{ORS} \cdot D$ with $K_B^{ORS} > K_S^{ORS}$. From that it is obvious that the SC is coordinated only if $K_B^{ORS} = K_{SC}^{*}$. From (27) and (4) it is easy to see that equality of multipliers is just given if the condition $(w - w_O) \cdot (1 - \Phi(1/K_S^{ORS})) = 0$ holds. This condition is only fulfilled if the following combination of contract parameters is given:

$$w = w_0 = c / \mu_Z . \tag{31}$$

A similar result is found in Yan et al. (2010). Since the parameter combination in (31) results in $K_B^{ORS} = K_{SC}^{*} > K_S^{ORS} = 1$, it guarantees SC coordination and fulfills the condition of case I. However, inserting this parameter set in the supplier's profit function (23) makes evident that this solution is combined with a supplier's profit of zero so that the participation constraints of SC interaction are violated. Summarizing, it turns out that SC coordination cannot be achieved if the buyer orders at a level above his demand.

In **case II** ($Y \le D$) when $Y^{ORS} = D$ it is obvious that SC coordination needs a supplier's multiplier that equals the SC's optimal one, i.e. $K_S^{ORS} = K_{SC}^*$. From (22) and (4) one finds that this condition is fulfilled if the parameter equation $p \cdot (c - w_0 \cdot \mu_Z) = c \cdot (w - w_0)$ holds. Since according to (30) this equation also fulfills the condition for $Y^{ORS} = D$, the SC will be coordinated under this combination of contract parameters *w* and w_0 that can be written as

$$w + \left(\frac{p}{c / \mu_Z} - 1 \right) \cdot w_0 = p \,. \tag{32}$$

From (32) it follows that $w - w_0 > 0$ so that for the buyer's multiplier in (27) we get: $K_B^{ORS} < K_S^{ORS} = K_{SC}^*$. Thus, under coordinating contract parameters the condition of case II, namely $Y \le D$, is assured.

Summarizing, we see that under a Pull-ORS contract SC coordination always will be achieved as long as contract parameters are set in a way that wholesale price *w* plus over-production reward w_0 times SC's relative contribution margin equal the retail price. Under these conditions the buyer just orders the demand size and the supplier chooses the SC optimal production level. The parameter choice combinations in (32) also enable an arbitrary split of the optimal SC profit under the two parties. From (23) it follows that the supplier's profit under coordination equals $\Pi_S^{ORS} = (w - w_0) \cdot \left[1 - \Phi(1 / K_{SC}^*) \right] \cdot D$. If this profit is divided by the SC optimal profit from (5) we get a supplier's profit share, denoted by α_S, that is simply

$$\alpha_S^{ORS} = \frac{w - w_0}{p} \,. \tag{33}$$

From this ratio and parameter condition (32) we find as limiting cases where the participation constraints no longer hold, the combinations $w = p, w_0 = 0$ (with $\Pi_B^{ORS} = 0$) and $w = w_0 = c / \mu_Z$ (with $\Pi_S^{ORS} = 0$). The latter situation is just what we found as condition for SC coordination in case I ($Y \ge D$).

When the SC parties agree upon a Push-ORS contract where over-production is connected with over-deliveries, the buyer is in the same situation as in case I ($Y \ge D$) since under this condition he will sell the minimum of the supplier's production output and external demand so that the profit function in (25) applies. Consequently, the outcome of the analysis for case I holds, namely that SC coordination cannot be achieved. So we see that the ability of the ORS contract to coordinate essentially depends on the specific variant of this contract type. This resembles the result that is also found in Yan and Liu (2009).

3.2.3. Penalty contract

If a penalty (PEN) contract is applied the supplier will bear a higher risk than under a simple WHP contract since she will be punished for under-delivery. The supplier is penalized by the buyer (in the amount of π) for each unit ordered that cannot be fulfilled because of insufficient production yield. Given the potential penalty the supplier is motivated to produce more than under the simple WHP contract which again opens the chance of aligning decisions to achieve SC coordination.

Supplier's optimal decision

Under the PEN contract, the profit to optimize by the supplier in addition to the WHP contract includes the penalty for under-delivery and is given by

$$\begin{aligned}\Pi_S^{PEN}(Q_S\,|\,Y) &= w\cdot E\left[\min(z\cdot Q_S,Y)\right]-\pi\cdot E\left[\max(Y-z\cdot Q_S,0)\right]-c\cdot Q_S\\ &= w\cdot\left[\int_0^{Y/Q_S} z\cdot Q_S\cdot\varphi(z)dz+\int_{Y/Q_S}^{1} Y\cdot\varphi(z)dz\right]-\pi\cdot\int_0^{Y/Q_S}(Y-z\cdot Q_S)\cdot\varphi(z)dz-c\cdot Q_S\ .\end{aligned} \tag{34}$$

The first-order derivative of this profit can be derived as

$$\frac{\partial}{\partial Q_S}\Pi_S^{PEN}(Q_S\,|\,Y)=(w+\pi)\cdot\int_0^{Y/Q_S} z\cdot\varphi(z)dz-c\ .$$

From the FOC the supplier's optimal production quantity can be obtained as

$$Q_S^{PEN}\qquad\text{from}\qquad\int_0^{Y/Q_S} z\cdot\varphi(z)dz=\frac{c}{w+\pi}\ . \tag{35}$$

which results in

$$Q_S^{PEN}=K_S^{PEN}\cdot Y\qquad\text{with}\qquad K_S^{PEN}>1\quad\text{and} \tag{36}$$

$$K_S^{PEN}\qquad\text{from}\qquad\int_0^{1/K_S} z\cdot\varphi(z)dz=\frac{c}{w+\pi}\ . \tag{37}$$

