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# Dual Sourcing Using Capacity Reservation and Spot Market: Optimal Procurement Policy and Heuristic Parameter Determination

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**Abstract.** This contribution focuses on the cost-effective management of the combined use of two procurement options: the short-term option is given by a spot market with random price, whereas the long-term alternative is characterized by a multi period capacity reservation contract with fixed purchase price and reservation level. A reservation cost, proportional with the reservation level, has to be paid for the option of receiving any amount per period up to the reservation level. A long-term decision has to be made regarding the reserved capacity level, and then it has to be decided - period by period - which quantities to procure from the two sources. Considering the multi-period problem with stochastic demand and spot price, the structure of the optimal combined purchasing policy is derived using stochastic dynamic programming. Furthermore, a simple heuristic procedure is developed to determine the respective policy parameters. Finally, we present a comprehensive numerical study showing that our heuristic policy performs very well.

**Key words:** Dual sourcing, capacity reservation, spot market, procurement policy, stochastic dynamic programming

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## 1. Introduction and Literature Review

Supply strategy is getting more important with the tendency of increased outsourcing and higher value of purchased materials and parts. With the increase of component production outsourcing, the share of purchasing cost increased up to 50 - 90% of the total revenue of manufacturing companies (de Boer et al., 2001), becoming a critical element of competitiveness. The number of available procurement alternatives also increased with a wide range of spot purchase opportunities and different contract based supply options. With this increased importance and complexity, the strategic sourcing attracted substantial managerial and quantitative research recently.

Spot market purchasing provides flexibility and a benefit in case of a low spot market price or insufficient reserved capacity. Around 30% of the memory chips are bought on spot markets according to the Gartner Group estimates (see Andren, 2000 and Seifert et al., 2004). With the growing importance of electronic commerce and global sourcing, the spot market is competing with contract based procurement. A 2002 survey shows that 57% of industrial buyers ranging from automotive to electronics increased spot purchases and reduced buying based on long term contracts (Ansberry, 2002).

Capacity reservation contract is used as an operational risk hedging for high spot market price incidents. The simple price contract has been extended to several contract forms including the *capacity reservation contract* extensively used for purchasing chemicals, commodity metals, semiconductors, and electric power (Kleindorfer and Wu, 2003). In electric power generation the so-called "tolling agreement" (see Woo et al., 2006) gives the local electricity distribution companies the right, but not the obligation, to dispatch a generation unit specified by the agreement. Typically, the capacity option contract needs to be fixed for a longer time horizon specifying the price and quantity and pay the reservation price up front. Short-term decisions are required in each period how much to order from the reserved capacity source up to the available capacity or purchase completely on spot market or use both sources.

Leading companies are combining different purchasing options strategically to reap the benefits of the alternative sources. Applications include chemicals, commodity metals, raw materials, oil, liquefied gas, and semiconductors. Reiner and Jammernegg (2010) examined

the practice of a chemical company buying raw materials using contracts and spot market. With simulation they showed the advantages of multiple sourcing including the benefit of speculative inventory purchased from spot market. A multiple sourcing strategy is also used in LNG (Liquefied Natural Gas) purchasing as it is reported in Yacef (2010). The combined forward contract and spot purchase strategy is applied in electricity market to combine the risk hedging and price benefits as it is discussed in Giacometti et al. (2010). Food packaging industry also uses the forward buying combined with spot purchase as it is reported in Vukina et al. (2009).

In our research we address a procurement problem of a component/material that is used for producing finished goods in a make-to-stock environment under stochastic demand. For the component we consider two sourcing options: the spot market and a capacity reservation contract. The capacity reservation contract is a real option contract where a reservation price, proportional with the reserved quantity, has to be paid for the option of receiving any amount per period for the contract price up to the reservation quantity. The combined strategic sourcing decision is quite challenging because it requires looking ahead for several periods that have stochastic demand and random spot market price fluctuations. A long-term decision has to be made regarding the reserved capacity level fixed with the long-term supplier. It should create sufficient protection for high spot market price incidents. Then it has to be decided - period by period - which quantities to procure from the different sources. The combined procurement strategy has to protect against risks of insufficient demand fulfillment and exploit the benefits of forward buying in periods with low spot price levels. The decision on capacity reservation has to take into account the short-term capacity utilization of each source which itself depends on the available long-term capacity. Thus, there is a highly complex interdependence of long-term and short-term decisions under uncertainties in demand and spot market price. In this context, the multi-period approach allows for integrating capacity reservation, forward buying and safety stock holding aspects in a single model.

We formulate a stochastic dynamic optimization model for the above problem and prove that, for random stationary demand and spot market price, the optimal procurement decisions can be made based upon a three-parameter policy. Two parameters are fixed numbers, the order-up-to level for ordering from the long-term supplier and the long-term capacity

reservation level. The third policy parameter, the order-up-to level for short-term spot market procurement, is a function of the spot price. It is very cumbersome to numerically calculate the optimal values of all policy parameters; therefore, we develop a fairly simple heuristic approach for determining all parameters including the parameter function.

Several papers consider capacity contracts or spot purchase as the only sourcing option. We refer to the capacity contracts papers of Kamrad and Siddique (2004), Burnetas and Ritchken (2005), Erkoc and Wu (2005), Hazra and Mahadevan (2009), Kirche and Srivastava (2010) that have the closest connection to our research. The spot purchase related papers by Geman (2005), Tapiero (2008), Arnold et al. (2009), and Guo et al. (2011) do not examine the trade-offs between the spot market and long-term contracts.

Dual sourcing and order splitting are also major research streams related to our problem. We refer to the papers of Minner (2003) and Thomas and Tyworth (2006) for a comprehensive review of this research area. Despite the considerable number of researches extending different dimensions of dual- or multiple-sourcing problems, only a few of them is dealing with combining spot market purchases with purchases made in advance from a specific long-term supplier. Henig et al. (1997) derived a three-parameter optimal policy without the consideration of uncertainty on the procurement side, which is a critical factor in practice. Bonser and Wu (2001) study the fuel procurement problem for electric utilities in which the buyer can use a mix of long-term and spot purchases.

Our problem was first defined and studied in the inventory literature in Serel et al. (2001). They considered the simple ($R,S$) capacity reservation – order up policy, but they disregarded the spot market price uncertainty. Wu et al. (2002) consider uncertainty in spot market prices and analyze the contracts for non-storable goods involving options executable at a predetermined price. Kleindorfer and Wu (2003) linked this literature to evolving B2B exchanges on the Internet. In Sethi et al. (2004) a situation with both demand and price uncertainty is taken into consideration, and a quantity flexibility contract is employed; however, no capacity reservation takes place. Seifert et al. (2004) also analyzed a single-period dual sourcing problem from the buyer's standpoint with changing levels of buyer's risk preferences. Using a similar single-period model, Spinler and Huchzermeier (2006) show that the combination of an options contract and a spot market is Pareto improving with respect to the other alternative market structures. Martínez-De-Albéniz and Simchi-Levi

(2005) address the dynamic supply contract selection problem using long-term and options contracts as well as the spot market. They assume that the demand is known before stock is replenished in contrast to our model which considers the uncertainty of demand at the time of ordering. Haksoz and Seshadri (2007) published a review paper in this topic. Talluri and Lee (2010) propose a methodology for optimal contract selection based on a mixed-integer programming approach. Arnold and Minner (2011) deal with a two-period problem with dual sourcing. Gallego et al. (2011) analyzed the dual sourcing using spot market and real option market from the point of revenue management.

