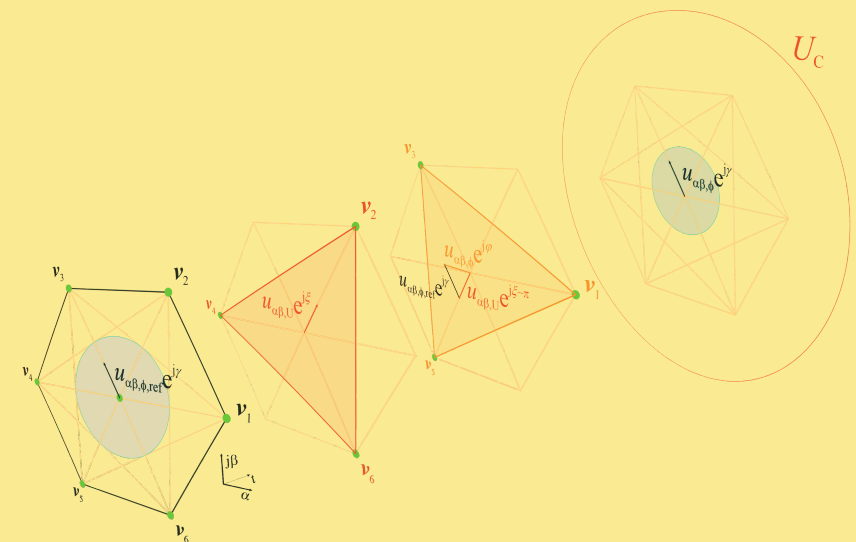


The increasing demand for clean and reliable electrical energy has intensified research into distributed generation systems that can efficiently integrate renewable sources. However, the extensive penetration of converter-based generation introduces challenges in maintaining voltage and frequency stability, particularly in low-inertia and islanded microgrids. This thesis proposes a family of impedance-source converters with reduced component count, based on the split-source inverter, along with associated control strategies for the coordinated integration of multiple low-power renewable sources, aimed at applications in microgrids and nanogrids. A detailed continuous- and discrete-time state-space analysis forms the basis for developing a model predictive control approach for switching control. Furthermore, a space vector analysis at a fixed switching frequency is employed to establish the operational limits of the converter and to guide its design and performance evaluation. Building upon the grid-supporting control concept, this thesis proposes a novel strategy that utilizes the energy stored in the DC bus capacitors to provide a fast frequency response, enabling coordination with larger storage systems. The proposed control framework and converter topologies are validated through a combination of software-in-the-loop simulations, hardware-in-the-loop experiments, and laboratory prototypes. The results demonstrate effective collective power injection from multiple independent sources and confirm the grid-supporting potential of inverter-based distributed resources, even when connected to larger power systems.

Gabriel Maier Cocco: Grid-Supporting Control of Multi-Source Power Converters

Gabriel Maier Cocco

Contributions to Distributed Generation: Grid-Supporting Control of Multi-Source Grid-Tied Power Converters



Contributions to Distributed Generation: Grid-Supporting Control of Multi-Source Grid-Tied Power Converters

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Gabriel Maier Cocco

**CONTRIBUTIONS TO DISTRIBUTED GENERATION:
GRID-SUPPORTING CONTROL OF MULTI-SOURCE GRID-TIED
POWER CONVERTERS**

Magdeburg, Germany
2025

Gabriel Maier Cocco

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Thesis submitted to the Graduate Program in Electrical Engineering, with concentration area in Energy Processing, at the Federal University of Santa Maria (UFSM, RS), under joint supervision with the Faculty of Electrical Engineering and Information Technology at Otto von Guericke University Magdeburg (OVGU), as a requirement for obtaining the degree of **Doctor in Electrical Engineering**.

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ABSTRACT

CONTRIBUTIONS TO DISTRIBUTED GENERATION: GRID-SUPPORTING CONTROL OF MULTI-SOURCE GRID-TIED POWER CONVERTERS

AUTHOR: Gabriel Maier Cocco
SUPERVISOR: Humberto Pinheiro
SUPERVISOR: Martin Wolter

The increasing demand for clean and reliable electrical energy has intensified research into distributed generation systems that can efficiently integrate renewable sources. As the use of nonrenewable energy becomes progressively constrained by environmental and policy factors, renewable technologies such as solar, wind, biofuel, hydropower, and green hydrogen are emerging as sustainable alternatives for modern power systems. However, the extensive penetration of converter-based generation introduces challenges in maintaining voltage and frequency stability, particularly in low-inertia and islanded microgrids. Among the converter topologies, DC–DC boost converters followed by DC-AC inverters are often found to interface renewable energy sources with the grid. Despite numerous advancements in multi-input converter topologies, achieving both flexible power integration and robust grid support remains a significant challenge. In particular, converters must provide dynamic services such as frequency and voltage regulation while ensuring high efficiency and reliability. This thesis proposes a family of impedance-source converters based on the split-source inverter, along with associated control strategies for the coordinated integration of multiple low-power renewable sources, aimed at applications in microgrids and nanogrids. The work emphasizes converter designs that enhance reliability, power quality, and resilience, while also contributing to grid-support functions. A detailed continuous- and discrete-time state-space analysis forms the basis for developing a Model Predictive Control approach for switching control. Furthermore, a space vector analysis at fixed switching frequency is employed to establish the operational limits of the converter and to guide its design and performance evaluation. Building upon the grid-supporting control concept, this thesis proposes a novel strategy that utilizes the energy stored in the DC bus capacitors to deliver a fast frequency response, enabling coordination with larger storage systems. The proposed control framework and converter topologies are validated through a combination of software-in-the-loop simulations, hardware-in-the-loop experiments, and laboratory prototypes. The results demonstrate effective collective power injection from multiple independent sources and confirm the grid-supporting potential of inverter-based distributed resources, even when connected to larger power systems.

Keywords: Hybrid Power Generation. Renewable Integration. Split-Source Inverter. Grid-Supporting Control. Predictive Control.

RESUMO

CONTRIBUIÇÕES PARA A GERAÇÃO DISTRIBUÍDA: CONTROLE COM SUPORTE À REDE DE CONVERSORES DE POTÊNCIA COM MÚLTIPLAS FONTES

AUTOR: Gabriel Maier Cocco
ORIENTADOR: Humberto Pinheiro
ORIENTADOR: Martin Wolter

A crescente demanda por energia elétrica limpa e confiável intensificou as pesquisas em sistemas de geração distribuída que possam integrar fontes renováveis de forma eficiente. À medida que o uso de energia não renovável se torna progressivamente limitado por fatores ambientais e políticos, tecnologias renováveis como solar, eólica, biocombustíveis, hidrelétrica e hidrogênio verde estão surgindo como alternativas sustentáveis para os sistemas de energia modernos. No entanto, a ampla penetração de geração baseada em conversores introduz desafios na manutenção da estabilidade de tensão e frequência, particularmente em microrredes com baixa inércia e operando em modo isolado. Dentre as topologias de conversores, conversores DC-DC *boost* seguidos por inversores DC-AC são comumente utilizados para conectar fontes renováveis à rede elétrica. Apesar de numerosos avanços nas topologias de conversores com múltiplas entradas, alcançar tanto uma integração de energia flexível quanto um suporte robusto à rede continua sendo um desafio significativo. Em particular, os conversores devem fornecer serviços dinâmicos como regulação de frequência e tensão, ao mesmo tempo em que garantem alta eficiência e confiabilidade. Esta tese propõe uma família de conversores fonte de impedância baseados no inversor *split-source*, juntamente com estratégias de controle associadas para a integração coordenada de múltiplas fontes renováveis de baixa potência, voltadas para aplicações em microrredes e nanorredes. O trabalho enfatiza projetos de conversores que aumentam a confiabilidade, a qualidade da energia e a resiliência, ao mesmo tempo em que contribuem para funções de suporte à rede. Uma análise em espaço de estados, em tempo contínuo e discreto, serve como base para o desenvolvimento de uma abordagem de Controle Preditivo por Modelo para o controle de comutação. Além disso, uma análise por vetor espacial em frequência de comutação fixa é empregada para estabelecer os limites operacionais do conversor e orientar seu projeto e avaliação de desempenho. Com base no conceito de controle com suporte à rede, esta tese propõe uma nova estratégia que utiliza a energia armazenada nos capacitores do barramento CC para fornecer uma resposta rápida de frequência, permitindo a coordenação com sistemas de armazenamento de maior porte. A estrutura de controle proposta e as topologias de conversores são validadas por meio de uma combinação de simulações com *software-in-the-loop*, experimentos com *hardware-in-the-loop* e protótipos de laboratório. Os resultados demonstram uma injeção coletiva eficaz de potência a partir de múltiplas fontes independentes e confirmam o potencial de suporte à rede dos recursos distribuídos baseados em inversores, mesmo quando conectados a sistemas de energia de grande porte.

Palavras-chave: Geração Híbrida. Integração de Renováveis. Inversor de Fonte Dividida. Controle de Suporte à Rede. Controle Preditivo.

KURZZUSAMMENFASSUNG

BEITRÄGE ZUR DEZENTRALEN ENERGIEERZEUGUNG: NETZDIENLICHE REGELUNG VON NETZGEKOPPELTEN LEISTUNGSUMRICHTERN MIT MEHREREN QUELLEN

AUTOR: Gabriel Maier Cocco
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Die steigende Nachfrage nach sauberer und zuverlässiger elektrischer Energie hat die Forschung im Bereich verteilter Erzeugungssysteme, die erneuerbare Quellen effizient integrieren können, intensiviert. Da der Einsatz konventioneller Energien zunehmend durch Umwelt- und politische Faktoren eingeschränkt wird, treten erneuerbare Technologien wie Solarenergie, Windkraft, Biokraftstoffe, Wasserkraft und grüner Wasserstoff als nachhaltige Alternativen für moderne Energiesysteme in den Vordergrund. Die weite Verbreitung leistungselektronisch gekoppelter Erzeuger bringt jedoch Herausforderungen hinsichtlich der Spannungs- und Frequenzstabilität mit sich, insbesondere in Systemen mit geringer Trägheit und Inselnetzen. Unter den verfügbaren Konvertertopologien werden DC-DC-Boost-Wandler gefolgt von DC-AC-Wechselrichtern häufig eingesetzt, um erneuerbare Energiequellen mit dem Netz zu koppeln. Trotz zahlreicher Fortschritte bei Multi-Input-Wandlerkonzepten stellt die gleichzeitige Erfüllung der Anforderungen an flexible Energieeinspeisung und robuste Netzunterstützung weiterhin eine große Herausforderung dar. Insbesondere müssen die Wandler dynamische Systemdienstleistungen wie Frequenz- und Spannungshaltung bereitstellen, während sie gleichzeitig hohe Effizienz und Zuverlässigkeit gewährleisten. In dieser Arbeit wird eine Familie von Impedanzquellenwandlern auf Basis des Split-Source-Inverters vorgestellt, ergänzt durch geeignete Regelungsstrategien zur koordinierten Integration mehrerer dezentraler Kleinerzeuger mit Fokus auf Anwendungen in Mikro- und Nanonetzen. Der Schwerpunkt liegt auf der Entwicklung von Wandlerarchitekturen, die Zuverlässigkeit, Spannungsqualität und Resilienz verbessern und gleichzeitig netzunterstützende Funktionen ermöglichen. Eine detaillierte Analyse im kontinuierlichen und diskreten Zustandsraum bildet die Grundlage für die Entwicklung eines modellprädiktiven Regelungsverfahrens zur Steuerung der Schaltvorgänge. Darüber hinaus wird eine Raumzeigeranalyse bei fester Schaltfrequenz verwendet, um die Betriebsgrenzen des Wandlers zu bestimmen und Auslegung sowie Leistungsbewertung zu unterstützen. Aufbauend auf dem Konzept der netzunterstützenden Regelung wird eine neuartige Strategie vorgeschlagen, bei der die im Zwischenkreiskondensator gespeicherte Energie für schnelle Frequenzstützung genutzt wird, wodurch koordinierte Zusammenarbeit mit größeren Speichersystemen ermöglicht wird. Das vorgeschlagene Regelungskonzept sowie die Konvertertopologien werden durch eine Kombination aus Software-in-the-Loop-Simulationen, Hardware-in-the-Loop-Experimenten und Laborprototypen validiert. Die Ergebnisse belegen eine effektive kollektive Energieeinspeisung aus mehreren unabhängigen Quellen und bestätigen das Potenzial von wechsellrichterbasierten, verteilten Ressourcen zur Netzunterstützung, selbst Anbindung an größere Energiesysteme.

Schlagwörter: Hybride Erzeugung. Integration erneuerbarer Energien. Split-Source-Wechselrichter. netzunterstützende Regelung. prädiktive Regelung.

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List of Symbols

Bold lowercase symbols denote vectors, while bold uppercase symbols denote matrices. Italic lowercase symbols represent instantaneous (time-varying) values, and italic uppercase symbols represent RMS values. Underlined symbols indicate complex-valued quantities.

Subscript indices in parentheses indicate a set of variables forming a sequence. Running indices are written in italic.

The Hadamard product " $\circ$ " represents an element-wise multiplication.

Notation

$\bar{X}$	Logical NOT	$\langle x \rangle$	Average value
$\bar{x}$	Continuous component	$\tilde{x}$	Alternating component
X	Matrix	$\hat{x}, \hat{X}$	Phasor
x	Vector	$\underline{x}, \underline{X}$	Complex value
$\dot{x}$	Derivative	x, X	Scalar
$\hat{x}$	Estimated value		

Superscript Indices

$*$	Complex conjugate	$\top$	Transposed
$'$	Related value		

Subscript Indices

$\alpha, \beta, 0$	$\alpha\beta 0$ reference frame	m, n, r, x	Integer control variable
Ω	Circular shift operation	ω	Frequency at the point of connection
ϕ	Phase	a, b, c	Natural coordinates
Θ	Rotation operation	A	Activation
ζ	Damping	BESS	Battery energy storage system
j	Switching state index	C	DC-link capacitor
k	Sampling instant		

c	Inverter	neg	Negative
d, q, 0	dq0 synchronous frame	o	Output
dem	Demagnetization	opt	Optimal
ELC	Electronic load controller	pos	Positive
f	Filter	P	Proportional
FFR	Fast frequency response	p	Predictive
G	Cost function	PV	Photovoltaic system
g	Generator or utility grid	ref	Reference value
h	High frequency	R	Recovery
I	Integral	r	Rated
i	Inverter current	set	Fixed set point
IR	Instantaneous reserve	S	Aggregate
lim	Limit	s	Sampling
l	Load	s	Scaled
LPF	Low-pass filter	sw	Switching
LP	Cutoff angular frequency	T	Transformer
mag	Magnetization	t	Time-related
max	Maximum	U	Input voltage
med	Mean value	u	Point of connection voltage
min	Minimum	vr	Virtual
MPP	Maximum power point	v	Vector

Symbols

k	Gains vector	ψ	Tuning factor
V	Switching vectors matrix	σ	Substitution variable
v	Switching vector	τ	Time constant
δ	Vector dwell time	θ	Synchronization angle
γ	Reference voltage angle	Γ	Scaling variable
κ	Adaptive gain	φ	Non-active component an- gle
ω	Angular frequency		

ξ	Active component angle	s	Complex frequency
ζ	Damping factor	t	Continuous time
B	Damping coefficient	U	Voltage
C	Capacitance	u	Voltage
D	Duty cycle	x, y, a, b, c	Auxiliary generic variable
d	Time-varying duty cycle	Y	Admittance
E	Energy	Z	Impedance
e	Error	$\mathbf{\Xi}, \mathbf{\Lambda}, \mathbf{\Phi}$	Auxiliary matrix for transformation
F	Low-pass filter function	$\mathbf{A}$	State matrix
f	Frequency	$\mathbf{B}$	Control input matrix
G	Plant transfer function	$\mathbf{C}$	Output matrix
g	Cost function	$\mathbf{D}, \mathbf{\Psi}$	Auxiliary matrix
H	Inertia constant	$\mathbf{d}$	Dwell times vector
h	Inertia coverage	$\mathbf{F}$	Disturbance input matrix
i	Electrical current	$\mathbf{I}$	Identity matrix
K	Gain	$\mathbf{M}$	Decomposition matrix
k	Factor	$\mathbf{s}$	Switching states vector
L	Inductance	$\mathbf{T}$	Transformation matrix
l	Euclidean distance	$\mathbf{u}$	Input or vector or voltage
N	Ratio	$\mathbf{w}$	Disturbance input vector
P	Active power	$\mathbf{x}$	State vector
Q	Reactive power	$\mathbf{y}$	System output vector
R	Resistance	T	Time period
S	Apparent power		
S	Logic switching state		

Other symbols

π	3.14159...	e	2.71828...
$\mathbf{1}$	All-ones column vector	j	$\sqrt{-1}$

List of Abbreviations

9S-MISSI	nine-switch multi-input split-source inverter
AC	alternating current
AG	aggregate
APS	Announced Pledges Scenario
BESS	battery energy storage systems
CCUS	carbon capture, utilisation and storage
CMV	common-mode voltage
DC	direct current
DC-MISSI	diode clamped multi-input split-source inverter
DCI	diode clamped inverter
DER	distributed energy resources
DSP	digital signal processor
DSTATCOM	distribution static synchronous compensator
ELC	electronic load controller
ESS	energy storage system
E-STATCOM	energy-STATCOM
FC-MISSI	flying capacitor multi-input split-source inverter
FCI	flying capacitor inverter
FCS	finite control set
FESS	flywheel energy storage systems
FFR	fast frequency response
FN	frequency nadir
FPGA	field programmable gate arrays
GFL	grid-following
GFM	grid-forming
HIL	hardware-in-the-loop
HVDC	high-voltage direct current transmission
IBR	inverter-based resources
Inc-Cond	incremental conductance algorithm

IR	instantaneous reserve
LPF	low-pass filter
MPC	Model Predictive Control
MPP	maximum power point
MPPT	maximum power point tracking
MISSI	multi-input split-source inverter
NZE	Net Zero Emissions by 2050 Scenario
PCC	point of common coupling
P&O	perturb and observe
PHES	pumped hydro energy storage
PI	proportional-integral
PLL	phase-locked loop
PoC	point of connection
PR	proportional-resonant
PV	photovoltaic
RES	renewable energy sources
RoCoF	rate of change of frequency
RT	real-time
SCES	supercapacitor energy storage
SCR	short-circuit ratio
SEIG	self-excited induction generator
SMES	superconducting magnetic energy storage
SSI	Split-Source Inverter
STEPS	Stated Policies Scenario
THD	total harmonic distortion
TRD	total rated-current distortion
VSI	voltage source inverter
VSM	virtual synchronous machine
ZOH	zero-order hold
ZSI	Z-Source Inverter

1 Introduction

The ability to harness, store, and consume energy reflects the technological development of a civilization [1]. Consistent evidence suggests that oxygen combustion was controlled about 1 million years ago, giving us heat, light, cooking, and early tool making [2]. Since that time, the manipulation of energy has been a driving force behind the progress of human civilization. Throughout the ages, technologies acquiring energy beyond human and animal power have continuously evolved and improved [3]. Currently, humanity has reached Type 0.7276 on the Kardashev Scale (from 0 to 3), a metric proposed in 1964 to gauge the correlation between the amount of energy that a civilization is able to use and its development [1], [3]. Still evolving, the general projections show that energy consumption by society is increasing. In this sense, forecasting energy consumption and production, as well as predicting and proposing technological advancements in efficient power processing and power supply reliability, provide instructions and means to consciously build the future structure of global energy management [4], [5].

Since pre-industrial times, the concentration of greenhouse gases in the atmosphere has increased unequivocally due to human activities, particularly the burning of fossil fuels [6]. Faced with environmental consequences and projections, the consensual path to net-zero emissions nowadays requires the swift and comprehensive deployment of the full range of clean and efficient energy technologies currently available [7]. Clean electricity generation, network infrastructure, and end-use sectors are key areas for increased investment, as increasingly cheaper renewable energy technologies give electricity a lead in this task.

Electrical power processing from direct current (DC) and alternating current (AC) power sources is mainly performed by power electronics equipment, which has become increasingly widespread in the electrical power system through distributed energy resources (DER), high-voltage direct current transmission (HVDC), battery energy storage systems (BESS), and electrical mobility. The constructive aspect and control of such converters are critical points concerning cost, efficiency, compatibility, and interaction with other systems to which they are connected.

Motivations for the development of this thesis will be briefly presented next, followed by a literature review in Chapter 2 that will address the definition of the research problem and methodologies employed in this work, also discussing distributed generation systems, highlighting impedance-source inverter topologies and control techniques for power injection, voltage regulation, harmonic mitigation, and frequency support.

1.1 Motivation

Global electricity demand is projected to increase significantly across all scenarios, primarily driven by population growth, rising income levels, and the ongoing electrification of various end-use sectors [4]. As illustrated in Figure 1.1, by 2050, electricity demand is expected to exceed current levels by more than 80% in the Stated Policies Scenario (STEPS), 120% in the Announced Pledges Scenario (APS), and 150% in the Net Zero Emissions by 2050 Scenario (NZE). This growth is largely attributed to emerging markets and developing economies, which collectively represent approximately three-quarters of the global increase in all three scenarios. Concurrently, the share of unabated fossil fuels in electricity generation declines markedly. The additional demand is predominantly met by low-emission electricity sources, including renewables, nuclear energy, fossil fuels with carbon capture, utilisation and storage (CCUS), hydrogen, and ammonia, thereby significantly increasing their contribution to the global electricity mix in each scenario.