Under the production decision in (36) with the multiplier from (37) the supplier's profit in (34) simplifies to

$$\Pi_S^{PEN}=\left[w-(w+\pi)\cdot\Phi(1/K_S^{PEN})\right]\cdot Y\ . \tag{38}$$

Buyer's optimal decision

The buyer, as Stackelberg leader, maximizes his profit which now is increased by the supplier's penalty payments for under-delivery

$$\Pi_B^{PEN}(Y\,|\,Q_S^{PEN})=p\cdot E\left[\min(z\cdot Q_S^{PEN},Y,D)\right]-w\cdot E\left[\min(z\cdot Q_S^{PEN},Y)\right]+\pi\cdot E\left[\max(Y-z\cdot Q_S^{ORS},0)\right]\ . \tag{39}$$

Like above, we have to distinguish the cases $Y\geq D$ and $Y\leq D$.

I. Case $Y\geq D$

The buyer's profit function under anticipation of the supplier' decision can here be written as

$$\Pi_B^{PEN}(Y\,|\,Q_S^{PEN}) = p\cdot\left[\int_0^{D/(K_S^{PEN}\cdot Y)} (z\cdot K_S^{PEN}\cdot Y - D)\cdot\varphi(z)dz + D\right] - w\cdot\left[\int_0^{1/K_S^{PEN}} (z\cdot K_S^{PEN}\cdot Y - Y)\cdot\varphi(z)dz + Y\right]$$
$$+\pi\cdot\int_0^{1/K_S^{PEN}} (Y - z\cdot K_S^{PEN}\cdot Y)\cdot\varphi(z)dz\ . \tag{40}$$

The buyer's optimal order decision can be derived from the FOC by using the derivative $\frac{\partial}{\partial Y}\Pi_B^{PEN}(Y\,|\,Q_S^{PEN}) = p\cdot K_S^{PEN}\cdot\int_0^{D/(K_S^{PEN}\cdot Y)} z\cdot\varphi(z)dz - \left[w + c\cdot K_S^{PEN} - (w+\pi)\cdot\Phi(1/K_S^{PEN})\right]$,

resulting in a policy where the buyer's order quantity is proportional to his external demand like for the WHP contract

$$Y^{PEN} = K_B^{PEN}\cdot\frac{D}{K_S^{PEN}} \quad \text{with a multiplier} \quad 1 < K_B^{PEN} < K_{SC}^{*} \quad \text{and} \tag{41}$$

$$K_B^{PEN} \quad \text{from} \quad \int_0^{1/K_B} z\cdot\varphi(z)dz = \frac{c}{p} + \frac{w-(w+\pi)\cdot\Phi(1/K_S^{PEN})}{p\cdot K_S^{PEN}}\ . \tag{42}$$

Note that for $\pi = 0$ the optimal decisions from the WHP contract emerge as $K_S^{PEN} = K_S^{WHP}$ and thus, $K_B^{PEN} = K_B^{WHP}$.

The result for case I ($Y \geq D$) only holds as long as $K_B^{PEN} > K_S^{PEN}$ is true. Otherwise, the second case must be analyzed.

II. Case $Y \leq D$

In this case the buyer's profit function from (39) turns out to become linear in *Y*, yielding

$$\Pi_B^{PEN}(Y\,|\,Q_S^{PEN}) = (p - w - \pi)\cdot\left[\int_0^{1/K_S^{PEN}} (z\cdot K_S^{PEN}\cdot Y - Y)\cdot\varphi(z)dz + Y\right] + (p-w)\cdot Y\ , \tag{43}$$

with a first-order derivative

$$\frac{\partial}{\partial Y}\Pi_B^{PEN}(Y\,|\,Q_S^{PEN}) = (p-w)\cdot\left[1-\Phi(1/K_S^{PEN})\right] + \left[\frac{(p-w-\pi)\cdot c}{w+\pi}\right]\cdot K_S^{PEN} + \pi\cdot\Phi(1/K_S^{PEN})\ . \tag{44}$$

Thus, the buyer's optimal decision is to increase his order to the upper level *D* if the derivative in (44) is positive, and to order nothing if not. So, under our general parameter restrictions, we find that

$$Y^{PEN} = D \quad \text{if} \quad p \geq w + \pi\ . \tag{45}$$

Interaction of buyer and supplier decisions

Like for the ORS contract, the supplier-buyer interaction and its impact on the potential for coordination will be carried out separately for the two cases $Y \geq D$ and $Y \leq D$.

In **case I** ($Y \geq D$) SC coordination can be achieved if $K_B^{PEN} = K_{SC}^{*}$. From (42) and (4) it is obvious that both multipliers are equal if the contract parameters fulfill the following condition

$$w = (w+\pi)\cdot\Phi(1/K_S^{PEN}) = 0\ . \tag{46}$$

Since this parameter combination results in $K_B^{PEN} = K_{SC}^* > K_S^{PEN} \geq K_S^{WHP}$, it guarantees SC coordination and follows the condition of case I. However, inserting the parameter combination from (46) in the supplier's profit function (38) under the PEN contract results in a profit of zero so that the participation constraints of SC interaction are violated. Summarizing, it turns out that just like under an ORS contract, SC coordination cannot be achieved if the buyer orders at a level above his demand. This is an insight that was also found by Gurnani and Gerchak (2007) where the following analysis of case II, however, is missing.