The analysis in Serel (2007) is the closest to ours. The main difference is that they consider a spot market with random capacity at a given price instead of a random price without capacity restriction. Furthermore, they assume that the spot capacity is not known when the ordering decision is made so that procurement decisions will not depend on the respective capacity level at the spot market. Under these circumstances, the optimal policy in Serel (2007) has a simple three-parameter structure, but is not capacity-contingent. Li et al. (2009) develop a stochastic dynamic programming model, as we do in our paper. They incorporate mixed strategies that include purchasing commitments and contract cancellations but they left out the inventory policy and replenishment decisions within each contract review period. In the paper of Fu et al. (2010) the buyer has three choices, either procure using fixed price contracts or option contracts or use spot purchases. However, they consider a single-period problem unlike our multi-period decision making framework. Zhang et al. (2011) consider two supply sources: one is the contract supplier from which the buyer orders over a specific contract period at a pre-agreed price, and the other is the spot market. However, when ordering from the contract supplier, the buyer must fulfill a pre-determined total order quantity, unlike in our problem where a downward flexibility is allowed but a reservation price must be paid ahead that is proportional with the reservation quantity.

The same problem environment as in our paper is also addressed in Inderfurth and Kelle (2011). There, however, a simple capacity reservation and base stock policy is considered where both short-term, spot market based and long-term, capacity reservation based purchasing decisions follow a single order-up-to level which does not depend on the spot market price. So procurement decisions are not conditioned on the current price situation, and forward buying is not taken into account.

In the current paper, our focus is on investigating the structure of the optimal purchasing and capacity reservation policy, including the optimal reaction on spot price changes. To this end we use a stochastic dynamic programming approach for analyzing and solving the problem. Besides the analytic results we provide a well performing heuristics for practical application.

Section 2 contains the model of the above practical sourcing problem and the analysis of the optimal procurement policy. In Section 3 we develop a fairly simple heuristic approach for determining all parameters and parameter functions. In Section 4, we present a comprehensive numerical study showing that our heuristic policy performs very well for a wide range of problem instances. Section 5 concludes the paper with discussions and provides future research directions.

## 2. Sourcing Model and Optimal Procurement Policy

### 2.1 Problem Description and Model Formulation

Before modelling the considered decision problem with multiple procurement sources and a capacity reservation option under several uncertain impact factors we first describe the role of the decision variables and add some more detailed assumptions. The management task is to fix a long-term capacity reservation level and to decide period-by-period how to combine the two supply options in order to profit from the cost savings of long-term procurement while still remaining flexible. Concerning the price variations on the spot market, this flexibility can be used to benefit from low short-term price levels through large procurement orders while procuring via the long-term contract is a means to hedge the risk of high spot market prices.

As it is common in the relevant literature, we consider independent and identically distributed (i.i.d) random product demand, $\tilde{x}$, and random component spot market price, $\tilde{p}$, per period. For a large spot market volume we can assume that our purchasing decision does not influence the spot market price, so there is no correlation between our demand and the spot price. Furthermore, we assume that the price mechanism is such that at a current spot price the procurement quantity is not restricted in any period.

In the sequel we use the following notation:

$F(x), f(x), \mu_x, \sigma_x$ cumulative distribution, density function, expected value and standard deviation of **demand** $\tilde{x}$ and

$G(p), g(p), \mu_p, \sigma_p$ the same distribution characteristics for the **spot market price** $\tilde{p}$.

We consider a periodic decision process involving different level of knowledge in time. The **first decision** is on

$R$ the capacity reservation quantity

that must be *fixed for a longer time horizon* based on the random demand and spot market price distribution and the following stationary cost factors:

$c$ the unit purchase price charged by the long-term supplier,

$r$ the capacity reservation price per period for a unit of capacity reserved,

$h$ the inventory holding cost per unit and period,

$v$ the shortage cost per unit and period.

The **next decision** is at the *beginning of each time period* about

$Q_{L,t}$ order quantity from the long-term supplier, and/or

$Q_{S,t}$ order quantity from the spot market

at the beginning of each period, $t$, knowing

$I_t$ inventory level at the beginning of the period and

$p_t$ the realized current spot market price,

but without knowing the realized demand for the period. The shipments are assumed to arrive before demand occurs. Both the finishing inventory level and the period's total cost are available after demand realizes at the end of the period. Unsatisfied demand is backordered.

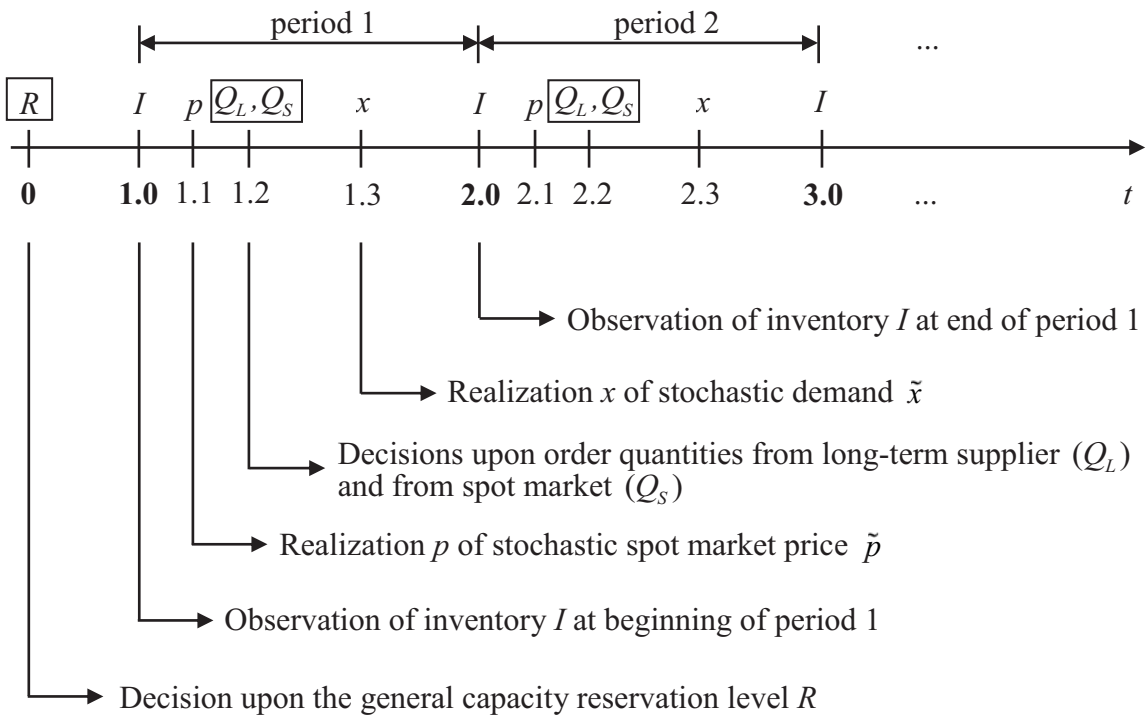


**Figure 1**. Timeline of the observations and decisions.

The timeline of the decisions is illustrated in Figure 1.

Costs are discounted at

$\beta$ the single-period discount factor ($0 < \beta \leq 1$).

We assume a planning horizon of $T$ periods. The overall objective is to choose the long-term capacity reservation level before the first period starts and thereafter to decide upon procurement quantities from both sources in each period of the planning horizon in such a way that the expected total cost, $C$, is minimized.