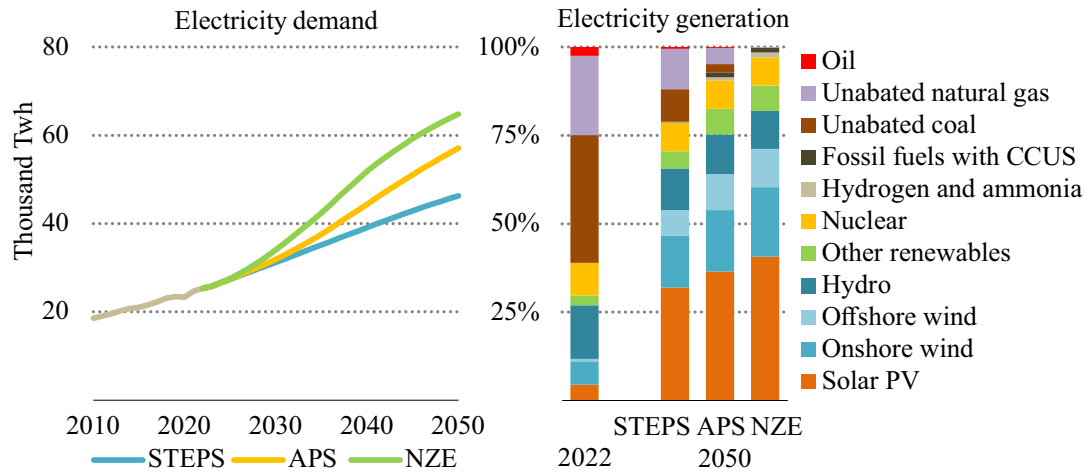


Figure 1.1: Global electricity demand and generation mix by scenario (adapted from [4])

Power generation systems that use non-renewable fuels are restricted. However, those that utilize renewable sources, such as solar and wind, have significant capacity for growth and cause less environmental damage. Globally, wind and solar energy generation currently account for more than 13.4% of the electricity consumption [8]. This statistic highlights the increasing importance of renewable energy solutions such as wind, solar, biomass, and hydropower.

Currently, modern bioenergy constitutes over half of the demand for renewable energy [4]. However, solar photovoltaic (PV) and wind power are poised for rapid expansion in the forthcoming years, particularly within the power sector. The installed capacity of all renewable power sources is projected to more than double by 2030 in both the STEPS

and the APS scenarios. In the NZE Scenario, this capacity is expected to triple by 2030. Notably, the decarbonization of the power sector progresses more rapidly than end-use electrification in each scenario, as illustrated in Figure 1.2. Nevertheless, both processes are fundamental pillars of the transition to a clean energy economy [4].

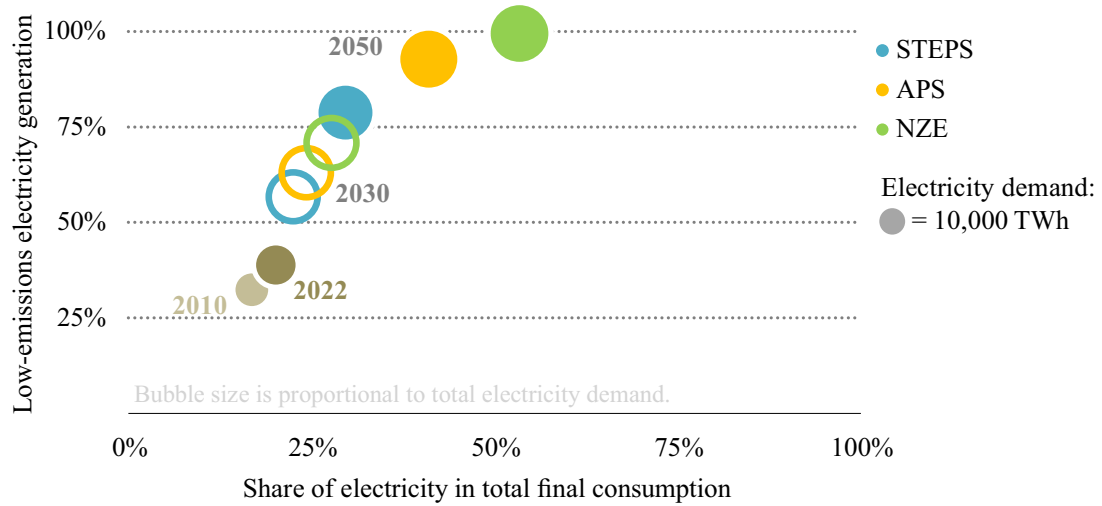


Figure 1.2: Electricity in total final consumption and low-emissions sources in electricity generation by scenario (adapted from [4])

Electric power systems are undergoing a rapid transition toward a higher share of generation from non-traditional sources, such as wind and solar, as well as energy storage devices, including batteries. Beyond their weather-dependent nature, renewable generation sources exhibit a broad size spectrum, spanning from residential-scale units to large utility-scale power plants. These sources are interconnected within the electric grid, with integration occurring at both the distribution and high-voltage transmission levels [9]. Figure 1.3 presents the transition to a grid with more inverter-based resources, highlighting challenges due to the operation of future power systems, which must be based on a combination of the physical properties and control responses of traditional, large synchronous generators, as well as those of numerous and diverse inverter-based resources.

Therefore, the increasing penetration of variable or inflexible low-carbon electricity generation underscores the need for power system flexibility. Electricity storage emerges as one viable option to provide this flexibility. Figure 1.4 offers a comprehensive overview of electricity storage applications, listed by [6], categorizing the 23 most common applications.

It can be seen that storing energy and using it to any convenience is increasingly important, giving the system desirable characteristics as power quality and reliability. Nonetheless, for instance, due to the high cost of electrically conductive materials that

directly store electrons, storing large amounts of electricity can be challenging. Energy storage technologies, as presented in Figure 1.5, are categorized into five forms in which energy is stored: chemical, thermal, mechanical, electrical, and based on the type of conversion reaction that takes place, the so-called electrochemical reactions.

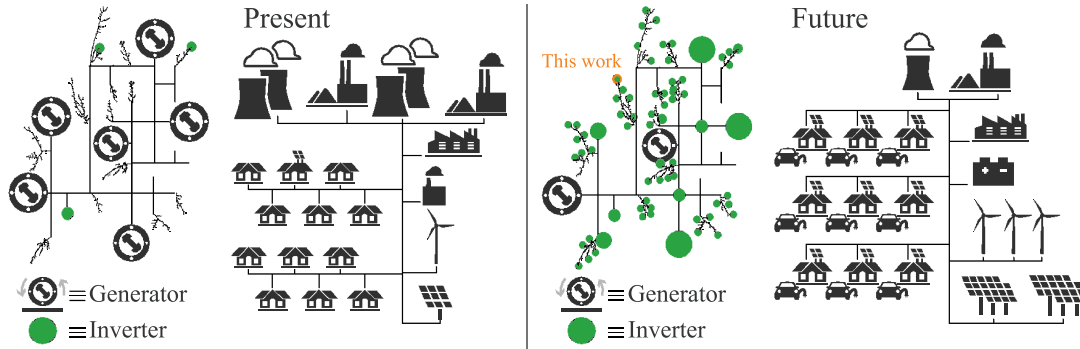


Figure 1.3: The present power system and future systems (adapted from [9])

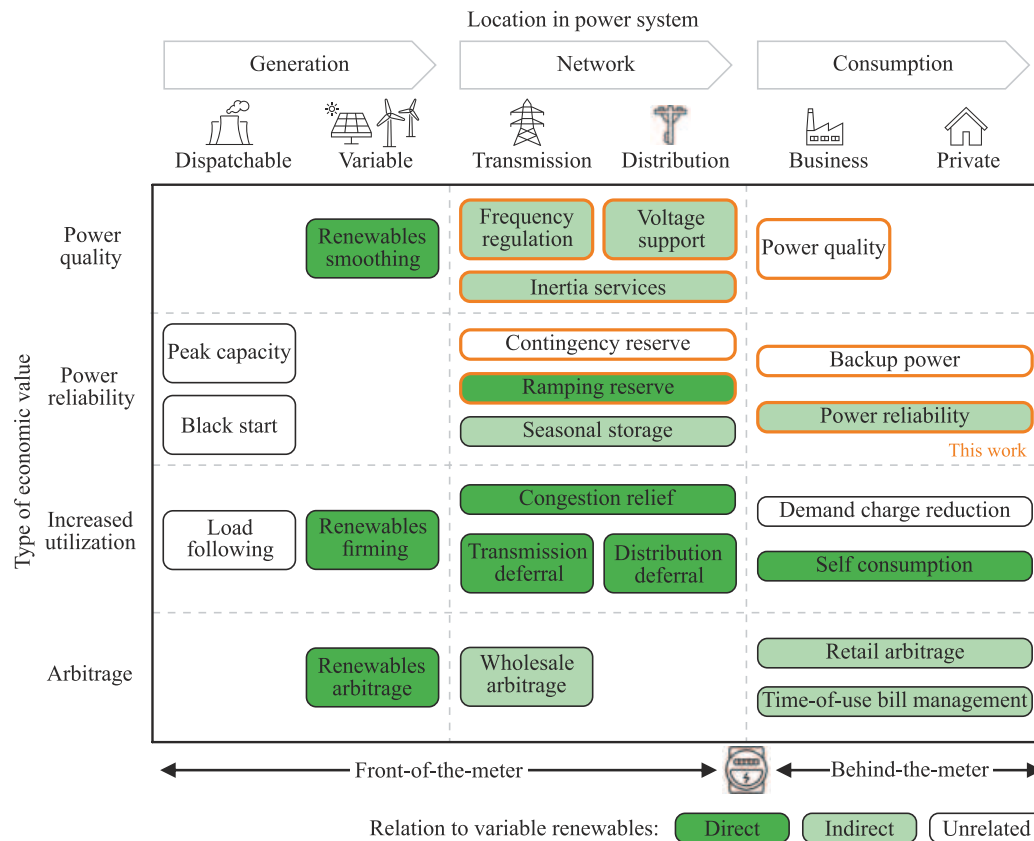


Figure 1.4: Overview of common electricity storage applications: location in power system, economic value, and relation to variable renewable electricity generation (adapted from [6])

Furthermore, energy storage technologies vary in their suitability for storage applications due to differences in cost and performance parameters, which are ultimately defined by underlying energy storage principles [6]. For instance, direct storage as electrical energy (capacitors, coils) enables rapid and frequent discharge by avoiding an energy conversion process and benefiting from the high cycle life of the technologies.

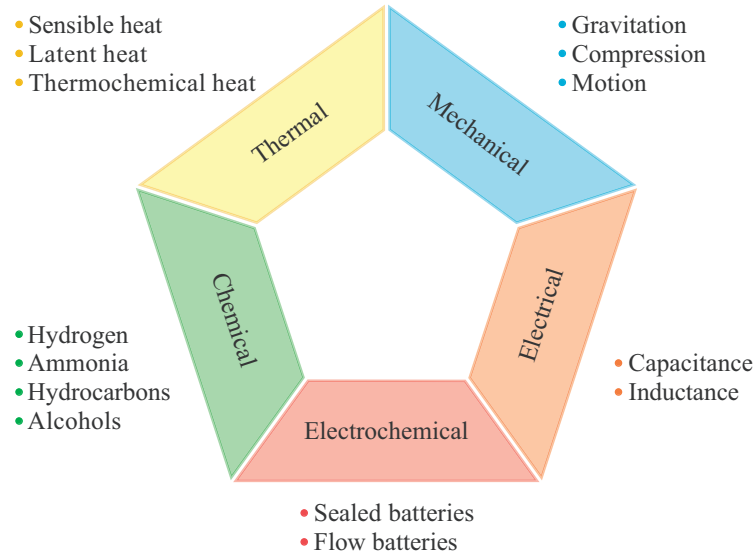


Figure 1.5: The five categories of energy storage are represented with corresponding energy storage concepts (adapted from [6])

Figure 1.6 illustrates the suitability of various electricity storage technologies across the five energy storage categories to meet the application requirements of discharge duration and electrical energy storage capacity. The five categories of energy storage are depicted with corresponding energy storage concepts, represented by colored shapes.

On the one hand, power converters can interface with such energy storage systems. On the other hand, an inherent energy storage system of power converters is their capacitors. According to [6], electricity storage technologies within the same category can often be suitable for similar use cases. Chemical storage is well-suited for large energy capacities and long discharge durations, while thermal, mechanical, and electrochemical storage are suitable for medium-sized energy capacities with durations ranging from short to long. Electrical storage is best suited for small energy capacities and short durations. An inverter-based system employing different energy storage technologies, therefore, has different time response capabilities. [10] points out that energy storage system (ESS) can enable the realization of grid-support capabilities in power electronic converters by providing the necessary energy to assist the grid when required. Figure 1.7 shows the suitability of the technology of superconducting magnetic energy storage (SMES), supercapacitor energy storage (SCES), flywheel energy storage systems (FESS), BESS,

and pumped hydro energy storage (PHES), with the following color meanings: green indicates suitability for service provision, orange indicates suitability with technological limitations, and red indicates unsuitability due to technical or economic reasons.

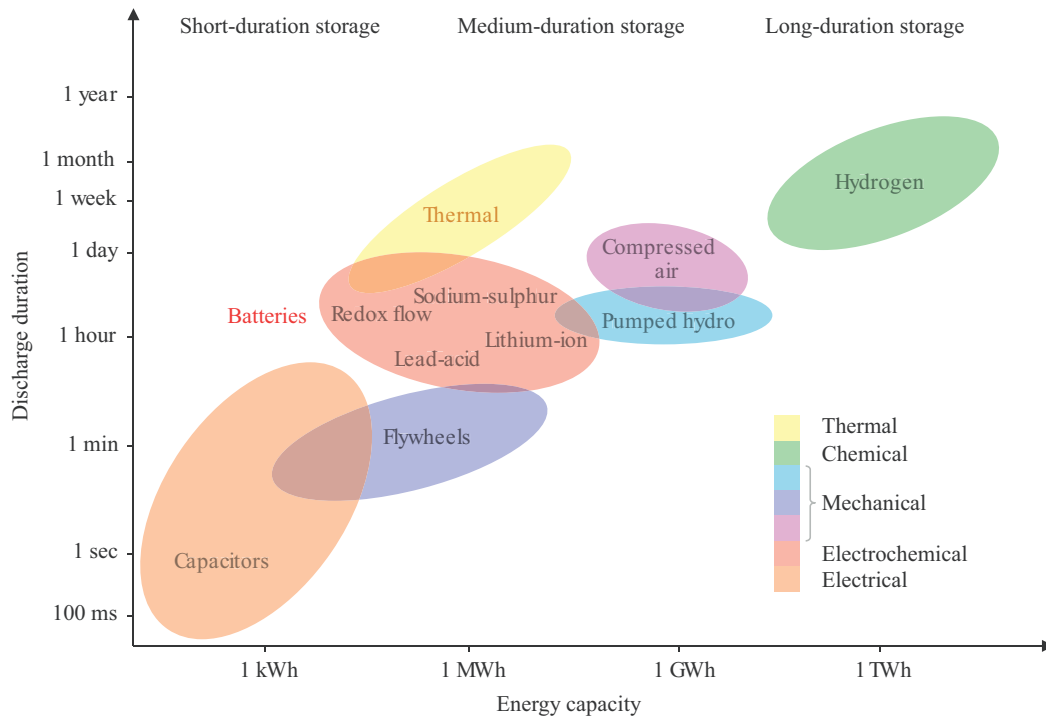


Figure 1.6: Suitability of electricity storage technologies for discharge duration and capacity requirements across five energy storage categories (adapted from [6])

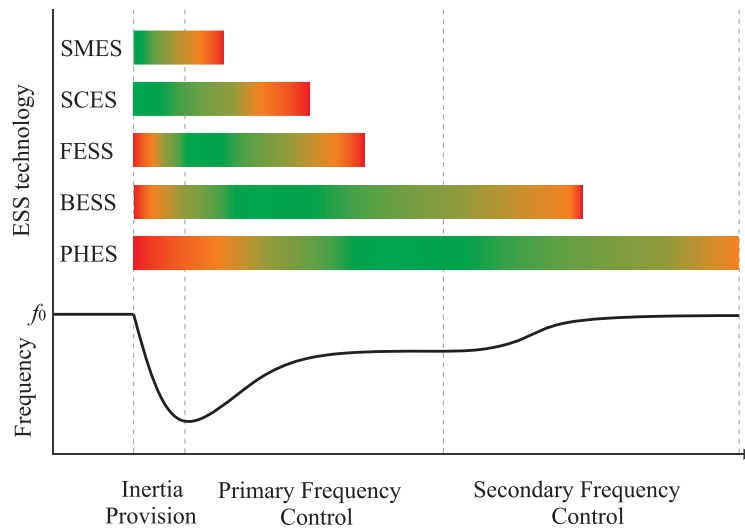


Figure 1.7: Energy storage technologies contribution during frequency disturbances and regulation services (adapted from [10])

To explore the connection of multiple energy sources to the power system, it is possible to modify or employ different converter topologies, which can also operate as distribution static synchronous compensator (DSTATCOM), helping to regulate the system. An example of topology is the split-source inverter, which, due to its impedance source characteristic, has boost capability without the need for additional active switches [11]–[13]. This topology allows for unified control and modulation concerning the energy conversion stages [14]. Thus, its characteristics make this topology an interesting option in terms of advantages in the design of low-power hybrid generation systems.

The contribution of this thesis lies in the development of grid-supporting strategies and multi-input impedance source converters for the processing and integration of renewables, based on the split-source topology. It also proposes control techniques to improve the dynamic security of the power system. In this context, several topologies are analyzed, and case studies are conducted, demonstrating the role of the DSTATCOM in voltage regulation within an isolated hybrid generation system, utilizing a self-excited induction generator (SEIG) and a generation system based on PV panel arrays. Additionally, simulations are performed on a low-power microgrid with energy storage systems, evaluating both types of ESS: the electronic load controller (ELC) and the BESS. The application of the proposed methods within both traditional and state-of-the-art control frameworks is evaluated. The potential contributions of the proposed methods to large-scale power system operation are also analyzed.

1.2 Research objective

From the prior discussion, it is evident that power converters play a vital role in power processing, and therefore, in the predicted global energy transition, as they convert DC input power to AC output, taking into account both the load requirements and the input conditions. Therefore, this thesis focuses on enhancing the capabilities of a typical family of these power inverters, known as split-source impedance source inverters, as well as improving the interaction of these converters with the power system. Consequently, the objective of this thesis is to address the following aspects:

- To design power converters with low component count to interconnect multiple sources and support the power system
- To investigate the possibility of introducing a new topology that overcomes the limitations of classical topologies
- To propose and model new inverter topologies and controllers to connect multiple sources and assess their performance

- To investigate the challenges associated with grid-tied and off-grid operation of the proposed family of single-stage DC-AC power converters

Specific objectives include:

- To extend the modeling of split-source impedance source inverter topology to accommodate multiple sources
- To develop control and modulation techniques for the proposed topologies
- To evaluate inverter performance in terms of dynamics and power processing, considering its degrees of freedom and constraints
- To develop control strategies to address power quality and support the power system
- To establish a reliable evaluation system to validate the proposed methods

1.3 Thesis outline

This thesis is structured as follows: Chapter 2 provides a comprehensive review of the current state of research relevant to the topics addressed in this thesis. Additionally, it outlines the methods and objectives of the research.

In Chapter 3, the characteristics of the split-source inverter circuit are introduced, along with the derivation and modeling of the new multi-input topology. It presents the mathematical modeling of the former and its multi-input generalization, evaluating the behavior on the DC and AC sides.

Chapter 4 focuses on obtaining the dynamic models of the elements that compose the generation system, with an emphasis on the AC side. State-space equations are derived in the $\alpha\beta$ and synchronous dq reference frames, and equations for the implementation of both linear and nonlinear controllers are also provided.

Chapter 5 introduces control strategies applied to the inverter for both grid-tied and off-grid operation. These strategies aim to regulate the amplitude and frequency of the voltage at the terminals of the SEIG, corresponding to the point of common coupling (PCC), and to mitigate harmonics. The chapter proposes Model Predictive Control (MPC) strategies for the internal loops and discusses considerations for their digital implementation. Additionally, it presents general considerations for the controllers applied in the inner and outer loops of the system. The application of ancillary controllers and model aggregation for assessment in a large-scale power system is discussed.

Chapter 6 presents the results obtained through computational simulations and exper-

imental tests using a developed prototype. Simulation and experimental results are presented to validate the performance of the system with the proposed control. The transient and steady-state behavior of the system variables under the variation of linear and nonlinear loads is evaluated. The power quality supplied by the system is also verified, with reference to the requirements of the IEEE Standards.

Chapter 7 presents the final considerations of the thesis, discussing briefly the results and contributions of this work. Moreover, the suggestions for future work and work-related publications are presented.

2 Literature Review

In this chapter, topics related to the methodology of the conducted research and development are presented, along with a literature review covering definitions and challenges of modern distributed generation systems, the state-of-the-art split-source impedance source inverters, and also discussing grid-supporting control, fast frequency response methods, and up-to-date testing and validation tools.

2.1 Hybrid systems in distributed generation

In power generation systems, the combination of more than one energy source aims to exploit the complementarity among them over time, considering that some renewable sources, such as photovoltaic solar and wind, are intermittent and can vary significantly depending on weather conditions [15]. Hybrid systems can operate either connected to the conventional electrical distribution grid or isolated, without the possibility of connecting to the grid, in the case of remote or difficult-to-access locations. In isolated areas, these systems become interesting strategies in terms of energy reliability. Due to complementarity, such systems can operate with a lower risk of interruption [16]. It is noteworthy that these systems can range from nano to microgeneration, mini-generation, and larger-scale generation. According to [17], they are often used in remote areas where electricity is not available or where public grid connection fees are prohibitively expensive, such as in remote villages, islands, and ship propulsion systems [16], [18]. In such systems, energy converters play a critical role in providing appropriate voltage levels to loads, which can be single-phase or three-phase, three-wire or four-wire, and either balanced or unbalanced.