In **case II** ($Y \leq D$) when $Y^{PEN} = D$, SC coordination is guaranteed if the supplier's multiplier equals the SC's optimal one, i.e. $K_S^{PEN} = K_{SC}^*$. From (37) and (4) one finds that this condition is fulfilled if the parameter equation $c/(w+\pi) = c/p$ holds. Since according to (45) this equation also fulfills the condition for $Y^{PEN} = D$, the SC will be coordinated under this combination of contract parameters w and π that are simply related as follows

$$w + \pi = p\,. \tag{47}$$

Summarizing, under a PEN contract SC coordination always will be achieved as long as contract parameters are set such that the sum of wholesale price and penalty charge are equal to the retail price. Then the buyer just orders the demand quantity and the supplier produces the SC optimal quantity. The parameter combinations in (47) also enable an arbitrary split of the optimal SC profit under the two parties. From (38) it follows that the supplier's profit under coordination equals $\Pi_S^{PEN} = \left[w - (w+\pi)\cdot\Phi(1/K_{SC}^*) \right]\cdot D$. If this profit is divided by the SC optimal profit from (5) we get a supplier's profit share α_S amounting to

$$\alpha_S^{PEN} = \frac{w - (w+\pi)\cdot\Phi(1/K_{SC}^*)}{p - p\cdot\Phi(1/K_{SC}^*)} \tag{48}$$

From (48) in combination with (42) it is obvious that for a profit fraction $\alpha_S > 0$ the buyer's multiplier fulfills the condition $K_B^{PEN} < K_S^{PEN} = K_{SC}^*$ so that under coordination indeed case II, i.e. $Y \leq D$, applies.

From the profit ratio in (48) and parameter condition (47) we find as limiting cases where the participation constraints no longer hold, the combinations $w = p, \pi = 0$ (with $\Pi_B^{PEN} = 0$) and $w = p\cdot\Phi(1/K_{SC}^*), \pi = p\cdot\left[1-\Phi(1/K_{SC}^*)\right]$ (with $\Pi_S^{PEN} = 0$). The latter case is identical to the situation we found as condition for SC coordination in case I ($Y \geq D$).

3.3. General insights

In a SC which is exposed to a supply situation with quantity uncertainty due to a production process with random yield, a specific risk situation occurs even if customer demand is deterministic. This situation is characterized by two types of risk stemming from under-production on the one hand and over-production on the other. Under conditions of central decision making both risks are coped with globally and are responded to by an optimal policy which determines the production quantity by inflating the external demand. The inflation factor or multiplier depends on the level of yield risk as well as on cost/price parameters.

In a decentralized SC with an independent supplier and buyer, both actors are exposed to the above risks. The SC interaction, however, strongly depends on how these risks are divided among the SC members. This division again depends on the type of contract that rules the terms of business between them. Concerning the policy structure, it turns out that under all contracts and risk distributions considered a linear demand/order inflation rule is valid, although with contract-specific multipliers.

Under a simple WHP contract the risk distribution is such that the combination of buyer and supplier decisions does not inflate the external demand to a sufficient level to reach the SC optimal production quantity. This type of double marginalization effect holds for every wholesale price that is acceptable for the buyer and the supplier under their goal to make profits. Thus, the WHP contract fails to achieve SC coordination. In this context it is interesting that – different from the corresponding SC situation with deterministic yield and random demand – the buyer-supplier interaction does not only come close to coordination if the wholesale price approaches the supplier's unit cost, but also if it gets near to the buyer's retail price. So, there exists some 'inner' wholesale price for which the coordination deficit reaches its maximum level.

SC coordination becomes possible if more sophisticated contracts than a WHP contract are installed. This results from an additional contract parameter which allows for a different sharing of risks. In particular, this is done by reducing the supplier's risk position under an ORS and by increasing it under a PEN contract. The flexibility of risk sharing built in into both contracts can be used to motivate the supplier to deviate from the WHP under-production decision. Interestingly, SC coordination is only achievable if contract parameters are fixed in such a way that the buyer is not motivated to inflate his order above end customer demand. Additionally, by adequate parameter choice it is possible to generate an arbitrary split of the SC optimal profit between the two SC members. Concerning the ORS contract, however, this is only true if the contract is arranged in such a way that the over-produced items remain with the supplier so that a Pull-ORS type is exercised. If these items, notwithstanding that they were not ordered, are shipped to the buyer at the reduced price according to a Push-ORS contract, the split of SC risks changes in such a way that coordination can only be accomplished at price parameter values which reduce the supplier's profit to zero. So it turns out that an ORS contract needs a non-over-delivery condition to facilitate SC coordination while a PEN contract does not have constraints like that.

These insights become more evident when the results of the preceding analysis are demonstrated by means of a numerical example. To this end, we assume that the yield rate *z* is uniformly distributed between 0 and 1, resulting in a density $\varphi(z)=1$, distribution $\Phi(z)=z$, and a mean yield rate of $\mu_z=0.5$. The condition for profitability translates to $p>w>2\cdot c$. For this case we can derive simple closed-form expressions for all multipliers *K* that describe the production and order policies within the SC. So, from (4) and (5) we get

$K_{SC}^{*}=\sqrt{p/(2\cdot c)}$ and $\Pi_{SC}^{*}=\left[p-\sqrt{2\cdot c\cdot p}\right]\cdot D$

as results for the optimal demand multiplier and maximum SC profit in the case of *centralized decision making.* The respective results for *decentralized decision making* under a WHP contract are

$K_{S}^{WHP}=\sqrt{w/(2\cdot c)}$ and $\Pi_{S}^{WHP}=\left[w-\sqrt{2\cdot c\cdot w}\right]\cdot Y$ for the supplier and

$K_{B}^{WHP}=\sqrt{\dfrac{p}{2\cdot(\sqrt{2\cdot c\cdot w}-c)}}$ and $\Pi_{B}^{WHP}=\left[p-\sqrt{p\cdot(\sqrt{8\cdot c\cdot w}-2\cdot c)}\right]\cdot D$ for the buyer, respectively.