Given this problem description, the optimization problem can be described as follows:

$$\text{Min } C = E_{\{p_1,\ldots,p_T\},\{x_1,\ldots,x_T\}} \left\{ \sum_{t=1}^{T} \beta^{t-1} \left( rR + cQ_{Lt} + p_t Q_{St} + h\left[I_{t+1}\right]^+ + v\left[-I_{t+1}\right]^+ \right) \right\} \tag{1}$$

with inventory balance equation $I_{t+1} = I_t + Q_{Lt} + Q_{St} - x_t$

and initial inventory $I_1 = \overline{I}$ .

## 2.2 Optimal Procurement Policy

For the problem under consideration, the structure of the optimal policy can be determined by using a stochastic dynamic programming approach. In order to develop the recursive equations of dynamic programming we introduce the following additional notation:

$D_t(I_t,R,p_t)$ minimum expected cost from period $t$ to $T$ for a starting inventory $I_t$ and a given capacity reservation level $R$, **after** realization of spot market price $p_t$,

$C_t(I_t,R)$ minimum expected cost from period $t$ to $T$ for a starting inventory $I_t$ and a given capacity reservation level $R$, **before** spot market price $p_t$ realizes.

$C_1(\bar{I},R)$ corresponds to the minimum cost from optimizing the procurement decisions over all periods under a given reservation level $R$. Thus, the optimal capacity level $R$ can be calculated by solving the single-variable optimization problem

$$\min\ C_1(\bar{I},R). \tag{2}$$

The $C_1(\bar{I},R)$ function results from the solution of the stochastic dynamic procurement problem. For determining the optimal procurement decisions in period $t=1,\ldots,T$, we evaluate the dynamic programming recursive relations which (suppressing the time index, $t$, for all variables for sake of simplicity) can be expressed by

$$D_t(I,R,p) = \min_{R\geq Q_L\geq 0, Q_S\geq 0}\left\{cQ_L + pQ_S + L(I+Q_L+Q_S) + \beta\cdot\int_0^\infty C_{t+1}(I+Q_L+Q_S-x,R)f(x)dx\right\} \tag{3}$$

with $C_t(I,R) = \int_0^\infty D_t(I,R,p)g(p)dp$

and $C_{T+1}(I,R)\equiv 0$ as final cost condition for all $I$ and $R$.

The function $L(I) = h\cdot\int_0^I (I-x)f(x)dx + v\cdot\int_I^\infty (x-I)f(x)dx$ describes the expected one-period holding and shortage costs.

From analyzing these relationships we can derive several propositions which characterize the optimal dual source procurement policy. Proofs are found in Appendix A.

***Proposition 1*:** Under a fixed capacity reservation level, $R$, for a finite planning horizon the optimal structure of the combined ordering decisions in each period $t$ follows an $(S_{L,t}, S_{S,t}(p))$ policy, characterized by an order-up-to level $S_{L,t}$ for procuring from the long-term supplier and a corresponding price-dependent level $S_{S,t}(p)$ for the spot market. In detail, the order policy in each period $t$ prescribes:

**(a)** if $p_t < c$, order only from spot market up to base stock level $S_{S,t}(p)$,

**(b1)** if $p_t \geq c$, only order from the long-term supplier up to a stock level $S_{L,t}$ if the reserved capacity, $R$, is sufficient.

**(b2)** if $p_t \geq c$ and the reserved capacity is not sufficient, order from spot market up to level $S_{S,t}(p)$ as long as this level is not yet exceeded by supply from the other source.

More formally, this policy can be described in the following way

**(a)** If $p_t < c$:

$$Q_{L,t} = 0 \quad \text{and} \quad Q_{S,t} = \begin{cases} S_{S,t}(p_t) - I_t & \text{if} \quad I_t \leq S_{S,t}(p_t) \\ 0 & \text{if} \quad I_t \geq S_{S,t}(p_t) \end{cases} \tag{4}$$

**(b)** If $p_t \geq c$:

$$Q_{L,t} = \begin{cases} S_{L,t} - I_t & \text{if } I_t \leq S_{L,t} \text{ and } S_{L,t} - I_t \leq R \\ R & \text{if } I_t \leq S_{L,t} \text{ and } S_{L,t} - I_t \geq R \\ 0 & \text{if } I_t \geq S_{L,t} \end{cases} \quad \text{and} \quad Q_{S,t} = \begin{cases} S_{S,t}(p_t) - R - I_t & \text{if } I_t \leq S_{S,t}(p_t) - R \\ 0 & \text{if } I_t \geq S_{S,t}(p_t) - R \end{cases} \tag{5}$$

For a finite horizon problem the order-up-to levels $S_{L,t}$ and $S_{S,t}(p_t)$ vary from period to period. For given policy parameters, this specific policy structure makes it fairly simple to apply the optimal decision rule in practice. Additionally, this structure can be exploited for reducing the computational effort in calculating the optimal policy parameter values.

***Proposition 2*:** The order-up-to level $S_{S,t}(p_t)$ for short-term procurement decreases with increasing spot price. This intuitive dependency is complemented by the property that this level will be smaller (larger) than the order-up-to level $S_{L,t}$ if the current spot market price $p_t$ is higher (lower) than the long-term procurement price $c$. In case of price equality both order-up-to levels coincide.

Formally described, we have the relationship:

$$S_{S,t}(p_t) \begin{cases} > S_{L,t} & \text{if } p_t < c \\ = S_{L,t} & \text{if } p_t = c \\ < S_{L,t} & \text{if } p_t > c \end{cases} . \tag{6}$$

***Proposition 3*:** The minimum cost function $C_1(\bar{I}, R)$ is convex in the capacity reservation level $R$ for each starting inventory $\bar{I}$. This property can be exploited for reducing the computational effort in calculating the optimal reservation level after the procurement policy has been optimized.

***Proposition 4*:** For the infinite horizon problem (i.e., $T \to \infty$) with discount factor $\beta$<1, the ($S_L$,$S_S(p)$) policy structure still remains optimal. In this stationary case the policy parameters are equal for each period.

The optimality of a stationary ($S_L$,$S_S(p)$) policy for a discounted cost criterion in the infinite horizon case does not necessarily ensure that this property also holds for the average cost criterion (where $\beta \to 1$). From a practical point of view, however, this policy can also be applied to minimize average period cost since discount factor $\beta$ can be chosen arbitrarily close to 1.

The above analysis does not only give insights into the structure of the optimal dual sourcing procurement policy including the optimal response to randomly fluctuating prices, it also can be used to calculate the optimal capacity reservation level and the optimal policy parameters for periodic procurement decisions for given problem data. However, different from the situation in Inderfurth and Kelle (2011) where a simplified policy structure with a single fixed order-up-to level was used, no simple closed-form expressions for determining the policy parameters can be derived. So a numerical procedure for parameter calculation is needed. To this end the dynamic programming recursive equations iteratively have to be evaluated for alternative capacity levels and, by the end, the optimal reservation level has to be determined under proper discretization of the state and decision space. Even though it is

possible to exploit the known policy structure and convexity properties of the relevant cost functions, numerical optimization will be a highly cumbersome computational task that only is practical for small problem instances. In order to offer an approach that can be applied to real-world problem sizes, in the next section we present a heuristic procedure for calculating all policy parameters with minor computational effort.

## 3. Heuristic Approach to Determine the Policy Parameters

### 3.1 Overview

Our heuristic approach is based on the optimal policy structure (as stated in **Proposition 1**) with price dependent order-up-to levels and we approximate the policy parameters so that they are near-optimal in most cases. For this purpose, we exploit further properties of the optimal policy, particularly the ones described in **Proposition 2**. The main advantage of this approach is that it does account for the impact of large price differences (e.g. when observing a low spot-market price realization compared to the average price) that makes it profitable to fill the demand of multiple periods with one spot market order. The presence of such forward buying behavior affects the usage and, thereby, the potential advantage of the reserved long-term capacity.