In order to take some examples of the sizing of such systems, from the Brazilian normative and regulatory framework, microgrids can be defined as a group of interconnected consumers (loads) operating in a coordinated manner as a single consumer unit, with an installed capacity greater than 75 kW and the contracted demand at the point of connection, equal to or less than 2,500 kW [19]. On the other hand, nanogrids are the smallest microgrids, limited to a single consumer unit of a distribution system, capable of connecting or disconnecting with other units through a gateway. As stated in [19], nanogrids can be defined as a single consumer unit, with an installed load of up to 75 kW, forming a microgrid equipped with distributed energy resources operating in a coordinated manner/controlled, with low voltage connection, with the possibility of operation connected to the distribution network or in an islanded mode, even if only momentarily, provided that it is properly dimensioned for that and it is in compliance with the Operating Agreement and the current technical standards.

Multi-sourcing ensures substantial redundancy and represents a well-established method deal with the unpredictability of renewable energy sources [20], [21]. A hybrid microgeneration system consisting of a hydropower source and photovoltaic source with batteries is presented in [22], as illustrated in Figure 2.1. Despite generating alternating current, the authors chose to rectify the voltage and inject it into the DC bus as in [23], to which the photovoltaic generation and the battery bank are connected. Meanwhile, a three-phase inverter processes and provides AC power to the loads. In this approach, the nominal power of the inverter must match the sum of the nominal powers of the loads.

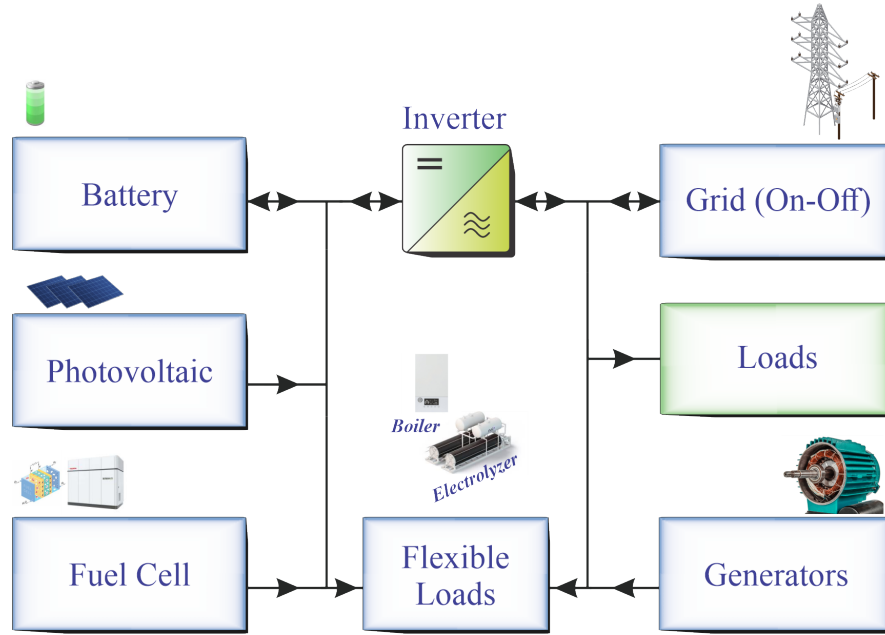


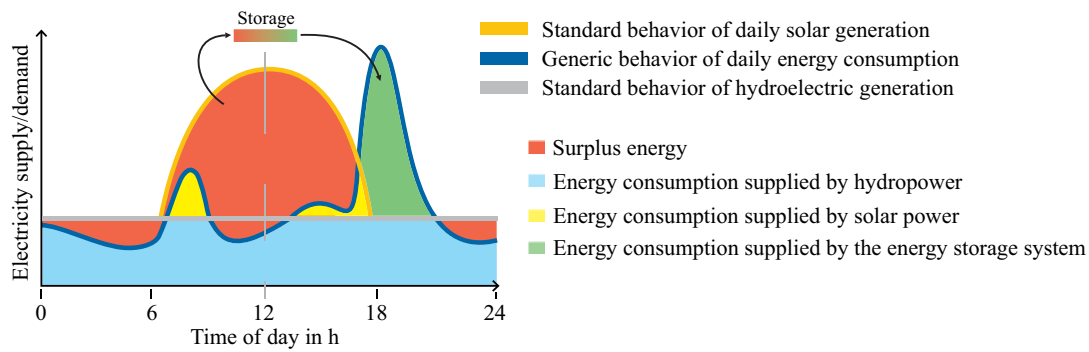
Figure 2.1: Modern distributed generation systems (adapted from [15])

Energy management is an important design issue in hybrid renewable energy systems, addressing dispatch decisions, surplus energy, generation or load curtailment, and energy storage [6], [24], [25]. For instance, the use of smart loads is an effective solution that allows controlling of the system demand, directing the energy for possible applications such as energy storage in batteries, as well as heat-storage systems, water pumping systems, and hydrogen conversion [26], [27].

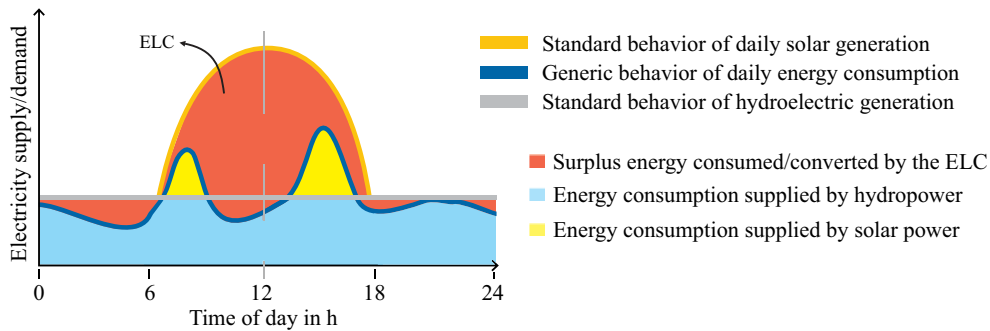
In a hybrid generation system, as presented in the diagram in Figure 2.2, the analysis of the complementarity of energies generated by different sources becomes crucial. The curves and regions in Figure 2.2(a) illustrate the operating moments of each generation element in the system throughout the day, based on a generic load curve. In this case, the need for an energy storage system, such as a bidirectional power flow battery bank, becomes apparent. This is because the peak demand occurs at a time when there is insufficient energy generation from the available sources. Thus, the surplus energy

generated by the system during the day is processed and stored to be used during peak demand instants [28].

On the other hand, some applications for these systems do not require the use of energy storage devices. Instead, the generation system is generally designed based on the load curve to ensure reliability and minimize costs. An example is the system needed to meet the demand illustrated in Figure 2.2(b). For this, an ELC is employed to consume the surplus generated energy. It can also be used to regulate the microgrid frequency, allowing the hydro-based SEIG system to operate in a constant power condition. Additionally, the energy delivered to the ELC can be utilized for various reuse applications and forms of energy storage, such as water heating in a reservoir and pumping [29] for re-electrification.



(a) With EES and/or re-electrification of ESS



(b) Without EES and/or re-electrification of ESS

Figure 2.2: Energy management in a hybrid generation system (adapted from [30])

Power converters are an essential part of the development of distributed generation systems based on renewable energy sources. This issue has gained great importance in recent years and has required the development of topologies, control techniques, and technological improvements in power electronics [31], [32].

For instance, in low-power renewable generation systems, the SEIG proves to be a good solution with low cost for hydro and wind power sources. However, systems based

on SEIG present voltage regulation problems under load variations and nonlinear load connections, making the applications of these systems less attractive [33], [34]. Due to the SEIG characteristics, it requires a reactive power supply to regulate the terminal voltages, being also interesting that the voltage distortions remain as minimal as possible. In this way, a voltage source inverter (VSI) as a DSTATCOM can be used [30], [35].

On the other hand, in several applications, the AC and DC voltage levels have to be matched, making it necessary to interface the AC and DC buses with conventional two-stage architectures [36]. The DC boost converter, plus a voltage source inverter to process the energy from several different power sources, is generally employed.

Multi-source renewable energy systems have become a prevalent focus in both academic and industrial research [20], [37]. A key advantage of these systems is the ability to transfer power independently from each source, which is particularly beneficial in scenarios such as partial or complete shading of individual PV modules. In practical applications, implementing independent maximum power point tracking (MPPT) and voltage boosting functions is strongly recommended to prevent the degradation of the overall MPPT performance caused by the presence of numerous series-connected PV modules. Additionally, multi-port inverters often reduce the component count compared to parallel configurations of single-port inverters.

In [20], it is emphasized that multi-source renewable energy systems, a subject extensively explored in academic and industrial research, are distinguished by their ability to independently transfer power from each source. This is particularly advantageous in scenarios like partial or complete shading of individual PV modules. Independent maximum power point (MPP) tracking and voltage-boosting are essential to prevent series-connected PV modules from impairing MPPT performance. Multi-port inverters are preferred, as they reduce component count compared to parallel single-port configurations. To minimize system cost, component count, and switching power losses, multi-port inverters have been the subject of increased investigation. As highlighted in [20], unlike conventional bus architectures, which use a dedicated converter for each connected source, multi-port inverters consolidate power conversion into a single-stage topology featuring multiple input ports, output ports, or both.

2.2 Single-stage inverters

In terms of topologies, the growth of distributed generation in recent years has driven the development of new converter topologies [38]. In various applications, the required alternating voltage exceeds the available direct voltage in the DC bus [39]. Therefore, conventional architectures use two stages, which include a DC-DC voltage boost

converter, plus a three-phase or single-phase DC-AC voltage inverter to process energy from the source, as illustrated in Figure 2.3(a). In this context, several single-stage topologies emerge as candidates for application in generation systems, compensation, or charging and discharging of energy storage systems [40], [41].

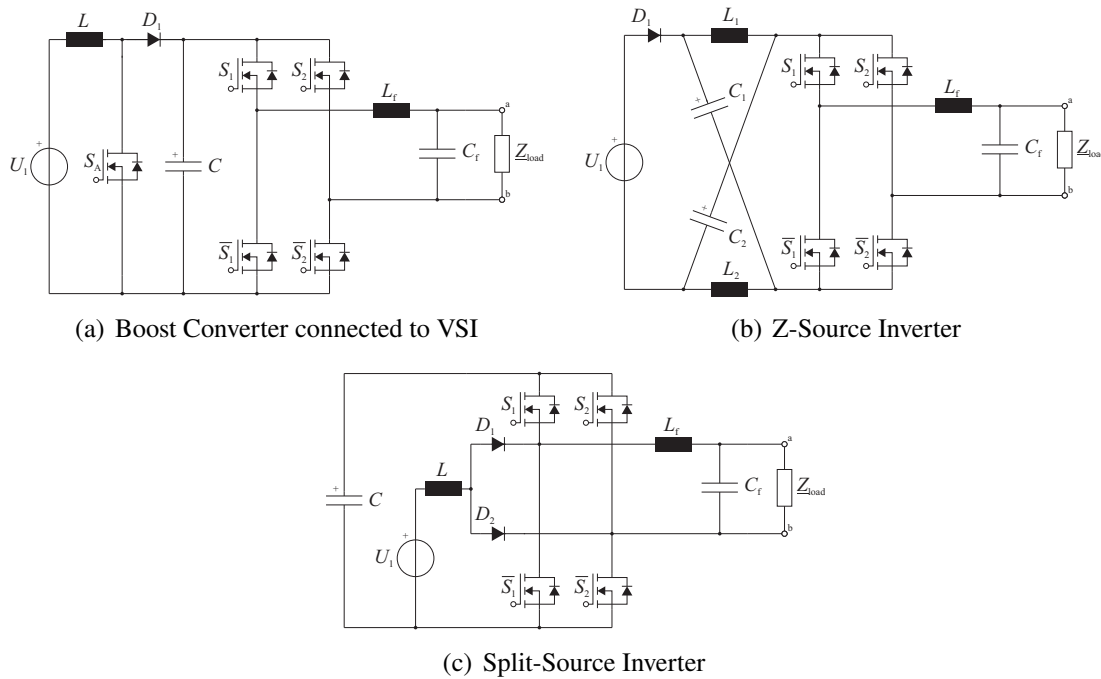


Figure 2.3: Single-phase DC-AC inverter topologies [42], [43]

Alternatively, the Z-Source Inverter (ZSI) [44], originally proposed by Peng [42] in 2002, is used to provide a reduced number of energy conversion stages with the ability to increase and decrease voltage on the load, using a reduced number of components, as shown in Figure 2.3(b). However, additional switching states are required to produce the so-called shoot-through state, in which two switches from the same inverter leg need to be conducting or in a short-circuit state simultaneously. Consequently, other impedance source topologies have been proposed [45]. The Quasi-Z-Source (qZSI) [46], the Y-source (YSI) [47], and the Buck-Boost Voltage Source Inverter (BBVSI) [11], for example, are alternatives that aim to reduce stress and losses in inverter elements, minimize the number of components, and simplify control strategies.

On the other hand, to overcome some of the drawbacks of ZSIs, the Split-Source Inverter (SSI) was recently proposed by Abdelhakim in 2015 [11], presenting interesting advantages when compared to the ZSI. This topology, depicted in Figure 2.3(c), features a single energy conversion stage and employs a conventional six-switch inverter with three additional diodes. It boasts reduced switching stress for higher voltage gains.

However, as drawbacks, the topology exhibits asymmetric current stress between the upper and lower switches of each leg, and also, substantial losses in the input diodes [48]. The continuous input current and DC-link voltage, low passive component count, use of standard PWM techniques as VSI, and low switching stresses are the major merits reported in the literature for split-source converters. However, it needs a dead-band circuit for protecting switches from short-circuit problems like VSI.

2.2.1 Multi-source inverters

Distributed power generation systems can require the interconnection of several independent energy sources demanding different converter topologies. In this sense, to process the energy from several different DC power sources the DC-DC boost converters plus voltage source inverter is generally employed [31], [44]. Over the last few years, several topologies and arrangements have been proposed focused on meeting the interconnection of renewable power sources in distributed power generation [49], [50], including multi-input converters [51], allowing the inclusion of multiple sources in on-grid and off-grid hybrid power generation systems [52]. In addition, the modulation and control of these converters comprise a diverse and vast amount of strategies [53]–[56], resulting in different dynamics and contributions to the grid side.

Over the last few years, several topologies and arrangements have been proposed focused on meeting the interconnection of renewable power sources in distributed power generation [50], [57], including multi-input converters [51], [58]–[60], allowing the inclusion of multiple sources in on-grid and off-grid hybrid power generation systems. Several topologies employ transformers, coupled inductors, cascaded and multilevel arrangements [51], [61].

For instance, [62] proposes a single-stage multi-port boost inverter for PV and energy storage applications, where continuous input current is drawn from both magnetically isolated input ports. The proposed inverter exhibits lower voltage and current stress, as well as higher voltage gain, compared to existing integrated multi-port DC–AC solutions [20]. According to the authors, the design offers several advantages: reduced cost, elimination of circulating current issues, no requirement for a low-frequency transformer or a DC-link capacitor, and inherent power-sharing capability. The converter also demonstrates improved performance in terms of reduced semiconductor component count and mitigated electrical stress. However, these benefits come at the expense of increased complexity in the power circuit design.

A dual-input extension of the conventional SSI is proposed and analyzed in [63] and [64] for single-phase and three-phase architectures, respectively. The modification involves

the addition of an extra active switch at the midpoint of the coupling diodes of one leg of the VSI [63], [64]. This dual-input configuration enables power transfer either simultaneously or individually from two separate PV modules. The features of the conventional SSI, such as continuous input current and voltage boosting capability, are preserved, while individual MPPT is also supported. The proposed topology reduces the number of active switches, thereby lowering current and voltage stress, and ultimately decreasing system cost, physical size, and control circuit complexity [20].

In [63], the proposed design is compared against existing topologies and a parallel configuration of two SSIs (2xSSI), confirming the advantage of a reduced number of active switches. Although the design introduces additional diodes, these are arranged in common-anode or common-cathode configurations, allowing their integration into a single package and effectively functioning as one switch. In [64], the three-phase dual-input version is evaluated, demonstrating similar capabilities in both single and dual-input operation modes, and confirming a lower count of active switches relative to comparable architectures. The findings in both studies are validated through simulation and experimental results.

To contribute to the development of the interconnection of multiple independent energy sources, a triple-input topology is introduced [52], called multi-input split-source inverter (MISSI), together with a predictive control strategy for the connection of the grid. The controller performs a discrete-time model of SSI to predict the finite future behavior of the input and output currents based on converter switching states [48], [65]. The control action is defined by minimizing a cost function. Moreover, a linear proportional-integral controller is used to regulate the DC-bus voltage. The three-phase MISSI with three DC inputs presents a single-stage power conversion and employs a conventional six-switch power inverter with six additional diodes.

2.2.2 State of the art of split-source inverters

From the ZSI, a variety of impedance-source topologies were proposed [44], presenting interesting features when compared to the ZSI. The SSI presents a single-stage power conversion and employs a conventional six-switch power inverter with three additional diodes. In addition, it presents reduced switch voltage stress for higher voltage gains [48]. Compared to BBVSI, which uses additional active semiconductor switches, SSI uses only diodes. Compared to ZSI and YSI, it reduces the number of passive components. However, the topology presents some drawbacks consisting of the asymmetrical current stress among the upper and lower switches and substantial commutation losses in the input diodes [11], [66].

The papers developed on the SSI emerged a range of modulation strategies, control techniques, topology improvements (see Figure 2.4), and applications [67], [68]. In order to reduce the significant losses in diodes and enable a wide range of AC output voltages operating in both buck and boost modes, a topological improvement to the SSI was proposed in [69]. Additionally, multilevel inverter topologies were derived from the SSI, such as the Flying Capacitor (FC-SSI) [12] and the Diode-Clamped (DC-SSI) [13] as well as modular schemes [70]. An alternative configuration of the SSI was also proposed, aiming to use common cathode diodes in a single device to reduce losses due to parasitic inductances during switching and achieve high efficiency (95.5 %) [43].

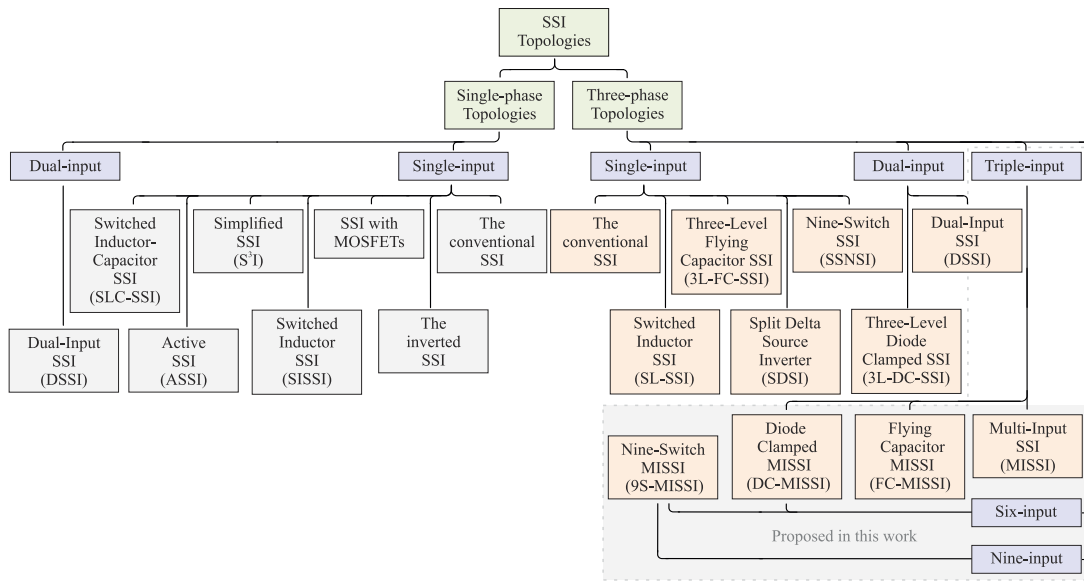


Figure 2.4: Main Split-Source Inverter topologies (adapted and expanded from [71])

The original SSI topology is designed to feed balanced three-phase three-wire loads or to connection to a three-wire utility grid. Nevertheless, unbalanced loads may also be connected to such systems. Consequently, circulating currents arise throughout the power system, potentially causing overheating of the neutral conductor, imbalance, and distortions in output voltages [17], [72]. To address these effects, a four-wire voltage inverter can be used by splitting the DC bus capacitors. This approach is straightforward, however, it may result in imbalances in the capacitor voltages and, therefore, lower utilization of their potential level, requiring elevated capacitor values to mitigate current ripples [73]. To enhance voltage utilization and reduce the size of the DC bus capacitor, the four-leg inverter is an attractive solution for four-wire three-phase systems [74]. For instance, a topology resembling SSI with four legs was proposed in [55] to process energy from an isolated system with unbalanced loads.

Furthermore, SSI also appears in recent literature where control approaches for islanded

AC microgrids have been developed [75], extension for multiphase [76], arrangements with other inverters as the nine-switch inverter [77], [78], application in AC-AC matrix converters [79], interface with other converters through the DC-bus [80], shunt active-filter [81], interleaved [82], automotive traction applications [83], dual-output [84] and also, contemplating semiconductor arrangements with inclusion of active switches [69], [85].