When we fix demand and cost/price data to be $D=100$, $c=1$ and $p=14$ we can calculate the impact of different values of the wholesale price (in the interval $2\cdot c\le w\le p$) as presented in Table 1.

w	Q_{SC}^{*}	Q_{S}^{WHP}	Y^{WHP}	Π_{S}^{WHP}	Π_{B}^{WHP}	$\Pi_{S}^{WHP}+\Pi_{B}^{WHP}$	Π_{SC}^{*}
2	265	265	265	0	871	871	871
3	265	220	179	99	763	862	871
4	265	196	138	162	684	847	871
5	265	180	114	209	622	831	871
6	265	173	100	254	569	823	871
7	265	187	100	326	513	839	871
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
13	265	255	100	790	80	870	871
14	265	265	100	871	0	871	871

Table 1: Impact of wholesale price value under the WHP contract

From Table 1 the interplay of production and order sizes for different wholesale price levels becomes visible, and it can be seen how the SC looses efficiency if this SC internal price deviates from both its minimum and maximum feasible levels. The highest efficiency loss of nearly 5% occurs if the wholesale price *w* is fixed such that the buyer just loses his incentive to increase his order size above the demand level of 100. As soon as the wholesale price w reaches a level that does no longer motivate the buyer to order more than externally demanded, further raising w results in an increased supplier's production so that the SC coordination deficit diminishes.

Under an ORS contract with parameters w and w_0 we find that in the case of uniformly distributed yield rate the supplier's multiplier and profit from (22) and (23) simplify to

$K_S^{ORS} = \sqrt{(w-w_0)/(2\cdot c-w_0)}$ and $\Pi_S^{ORS} = \left[(w-w_0)-\sqrt{(w-w_0)\cdot(2\cdot c-w_O)}\right]\cdot Y$, respectively.

Given that the non-over-delivery ORS contract variant is applied, the parameter combinations for w and w_0 in (32) guarantee SC coordination and result in a supplier's profit of

$$\Pi_S^{ORS} = (w-w_0)\cdot\left[1-\sqrt{2\cdot c/p}\right]\cdot D.$$

As coordination can be achieved, the buyer will consequently make a profit of

$$\Pi_B^{ORS} = \Pi_{SC}^{*} - \Pi_S^{ORS}.$$

With demand and cost/price data as for Table 1, the following Table 2 presents the coordinating contract parameter combinations from (32) when the wholesale price is varied in its feasible range. It gives evidence about how the total SC profit is split among buyer and supplier for different contract parameter sets.

w	w_O	$Q_S^{ORS}=Q_{SC}^{*}$	Y^{ORS}	Π_S^{ORS}	Π_B^{ORS}	$\Pi_S^{ORS}+\Pi_B^{ORS}=\Pi_{SC}^{*}$	α_S^{ORS}
2	***2.00***	265	100	0	871	871	0%
3	***1.83***	265	100	73	798	871	8%
4	***1.67***	265	100	145	726	871	17%
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
12	***0.33***	265	100	726	145	871	83%
13	***0.17***	265	100	798	73	871	92%
14	***0.00***	265	100	871	0	871	0%

Table 2: Impact of parameter setting on profit distribution under ORS contract

The above table proves that, as shown in the previous analysis, either party yields zero profit when it faces price/cost identity ($w=w_0=c/\mu_Z$ with $\Pi_S^{ORS}=0$ and $w=p, w_0=0$ with $\Pi_B^{ORS}=0$). In between those boundaries the profit can be split arbitrarily. Furthermore, it becomes evident that coordination is only enabled if the buyer orders at the demand level and the supplier alone accounts for the yield uncertainty by equivalently raising her production quantity.

Finally, considering the PEN contract with parameters w and π it can be derived from (37) and (38) that the supplier's multiplier and profit under uniform distribution can be written as

$K_S^{PEN} = \sqrt{(w+\pi)/(2\cdot c)}$ and $\Pi_S^{PEN} = \left[w - \sqrt{2\cdot c\cdot(w+\pi)}\right]\cdot Y$, respectively.

If the condition for coordinating contract parameters w and π from (47) is met, the supplier's profit will be

$$\Pi_S^{PEN} = \left[w - (w+\pi)\cdot\sqrt{2\cdot c/p}\right]\cdot D \ .$$

Due to coordination, the buyer's profit can accordingly be given as

$$\Pi_B^{PEN} = \Pi_{SC}^{*} - \Pi_S^{PEN} \ .$$

As for the ORS contract, these results can be used to demonstrate how the profit split depends on choice of the wholesale price and the coordinating penalty charge. Assuming demand and price/cost data as for Table 1, Table 3 gives an impression of these dependencies.

w	π	$Q_S^{PEN} = Q_{SC}^{*}$	$Y^{PEN} = D$	Π_S^{PEN}	Π_B^{PEN}	$\Pi_S^{PEN} + \Pi_B^{PEN} = \Pi_{SC}^{*}$	α_S^{PEN}
5.3	***8.7***	265	100	0	871	871	0%
6	***8***	265	100	71	800	871	8%
7	***7***	265	100	171	700	871	20%
8	***6***	265	100	271	600	871	31%
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
12	***2***	265	100	671	200	871	77%
13	***1***	265	100	771	100	871	89%
14	***0***	265	100	871	0	871	100%

Table 3: Impact of parameter setting on profit distribution under PEN contract

Different from the ORS contract, a wholesale price $w < 5.3$ is not feasible, since according to condition (48) this would result in a negative profit for the supplier.