Since all policy parameters interact with one another, we propose an iterative parameter determination where each iteration consists of two consecutive steps. We start with an initial long-term capacity reservation quantity, $R$. In the first iteration step, we determine all spot market order-up-to levels, $S_S(p)$, and the long-term procurement order-up-to level, $S_L$, for fixed level of $R$=0. Here we account for the potential gain of forward buying in case of low spot-market prices as well as for the single-period protection against the risk of large demand in case of high spot prices. In the second iteration step, we update the long-term capacity reservation quantity, $R$, in order to incorporate the effects of forward buying. The iterative procedure terminates if there is no (substantial) change in $R$. In the next subsections we describe the parameter determination in detail. Although we could not formally prove the convergence of the algorithm, in all instances of our numerical performance study only a few iterations were required.

### 3.2 Determination of order-up-to levels $S_L$ and $S_S(p)$

From **Proposition 2** we know that the optimal long-term order-up-to level, $S_L$, is equal to the short-term spot-market order-up-to level, $S_S(p)$, for $p=c$. Thus, $S_L$ is not treated explicitly, but can be derived from the respective short-term order-up-to level $S_S$(c). For determining the whole set of order-up-to levels, $S_S(p)$, two stock-keeping motives are considered: the price differences yielding a forward buying speculation type of stock and the demand uncertainty requiring a safety stock. These motives interact with each other in a rather complex way, e.g. when carrying a large speculation stock, the demand risk is less severe during the current replenishment cycle. On the other hand, when a high safety stock is required the correspondent order can behave like a forward buy for a number of periods. For simplifying our heuristic, we separate two spot-market price regions. In one region the speculation motive, in the other one the safety motive is dominant. This allows us to deal with each motive separately and to determine the corresponding order-up-to level candidate first. Afterwards, a simple rule is used to decide for each price, $p$, which candidate to select.

#### *3.2.1 Consideration of the price speculation motive*

For a small spot price, $p$, *price speculation* is the dominating motive for stock-keeping. We determine the corresponding forward buy (FB) order-up-to levels $S_S^{FB}(p)$ based upon an average forward buying period, $m(p)$, i.e. the number of future periods for which demand should be satisfied from a spot market order placed in the current period. For each time span $n \in \{1,...,n^+\}$, where depending on the maximum price level an upper limit $n^+$ is established, we first estimate the probability $\pi(n)$ that it is beneficial to satisfy the demand of each period up to period $t + n$ from procuring in the current period $t$. Such forward buying is only beneficial if during each one of the next $n$ periods the spot-market price increases at a larger rate than the incurred holding cost, yielding

$$\pi(n) = \prod_{i=1}^{n} \left(1 - G(p + ih)\right). \tag{7}$$

If in any period, $t + n$, the long-term procurement cost, $c$, would be exceeded by the corresponding spot-market price $p + nh$, the spot purchase is not relevant and the probability, $\pi(n)$, is reduced in order to account for the long-term source with capacity R. This results an adjusted probability $\pi_R(n)$ for $n=1,\ldots,n^+$

$$\pi_R(n) = \begin{cases} \pi(n) & \text{for } p + nh \le c \\ \left(1-\alpha(R)\right)\pi(n) & \text{otherwise} \end{cases}, \tag{8}$$

where the weight $\alpha(R) = \min\left(R / \mu_X, 1\right)$ in a simple way approximates the fraction of demand that on average can be filled under contract price $c$ under consideration of capacity limitation. Note that by definition $\pi_R(n) = 0$ for $n=n^+ +1$.

Finally, we determine the expected forward buying period, $m(p)$, by calculating the mean $m(p) = \sum_{n=1}^{n^+} n \cdot \left[\pi_R(n) - \pi_R(n+1)\right]$. Thus *m(p)* can be expressed by

$$m(p) = \sum_{n=1}^{n^+} \pi_R(n) . \tag{9}$$

The resulting order-up-to level is calculated as average forward buying period plus 1 (for the current period) multiplied by the average demand

$$S_S^{FB}(p) = \left(m(p)+1\right) \cdot \mu_X . \tag{10}$$

### *3.2.2 Consideration of the single-period demand uncertainty*

For high values of the stock-market price, $p$, the stock-keeping is dominated by *single-period demand uncertainty* covered by a safety stock rather than by forward buying. We apply a single-period newsvendor approach to obtain the corresponding safety (SF) order-up-to levels, $S_S^{SF}(p)$. However, for valuing the end-of-period excess inventory/shortage the price difference between the contract price and next period's expected sourcing cost is considered that can be either positive or negative.

Estimating the next period's expected sourcing cost, $\bar{q}(p,R)$, we must account for the two sources with random and fixed price, respectively and the limited capacity, $R$, of the long-

term contract. Also, we need to consider the interaction of the current and next period's price. We distinguish between two cases whether the current spot-market price, *p,* is smaller or equal than long-term procurement cost, *c*, or larger. As a simplifying assumption we presume that long-term reservation capacity is not larger than expected demand what is reasonable under a long-term perspective and is regularly experienced in our numerical study.

- For $p \le c$ in the current period, the order-up-to level, $S_S(p)$, exceeds $S_L$ (**Proposition 2**), and it is unlikely that the undershoot of $S_L$ at the end of the period will be that high that the capacity restriction will always be binding. In that case we must count with the possibility of using both procurement sources in the next period. If next period's spot-market price $\tilde{p}$ is smaller than *c*, only the spot price is relevant. If $\tilde{p} > \mathrm{c}$ in the next period, the long term supplier is preferred but we need to consider the expensive spot market purchase if the limited long-term procurement capacity, *R,* is exceeded. So in this case the expected sourcing cost is the weighted average of the long-term price and the spot-market price. Like in (8), we use the weight $\alpha(R) = \min\left(R / \mu_X, 1\right)$ that approximates the fraction of demand that in the average can be filled using the contract price, *c*. Thus, the next period's expected sourcing cost $\bar{q}(p,R)$ is approximated as follows

$$\begin{aligned}\bar{q}(p,R) &\approx E_{\tilde{p}}\left\{\min\left(\tilde{p}, \alpha(R)c + (1-\alpha(R))\tilde{p}\right)\right\} \\ &= \int_0^c \tilde{p} \cdot g(\tilde{p})d\tilde{p} + \int_c^\infty \left(\alpha \cdot c + (1-\alpha)\tilde{p}\right) \cdot g(\tilde{p})d\tilde{p} \qquad \text{for} \quad p \le c. \end{aligned} \tag{11}$$

For extreme values of *R*, the approximation yields exact results: $\bar{q}(p,0) = \mu_p$ and $\lim_{R\to\infty} \bar{q}(p,R) = E_{\tilde{p}}\left\{\min\left(\tilde{p}, c\right)\right\}$.