Over the years, more topologies were proposed and analyzed, inspired by the SSI, as well as patent applications with innovative solutions in low-power applications for vehicle powertrain [86], fuel cell applications [87], hydrogen production [88], grid-connected for distributed power generation [89]–[91], permanent magnet synchronous motor (PMSM) electric driver [92], and photovoltaics [93].

The main topologies derived from the SSI, are depicted in Figure 2.4. As a contribution of this thesis work, the number of inputs for three-phase inverters is increased by proposing new arrangements.

2.3 Power system stability and ancillary services

Due to the high inverter penetration in modern power systems by means of renewable energy sources (RES) through the massive integration of PV and wind power generation units, the lack of inertia becomes a real problem [9], [94]. In addition, the transients are becoming faster and more extreme. The monitoring of the power system status is becoming a crucial essential task and a topic in constant research. Furthermore, when a frequency change occurs, the rotational inertia helps to dampen the frequency variation [95]. Appropriate levels of rotational inertia are required to ensure the reliable and secure operation of the power system. On the other hand, energy storage systems are also being integrated to achieve the capability to provide inertia, among other ancillary services, which can be defined as services that support the generation and transmission/distribution of electricity [6].

In this context, Figure 2.5 presents the definitions of power system stability, regarding modern systems with high penetration of inverter-based resources (IBR). Recently, two new stability classes have been introduced, namely “Converter-driven stability” and “Resonance stability”, motivated by the increased use of converter-interfaced generation (CIGs) [96], [97].

From the processing of electrical energy from renewable energy sources through inverters, distributed generation can contribute to the injection of active power when requested by the electrical grid, even in the presence of a disturbance [98], [99]. However, commercial converters widely used in real systems, generally use grid-following (GFL)

control, feeding the grid through tracking the grid voltage by phase-locked loop (PLL) techniques, for instance. Therefore, these converters cannot autonomously generate appropriate voltage levels at the connection point without taking the PCC voltages as a reference [100].

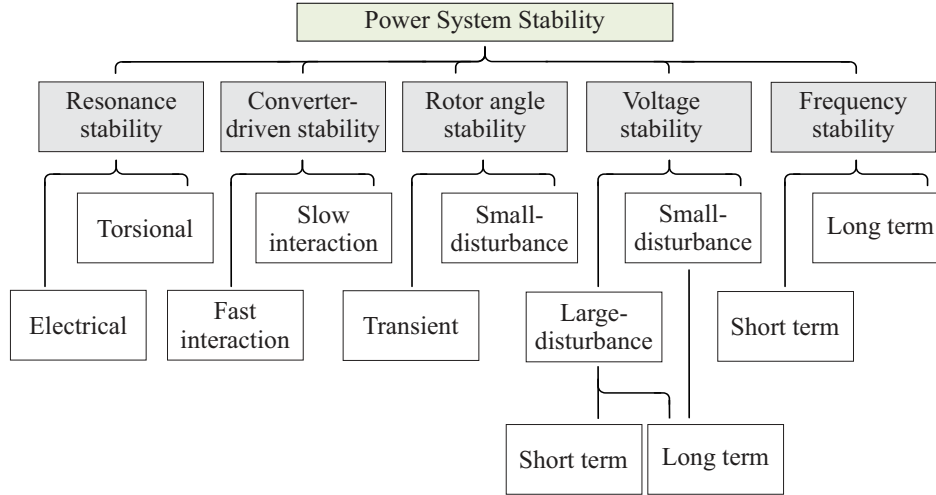


Figure 2.5: Classification of power system stability (adapted from [101], [102])

On the other hand, grid-forming (GFM) control can feed stand-alone loads as it has voltage source characteristics [103]. In addition, several control techniques have been proposed in the literature to provide synthetic inertia and power-sharing [104].

Grid-supporting control can be represented by a GFL or GFM topology. It provides ancillary services such as transient support due to sudden frequency dips, reactive support during voltage dips, harmonic compensation, and islanding capabilities [105]. Therefore, these inverters must remain connected when detecting some grid contingency, providing support [106], [107]. Figure 2.6 presents the differences between GFM and GFL with and without grid-supporting techniques.

While these paradigms are widely adopted, their definitions can vary across research communities. Traditionally, in power electronics, a GFM VSC is defined as a voltage source that imposes an AC voltage with fixed nominal amplitude and frequency, whereas a GFL VSC is treated as a current or power source that injects controllable power [108]. More recent interpretations describe GFM converters as those that regulate AC voltage with a frequency adapted for synchronization and grid support, while GFL converters typically rely on phase-locked loops for synchronization and current control, regardless of additional grid support functionalities [109].

Alternative definitions in the power systems literature consider characteristics such as the presence of virtual inertia, islanded operation capability, or the ability to suppress

frequency oscillations. Although GFM and GFL classifications generally refer to the AC terminal behavior, it is also meaningful to distinguish between DC-GFM and DC-GFL based on whether the converter imposes or relies on a stable DC voltage, respectively. In this work, a VSC is AC-GFM (or DC-GFM) if it imposes a stable AC (or DC) voltage at its terminal, and as AC-GFL (or DC-GFL) if it depends on the presence of other units to ensure a well-defined AC (or DC) terminal voltage [108], [110]. Therefore, taking into account the DC side and the controlled variables, the control structure of the grid-tied VSC can be depicted as shown in Figure 2.7.

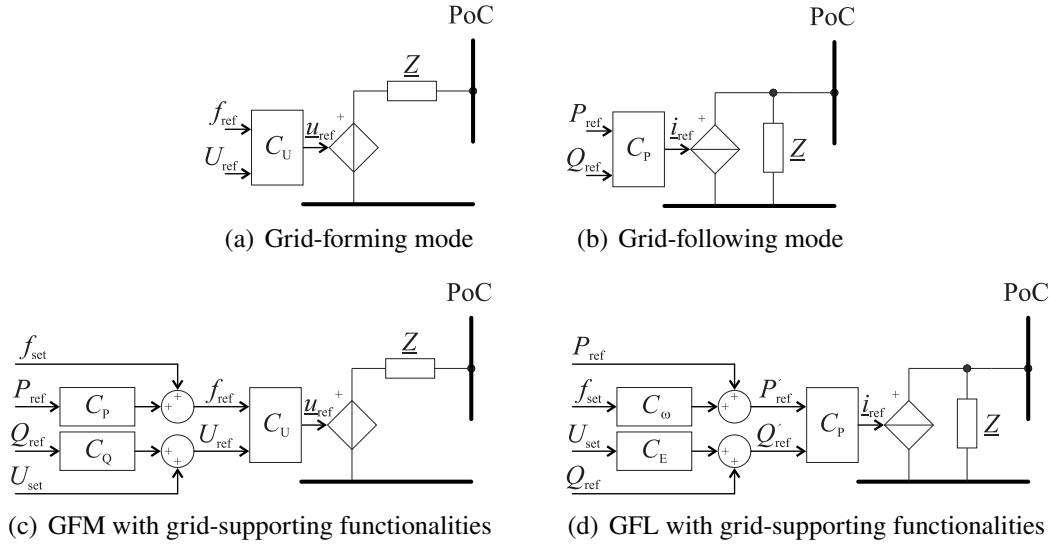


Figure 2.6: Representation of grid-connected power converters and control techniques [100], [111], [112]

For instance, in [113], considering a 12-bus power system, contributions of smart-inverters control strategies to voltage profile regulation in a low voltage distribution network are presented, such as variable power factor and apparent power limit in 1 p.u., 1.2 p.u., fixed power factor ($\text{PF} = 0.9$), limited and variable power factor ($\text{PF} > 0.9$), fixed apparent power ($S = 1$). It is concluded that an overall best solution to solve both overvoltage and undervoltage issues does not exist. However, in the distribution system studied in this work, the utilization of a hybrid solution is a promising alternative that can make smart inverters more efficient against under- and overvoltage in both day and night periods.

Furthermore, wind power plants are contributing a significant share of the electrical power supply in the modern power system [114]. These systems largely employ induction generators that need proper reactive power control [115]. In [116], a frequency control module is proposed to improve the virtual inertia and monitor the primary frequency responses. The proposed plant-level control module is simulated in a test microgrid

with and without the support of a conventional power plant. The approach optimizes the stable de-loading operations of the turbines and ultimately improves the wind power plant capacity rate.

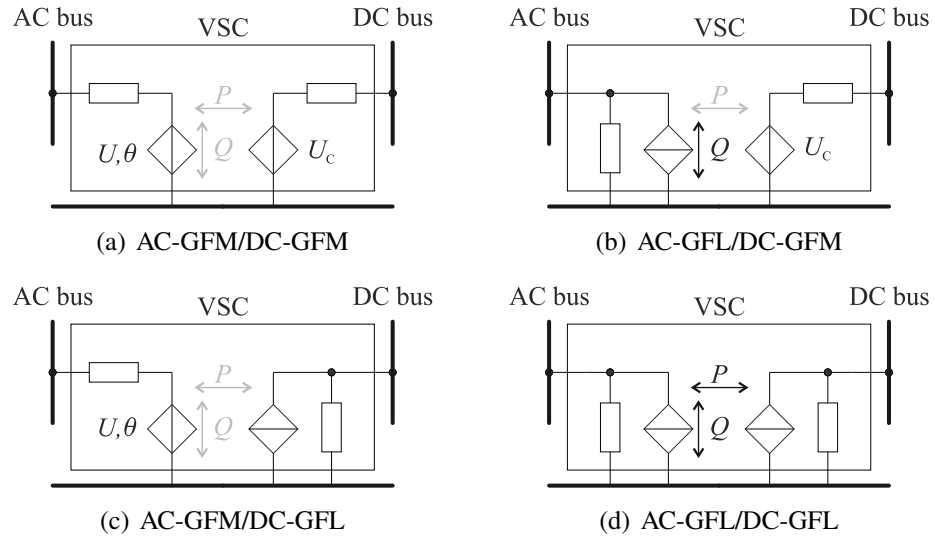


Figure 2.7: Signals imposed/controlled by the VSC [108], [109], [117]

The studies aforementioned are among those that seek to develop grid-supporting controllers. To illustrate this, the main virtual inertia strategies discussed in the literature are depicted in Figure 2.8.

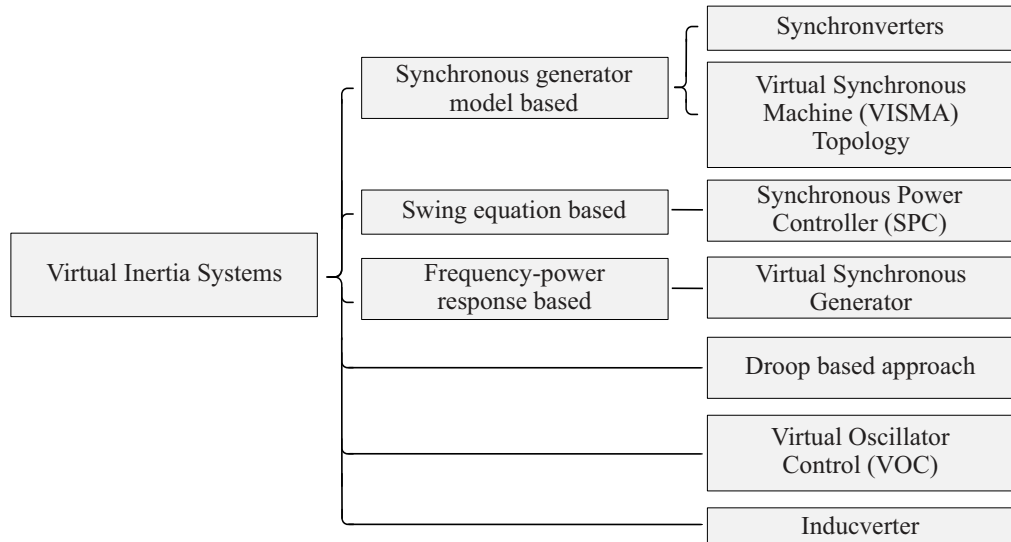


Figure 2.8: Main virtual inertia control systems (adapted from [111], [118])

For the electrical power system, the balance between generation and demand is a substantial condition. Consequently, the online detection of the sudden disconnection

of one or more generation units becomes critical in defining emergency operation plans. [119] proposes the application of graph theory as a tool for reconfiguration of the distribution network, to increase the reliability of the network and reduce the time of power outages for consumers.

The international standards and local agency statements govern the requirements for distributed energy resources connected to the power system. For instance, in a system operating connected to the grid, the normal frequency operation must be between 59.9 Hz to 60.1 Hz. Intentionally or through the act of protection, the system is disconnected automatically if the frequency reaches out of range 56.5 Hz to 66 Hz [19], [120].

The main international standards widely accepted for microgrids are:

- IEEE 2030: specification and recommended practice for microgrids
- IEEE 1547: standard for design, operation, and integration of distributed energy resources
- IEC TS 62898: guidelines for microgrid projects planning, specification, and operation

It can be noted from Figure 2.9 that the requirements for system operation in islanded mode are relaxed [121]. In addition, systems operating out of the operational range should be suddenly disconnected from the system. This contingency on loads, generators, or other equipment can be harmful to the dynamic security of low-power systems, or even large-power systems facing a large number of occurrences, making the system situation even worse. Then, it should be avoided, calling attention to the need for further control strategies.

2.3.1 Fast frequency response

Facing the upcoming challenges in smart energy systems, the interest in strategies to contribute to the stability of the power system has grown noticeably both on the grid side [115], [119], [122] and on the converter side [116], [123]–[125]. It also applies to small islanded power systems, where quick disturbances can lead to system collapse due to their low-power and low-inertia characteristics. It can be caused by generator disconnection, fault, overload, and nonlinear loads.

Ancillary service control can be designed against system contingencies, [126]–[128]. In [114], [129], generator emulation control (GEC) and inertia emulation control (INEC) were applied to provide synthetic inertia to the system, the DC-link capacitors were used to provide the needed energy from a permitted voltage deviation regarding the stable

operation. Moreover, grid-friendly restoration is an interesting feature to be achieved to avoid a secondary frequency drop (SFD). The strategy proposed in [130] prolonged the recovery time to smooth the recovery process and reduce the SFD.

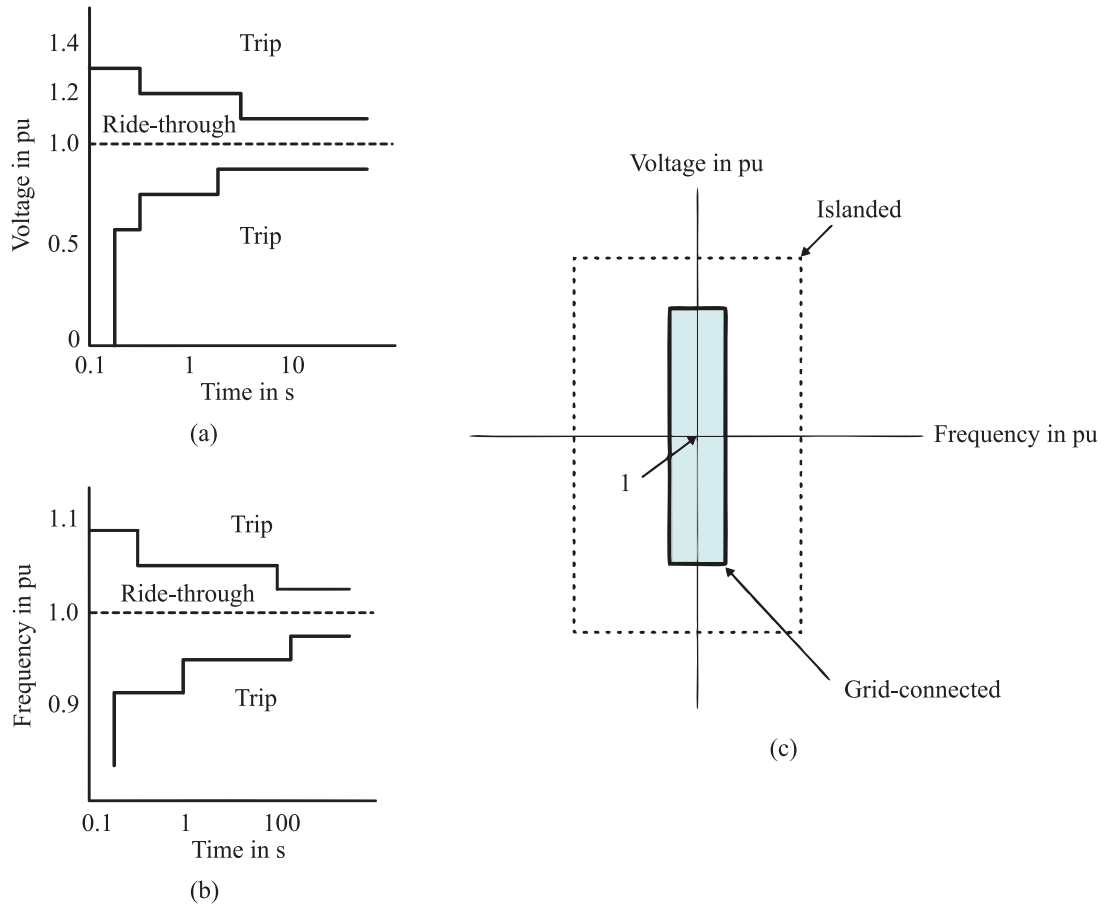


Figure 2.9: (a) Generic operational voltage ranges—grid-connected mode. (b) Generic operational frequency ranges—grid-connected mode. (c) Voltage and frequency ranges in grid-connected and islanded modes [121]

Considering the appropriate stable control operation limit, and also that inverters are usually designed with a spare voltage capacity, the control strategy can be drawn [98]. The controller arrangement can prioritize the system frequency restoration, seeking to avoid premature contingencies such as equipment shutdown when the frequency exceeds the threshold. It can be achieved through an fast frequency response (FFR) strategy [131], [132].

FFR is a response that needs to be deployed automatically and provide a full response as fast as possible after the frequency drops below a preset threshold, for instance, within 0.5 second (Figure 2.10) and a threshold of 59.8 Hz, according to the proposed FFR requirement in the Electric Reliability Council of Texas (ERCOT) ancillary service

market design [133], [134]. In [133], the benefit of using FFR on the ability of a PV power plant to arrest the rate of frequency decline during contingencies was demonstrated.

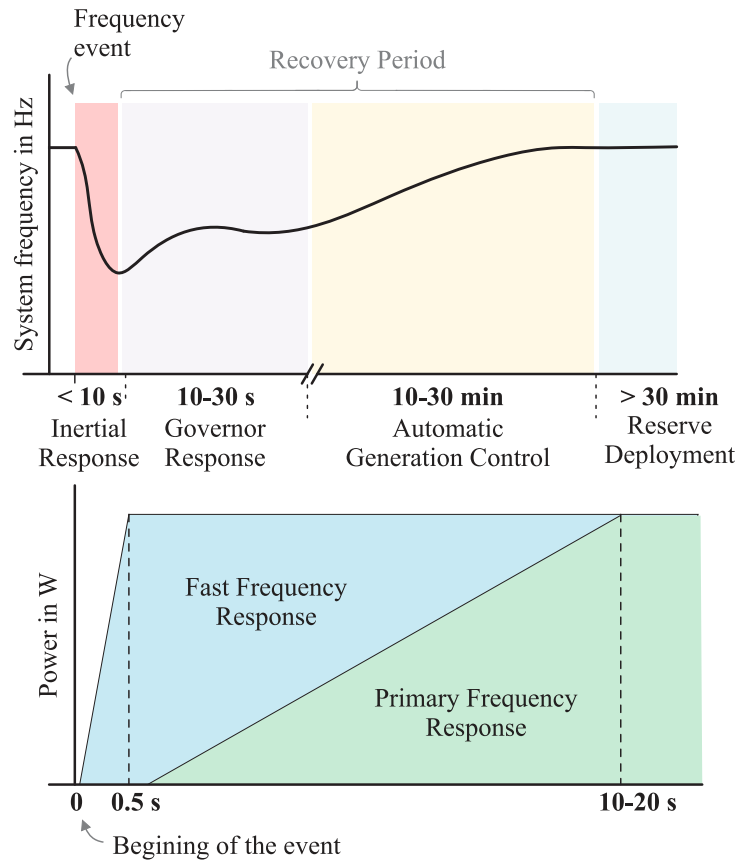


Figure 2.10: Multiple time-frame frequency response in a bulk power system following a frequency event, and FFR time frame (adapted from [118], [132], [133])

In power systems, the FFR services can be activated using different forms of responses which can be summarised as follows [132], [135]:

- Proportional response: active power injection or controlled load reduction in proportion to a measured frequency deviation
- Derivative response: active power injection or controlled load reduction in proportion to calculated rate of change of frequency (RoCoF)
- Step response: injection of a constant amount of active power or controlled reduction of a constant amount of load once a frequency preset trigger point or a preset RoCoF is reached

Therefore, FFR also helps to reduce the frequency nadir (FN), which represents the lowest frequency point after a disturbance occurs and serves as an important indicator

of the post-disturbance frequency stability of power systems. In this context, one of the contributions of this thesis lies in the proposal of an ancillary controller that can be easily extended and applied to various IBR topologies employing DC-GFM. The controller, which features an integral control action, is designed to prioritize maintaining the system frequency within the defined operating range for as long as possible, based on the power and energy available in the inverter.