Similar to the results of the ORS contract, the parties yield zero profits at the limiting parameter combinations of w and π: (i) $w = p\cdot\Phi(1/K_{SC}^{*}), \pi = p\cdot\left[1-\Phi(1/K_{SC}^{*})\right]$ resulting in $\Pi_S^{PEN} = 0$ and

(ii) $w = p, \pi = 0$ with $\Pi_B^{PEN} = 0$. In between these boundaries an arbitrary profit split is possible. Again, it becomes evident that coordination is only enabled if the buyer orders according to the demand level and the supplier appropriately raises the production quantity to account for the yield uncertainty.

In the next section it is evaluated to which extent the existence of an emergency source for procurement will influence the SC decisions and interaction in connection with yield uncertainty.

4. Contract analysis with emergency procurement

In this section, we deviate from the assumption that only a single opportunity exists to supply the quantities demanded by the buyer. Instead, we consider the case that in addition to regular production the supplier is able to procure any quantity in time from an alternative source once production yields have materialized. As mentioned before, the emergency source is not subject to risk or capacity restrictions. Under these circumstances we face a sequential procurement decision. First, a regular production quantity is determined and then, after yield realization, an emergency procurement decision is made. As a result, under-delivery can be avoided, and external demand can be fulfilled completely. A dual strategy with both procurement sources needs only to be considered if the unit cost of emergency procurement, denoted by c_E, is higher than the expected cost of regular production. So, we assume that $c_E > c / \mu_Z$ holds. In this section's problem setting, the relevant decisions and profits are indicated by a circumflex ($\hat{Q}$, $\hat{Y}$, $\hat{\Pi}$).

4.1. Centralized vs. decentralized decision making

The centralized decision maker in this sequential decision making process first decides on the production input quantity $\hat{Q}_{SC}$ and second on the amount to be procured from the emergency source, denoted by $\hat{Q}_{SC}^E$. As long as the retail price is sufficiently high, namely $p > c_E$, the optimal emergency decision will always be to increase the available production output to the demand level if necessary, i.e. $\hat{Q}_{SC}^E = \max(D - z \cdot \hat{Q}_{SC}, 0)$. Anticipating this second-step quantity adjustment, the decision regarding regular production is made in order to maximize the following profit function $\hat{\Pi}_{SC}$ which accounts for fulfillment of total demand D

$$\hat{\Pi}_{SC}(\hat{Q}_{SC}) = p \cdot D - c \cdot \hat{Q}_{SC} - c_E \cdot E\left[\max(D - z \cdot \hat{Q}_{SC}, 0)\right] = p \cdot D - c \cdot \hat{Q}_{SC} - c_E \cdot \int_0^{D/\hat{Q}_{SC}} (D - z \cdot \hat{Q}_{SC}) \cdot \varphi(z) dz \,. \tag{49}$$

The profit maximizing production input quantity can then be derived from the FOC as

$$\hat{Q}_{SC}^* \qquad \text{from} \qquad \int_0^{D/\hat{Q}_{SC}} z \cdot \varphi(z) dz = \frac{c}{c_E}$$

so that

$\hat{Q}_{SC}^{*} = \hat{K}_{SC}^{*} \cdot D$ with $\hat{K}_{SC}^{*} > 1$ and (50)

$$\hat{K}_{SC}^{*} \quad \text{from} \quad \int_{0}^{1/\hat{K}_{SC}} z \cdot \varphi(z) dz = \frac{c}{c_E} \,. \tag{51}$$

In case of $c_E < c / \mu_z$, the regular production quantity would obviously be equal to zero, and only the emergency source would be utilized.

The first interesting result from this analysis is that the SC optimal production input quantity is not dependent on the retail price anymore, given that $p > c_E$ holds. But still, demand is inflated by a factor which now depends solely on the costs for regular production and emergency procurement. Having analyzed the centralized SC problem, this result can again be used as a benchmark for decentralized decision making in a SC. As we will see, in this setting it is sufficient to restrict the analysis to the WHP contract.

Wholesale price contract

Under the simple WHP contract, given the buyer's order $\hat{Y}$, the supplier first chooses her production input level and then the emergency procurement quantity, if necessary. In this case, emergency procurement will be $\hat{Q}_S^E = \max(\hat{Y} - z \cdot \hat{Q}_S, 0)$ if the wholesale price permits profitability for the supplier, i.e. if $c_E < w$. Prior to utilizing the reliable source, the supplier chooses the quantity for the less expensive regular production process. Here, the profit to maximize for the supplier is

$$\hat{\Pi}_S^{WHP}(\hat{Q}_S \left| \hat{Y} \right.) = w \cdot \hat{Y} - c \cdot \hat{Q}_S - c_E \cdot E\left[\max(\hat{Y} - z \cdot \hat{Q}_S, 0)\right] = w \cdot \hat{Y} - c \cdot \hat{Q}_S - c_E \cdot \int_{0}^{\hat{Y}/\hat{Q}_S} (\hat{Y} - z \cdot \hat{Q}_S) \cdot \varphi(z) dz \,. \tag{52}$$

The optimal decision with respect to regular production is then derived from the FOC as

$\hat{Q}_S^{WHP}$ from $\int_{0}^{Y/\hat{Q}_S} z \cdot \varphi(z) dz = \frac{c}{c_E}$, resulting in a linear production policy

$\hat{Q}_S^{WHP} = \hat{K}_S^{WHP} \cdot \hat{Y}$ with a multiplier $\hat{K}_S^{WHP} = \hat{K}_{SC}^{*}$ being equal to that of the centralized solution in (51). In contrast to the case without emergency procurement, the optimal production input choice of the supplier is independent of the wholesale price and identical to the SC optimal production quantity if the buyer's order $\hat{Y}$ equals the external demand *D*.