- For $p > c$ in the current period, the order-up-to level, $S_S(p)$, is smaller than $S_L$. In this case the long-term capacity *R* is assumed to be binding for reaching the order-up-to level, $S_L$, in the next period in case of a high spot-market price. Thus, the marginal overage/underage is assumed to be compensated only by using the spot market. The next period's expected sourcing cost in this case is approximated as average spot market price

$$\bar{q}(p,R) = \mu_p \quad \text{for} \quad p > c\,. \tag{12}$$

Appropriate underage and overage cost are then given as follows

$$c_u = v - (p - \bar{q}(p,R)) \text{ and } c_o = h + (p - \bar{q}(p,R)), \tag{13}$$

and the critical ratio becomes

$$cr_S^{SF}(p,\bar{q}(p,R)) = \frac{v - p + \bar{q}(p,R)}{h+v} \quad . \tag{14}$$

The critical ratio $cr_S^{SF}$ can approach and even surpass 1 for sufficiently small values of $p$. This might yield an unreasonably large order-up-to level and actually results in forward buying which, however, is not punished by multi-period holding costs. For that reason the order-up-to level for single-period risk protection is limited by an upper bound $S_{\max}$. Thus, the short-term order-up-to level $S_S$ depends on $R$ and is determined as follows

$$S_S^{SF}(\bar{q}(p,R)) = \begin{cases} -\infty & \text{for } cr_S^{SF}(\bar{q}(p,R)) < 0 \\ \min\left(F^{-1}\left(cr_S^{SF}(\bar{q}(p,R))\right), S_{\max}\right) & \text{for } 0 \le cr_S^{SF}(\bar{q}(p,R)) < 1 \\ S_{\max} & \text{otherwise} \end{cases} \tag{15}$$

For reasonable determination of the upper bound, we suggest to select $S_{\max}$ by comparing the single-period order-up-to level with another one that is determined by a newsvendor approach under consideration of a two-period demand risk (reflecting a one-period forward buy). Similar considerations as above yield a critical ratio $cr_S^{2\,periods}(p,\bar{q}(p,R)) = \dfrac{v - p + \bar{q}(p,R)}{2h+v}$ which is smaller than $cr_S^{SF}(p,\bar{q}(p,R))$. It can easily be found that (in case of an unbounded demand distribution like the normal or gamma distribution) there is an intersection point of single-period and two-period order-up-to levels as the single-period critical ratio approaches 1. We approximate the intersection point by inserting the spot-market price $p$ at which $cr_S^{SF}(p,\bar{q}(p,R))$ is equal to one into $cr_S^{2\,periods}(p,\bar{q}(p,R))$ yielding an order-up-to level

$$S_{\max} = F_2^{-1}\left(\frac{h+v}{2h+v}\right). \tag{16}$$

where $F_2^{-1}$ denotes the inverse of the two period cumulative demand distribution function.

### *3.2.3 Integration of stock-keeping motives*

Finally, the calculated critical ratio $cr_S^{SF}(p,\bar{q}(p,R))$ is used to decide which one of the order-up-to levels, $S_S^{FB}(p)$ or $S_S^{SF}(p)$, to select. In the case of a critical ratio that equals or exceeds 1, the price speculation motive is considered to be dominant, otherwise the demand uncertainty motive is taken as prevalent. Thus, the spot market order-up-to levels $S_S(p)$ are determined as follows

$$S_S(p) = \begin{cases} -\infty & \text{for } cr_S^{SF}(p,\bar{q}(R)) < 0 \\ S_S^{SF}(p) & \text{for } 0 \le cr_S^{SF}(p,\bar{q}(R)) < 1 \\ S_S^{FB}(p) & \text{otherwise} \end{cases}. \tag{17}$$

Note that $S_S(p)$ depends on the capacity reservation level $R$ which is fixed as input in the respective step of iteration.

## 3.3 Determination of the Capacity Reservation Level *R*

For obtaining the long-term capacity reservation level, we use a modified version of the simple base stock policy method proposed in Inderfurth and Kelle (2011). There, in a single period newsvendor approach, the overage cost was given by the reservation unit cost, $r$. The underage cost was set to δ which denotes the conditional expected gain of having the fixed price, c, in case of higher spot price (p > c), i.e.

$$\delta = E[\pi - c \mid \pi > c] = \int_c^{\infty} (p-c)g(p)dp. \tag{18}$$

In this approach, procurement activities take place in every period, as identical order-up-to levels for both procurement options are presumed.

In our case, however, the forward buying activities reduce the need to order in every period, which particularly reduces the probability to procure at a high spot-market price. This issue is accounted for by multiplying the conditional expected gain of sourcing from the long-term supplier, δ, by a compensation factor $1/(1+\bar{m})$ that approximates the fraction of periods in which the long-term supplier is used under consideration of forward buying. In the

compensation factor, $\bar{m}$ stands for the length of the average forward buying period that is approximated by considering the actual spot market order-up-to levels, $S_S(p)$, as follows

$$\bar{m} = \int_0^\infty \max\left\{0, \frac{S_S(p)}{\mu_X} - 1\right\} \cdot g(p)dp \,. \tag{19}$$

Thus, the long-term capacity reservation quantity, $R$, is given by

$$R = F^{-1}\left(1 - \frac{r}{\frac{1}{1+\bar{m}}\delta}\right). \tag{20}$$

$\bar{m}$ in (19) is a function of all $S_S(p)$ values at a specific iteration step. An incorporation of $\bar{m}$ into the $R$ calculation results in a possible adjustment of the $R$ value which is used as an input in the next iteration.

## 4. Numerical Optimization and Performance of the Heuristic

In this Section we describe the numerical optimization procedure, the experimental design, and check the performance of the heuristic.

The numerical optimization method is based on the value iteration of stochastic dynamic programming with discretized state space and linear approximation of the value function for extremely high or low net inventory levels. Demand and price distributions are discretized in the $\mu \pm 3\sigma$ interval. The level of expected demand and price is scaled such a way that a numerical optimization takes a reasonable time. It is likely that the results will not differ much if the demand data are scaled up or prices are discretized in more detail. Demand and price values are chosen to be integers. The enumeration procedure is exploiting the policy structure provided in **Proposition 2** as well as the convexity property regarding long term reservation capacity given in **Proposition 3**. The iterative optimization procedure can be sketched as follows. For a given capacity reservation level $R$, we solve a single-period problem and determine the corresponding order-up-to levels. In subsequent iterations, the number of time periods is increased by one until both the order-up-to levels no longer change and the difference in average cost per period between the current and the previous iteration falls below a given small number. Finally, the procedure is repeated for a capacity reservation

level which is reduced or increased by one until there is no further improvement to the objective. For computational efficiency we implemented the optimization procedure using the C programming language.

We executed a full factorial design with 3 levels for all relevant parameters except of average demand and long-term procurement cost that are fixed. So we were considering $3^6 = 729$ instances. The random demand and spot price values were drawn from gamma distributions. Cost and price parameters are chosen in such a way that

- the long-term contract is less costly than the spot market option, on the average,
- the spot price is lower than the contract purchase price in a considerable number of periods
- the price variability is sufficiently high and holding cost is sufficiently low that forward buying will occur quite often, and
- the shortage cost plays such a role that safety stock are needed, especially in periods without forward buying.

In that way we tried to capture all relevant scenarios that are of interest for testing our heuristic. The detailed parameter selection is included in Table 1.

**Table 1.** The selected parameters for the experimental design.

| **Parameters** | | **Levels** | | |
|---|---|---|---|---|
| | | low | mid | high |
| Contract price | $c =$ | 8 (fixed) | | |
| Reservation price | $r =$ | 0.5 | 1.0 | 2.0 |
| Holding cost factor | $h =$ | 0.5 | 1.0 | 2.0 |
| Shortage cost factor | $v =$ | 2 | 4 | 8 |
| Expected demand/period | $\mu_x =$ | 10 (fixed) | | |
| Demand standard deviation | $\sigma_x =$ | 1 | 2 | 4 |
| Expected spot price | $\mu_p =$ | 10 | 12 | 14 |
| Spot price standard deviation | $\sigma_p =$ | 1 | 2 | 4 |

For each instance we determine the relative expected cost deviation $\Delta C_{rel}$ of our heuristic approach from the optimal solution. Figure 2 provides a box plot diagram summarizing the general behavior of the heuristic. The first column shows the percent cost increase if the heuristic decision values are used instead of the optimal ones. The box plot specifies (in increasing order) the minimum, first quartile, median, third quartile and maximum of the percent cost increase. The average percent increase is in parentheses below the description of the graph (the numerical values of the graphs for Figure 2, and also for the subsequent graphs, are included in Appendix B). For the 729 instances, the average cost penalty of using the heuristic is about 1%. The worst case cost increase is 7% what is considered to be still acceptable for practical applications.