2.4 Model predictive control

Emergently, MPC has been successfully used for the control of voltage source converters [65], [136], [137]. This control strategy can be considered as an algorithm that uses a model of the system to predict its future behavior and selects the most appropriate control action based on an optimality criterion [48]. The finite control set (FCS) MPC takes advantage of the discrete nature of power converters by relating the switching state directly to the control error [138]. In addition, since the switching state is directly chosen from the cost function minimization, no linear controller and modulators are necessary. The optimization problem is limited to predicting the system behavior for those possible switching states over a short horizon. On the other hand, the predictive model can also be used to compensate for delays in the control action depending on the implementation characteristics of the system [136]. The FCS-MPC has been successfully used with impedance source converters [139] as well as with SSI converters [48], [55], [140], [141] where the inductor current, capacitor voltage, and output voltages are examples of main control objectives.

As presented in the literature, to solve the SEIG voltage regulation problems, a VSI as a DSTATCOM can be used [35]. Generally, proportional-integral (PI) type controllers are used [142], consisting of a simple control method with good performance for continuous signals. On the other hand, proportional-resonant (PR) controllers were employed in [143], [144] for tracking alternating time signals, however, once the signals are included in the controller, unmodeled signals and frequency deviations in the system can degrade the performance of the control system. In this sense, MPC techniques are developed in [54], [145], [146] achieving fast dynamics and tracking of non-linearities.

Furthermore, MPC can be employed to reach good performance since it considers the limitations of the power converter in the problem formulation. Approaches have been presented to deal with computational burden in multilevel topologies or in extending the prediction horizon [137], [147], [148]. In addition, MPC also proves to be a good method to analyze and control converters with a restricted degree of freedom [52], [60].

As presented in [149], when considering power electronics applications MPC can be classified as:

- Continuous Control Set Model Predictive Control (CCS-MPC)
- FCS-MPC also known as Optimal Switching Vector Model Predictive Control (OSV-MPC)
- Optimal Switching Sequence Model Predictive Control (OSS-MPC)

It can be highlighted, among the main advantages of using the predictive controller, the ability of the controller to quickly track variations of the reference signal, even if they are strongly nonlinear discontinuous [65], [150]. Current and voltage variables are commonly used as a control objective, also the instantaneous power theory or other decomposition techniques [151], [152] can be used to generate active and reactive power references. In [54], for instance, the control strategy predicts the next optimal state among the inverter states and uses references to apply the necessary compensations.

2.5 Testing and validation

To achieve future carbon-neutral power systems [7], several problems must be overcome, one of which is the development of advanced simulation platforms, capable of simulating the real-time (RT) dynamic performance of complex systems [153]. In this sense, it concerns several industrialists, developers, and scientific researchers worldwide, who are currently working on building flexible real-time simulation environments [154], [155].

The implementation of joint simulations for power systems and power electronics depends on the integration of different control systems and on the depth of the model, which directly influences the computing time [156]. Possible challenges may emerge in model selection related to scalability. Therefore, a proper choice of model depending on the study of the power system is crucial. The different ways to model power converters are depicted in Figure 2.11. The models basically vary in the amount of information about the system.

Given the need for further research into the technical impacts on distribution networks, major studies will require the use of very fast, flexible [157], and scalable real-time simulators [153]. The complex and interactive development and testing of the power network are generally conducted in RT simulators due to their robustness [158]. This improves commissioning time for on-site installation, minimizing outage time, with debugging ability. The grid-related challenges can be emulated and addressed in RT by creating virtual events as faults, and service interruptions in the simulation triggered by

real loads [154]. This kind of platform also supports the integration and testing of any power electronics-based loads and sources as energy storage systems [159], electrical vehicles [160], PV inverters [161], and motor drives, under different scenarios. It is desirable to ensure cross-platform compatibility and model response consistency in order to minimize the uncertainty in the simulations across different hardware architectures.

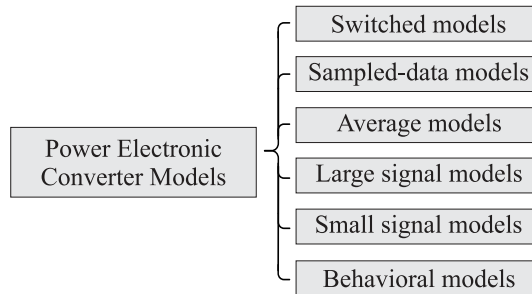


Figure 2.11: Model types in power electronics (adapted from [162])

In this sense, the tools used to validate the methods analyzed in this work are depicted in Figure 2.12. The different testbeds are widely utilized in academia and industry to validate and develop new hardware and software solutions. All the methods include at least the use of the embedded code implementable in a commercial digital signal processor (DSP).

Whereas the choice of which tool to use for a desired application is predicated by the capability, compatibility, and usability of each platform, the study in [163] reported a simplified cross-platform compatible real-time simulation model for Li-ion batteries. The real-time simulation results of the model on Typhoon and Opal-RT were found to be identical thanks to the implementation approach adopted. The preliminary testing of the model was successfully carried out by connecting it to a microgrid system.

The platforms offer similar levels of capability in their ability with differences coming down to the hardware specifications, such as usability in certain cases. On the other hand, the performance of Typhoon HIL is better than Opal-RT regarding generating accurate gate pulses at higher fundamental frequencies [164]. As presented in [165], it shows preliminary results of implementing a cyber-physical testbed co-simulation in real-time dedicated to analyzing system frequency response. The testbed is a system where the physical layer is represented by two real-time simulators in a closed loop: Typhoon HIL 604 and Opal-RT OP4510. In this sense, real-time simulation results show the flexibility of the proposed testbed and its adequacy for system frequency response studies.

Moreover, in [160], the hardware-in-the-loop (HIL) environment was used to accelerate the controller development and was tested without the risk of damaging actual hardware.

The HIL-based converter was integrated into an external real-time grid simulator to study its impact on the operation of a complex grid model. Using an interface card to connect the DSP to Typhoon HIL 402, the controller was implemented and tested using a virtual environment. The onboard charger, implemented in Typhoon HIL 402, is integrated into a real-time grid simulator using Opal-RT OP5600. The grid simulator sends a grid voltage signal via the Opal-RT to the converter and receives instantaneous current feedback from the Typhoon HIL device to determine power consumption from that particular node. The entire system was tested with the converter operating at constant rated power, and the drop in node voltage was observed.

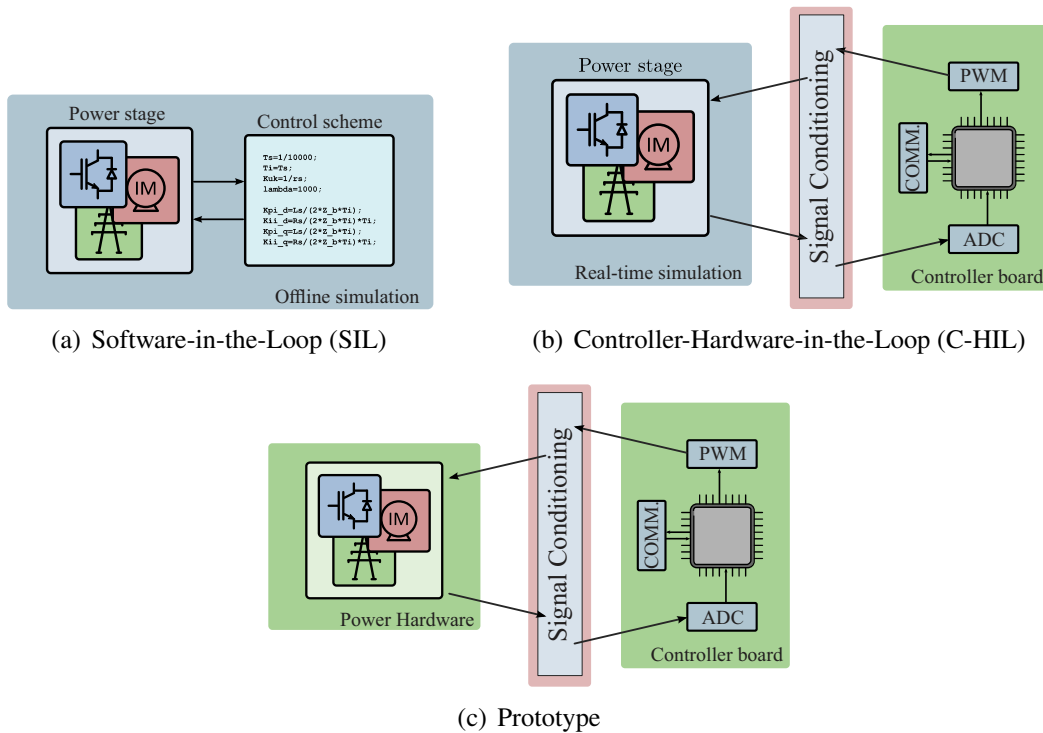


Figure 2.12: Validation methodology: power testbed types (adapted from [166])

Therefore, facing the upcoming challenges in smart energy systems, researchers are creating new methods for the validation of control, interoperability, reliability, and modern power equipment for applications covering power system stability, operation, control, and cybersecurity. Methods for laboratory testing of electrical power systems incorporate novel simulation techniques of real-time simulation and co-simulation technologies. These methods directly support the acceleration of electrical systems and power electronics research by validating technological solutions in high-fidelity environments.

Upon the state-of-the-art discussed, the following chapter will dive into the modeling

of the split-source inverter and explore its extension for a concept that enables the connection of multiple voltage sources in the input terminals of the inverter. Moreover, an analysis based on space vector modulation is proposed to analyze the degrees of freedom of the converters conceived, together with the extension of the concept to multilevel and bidirectional topologies. Simulations using MPC validate the modeling developed.

3 Three-Phase Split-Source Inverter

In this chapter, the fundamental characteristics of the split-source inverter are presented, along with the derivation and modeling of a novel multi-input concept. The discussion highlights its operating limits, advantages, and potential applications in modern power processing systems.

Figure 3.1 shows the three-phase split-source inverter topology, with an inductive filter connected to the AC output. The inverter uses the traditional and widespread VSI with the addition of three diodes with cathodes distributed between the phases and anodes connected together to an inductor, interfacing a DC voltage source.

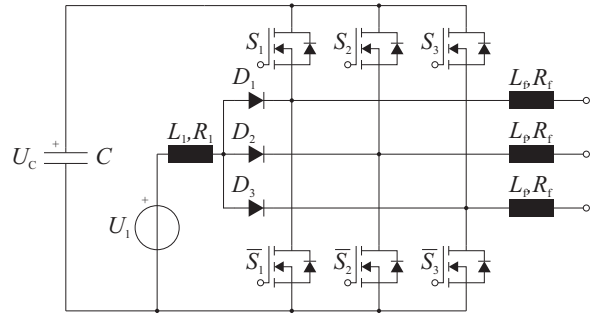


Figure 3.1: Three-phase split-source inverter [11]

The mid-point connection of each leg of the inverter has two state possibilities, being ON when the mid-point is connected to the positive pole of the DC bus capacitor, and OFF when the mid-point is connected to the negative pole. Then, considering the three legs, for this inverter, there are 2^3 possible states. Figure 3.3 shows the circuit diagrams for the eight switching states of the power converter. It can be seen that when at least one of the source interface diodes is connected to a leg of the inverter in the OFF state, the inductor magnetizes according to Figure 3.2(a). For the second possible input side state, the inductor is demagnetizing according to Figure 3.2(b).

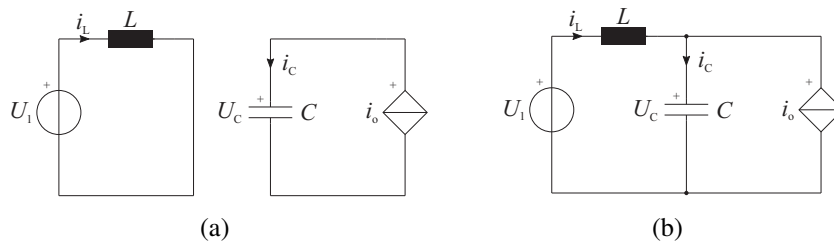


Figure 3.2: Inverter circuits for each source according to the switching states: (a) Magnetization of L , (b) Demagnetization of L

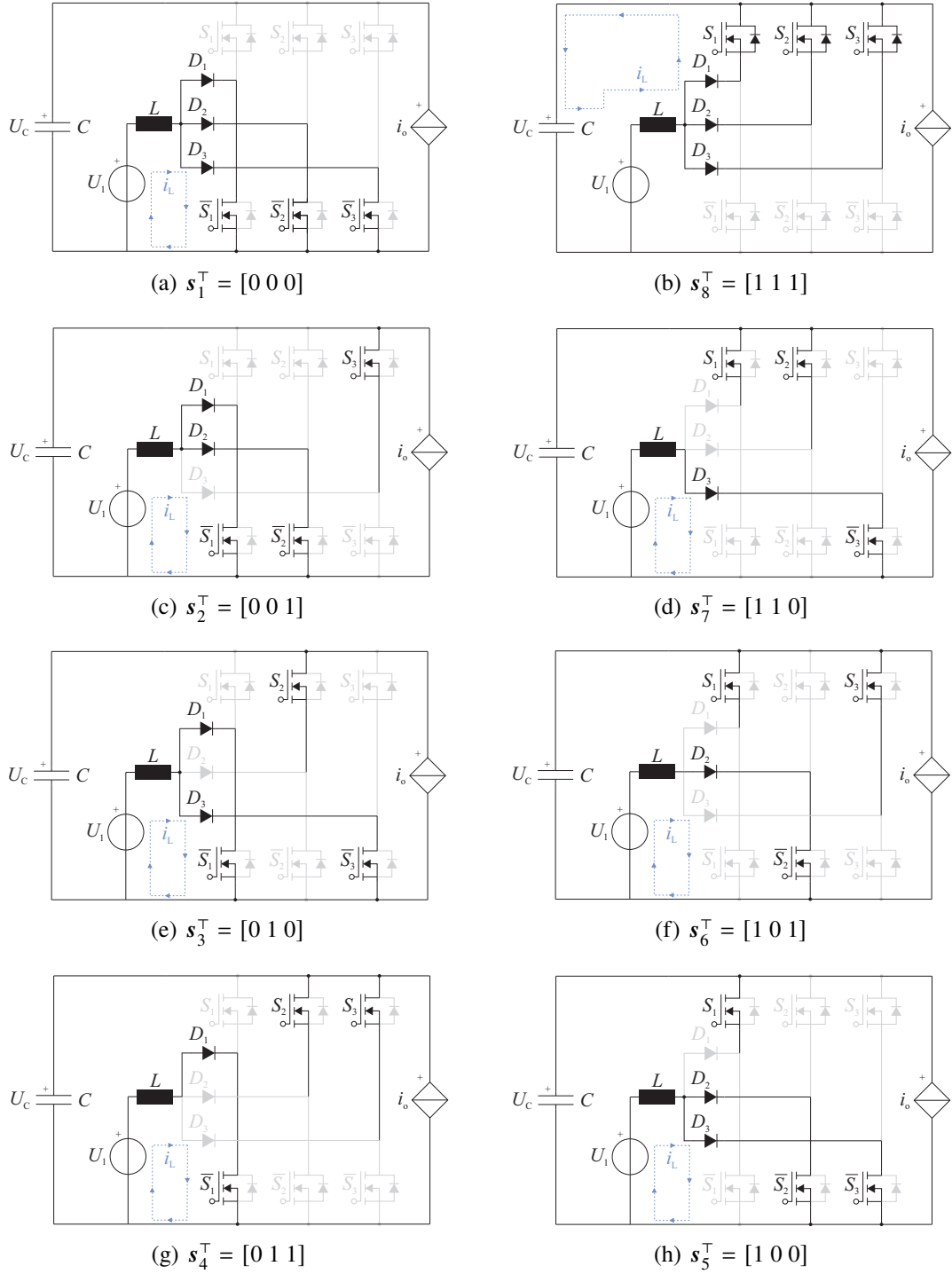


Figure 3.3: Equivalent circuit of the SSI for each switching state [11]

Each state corresponds to one output voltage vector in abc coordinates. Considering the three-phase three-wire system, these output voltage vectors can be represented in the

$\alpha\beta$ reference frame, further explained in Appendix A, by means of the power invariant Clarke transformation, written as

$$\mathbf{T}_{\alpha\beta} = \sqrt{\frac{2}{3}} \begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix}, \quad (3.1)$$

and the voltage levels can be represented in terms of the DC bus voltage by

$$\mathbf{u}'_{\alpha\beta,c} = \mathbf{u}_{\alpha\beta,c}/U_C. \quad (3.2)$$

Table 1 shows the eight switching states of the power converter presented in Figure 3.4, where each state of the SSI corresponds to an output voltage, a voltage across the input inductor, and a current in the DC bus capacitor. It should be noted that only in state 8, the DC bus capacitor is being charged by the input current from the DC source. Moreover, states 8 and 1 result in a zero output voltage level.

Table 1: SSI Voltages and Currents for Each Switching State

Index j	Switching state $\mathbf{s}^T = [S_1 \ S_2 \ S_3]$	Output voltages $\mathbf{u}'_{\alpha\beta,c} = [U'_{\alpha,c} \ U'_{\beta,c}]^T$	Inductor voltage U_L	Capacitor current i_C
1	[0 0 0]	[0 0]	U_1	i_o
2	[0 0 1]	$[-\sqrt{1/6} \ -1/\sqrt{2}]$	U_1	i_o
3	[0 1 0]	$[-\sqrt{1/6} \ 1/\sqrt{2}]$	U_1	i_o
4	[0 1 1]	$[-\sqrt{2/3} \ 0]$	U_1	i_o
5	[1 0 0]	$[\sqrt{2/3} \ 0]$	U_1	i_o
6	[1 0 1]	$[\sqrt{1/6} \ -1/\sqrt{2}]$	U_1	i_o
7	[1 1 0]	$[\sqrt{1/6} \ 1/\sqrt{2}]$	U_1	i_o
8	[1 1 1]	[0 0]	$U_1 - U_C$	$i_L + i_o$

Now the inverter output voltages can be represented in the $\alpha\beta$ plane, as presented in Figure 3.4(a), where $U_{\phi,c}$ is the maximum voltage amplitude synthesized by the inverter, $U_{\phi,c,l}$ is the maximum value bounding the linear region, which represents the maximum amplitude that a sinusoidal wave, without harmonics, can be reproduced. The states responsible for each position in the plane are shown in Figure 3.4(b).

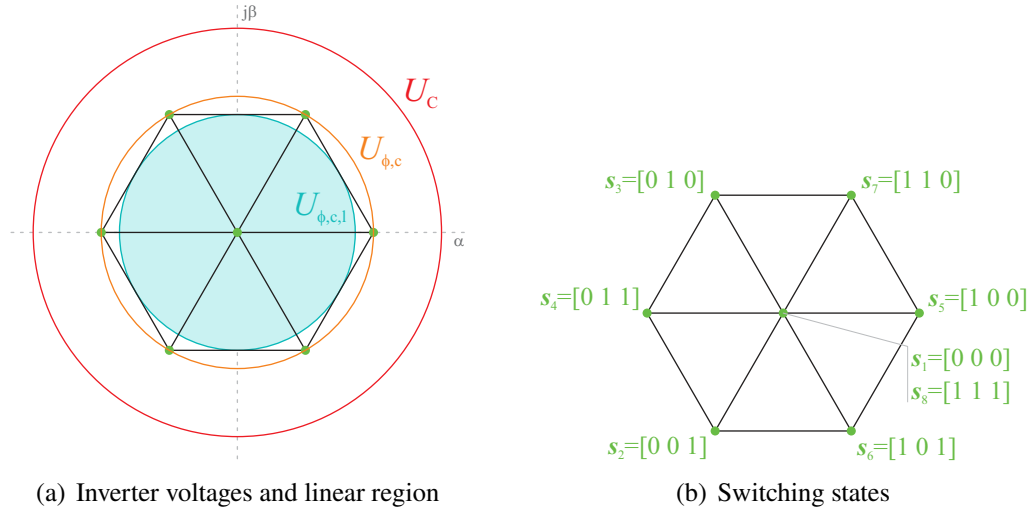


Figure 3.4: Voltages synthesized by the inverter in $\alpha\beta$ frame

3.1 Three-phase multi-Input SSI ¹

Similarly to SSI, the MISSI uses the same eight standard states of a conventional VSI, as depicted in Figure 3.5. Therefore, this converter can synthesize the same output voltages. Once three sources in the three-phase three-leg configuration are employed, the possible electrical schematics for the eight switching states are shown in Table 2.

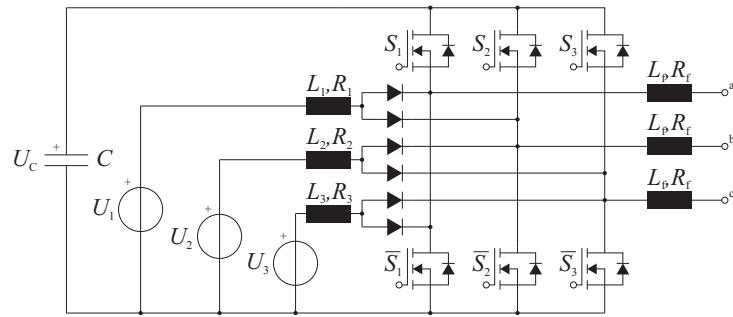


Figure 3.5: Three-phase multi-input split-source inverter [52]

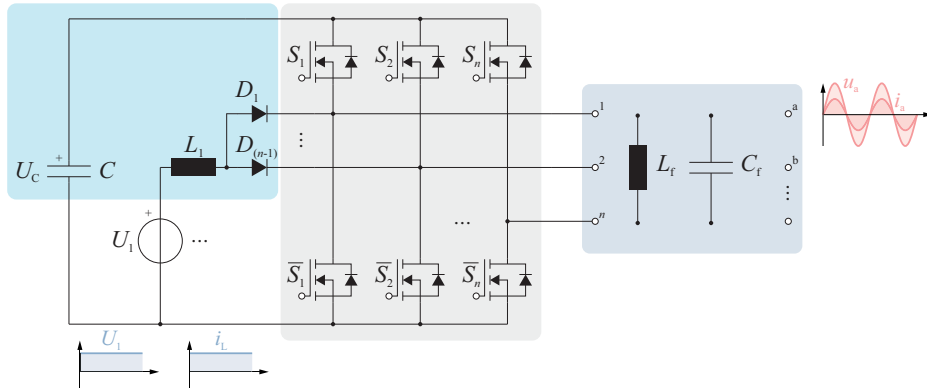
For DC sources connected to the inverter, there are two possible switching states. Figure 3.6 shows a generic schematic for the MISSI, where the individual characteristics for the connection of each DC source are considered for multi-phase systems. In Figure 3.7(a), the equivalent circuit of the first switching state possibility is presented, where the inductor L_x is magnetized. On the other hand, Figure 3.7(b) shows the

¹This section has been published in a similar form in [52].