This turns out to be just the case if an emergency procurement option exists which is profitable for the supplier. Due to the fact that the supplier will procure missing quantities after regular production, the delivery quantity received by the buyer is not random anymore. Under the WHP contract, this motivates the buyer to order exactly the demand size in order to maximize his profit, i.e. $\hat{Y}^{WHP} = D$. Now, it is clear that $\hat{Q}_S^{WHP} = \hat{K}_S^{WHP} \cdot \hat{Y}^{WHP} = \hat{K}_{SC}^{*} \cdot D$ which is exactly the SC optimal decision from (50). Accordingly, the decisions on emergency procurement are also identical (i.e. $\hat{Q}_S^E = \hat{Q}_{SC}^E$). From the production decision in (50) and the multiplier in (51) the profit of the supplier results in

$$\hat{\Pi}_S^{WHP} = \left[w - c_E \cdot \Phi(1/\hat{K}_{SC}^*) \right] \cdot D \; . \tag{53}$$

Thus, we come to the interesting conclusion that the simple WHP contract is sufficient to guarantee SC coordination if a non-random emergency procurement option with economic attractiveness for the supplier exists. The analysis provided by He and Zhao (2011) for the case of additional demand randomness confirms this result. Due to this finding it is not necessary to concern oneself with more complex contracts like an ORS or PEN contract under the perspective of SC coordination. However, it has to be noted that things are different when the emergency procurement source can be utilized by the buyer in place of the supplier. Under these circumstances, the supplier is in a position like in the WHP case without emergency procurement and produces according to (8). Thus, the production decision is affected by the wholesale price. Because of his alternative procurement source, the buyer will order less than described in (16). Altogether, compared to the SC optimal production level this results in a shortfall of production which is even larger than in the situation without emergency production and will make SC coordination impossible under a WHP contract. So it is evident that it essentially matters for SC coordination which one of the SC actors has access to an emergency procurement option.

4.2. Wholesale price determination with and without emergency procurement

From the above analysis it is evident that the emergency procurement option will only be exercised if its unit cost does not exceed the respective sales price. For $c_E > p > w$, utilizing the emergency source is not profitable for the supplier and the whole SC so that the resulting situation is identical to that without emergency procurement where, however, a simple WHP contract will not enable SC coordination. In this section we will discuss why it nevertheless might make sense to use an emergency procurement opportunity in case of $c_E > p > w$ when the SC actors abstain from complex multi-parameter contracts and only agree on implementing a simple WHP contract. In this context, we consider a SC situation where the supplier is fixing a wholesale price under anticipation of a fixed minimum profit requirement by the buyer.

Under decentralized decision making, a benefit can arise from utilizing the emergency option even under $c_E > p$ if the SC profit deficit resulting from individual optimization under the WHP contract without emergency procurement can be reduced. Such a reduction is always possible if the emergency cost c_E does not exceed the retail price p too much so that the loss in SC efficiency caused by diseconomies from emergency procurement is smaller than the SC deficit from implementing a simple WHP contract. If the supplier chooses her wholesale price such that the potential deficit reduction by exercising the emergency option is shared among the SC actors, a Pareto improvement (as second best solution) can be achieved, i.e. the interaction can improve all parties' profits without being the first best solution. However, the profitability of the interaction as well as the distribution of profits depends on the parameter setting applied.

The described potential for improving SC coordination under a WHP contact even for $c_E > p$ will be illustrated by a numerical example. In order to exploit closed-form solutions we again assume a uniformly distributed yield rate in $[0,1]$ with mean $\mu_z = 0.5$ like in Section 3.

Under *centralized decision* making the SC optimal demand multiplier and profit are given as

$\hat{K}^*_{SC} = \sqrt{c_E/(2\cdot c)}$ and $\hat{\Pi}^*_{SC} = \left[p - \sqrt{2\cdot c\cdot c_E}\right]\cdot D$, respectively. (54)

When decision making is *decentralized* and the parties agree on the WHP contract, the supplier's multiplier and profit are

$\hat{K}^{WHP}_S = \sqrt{c_E/(2\cdot c)} = \hat{K}^*_{SC}$ and $\hat{\Pi}^{WHP}_S = \left[w - \sqrt{2\cdot c\cdot c_E}\right]\cdot D$, respectively. (55)

As coordination is achieved the buyer's profit is simply

$\hat{\Pi}^{WHP}_B = \hat{\Pi}^*_{SC} - \hat{\Pi}^{WHP}_S$.

In Table 4, we provide a numerical example in order to illustrate the development of profits with increasing values for the emergency cost c_E. Assuming demand and price/cost data as for Table 1, we proceed from the parameter combination which yielded the largest profit deficit under the WHP contract without emergency option (i.e. $w = 6$). From Table 1, recall the accompanying profits of $\Pi^{WHP}_S = 254$ and $\Pi^{WHP}_B = 569$ which result in a SC deficit of Π^*_{SC} -($\Pi^{WHP}_S + \Pi^{WHP}_B$) $= 871 - 823 = 48$.