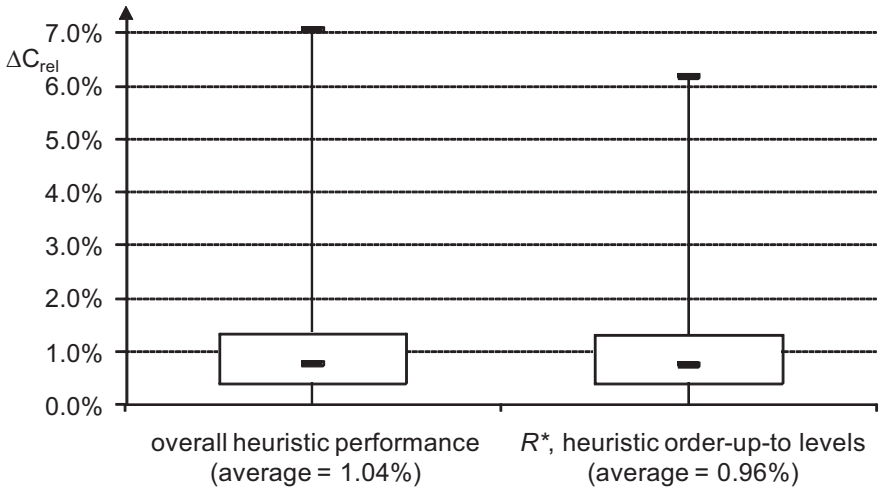


**Figure 2.** Overall performance statistics for the heuristic and influence of the choice of the capacity reservation level $R$

In the heuristic iteration procedure, a critical factor is the selection of the capacity reservation quantity, $R$. In the second graph of Figure 2, we show the effect if instead of updating $R$ by the two-step heuristic (described in Section 3.1), the optimal $R^*$ is used to calculate the order-up-levels, $S_L$, and $S_S(p)$ applying only the second step of the heuristic. There is only a minor improvement that demonstrates the effectiveness of the heuristic iteration procedure for determining $R$.

Next we examine the difference between the optimal values for the decision variables (which were considered as integer numbers) and the values resulted by the heuristic. The first column of Figure 3 illustrates the statistics of the difference for the reservation quantities, $R$. The heuristic $R$ has a slight tendency of overestimating the optimal $R^*$. However the detailed comparisons show that the heuristic $R$ is correct in 40.1% of all instances, the difference is not larger than one unit in 78.2% of all instances and not larger than two units in 89.8% of all instances. The second column of Figure 3 shows the statistics of the errors for long-term order-up level, $S_L$. The heuristic $S_L$ has a slight tendency of underestimating the optimal $S_L^*$. Here the detailed comparisons show that the heuristic $S_L$ is correct in 31.4% of all instances; there is maximum difference of one unit in 65.4% of all instances and of two units in 80.7% of all instances.

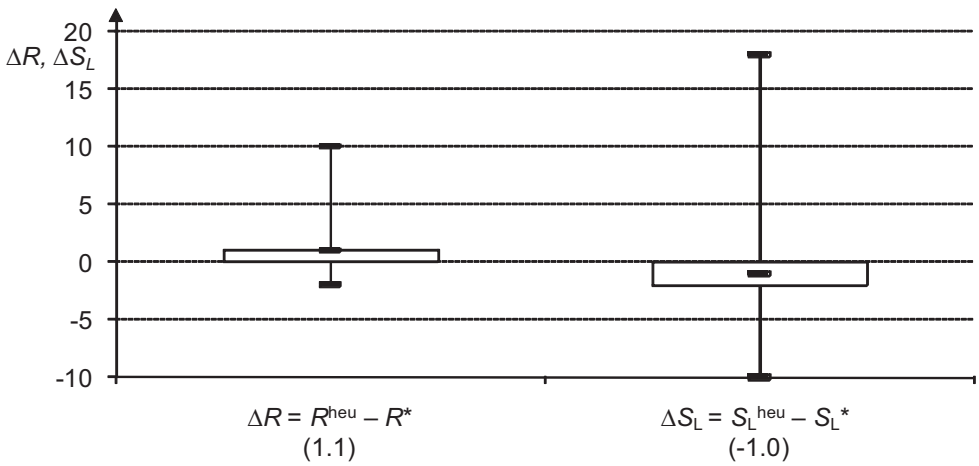


**Figure 3.** Errors of heuristic selection of policy parameters $R$ and $S_L$

The statistical comparison of the price dependent short-term order-up-levels is more challenging. However, in most cases the separate consideration of price speculation and demand uncertainty seems to perform well, as it is illustrated in Figure 4 for the instance characterized by the mid-values for all parameters in Table 1. For this instance the optimal and heuristic capacity reservation levels are the same ($R^* = R^{heu} = 11$) and the relative cost deviation is 0.3%.

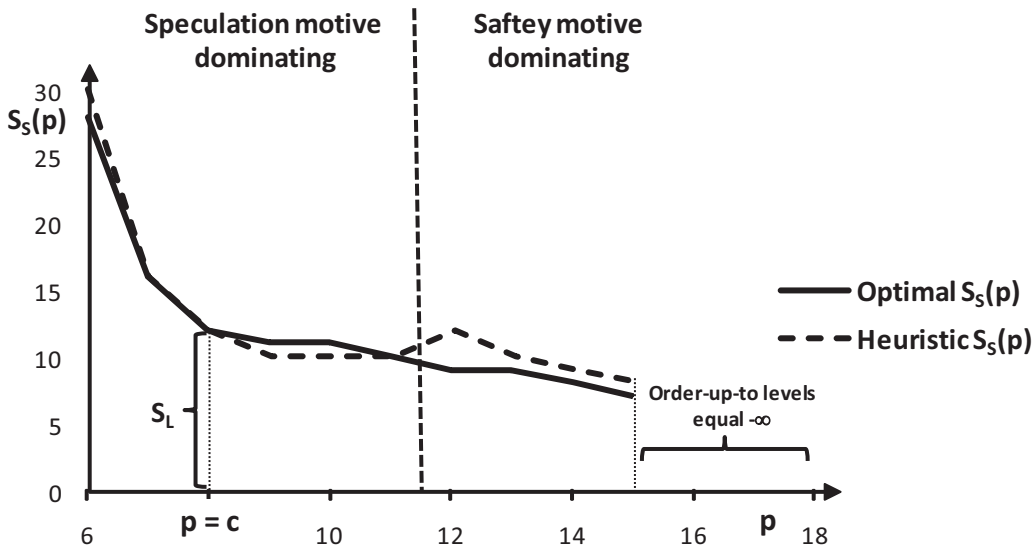


**Figure 4.** Comparison of optimal and heuristic order-up-to levels for the instance of taking the mid-values for all parameters in Table 1.

Generally, the heuristic is successful in approximating the spot market order-up-to level, $S_S(p)$, for the most price levels, p. In the two price ranges, where the speculative or the safety motive dominates, the respective approximations work very well. Major deviations are only observed at the transition between the two price ranges as it is illustrated in Figure 4.