Table 2: MISSI Voltages and Currents for Each Switching State

Index j	Switching state $s^T = [S_1 S_2 S_3]$	Output voltages $u'_{\alpha\beta,c} = [U'_{\alpha,c} U'_{\beta,c}]^T$	Inductor voltage			Capacitor current i_C
1	[0 0 0]	[0 0]	U_1	U_2	U_3	i_o
2	[0 0 1]	$[-\sqrt{1/6} \quad -1/\sqrt{2}]$	U_1	U_2	U_3	i_o
3	[0 1 0]	$[-\sqrt{1/6} \quad 1/\sqrt{2}]$	U_1	U_2	U_3	i_o
4	[0 1 1]	$[-\sqrt{2/3} \quad 0]$	U_1	$U_2 - U_C$	U_3	$i_{L,2} + i_o$
5	[1 0 0]	$[\sqrt{2/3} \quad 0]$	U_1	U_2	U_3	i_o
6	[1 0 1]	$[\sqrt{1/6} \quad -1/\sqrt{2}]$	U_1	U_2	$U_3 - U_C$	$i_{L,3} + i_o$
7	[1 1 0]	$[\sqrt{1/6} \quad 1/\sqrt{2}]$	$U_1 - U_C$	U_2	U_3	$i_{L,1} + i_o$
8	[1 1 1]	[0 0]	$U_1 - U_C$	$U_2 - U_C$	$U_3 - U_C$	$i_{L,1} + i_{L,2} + i_{L,3} + i_o$

equivalent circuit for the second possibility, when the inductor L_x is demagnetized and the capacitor C is charged.

**Figure 3.6:** Generic scheme for source connection in multi-phase systems

3.1.1 Input sources modeling

In continuous time, the dynamic equations for the inductor voltage $U_{L,x}$ and the capacitor current i_C , when the inductor magnetizes, are given according to

$$\frac{di_{L,x}}{dt} = \frac{U_x}{L_x}, \quad \frac{dU_C}{dt} = \frac{i_o}{C}. \quad (3.3)$$

On the other hand, the dynamic equations for the voltage inductor $U_{L,x}$ and the current

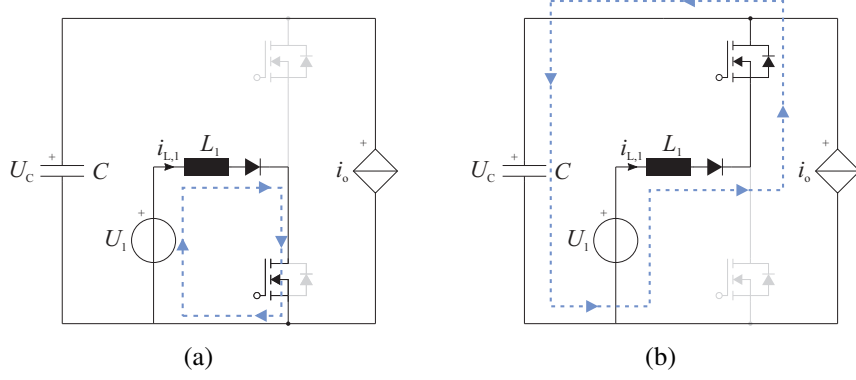


Figure 3.7: (a) Equivalent circuit during the magnetization of L_1 . (b) Equivalent circuit during the demagnetization of L_1

capacitor i_C when the inductor demagnetizes are

$$\frac{di_{L,x}}{dt} = \frac{U_x - U_C}{L_x}, \quad \frac{dU_C}{dt} = \frac{i_o + i_{L,x}}{C}. \quad (3.4)$$

Therefore, using forward Euler approximation, better described in Chapter 4, the discrete-time model for (3.3) can be written as

$$\begin{bmatrix} \hat{i}_{L,x,k+1} \\ \hat{U}_{C,k+1} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} i_{L,x,k} \\ U_{C,k} \end{bmatrix} + \begin{bmatrix} \frac{T_s}{L_x} & 0 \\ 0 & \frac{T_s}{C} \end{bmatrix} \begin{bmatrix} U_{x,k} \\ i_{o,k} \end{bmatrix}, \quad (3.5)$$

and for (3.4) is

$$\begin{bmatrix} \hat{i}_{L,x,k+1} \\ \hat{U}_{C,k+1} \end{bmatrix} = \begin{bmatrix} 1 & -\frac{T_s}{L_x} \\ \frac{T_s}{C} & 1 \end{bmatrix} \begin{bmatrix} i_{L,x,k} \\ U_{C,k} \end{bmatrix} + \begin{bmatrix} \frac{T_s}{L_x} & 0 \\ 0 & \frac{T_s}{C} \end{bmatrix} \begin{bmatrix} U_{x,k} \\ i_{o,k} \end{bmatrix}, \quad (3.6)$$

where i_o can be written as

$$i_{o,k} = (1 - 2s_k^T) \mathbf{i}_{abc,k}, \quad (3.7)$$

where $\mathbf{i}_{abc,k} = [i_{a,k} \ i_{b,k} \ i_{c,k}]^T$ contains the inverter phase currents in natural coordinates at the k th instant.

The current ripple in the input inductors L_x and its maximum operation frequency, as well as the static gain, can be found from the predictive model (3.3) and (3.4) by considering a constant average value for the inductor current in steady state, according to Figure 3.8, where the time for the two possible input-side switching states are defined as inductor magnetization T_{mag} and demagnetization T_{dem} .

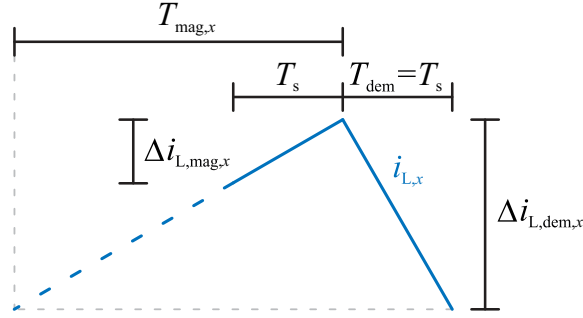


Figure 3.8: Steady state current ripple in the inductor L for a constant value current reference

The current ripple over a sampling period T_s is given by

$$\begin{aligned}\Delta i_{L,mag,x} &= \frac{T_s}{L_x} U_{x,k}, \\ \Delta i_{L,dem,x} &= -\frac{T_s}{L_x} U_{C,k} + \frac{T_s}{L_x} U_{x,k}.\end{aligned}\tag{3.8}$$

From the inductor volt-second balance principle

$$U_{L,mag,x} T_{mag,x} + U_{L,dem,x} T_s = 0.\tag{3.9}$$

Relating the current and time axes

$$\frac{|\Delta i_{L,dem,x}|}{\Delta i_{L,mag,x}} \propto \frac{T_{mag,x}}{T_s}.\tag{3.10}$$

Therefore, the static gain of each input can be written as

$$\frac{U_{C,k}}{U_{x,k}} = 1 + \frac{|\Delta i_{L,dem,x}|}{\Delta i_{L,mag,x}}.\tag{3.11}$$

In addition, the maximum operating frequency of each inductor will depend on the voltage of each source connected to the input of the inverter. Thus, it can be expressed by

$$f_{L,max,x,k} = \frac{U_{x,k}}{T_s U_{C,k}}.\tag{3.12}$$

3.2 Selected space vector analysis

For employment in an application as presented in Figure 3.9, and converter design, a very important point is to find what is the maximum phase voltage that the inverter is able to synthesize in its outputs, considering the DC bus voltage and the voltages of the input sources. For this, a space-vector analysis can be developed, selecting vectors involved in a hierarchical sequence of targets, which will define the operation priorities of the converter. Note that, each switching state can be represented as a switching vector as depicted in Figure 3.10 [167].

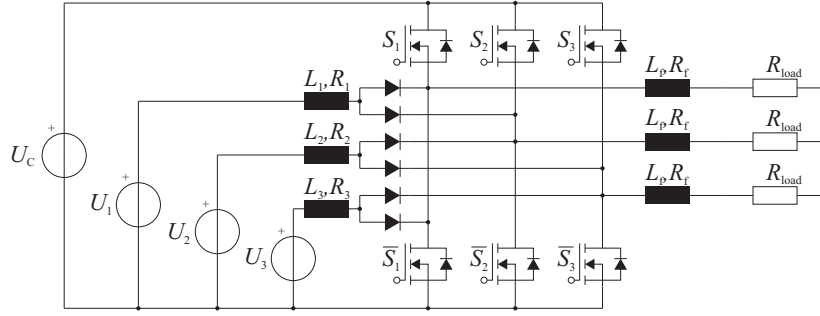


Figure 3.9: Three-phase multi-input split-source inverter supplying a resistive load

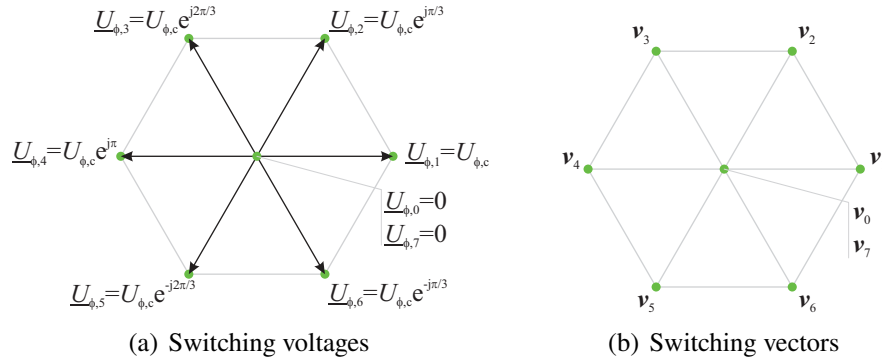


Figure 3.10: Voltage vectors synthesized by the inverter in $\alpha\beta$ frame

For instance, considering an optimization control strategy that prioritizes the input currents, in this example, setting up a weighing factor in a cost function as $\lambda \rightarrow \infty$, such that $\lambda \gg 1$, further explained in Chapter 5, the converter output voltage can be intentionally saturated. Figures 3.11(b),(d),(c),(f) show the simulated converter output voltages, given the input voltage levels. While 3.11(a) and (c) depict the linear region behavior.

Due to the magnitude of the input voltages, the voltage unbalance of the input sources, and the number of input sources employed, the feasible region will assume different positions and shapes.

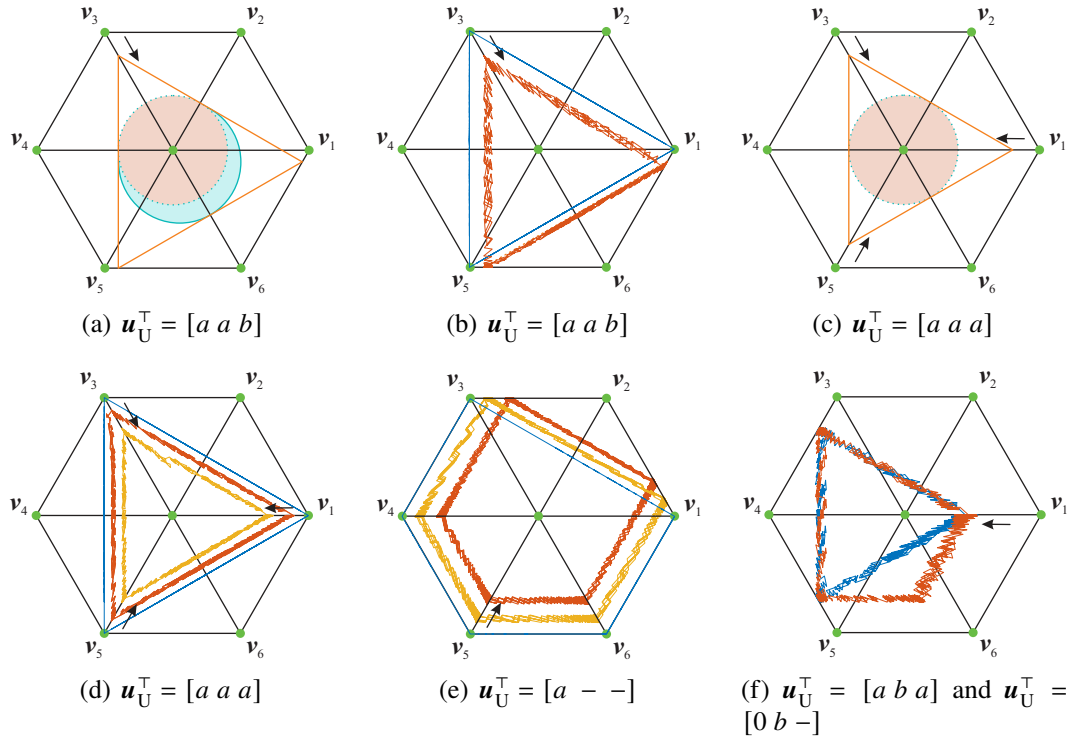


Figure 3.11: Simulated behavior of the linear region for different input voltages, where the value of the DC input voltage $b > a$

According to [168], five steps can be pointed out to implement the space vector modulation for voltage-source inverters [169]:

1. Definition of the possible switching vectors in the output voltage space
2. Identification of the separation planes between the sectors in the output voltage space
3. Identification of the boundary planes in the output voltage space
4. Obtaining of the decomposition matrices
5. Definition of the switching sequence

Defining a generic switching sequence, free from experimental implementation concerns, since it can be chosen depending on the application. For the MISSI, as seen in Table 2, the switching states related to the demagnetization period of the input sources no longer produce zero voltage in the output, as the classical SSI. For a MISSI with three input DC sources, three switching vectors are committed with demagnetization times; they are: $\mathbf{v}_2$, $\mathbf{v}_4$, and $\mathbf{v}_6$. Figure 3.12(a) illustrates these vectors in the $\alpha\beta$ plane. Then, the remaining vectors can be chosen to compose the switching sequence for a given sampling period T_s .

Figure 3.12(b) presents the switching sequence composed of 6 switching vectors.

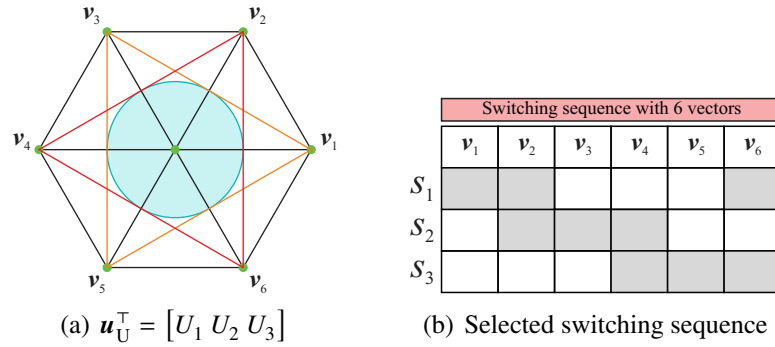


Figure 3.12: Space vector analysis for three-phase MISSI with three input sources

For the sake of simplicity, the null vectors (v_0 and v_7) are neglected. Moreover, it will potentially reduce the common-mode voltage (CMV) as described and developed in [170], where it is stated that depending on the parasitic path, the CMV synthesized by the inverter can result in significant leakage currents, which must be limited for safety reasons.

Considering that injecting power from the input sources is the main objective of the converter, it is therefore inevitable that at each sampling period the active vectors related to the input sources must operate. Once the task of achieving the required duty cycle for each input source has been accomplished, this leads to a residual voltage.

Defining the overall duty cycle as

$$D_U = t_{s\phi} / (t_{sU} + t_{s\phi}), \quad (3.13)$$

in steady-state, where

$$t_{sU} + t_{s\phi} = T_s, \quad (3.14)$$

the static gain for each input stage can be expressed as [14]:

$$\frac{U_C}{U_x} = \frac{1}{1 - D_x}, \quad (3.15)$$

where U_x and U_C represent the input voltage and the DC bus voltage in steady state, respectively, $t_{s\phi}$ represents the time in which active vectors are applied for all the input sources, while t_{sU} is the demagnetizing time associated with the input source inductors.

The dwell time of the vector related to each input

$$\delta_{t,y} = 1 - D_x, \quad (3.16)$$

corresponds directly to the demagnetization of the input inductor of that source. The decomposition matrix can be written as

$$\begin{bmatrix} u_{\alpha,U} \\ u_{\beta,U} \\ 1 \end{bmatrix} = \begin{bmatrix} \nu_2 & \nu_4 & \nu_6 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} \delta_{t,2} \\ \delta_{t,4} \\ \delta_{t,6} \end{bmatrix}, \quad (3.17)$$

and the average value of the resulting voltage

$$\langle u_{\alpha\beta,U} \rangle_{T_s} = \frac{1}{T_s} \left[\int_0^{t_1} \underline{U}_{\phi,2} dt + \int_{t_1}^{t_2} \underline{U}_{\phi,4} dt + \int_{t_2}^{t_{sU}} \underline{U}_{\phi,6} dt \right] = u_{\alpha\beta,U} e^{j\xi}, \quad (3.18)$$

where

$$\begin{aligned} \delta_{t,2} &= (t_1 - 0), \\ \delta_{t,4} &= (t_2 - t_1), \\ \delta_{t,6} &= (t_{sU} - t_2). \end{aligned} \quad (3.19)$$

The output voltages can be represented by joining the decomposition matrix for the remaining switching vectors, responsible for adjusting and finally synthesizing the output voltage with (3.20).

$$\begin{bmatrix} u_{\alpha,\phi} \\ u_{\beta,\phi} \\ 1 \end{bmatrix} = \begin{bmatrix} \nu_1 & \nu_3 & \nu_5 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} \delta_{t,1} \\ \delta_{t,3} \\ \delta_{t,5} \end{bmatrix} + \begin{bmatrix} u_{\alpha,U} \\ u_{\beta,U} \\ 1 \end{bmatrix}. \quad (3.20)$$

Reorganizing the expression and solving for the dwell time of the output voltage control vectors yields:

$$\begin{bmatrix} u_{\alpha,\phi} \\ u_{\beta,\phi} \\ 1 \end{bmatrix} = \mathbf{M}_{\phi} \begin{bmatrix} \delta_{t,1} \\ \delta_{t,3} \\ \delta_{t,5} \end{bmatrix} + \mathbf{M}_U \begin{bmatrix} \delta_{t,2} \\ \delta_{t,4} \\ \delta_{t,6} \end{bmatrix}, \quad (3.21)$$

$$\begin{bmatrix} \delta_{t,1} \\ \delta_{t,3} \\ \delta_{t,5} \end{bmatrix} = \mathbf{M}_{\phi}^{-1} \left(\begin{bmatrix} u_{\alpha,\phi} \\ u_{\beta,\phi} \\ 1 \end{bmatrix} - \mathbf{M}_U \begin{bmatrix} \delta_{t,2} \\ \delta_{t,4} \\ \delta_{t,6} \end{bmatrix} \right). \quad (3.22)$$

Therefore, the modulation strategy can be written as (3.23). Figure 3.13 presents the timeline of the application of the switching vectors sets.

$$\langle u_{\alpha\beta,\phi} e^{j\varphi} \rangle_{T_s} = \langle u_{\alpha\beta,U} e^{j\xi-\pi} \rangle_{T_s} + \langle u_{\alpha\beta,\phi,\text{ref}} e^{j\gamma} \rangle_{T_s}. \quad (3.23)$$

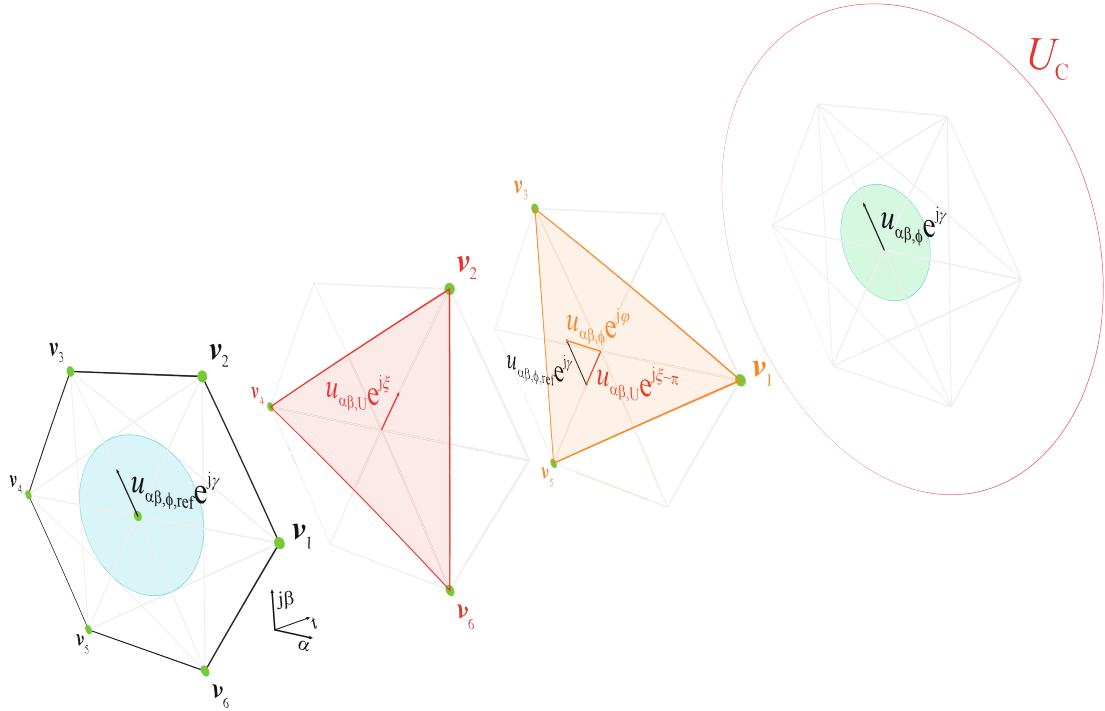


Figure 3.13: Space vector modulation strategy

Considering fixed and continuous input voltages, the last applied vectors will compose a new feasible boundary and therefore a new linear region for the output voltages in inverter operation. Figure 3.14 presents this behavior, where (a) is the solution for balanced input voltages, (b) and (c) are solutions for unbalanced input voltage values. It can be noted that the DC voltage is increasingly reduced, and the maximum synthesizable phase voltage is defined by the shortest distance to the boundary of the feasible region.