Table 4 now displays the results emerging from a supplier's offer of a WHP contract under emergency procurement with different c_E cost levels where the buyer will be guaranteed to receive complete delivery of all ordered units so that his order equals external demand, i.e. $\hat{Y}^{WHP} = D = 100$.

c_E	$\hat{Q}^{WHP}_S = \hat{Q}^*_{SC}$	$\hat{Y}^{WHP} = D$	$\hat{\Pi}^{WHP}_S$	$\hat{\Pi}^{WHP}_B$	$\hat{\Pi}^{WHP}_S + \hat{\Pi}^{WHP}_B = \hat{\Pi}^*_{SC}$	$\hat{\alpha}^{WHP}_S$
6	173	100	254	800	1.054	24%
7	187	100	226	800	1.026	22%
8	200	100	200	800	1.000	20%
⋮	⋮	⋮	⋮	⋮	⋮	⋮
14	265	100	71	800	871	8%
15	274	100	52	800	852	6%
16	283	100	34	800	834	4%
17	292	100	17	800	817	2%

Table 4: Impact of emergency cost values on profits

The results in Table 4 at first illustrate that for an emergency cost c_E which is smaller than the retail price $p = 14$, the SC profit is always higher than the sum of profits under the WHP contract without emergency production which from Table 1 equals 823 for $w=6$. Second, it turns out that, even if the retail price is exceeded (i.e. $c_E > 14$), utilizing the emergency option can be reasonable. For $c_E = 15$ and $c_E = 16$ the SC faces a reduction in the maximal profit $\Pi^*_{SC} = 871$ without emergency procurement. This loss, however, is smaller than the coordination deficit of 48. Hence, from a SC perspective it is profitable to utilize emergency procurement even if the respective cost exceeds the price gained per unit. Furthermore, it can be shown that exercising the emergency option is not profitable for the supplier under all circumstances. From (55) it follows that the supplier's profit will only be positive if $c_E < w^2/(2 \cdot c)$ holds. This means that, if $c_E \geq 18$ in our numerical example, the supplier's participation constraint no longer holds as $\Pi_S^{WHP} \leq 0$.

Nevertheless, the results also show that for some cases the supplier is worse off than under the WHP contract without emergency production while the buyer always benefits. In order to guarantee beneficial profit sharing for all parties, contract terms can be formulated appropriately. If the supplier is in the position to determine the wholesale price and has to guarantee the buyer a minimum profit of e.g. 569, parameter combinations can be found which assure higher profits for all parties compared to the situation without emergency production. For $p=14$ and $c_E = 15$ (i.e. $c_E > p = 14$), Table 5 illustrates under which wholesale prices both, buyer and supplier, benefit from the emergency option and how the profit is split among the actors.

w	$\hat{Q}_S^{WHP} = \hat{Q}^*_{SC}$	$\hat{Y}^{WHP} = D$	$\hat{\Pi}_S^{WHP}$	$\hat{\Pi}_B^{WHP}$	$\hat{\Pi}_S^{WHP} + \hat{\Pi}_B^{WHP} = \hat{\Pi}^*_{SC}$	$\hat{\alpha}_S^{WHP}$
8.10	274	100	262	590	852	31%
8.20	274	100	272	580	852	32%
8.30	274	100	282	570	852	33%

Table 5: Profit split for different WHP values under emergency option

Note that from Table 5 we find that the starting point for our consideration (based on $w=6$) is $\Pi_S^{WHP} = 254$, $\Pi_B^{WHP} = 569$ and $\Pi_{SC}^{WHP} = 823$. Thus, Table 5 describes contract terms that result in a win-win situation.

4.3. General insights

It is surprising to realize that the mere existence of a second procurement option without randomness in supply makes the design of sophisticated contracts for facilitating SC coordination unnecessary. This, at least, holds as long as the emergency option is economically viable for the SC or the supplier, respectively. The reason behind these facts is that the double marginalization effect vanishes as both actors' decisions do not depend on the wholesale price. It is interesting to see that, like under the coordinating ORS and PEN contract, SC coordination is achieved along with contract conditions that generate an incentive for the buyer to order exactly the firm demand size, thus preventing a distortion of customer demand information.

The coordinating property of the WHP contract in case of reliable emergency procurement relies on its ability to shift the whole SC risk to the supplier who is in a position to guarantee complete delivery of the buyer's order volume. This instance can even be exploited if the emergency option is not economically attractive for the SC. Utilizing emergency procurement in this case can help the supplier to offer a contract which is beneficial for all parties compared to a WHP contract with a single (random) procurement source.

Concerning the role of the emergency option for SC coordination, the question arises to which extent the described coordinating property in case of regular production with random yields can be generalized. Unfortunately, it turns out that this property is closely associated with the existence of certainty regarding external customer demand. If in addition to production yield also demand is random, it is easy to show (without going into details) that a WHP contract loses its coordinating power even if an emergency procurement option exists. If demand is stochastic with $cdf\ F(\cdot)$, the buyer's decision problem in a decentralized setting is completely identical to that of a simple newsvendor so that $\hat{Y}^{WHP} = F^{-1}\left[(p-w)/p\right]$ holds. The supplier herself has to solve exactly the problem described in Section 4.2 so that the production decision equals $\hat{Q}_S^{WHP} = \hat{K}_S^{WHP} \cdot \hat{Y}^{WHP}$ with $\hat{K}_S^{WHP}$ from $\int_0^{1/\hat{K}_S} z \cdot \varphi(z) dz = c / c_E$ (for a detailed analysis see He and Zhang (2008)). From this interaction it becomes clear that under a WHP contract the buyer's order size depends on the wholesale price, the same holds for the supplier's production quantity. Since under centralized planning the optimal production decision will not be influenced by the (SC internal) wholesale price it is evident that $\hat{Q}_{SC}^*$ and $\hat{Q}_S^{WHP}$ will not coincide. As a consequence, the coordinating property of the WHP contract with emergency procurement option will not hold in the case of stochastic customer demand. Furthermore, we found that this property gets lost already in the deterministic demand case if the emergency procurement option is in the ownership of the buyer in place of the supplier.