Next, we review the impact of different parameter settings on the behavior of the heuristic procedure (the underlying statistical data is summarized in Appendix B). As mentioned above, the overall average cost deviation of the heuristic is near to 1%. The error statistics of the heuristic gets slightly worse with the increase of the capacity reservation price, r. The effect of the holding and shortage cost (h and v), and the average spot market price, $\mu_p$, are similar. An increase in demand variability results in a slight average performance loss, while rising price volatility has no distinct impact on the performance (see Figure 5). Concerning worst case behavior, it is obvious that a serious deviation of up to 7% only occurs when price variability is considerably high (with a coefficient of variation of 40%), This might be due to the fact that an increasing extent of forward buying in this case makes it more difficult for the heuristic to fix the capacity reservation level *R* in a satisfactory manner. For lower price variability, however, even the worst case cost deviation will not exceed a 4% level. Thus, we

can conclude that our heuristic parameter determination approach turns out to work very well.

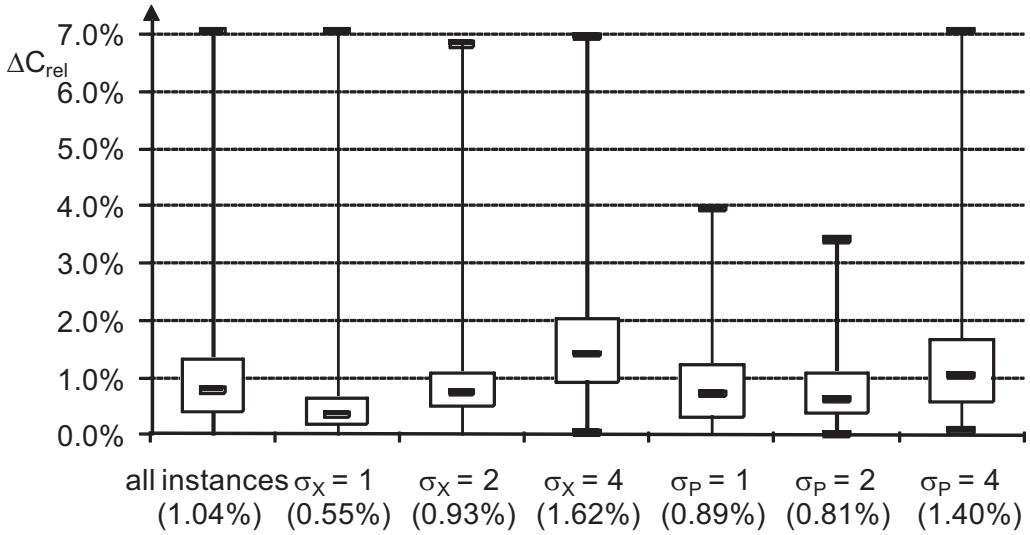


**Figure 5.** Influence of demand and price uncertainty on the performance of the heuristic.

## 5. Summary and Extensions

We discussed the important and challenging dual sourcing problem of using capacity reservation contract and spot market jointly. A long-term decision has to be made regarding the reserved capacity level, and then it has to be decided - period by period - which quantities to procure from the two sources. For the long-term supplier, a reservation cost, proportional with the reservation level, has to be paid for the option of receiving any amount per period up to the reservation level. The contract price is fixed; the spot market has random price but no capacity restriction.

For the above problem we derived the optimal policy structure and provided a numerical procedure to find the optimal parameters including a parameter function. In the decision on long-term capacity reservation we took into account the short-term capacity utilization of each source which itself depends on the available long-term capacity. We considered this highly complex interdependence of long-term and short-term decisions under uncertainties in demand and spot market price and our multi-period approach allows for integrating capacity reservation, forward buying and safety stock holding aspects in a single model.

The numerical method is based on the value iteration of stochastic dynamic programming with discretized state space. Besides the analytic results, we developed a simple heuristic procedure for practical application to approximate the optimal policy parameters. We presented a comprehensive numerical study showing that using these heuristic policy parameters we get very close to the optimum.

The next steps may include a detailed managerial analysis of the dual sourcing policy and comparisons with the options when only a single sourcing is used. The exact optimal policy can also be compared to the simplified policy that has only one static order-up-to level instead of the price-dependent spot market order-up-level function.

The current sourcing problem can be extended in many directions. So it would be interesting to analyze if a simple policy structure still is optimal when additional procurement options like fixed commitment contracts or forward contracts are incorporated. Further extensions can also include more sophisticated spot price models from the finance area. Finally, the issue of long-term contract negotiation regarding the cost and capacity parameters can be considered in the context of contract analysis and design for supply chain coordination among the long-term supplier and the buyer.

## Appendix A

***Proof of Propositions 1-3:***

The starting point is given by the dynamic programming recursive relations for $t = 1,...,T$ :

$$D_t(I,R,p) = \min_{R \geq Q_L \geq 0, Q_S \geq 0} \left\{ cQ_L + pQ_S + L(I + Q_L + Q_S) + \beta \cdot \int_0^\infty C_{t+1}(I + Q_L + Q_S - x, R) f(x) dx \right\}$$

with $C_t(I,R) = \int_0^\infty D_t(I,R,p) g(p) dp$ and $C_{T+1}(I,R) \equiv 0$.

The **major steps of the proof** include

- proving the optimality of the ($S_L$,$S_S(p)$) policy by complete induction,
- proving that this policy holds for any $t$ if $C_{t+1}(I,R)$ is convex,
- proving that $D_t(I,R,p)$ is convex if this policy is applied,

- proving that this holds for the final period $t=T$,
- proving that $C_1(\bar{I}, R)$ is a convex function.

The optimization problem in period $t$ can be reformulated as

$$D_t(I,R,p) = \min_{R \geq Q_L \geq 0, Q_S \geq 0} \{cQ_L + pQ_S + H_t(I + Q_L + Q_S, R)\}$$

with $H_t(I + Q_L + Q_S, R) \equiv L(I + Q_L + Q_S) + \beta \cdot \int_0^\infty C_{t+1}(I + Q_L + Q_S - x, R) f(x) dx$

By assumption $C_{t+1}(I,R)$ is convex in $I$ and $R$, *thus* $H_t(I,R)$ is also convex in $I$ and $R$ due to well-known convexity of $L(I)$. So, under the assumption that a $(S_L, S_S(p))$ policy holds, we can analyze the properties of minimum cost functions $D_t(I,R,p)$ and $C_t(I,R)$

**(i)** in case of $p \leq c$:

$$D_t(I,R,p) = \begin{cases} p \cdot (S_S(p) - I) + H_t(S_S(p), R) & \text{if} \quad I \leq S_S(p) \\ H_t(I,R) & \text{if} \quad I \geq S_S(p) \end{cases}$$

**(ii)** in case of $p \geq c$:

$$D_t(I,R,p) = \begin{cases} c \cdot R + p \cdot (S_S(p) - I - R) + H_t(S_S(p), R) & \text{if} \quad I \leq S_S(p) - R \\ c \cdot R + H_t(I + R, R) & \text{if} \quad S_S(p) - R \leq I \leq S_L - R \\ c \cdot (S_L - I) + H_t(S_L, R) & \text{if} \quad S_L - R \leq I \leq S_L \\ H_t(I,R) & \text{if} \quad I \geq S_L \end{cases}$$

We can easily show that $D_t(I,R,p)$ is twice continuously differentiable in $I$ and $R$. Due to convexity of $H_t(I,R)$ we have:

$$\frac{\partial^2}{\partial I^2} H_t(I,R) \geq 0, \quad \frac{\partial^2}{\partial R^2} H_t(I,R) \geq 0 \quad \text{and} \quad \frac{\partial^2}{\partial I^2} H_t(I,R) \cdot \frac{\partial^2}{\partial R^2} H_t(I,R) - \frac{\partial^2}{\partial I \partial R} H_t(I,R) \cdot \frac{\partial^2}{\partial R \partial I} H_t(I,R) \geq 0$$

So the Hessian of $D_t(I,R,p)$ is nonnegative definite for each $p$, $D_t(I,R,p)$ is convex in $I$ and $R$ for each $p$, and $C_t(I,R) = \int_0^\infty D_t(I,R,p) g(p) dp$ is convex in $I$ and $R$ due to $g(p) \geq 0$.