From the known expression for the distance between a point and a straight line in Euclidean space [171], the following is obtained:

$$l_v = \frac{|ax + by + c|}{\sqrt{a^2 + b^2}}, \quad (3.24)$$

with $(x_0, y_0) = (0, 0)$ the following is derived:

$$l_v = \frac{|(x_2 - x_1) y_1 - (y_2 - y_1) x_1|}{\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}}. \quad (3.25)$$

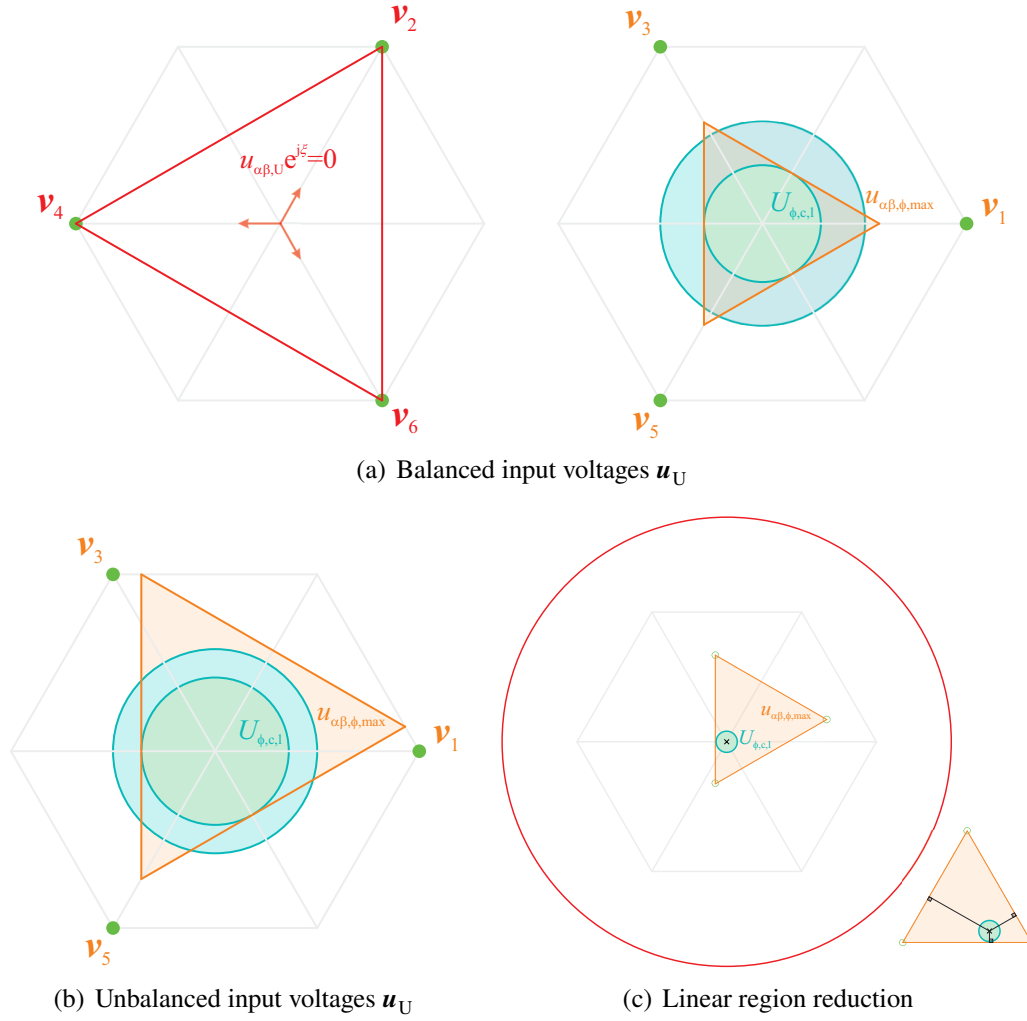


Figure 3.14: Representation of the behavior for the linear region for different input voltages

Organizing the terms to use (3.25) with the switching vectors, it follows that

$$l_v = \frac{|\mathbf{v}_1^\top \mathbf{T}_\Theta^\top (\mathbf{v}_2 - \mathbf{v}_1)|}{\|\mathbf{v}_2 - \mathbf{v}_1\|_2}, \quad (3.26)$$

where $\mathbf{T}_\Theta$ can be called the rotation matrix

$$\mathbf{T}_\Theta^\top = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}, \quad (3.27)$$

and l_v is directly related to the output voltage.

For a general approach, it is useful to organize the equations as follows. The dwell times

vector can be defined as

$$\mathbf{d}_U^\top = \frac{1}{U_C} \begin{bmatrix} U_1 & U_2 & U_3 \end{bmatrix} = \frac{1}{U_C} \mathbf{u}_U^\top. \quad (3.28)$$

Considering the remaining time in a sampling period, it holds that

$$\mathbf{D}_\phi = \mathbf{I} - \mathbf{I} \|\mathbf{d}_U\|_1. \quad (3.29)$$

The selected space vectors can be grouped into two groups, being

$$\mathbf{V}_\phi = \begin{bmatrix} \mathbf{v}_1 & \mathbf{v}_3 & \mathbf{v}_5 \end{bmatrix}, \quad (3.30)$$

and

$$\mathbf{V}_U = \begin{bmatrix} \mathbf{v}_2 & \mathbf{v}_4 & \mathbf{v}_6 \end{bmatrix}. \quad (3.31)$$

The new vertices of the feasible region can be found with

$$\mathbf{V}_{\alpha\beta} = \mathbf{V}_\phi \mathbf{D}_\phi + \mathbf{V}_U \mathbf{d}_U. \quad (3.32)$$

To find the maximum amplitude within the linear region, similarly to (3.26), the matrices are used, reorganizing the terms as

$$\boldsymbol{\Psi}_{\alpha\beta} = \mathbf{V}_{\alpha\beta} \mathbf{T}_\Omega - \mathbf{V}_{\alpha\beta}, \quad (3.33)$$

where $\mathbf{T}_\Omega$ represents a circular shift operation

$$\mathbf{T}_\Omega = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}. \quad (3.34)$$

Solving for $U_{\phi,c,l}$ finally leads to

$$U_{\phi,c,l} = \min \left\{ \boldsymbol{\Psi}_{\alpha\beta}^\top \mathbf{T}_\Theta \mathbf{V}_{\alpha\beta} \circ \left(\boldsymbol{\Psi}_{\alpha\beta}^\top \boldsymbol{\Psi}_{\alpha\beta} \right)^{\circ - \frac{1}{2}} \circ \mathbf{I} \mathbf{1} \right\}. \quad (3.35)$$

Relating to natural coordinates, the maximum sinusoidal phase voltage that can be

synthesized by the inverter is

$$U_{(a,b,c),c,l} = \sqrt{\frac{2}{3}} U_{\phi,c,l}, \quad (3.36)$$

which means that, for instance, the converter can be connected to a power system with phase voltages lower than

$$\underline{U}_a = \frac{U_{\phi,c,l}}{\sqrt{3}} e^{j0}. \quad (3.37)$$

Solving for a three-phase MISSI with three sources, the maximum phase voltage amplitude in $\alpha\beta$ -frame can be determined by

$$U_{\phi,c,l} = \min \left\{ \left[\begin{array}{l} \frac{\sqrt{6}(\delta_{t,1}+\delta_{t,2}+\delta_{t,3}-1)\left(\frac{\delta_{t,1}}{3}-\frac{\delta_{t,2}}{6}+\frac{\delta_{t,3}}{3}-\frac{1}{6}\right)}{|\delta_{t,1}+\delta_{t,2}+\delta_{t,3}-1|} \\ -\frac{\sqrt{6}(\delta_{t,1}+\delta_{t,2}+\delta_{t,3}-1)(2\delta_{t,1}-4\delta_{t,2}-4\delta_{t,3}+2)}{12|\delta_{t,1}+\delta_{t,2}+\delta_{t,3}-1|} \\ \frac{\sqrt{6}(\delta_{t,1}+\delta_{t,2}+\delta_{t,3}-1)(4\delta_{t,1}+4\delta_{t,2}-2\delta_{t,3}-2)}{12|\delta_{t,1}+\delta_{t,2}+\delta_{t,3}-1|} \end{array} \right] \right\}. \quad (3.38)$$

The approach can also be extended to analyze multiphase and multilevel systems, as exemplified in Figure 3.15. Now, the same simulation discussed at the beginning of this section can be performed, comparing the calculated feasible region with the simulated behavior. Figure 3.16 presents both the saturated output voltage and the calculated feasible boundary $u_{\alpha\beta,\phi,\max}$. Reducing the weighting factor, Figure 3.17 presents the behavior of the input currents and output voltages. As λ decreases, the converter tends to use the switching vectors to better synthesize the output voltages, while the ripple of the input currents increases.

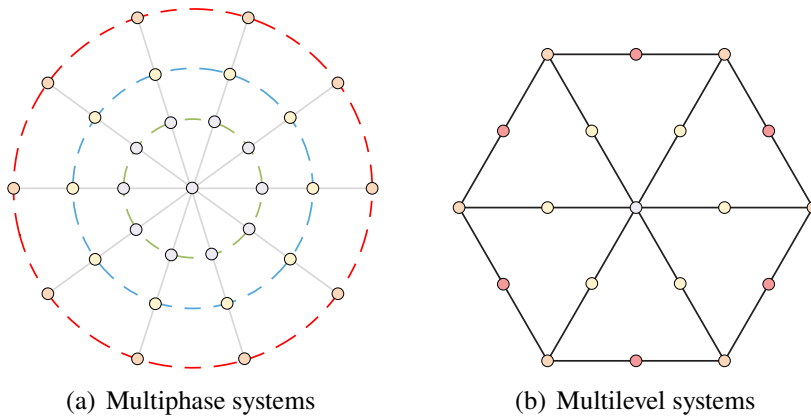


Figure 3.15: Voltage vectors synthesized by the inverter in $\alpha\beta$ frame

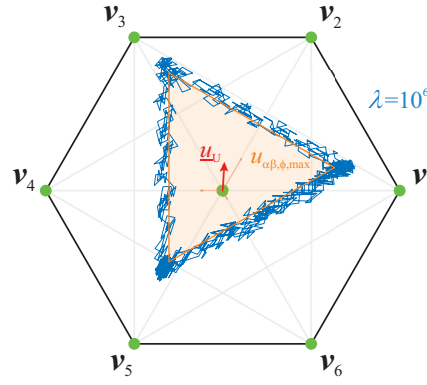


Figure 3.16: Simulated and calculated feasible region in $\alpha\beta$ reference frame

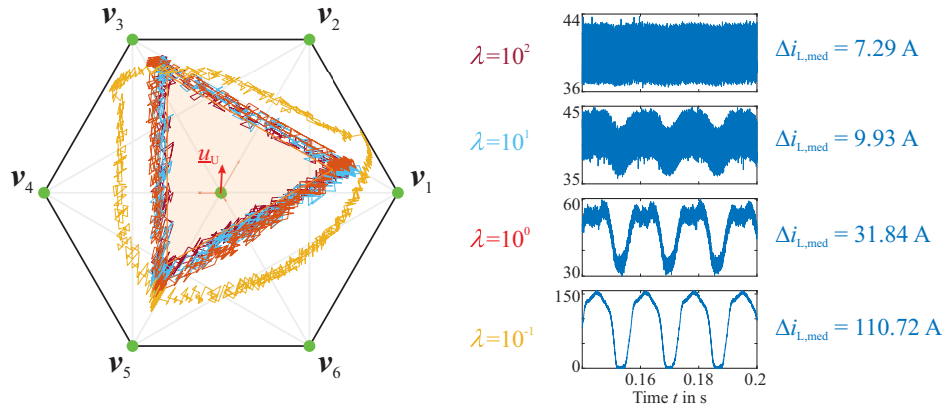


Figure 3.17: Simulated voltages and currents in $\alpha\beta$ reference frame for weighting factor variation

With the voltage value $U_{\phi,c,l}$, it is possible to find the point at which the converter operates close to its limits, as exemplified in Figure 3.18. In Figure 3.18(a), the MISSI operates with a single source, and its maximum synthesizable voltage is shown in purple. By gradually reducing the output voltage reference (dark blue) to a suitable value, a linear operating region can be achieved (light blue). The shape and displacement of the feasible region depend on the magnitude of the input voltage and its placement within the topology. Figure 3.18(b) illustrates similar behavior for three balanced input voltages, while Figure 3.18(c) presents the case for three unbalanced input voltages. It is noteworthy that an unbalance among the input voltages significantly reduces the linear operating region. To synthesize the same output voltage under unbalanced conditions, a higher DC-bus voltage magnitude is required.

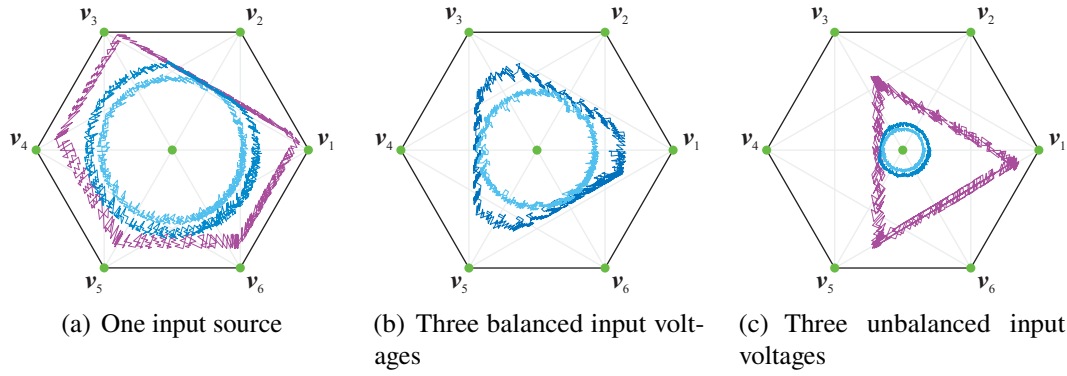


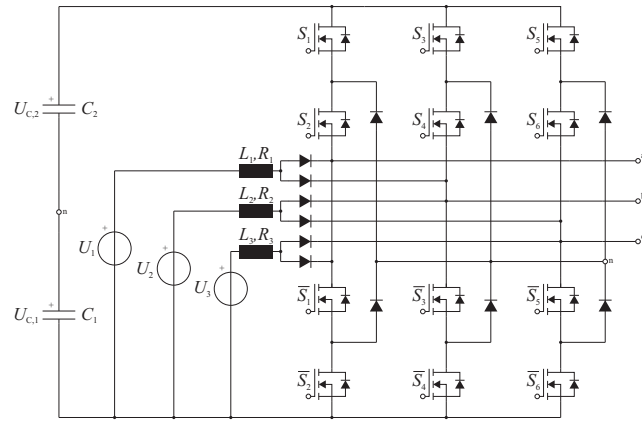
Figure 3.18: Simulated output voltages for the MISSI in $\alpha\beta$ reference frame for different operating conditions

3.3 Multilevel topologies

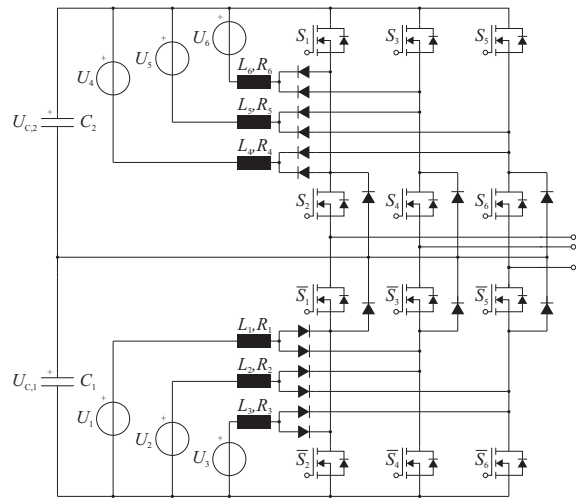
From the already discussed methods, a set of other topologies can be derived and controlled. The three-level diode clamped inverter (DCI) can accommodate the input stage in different ways. This topology is interesting because it provides lower harmonic content of the output voltages. Figure 3.19 shows different topological arrangements for the diode clamped multi-input split-source inverter (DC-MISSI), wherein (a) the input diodes are connected to the mid-point of the inverter legs, while in (b) the inputs are connected to the same point as the inverter clamping diodes, allowing the connection of up to six input sources. This topology enables voltage balancing across the inverter capacitors, even in the presence of unequal input DC sources, and also presents lower current stresses on the employed switches [13].

Applying a simulated voltage saturation, it can be seen from Figure 3.20 the results for the feasibility of the output voltage synthesis, where (a) and (b) show the results for the mid-point connection with three unbalanced and balanced voltage inputs, respectively. In Figure 3.20(c), the results corresponding to the connection with balanced voltages for the topology in Figure 3.19(b) are presented. It is worth noting that with different configurations, different switching vectors are committed with the input voltages. The numbers in Figure 3.20(c) indicate (1) only lower input sources connected, (2) only upper sources connected, and (3) six input sources connected.

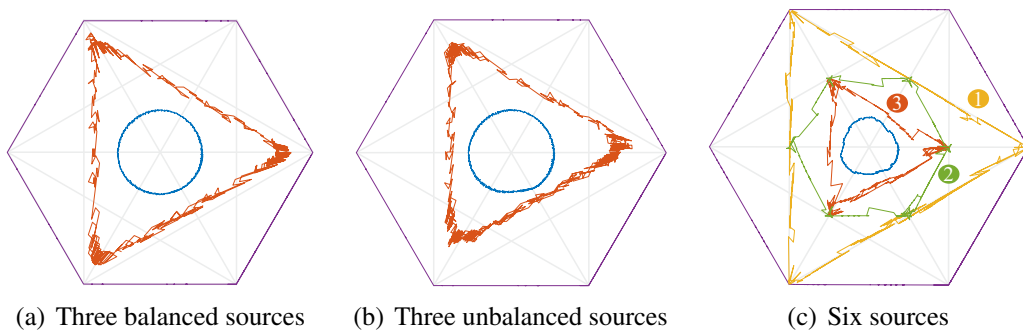
Even if the DCI has a greater number of switching vectors, the presence of multiple input sources will limit its operation. In the same way, the flying capacitor inverter (FCI) can be used to inject power from multiple sources. The flying capacitor multi-input split-source inverter (FC-MISSI) is depicted in Figure 3.21, as well as its voltage saturation behavior for unbalanced voltage inputs in Figure 3.22.



(a) DC-MISSI with three sources



(b) DC-MISSI with six sources

Figure 3.19: DCI-based multi-input multilevel converters**Figure 3.20:** Simulation of the saturation of the output voltages for the DC-MISSI

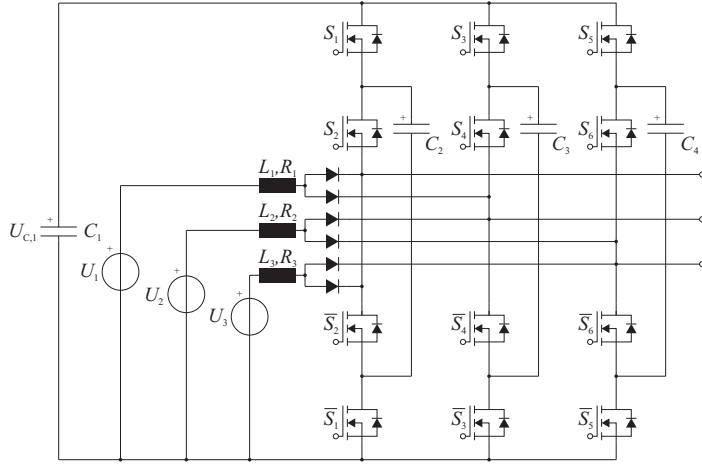


Figure 3.21: FC-MISSI with three inputs

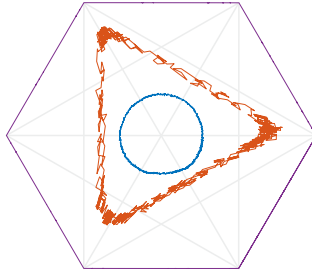


Figure 3.22: Simulation of the saturation of the output voltages for the FC-MISSI

3.4 Nine-switch topology

The nine-switch inverter offers potential cost savings and reduced system complexity compared to conventional multi-inverter systems [77], [172]. With six AC terminals, this topology has been increasingly employed to provide flexibility to inverter-based systems, allowing them to feed different AC loads or even process energy as a rectifier and inverter simultaneously [173], [174]. Such a topology was also employed using the split-source inverter concept [14], and it is a potential solution to increase the number of independent inputs [52]. Figure 3.23 presents the six-input configuration for the nine-switch multi-input split-source inverter (9S-MISSI). The DC-bus voltage, power management from the variable DC input voltage sources, and grid support on the PCC are control targets.