5. Conclusion and further research

The previous analysis revealed interesting insights into the field of SC coordination through contracts in the case of random production yields and deterministic demand. Without an option to utilize an emergency source the simple WHP contract fails to coordinate the SC due to the double marginalization effect. Interestingly, for the considered ORS contract it depends on the definition of the contract whether coordination can be achieved or not. Under a Pull-variant of the contract (no over-deliveries) coordination is possible and an arbitrary profit split can be generated. In contrast, under a Push-ORS contract (with over-deliveries), coordination cannot be enabled as the coordinating parameter setting violates the supplier's participation constraint.

For the PEN contract, however, it can be shown that the design enables SC coordination and, depending on the parameter setting, guarantees an arbitrary distribution of profits among the actors. For both contract types, Pull-ORS and PEN, it has been illustrated that only in case the buyer orders exactly at demand level, coordination is achieved

The situation is different when an emergency source with perfect yield is available to the supplier. Here, the WHP contract can achieve coordination as the buyer always orders what is demanded and the supplier's production decision does not depend on the wholesale price. Under these circumstances, it is not necessary to employ more complex contracts because the simple WHP contract suffices.

As coordination can be achieved by the analyzed contracts in the considered SC settings, further research should focus on the question to which extent the above results carry over to modified settings. An important aspect in this context is the extension from a serial to a converging SC. As mentioned in Section 1, for settings with such a SC structure as well as with stochastic customer demand some results from research contributions are already available. Almost no research is available for different types of yield processes that could be addressed like an all-or-nothing type of yield realization, also known as disruption risk (see Xia et al. (2011)), which can be seen as a border case of a stochastically proportional yield model. Furthermore, the assumption of stochastically proportional yield itself might be questioned. In some cases it is more realistic to suppose that a binomial yield process applies (see Yano and Lee (1995)). A further promising field for future research would be to investigate if the theoretical results from optimizing supplier's and buyer's SC decisions coincide with real-world behavior. In other fields of SC management there exist already a lot of research contributions concerning behavioral operations (see Bendoly et al. (2010)). Up to now, Gurnani et al. (2011) are the only ones to present insights into actual human behavior in stochastic yield scenarios by experimental research. SC interaction under random yield, however, has not yet been investigated by this type of research.

Appendix

Proof of concavity of $\Pi_{SC}(Q_{SC})$ ***in (1) :***[1]

For the first-order and second-order derivative of $\Pi_{SC}(Q_{SC})$ we get

$$\frac{\partial}{\partial Q_{SC}}\Pi_{SC}(Q_{SC}) = p \cdot \int_0^{D/Q_{SC}} z \cdot \varphi(z)dz - c \qquad \text{and} \qquad \frac{\partial^2}{\partial Q_{SC}^2}\Pi_{SC}(Q_{SC}) = -p \cdot \frac{D^2}{Q_{SC}^3} \cdot \varphi\left(\frac{D}{Q_{SC}}\right) < 0 .$$

Thus, Π_{SC} is concave in Q_{SC}. Setting the first-order derivative equal to zero, i.e. $\partial \Pi_{SC} / \partial Q_{SC} = 0$, equation (2) can immediately be derived.

Proof of concavity of $\Pi_B^{WHP}(Y \mid Q_S^{WHP})$ ***in (11) :***[2]

For the first-order and second-order derivative of $\Pi_B^{WHP}(Y \mid Q_S^{WHP})$ we get

$$\frac{\partial}{\partial Y}\Pi_B^{WHP}(Y \mid Q_S^{WHP}) = p \cdot K_S^{WHP} \cdot \int_0^{D/(K_S^{WHP} \cdot Y)} z \cdot \varphi(z)dz - w \cdot \left[K_S^{WHP} \cdot \int_0^{1/K_S^{WHP}} z \cdot \varphi(z)dz - \Phi(1/K_S^{WHP}) + 1 \right] \text{ and}$$

$$\frac{\partial^2}{\partial Y^2}\Pi_B^{WHP}(Y \mid Q_S^{WHP}) = -p \cdot \frac{D^2}{(K_S^{WHP})^2 \cdot Y^3} \cdot \varphi\left(\frac{D}{K_S^{WHP} \cdot Y}\right) < 0$$

Thus, $\Pi_B^{WHP}(Y \mid Q_S^{WHP})$ is concave in Y. Setting the first-order derivative equal to zero, i.e. $\frac{\partial}{\partial Y}\Pi_B^{WHP}(Y \mid Q_S^{WHP}) = 0$, and exploiting $\int_0^{1/K_S^{WHP}} z \cdot \varphi(z)dz = \frac{c}{w}$ from equation (9) yields the respective optimality condition for Y^{WHP} in (12).

[1] The profit function structure and the way of proving concavity and exploiting the FOC is identical for supplier's profit functions under all following contracts. For that reason the respective derivations are not repeated hereafter.

[2] The profit function structure and the way of proving concavity and exploiting the FOC is identical for buyer's profit functions under all following contracts. For that reason the respective derivations are not repeated hereafter.

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