**Steps of induction:**

**For each *t* < *T*** the following holds: From convexity of $C_{t+1}(I,R)$ it follows that also $C_t(I,R)$ is convex in *I* and *R, so* $H_{t-1}(I,R)$ is also convex in *I* and *R* and consequently for each *R* a ($S_L$,$S_S(p)$) policy is optimal also for *t*-1.

**For *t*=*T* (start of induction)** we have: $H_T(I,R) = L(I)$ independent of *R, thus* $H_T(I,R)$ is convex in *I* and a ($S_L$,$S_S(p)$) policy is optimal for *t*=*T*.

**General Conclusions**

***Policy Structure***: For each *R* a ($S_L$,$S_S(p)$) policy is optimal for each $1 \le t \le T$.

- Policy parameter $S_{L,t}$ is calculated from: $\frac{\delta H_t(S,R)}{\delta S} + c = 0$ for each *R*.
- Policy parameter $S_{S,t}(p)$ is calculated from: $\frac{\delta H_t(S,R)}{\delta S} + p = 0$ for each *R* and *p*.
- Policy parameter *R* is calculated from: $\frac{\delta C_1(\bar{I},R)}{\delta R} = 0$ for a given initial inventory $\bar{I}$.
- Functions $C_1(\bar{I},R)$ and $H_t(I,R)$ are convex.

From unconstrained optimization we get as optimal inventory levels

- after $Q_L$-optimization : $S_{L,t}(R)$ from: $\frac{\delta H_t(S,R)}{\delta S} + c = 0$
- after $Q_S$-optimization : $S_{S,t}(p_t,R)$ from: $\frac{\delta H_t(S,R)}{\delta S} + p_t = 0$.

Due to $\frac{\delta}{\delta Q_L} H_t(I+Q_L+Q_S,R) = \frac{\delta}{\delta Q_S} H_t(I+Q_L+Q_S,R)$, and due to restrictions $0 \le Q_L \le R$ and $0 \le Q_S$ we get the policy structure described in *Proposition 1* as well as the convexity property stated in *Proposition 3*.

From convexity of $H_t(S,R)$ and respective optimality conditions for the order-up-to levels it immediately follows that

$$S_{S,t}(p_t) \begin{cases} > S_{L,t} & \text{if } p_t < c \\ = S_{L,t} & \text{if } p_t = c \\ < S_{L,t} & \text{if } p_t > c \end{cases}$$

This is just the relationship described in *Proposition 2*.

***Proof of Proposition 4:***

For the infinite horizon problem the functional equations of dynamic programming have to fulfil

$$C(I,R) = \int_0^\infty D(I,R,p)g(p)dp \text{ and}$$

$$D(I,R,p) = \min_{R \geq Q_L \geq 0, Q_S \geq 0} \left\{ cQ_L + pQ_S + L(I + Q_L + Q_S) + \beta \cdot \int_0^\infty C(I + Q_L + Q_S - x, R) f(x)dx \right\}$$

Due to the stationary environment and the infinite horizon the decision problem for ordering is the same in each period. Now, Theorem 8-14 of Heyman and Sobel (1984) can be used to prove that the above functional relationship is satisfied by

$C(\bar{I},R) = \lim_{T \to \infty} C_1(\bar{I},R)$ and $D(\bar{I},R,p) = \lim_{T \to \infty} \mathrm{D}_1(\bar{I},R,p)$ where $C_1(\bar{I},R)$ and $D_1(\bar{I},R,p)$ are defined as in Section 2. In the very same way as it is done in Serel (2007) for the three-parameter policy in case of spot market capacity uncertainty, it can be shown that the conditions a to d of Theorem 8-14 hold in our case because the single-period costs and optimal order levels are bounded.

It follows that all convexity properties of the respective cost functions also hold in the infinite horizon case. The order-up-to levels $S_{L,1}$ *and* $S_{S,1}(p)$ converge to the stationary ones $S_L$ and $S_S(p)$ and can be calculated using the stationary cost function $H(I,R)$ and the optimality conditions from Section 2. The optimal capacity reservation level $R$ is calculated from minimizing $C_1(\bar{I},R)$ with respect to $R$.

**Appendix B**

**Table 2.** Box plot data and influence of other parameters

| | Min | Q1 | Median | Q3 | Maximum | Mean |
|---|---|---|---|---|---|---|
| **Figure 2** | | | | | | |
| overall heuristic performance | 0.00% | 0.41% | 0.77% | 1.33% | 7.06% | 1.04% |
| optimal *R*, heuristic order-up-to-levels | 0.00% | 0.39% | 0.73% | 1.31% | 6.18% | 0.96% |
| **Figure 3** | | | | | | |
| $\Delta R$ | -2 | 0 | 1 | 1 | 10 | 1.07 |
| $\Delta S_L$ | -10 | -2 | -1 | 0 | 18 | -1.01 |
| **Figure 5** | | | | | | |
| $\sigma_X = 1$ | 0.00% | 0.18% | 0.35% | 0.63% | 7.06% | 0.55% |
| $\sigma_X = 2$ | 0.00% | 0.49% | 0.75% | 1.10% | 6.82% | 0.93% |
| $\sigma_X = 4$ | 0.04% | 0.93% | 1.41% | 2.03% | 6.95% | 1.62% |
| $\sigma_P = 1$ | 0.00% | 0.31% | 0.71% | 1.21% | 3.95% | 0.89% |
| $\sigma_P = 2$ | 0.01% | 0.38% | 0.62% | 1.07% | 3.41% | 0.81% |
| $\sigma_P = 4$ | 0.07% | 0.57% | 1.03% | 1.69% | 7.06% | 1.40% |
| **Influence of other parameters** | | | | | | |
| r = 0.5 | 0.02% | 0.27% | 0.52% | 0.85% | 3.04% | 0.63% |
| r = 1.0 | 0.00% | 0.41% | 0.79% | 1.37% | 7.06% | 1.09% |
| r = 2.0 | 0.00% | 0.64% | 1.17% | 1.73% | 6.31% | 1.39% |
| h = 0.5 | 0.06% | 0.49% | 0.98% | 1.74% | 7.06% | 1.39% |
| h = 1.0 | 0.05% | 0.36% | 0.67% | 1.15% | 4.42% | 0.83% |
| h = 2.0 | 0.00% | 0.34% | 0.71% | 1.22% | 3.95% | 0.88% |
| v = 2 | 0.00% | 0.44% | 0.79% | 1.45% | 7.06% | 1.15% |
| v = 4 | 0.00% | 0.40% | 0.74% | 1.24% | 5.14% | 0.95% |
| v = 8 | 0.00% | 0.38% | 0.77% | 1.28% | 5.27% | 1.00% |
| $\mu_P = 10$ | 0.00% | 0.30% | 0.59% | 1.18% | 7.06% | 0.99% |
| $\mu_P = 12$ | 0.06% | 0.45% | 0.74% | 1.24% | 6.31% | 0.97% |
| $\mu_P = 14$ | 0.07% | 0.56% | 0.95% | 1.58% | 3.95% | 1.14% |

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