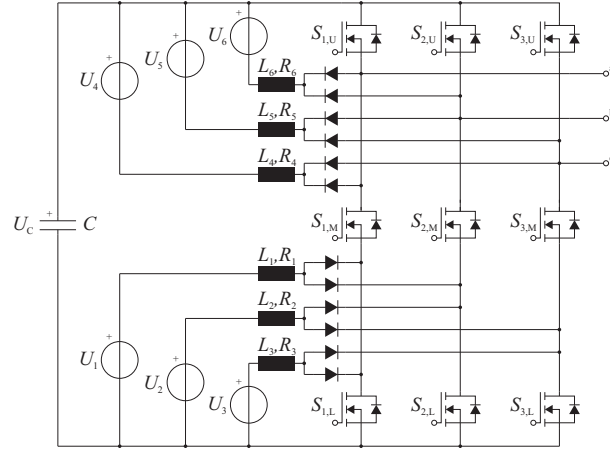


Figure 3.23: 9S-MISSI with six inputs

3.5 Bidirectional topologies ²

Bidirectionality can be easily achieved using the nine-switch topology. Figure 3.24 shows a configuration with six common grounded input sources, where half of them are bidirectional. This topology can be suited to the integration of different low-voltage low-power DC sources, such as battery, PV, and fuel cell, where the power transfer from one source to another can occur with a reduced path through the converter.

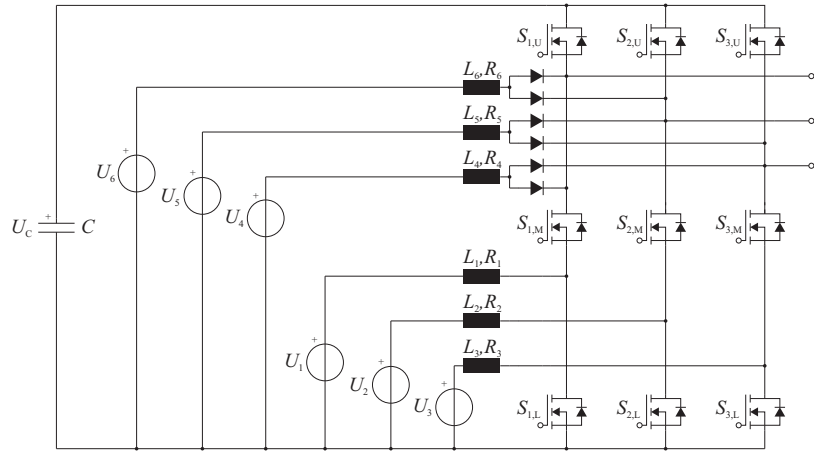


Figure 3.24: Bidirectional 9S-MISSI with common-grounded six inputs

Figure 3.25 shows the simulation results for the converter feeding an inductive-resistive load with the output voltage reference $\underline{U}_{a,\text{ref}} = 127$ V. This test shows the operational limits of the converter, specifically the voltage limits that the converter can synthesize with $\mathbf{u}_U^\top = [35 \ 35 \ 95 \ 35 \ 35 \ 95]$ V. Figure 3.25(a) shows that the calculated feasible

²This section has been published in a similar form in [175].

region corresponded well with the simulated feasible region. Based on this calculation, it is possible to determine an operating point where the output voltage can be synthesized within the linear region as $\underline{U}_a = 87.2$ V. In this case, the effect of the reduction of the linear region is observable. On the other hand, Figure 3.25(b) shows that with balanced input voltages $\mathbf{u}_U^\top = [35 \ 35 \ 35 \ 35 \ 35 \ 35]$ V (orange), the same output voltage can be well synthesized and $\underline{U}_a = 139$ V. Figure 3.25(c) presents the behavior of the weighting factor variation with $\mathbf{u}_U^\top = [45 \ 35 \ 55 \ 35 \ 65 \ 85]$ V and $\underline{U}_a = 68.36$ V. When this factor is decreased, the importance of the output voltages increases. Consequently, the ripple resulting in the input sources increases, and the tendency of the output voltages is to move away from the calculated linear region, prioritizing the output references.

Regarding the DC bus capacitor design, it can be considered the worst-case scenario for the voltage ripple ΔU_C , which will depend on the control or modulation strategy employed [11]. For unidirectional and bidirectional input sources with different parameters, it can be expressed as

$$C \geq \frac{T_s}{\Delta U_{C,h}} \sum_{x=1}^6 \frac{P_x}{U_x}. \quad (3.39)$$

The nine-switch inverter offers a high degree of freedom as it enables the possibility of using nine input sources, as demonstrated in Appendix C. An alternative topological change to make the two-level three-phase MISSI bidirectional is also evaluated.

With the structural and operational aspects of the split-source inverter and its multi-input extension established, Chapter 4 moves toward the dynamic analysis of the system. It develops the mathematical models in both $\alpha\beta$ and synchronous dq reference frames and derives the state-space equations necessary for system control and stability analysis.

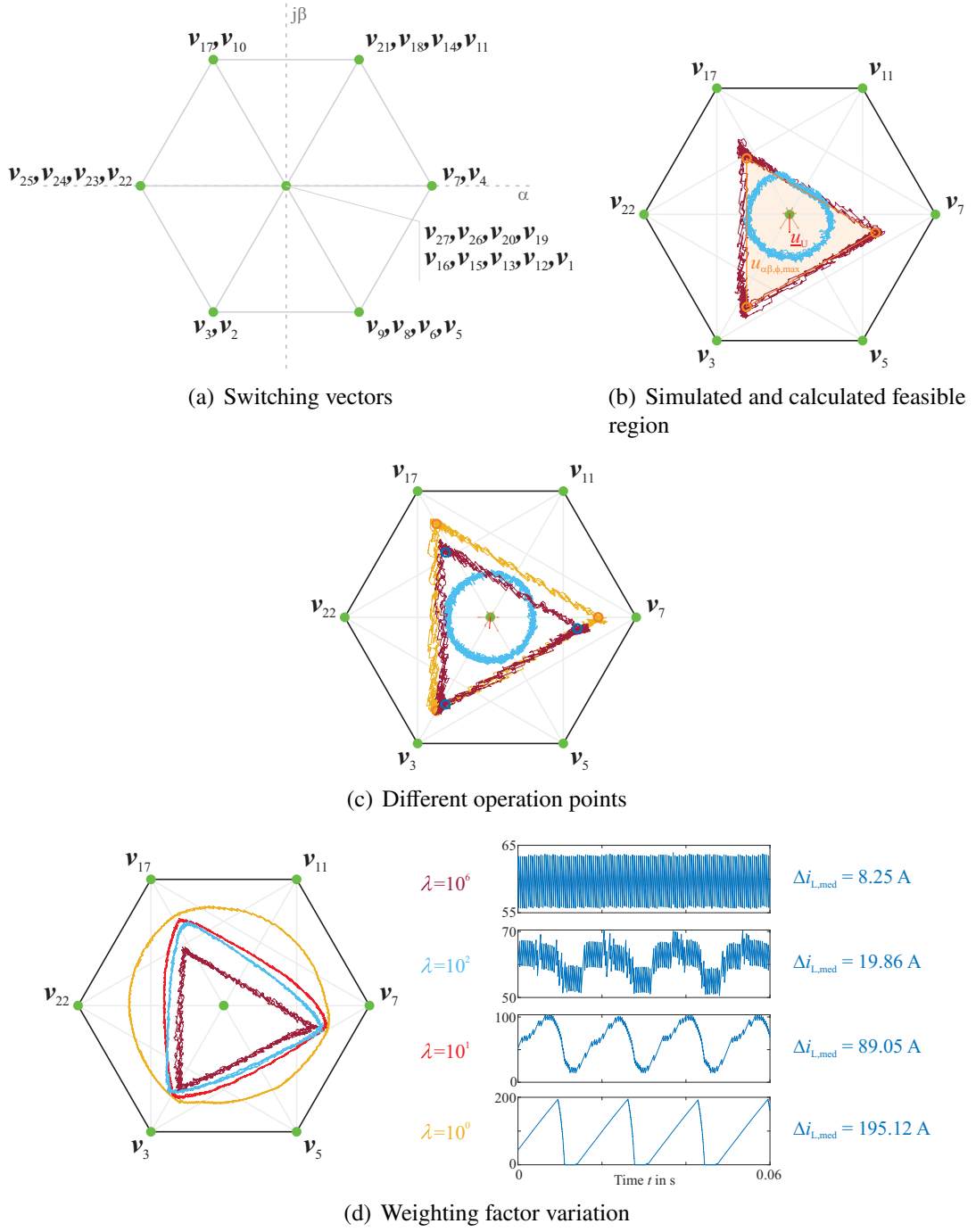


Figure 3.25: Simulated output voltages in $\alpha\beta$ reference frame and input current $i_{L,4}$

4 System Description and Modeling

This chapter outlines the overall system model for the inverter application and operation connected to the grid, SEIG-based power systems, and for supplying isolated loads. It focuses on developing the dynamic models of the components that make up the generation system, with particular emphasis on the AC side. It derives state-space representations in both the $\alpha\beta$ and synchronous dq reference frames, forming the foundation for control design. Additionally, the chapter provides equations required for implementing both linear and nonlinear control strategies.

4.1 Output current modeling

Looking like the utility grid connection point model, according to [143], the steady-state model of SEIG can be considered and a Thévenin equivalent can be found. Then, to simplify the system modeling, the steady-state model of the SEIG is represented as an equivalent of a voltage source in series with an inductor (L_g, R_g), as shown in Figure 4.1, together with a 3-Leg VSI represented as controlled voltage sources output filters.

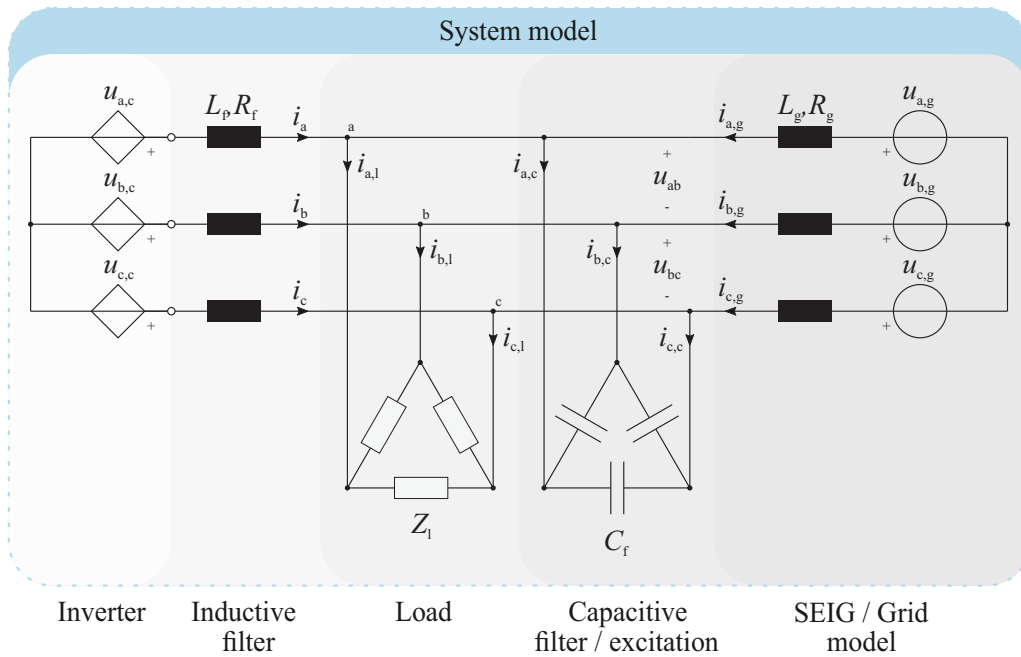


Figure 4.1: Three-phase equivalent circuit for SEIG shunt compensation

In applications involving connection to three-phase electric machines or the grid, it is customary to develop the control system using $\alpha\beta 0$ coordinates and synchronous dq0 coordinates, whereby the tracking problem is transformed into a regulation problem.

The decomposition is carried out based on the state-space equations in stationary abc coordinates, employing appropriate transformation matrices. Thus, by representing output currents in a stationary $\alpha\beta$ frame, the state-space model is obtained using the power-invariant Clarke transformation. Given a three-phase three-wire circuit, the 0 axis will not provide a path for common-mode current in the inverter. Therefore, the 0 coordinate axis can be disregarded [143]. The representation of the plant in state space using $\alpha\beta$ coordinates yields:

$$\begin{bmatrix} \dot{i}_\alpha \\ \dot{i}_\beta \end{bmatrix} = \begin{bmatrix} -\frac{R_f}{L_f} & 0 \\ 0 & -\frac{R_f}{L_f} \end{bmatrix} \begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix} + \begin{bmatrix} \frac{1}{L_f} & 0 \\ 0 & \frac{1}{L_f} \end{bmatrix} \begin{bmatrix} u_{\alpha,c} - u_\alpha \\ u_{\beta,c} - u_\beta \end{bmatrix}, \quad (4.1)$$

$$\begin{bmatrix} \dot{u}_\alpha \\ \dot{u}_\beta \end{bmatrix} = \begin{bmatrix} -\frac{1}{Z_1 C_f} & 0 \\ 0 & -\frac{1}{Z_1 C_f} \end{bmatrix} \begin{bmatrix} u_\alpha \\ u_\beta \end{bmatrix} + \begin{bmatrix} \frac{1}{C_f} & 0 \\ 0 & \frac{1}{C_f} \end{bmatrix} \begin{bmatrix} i_\alpha + i_{\alpha,g} \\ i_\beta + i_{\beta,g} \end{bmatrix}. \quad (4.2)$$

From the model, the dynamics of the output currents through the inductive filter in $\alpha\beta$ -frame are given by

$$\dot{\mathbf{i}}_{\alpha\beta}(t) = \mathbf{A} \mathbf{i}_{\alpha\beta}(t) + \mathbf{B} \left(\mathbf{u}_{\alpha\beta,c}(t) - \mathbf{u}_{\alpha\beta}(t) \right), \quad (4.3)$$

where

$$\mathbf{A} = -\frac{R_f}{L_f} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \text{ and } \mathbf{B} = \frac{1}{L_f} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad (4.4)$$

$\mathbf{u}_{\alpha\beta}(t) = [u_\alpha(t) \ u_\beta(t)]^\top$ are grid voltages in $\alpha\beta$ -frame and $\mathbf{u}_{\alpha\beta,c}(t)$ are the inverter output voltages in $\alpha\beta$ -frame.

The dynamics of the output voltages on the output capacitor filter in $\alpha\beta$ -frame are given by

$$\dot{\mathbf{u}}_{\alpha\beta}(t) = \mathbf{A}_u \mathbf{u}_{\alpha\beta}(t) + \mathbf{B}_u \left(\mathbf{i}_{\alpha\beta}(t) - \mathbf{i}_{\alpha\beta,g}(t) \right), \quad (4.5)$$

where

$$\mathbf{A}_u = -\frac{1}{Z_1 C_f} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \text{ and } \mathbf{B}_u = \frac{1}{C_f} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}. \quad (4.6)$$

The continuous-time linear system generic solution for the state-space model is

$$\mathbf{x}(t) = e^{\mathbf{A}t} \mathbf{x}(0) + \int_0^t e^{\mathbf{A}(t-\tau)} \mathbf{B} \mathbf{u}(\tau) d\tau, \quad (4.7)$$

and the discretization can be given using the integral approximation method, considering

a zero-order hold (ZOH) for the input $\mathbf{u}$, being constant during a fixed given sampling period T_s . Then it leads to

$$\mathbf{x}_{kT_s} = e^{AkT_s}\mathbf{x}(0) + \int_0^{kT_s} e^{A(kT_s-\tau)} \mathbf{B}\mathbf{u}(\tau) d\tau, \quad (4.8)$$

$$\mathbf{x}_{(k+1)T_s} = e^{AT_s} \left[e^{AkT_s}\mathbf{x}(0) + \int_0^{kT_s} e^{A(kT_s-\tau)} \mathbf{B}\mathbf{u}(\tau) d\tau \right] + \int_{kT_s}^{(k+1)T_s} e^{A[(k+1)T_s-\tau]} \mathbf{B}\mathbf{u}(\tau) d\tau, \quad (4.9)$$

using substitution [176] and defining $\mathbf{x}_{kT_s}$ as $\mathbf{x}_k$, it leads to

$$\mathbf{x}_{k+1} = e^{AT_s}\mathbf{x}_k - \int_{\sigma(kT_s)}^{\sigma([k+1]T_s)} e^{A\sigma} d\sigma \mathbf{B}\mathbf{u}_k, \quad (4.10)$$

and therefore:

$$\mathbf{x}_{k+1} = e^{AT_s}\mathbf{x}_k + A^{-1} \left(e^{AT_s} - \mathbf{I} \right) \mathbf{B}\mathbf{u}_k. \quad (4.11)$$

Considering the systems analyzed in this work, for digital implementation in a DSP, the discussed discretization presents issues due to the heavy matrix exponential and integral operations involved. On the other hand, it is easier and faster to calculate an approximate discrete model with a suitable precision, based on small time steps. Then, according to the definition of the derivative, for the continuous time equation:

$$\dot{\mathbf{x}}(t) = \frac{d\mathbf{x}}{dt} = \lim_{\Delta t \rightarrow 0} \frac{\mathbf{x}(t + \Delta t) - \mathbf{x}(t)}{\Delta t}. \quad (4.12)$$

When the sample period is quite small, or at least small enough for the application, another well-suited solution can be taken into account. If calculated $\mathbf{x}(t)$ from

$$t = kT_s, \quad (4.13)$$

to

$$t = (k + 1)T_s, \quad (4.14)$$

for $k = 0, 1, \dots$, then the discrete-time form of the currents model (4.3) for a sampling time T_s can be used to predict the future value of the output currents by the forward Euler approximation as

$$\frac{d\mathbf{x}}{dt} \approx \frac{\Delta \mathbf{x}}{\Delta t} = \frac{\mathbf{x}_{k+1} - \mathbf{x}_k}{T_s}, \quad (4.15)$$

then:

$$\hat{\mathbf{i}}_{\alpha\beta,k+1} = (\mathbf{I} + \mathbf{A}T_s) \mathbf{i}_{\alpha\beta,k} + \mathbf{B}T_s (\mathbf{u}_{\alpha\beta,c,k} - \mathbf{u}_{\alpha\beta,k}), \quad (4.16)$$

where $\mathbf{I}$ is a identity matrix of order 2. $\mathbf{u}_{\alpha\beta,k} = [u_{\alpha,k} \ u_{\beta,k}]^T$ are grid voltages in $\alpha\beta$ frame, $\mathbf{u}_{\alpha\beta,c,k}$ inverter output voltages in $\alpha\beta$ frame. In addition, considering the control of a power converter, a step forward can be included to compensate for the time for digital implementation, according to [136].

Considering controlling continuous variables as a design definition for the outer loop, the transformation of the stationary frame abc to the synchronous frame dq (Park transform) can be applied in the model, resulting in (4.17), alternatively. The state-space voltages in the time domain can be written as

$$\dot{\mathbf{u}}_{dq}(t) = \mathbf{A}_{dq} \mathbf{u}_{dq}(t) + \mathbf{B}_{dq} (\mathbf{i}_{dq}(t) + \mathbf{i}_{dq,g}(t)), \quad (4.17)$$

where

$$\begin{bmatrix} \dot{u}_d \\ \dot{u}_q \end{bmatrix} = \begin{bmatrix} -\frac{1}{Z_l C_f} & \omega_u \\ -\omega_u & -\frac{1}{Z_l C_f} \end{bmatrix} \begin{bmatrix} u_d \\ u_q \end{bmatrix} + \frac{1}{C_f} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} i_d + i_{d,g} \\ i_q + i_{q,g} \end{bmatrix}. \quad (4.18)$$

For controller design purposes, the transfer function can be found by

$$\mathbf{G}_{dq}(s) = \mathbf{C}_{dq} (s\mathbf{I} - \mathbf{A}_{dq})^{-1} \mathbf{B}_{dq}, \quad (4.19)$$

for specific output matrices $\mathbf{C}_{dq}$.

4.2 DC bus capacitor modeling

The DC bus model can be found by relating the active power through the DC to AC side by Tellegen's theorem, which states that the sum of the instantaneous power in all segments of a circuit is equal to zero:

$$C \frac{dU_C}{dt} U_C = -P_c = - (u_\alpha i_\alpha + u_\beta i_\beta) = - (u_d i_d + u_q i_q), \quad (4.20)$$

where P_c is the active power delivered to the grid by the converter. For the DC bus modeling, the power relation from the energy equation can also be used. Moreover, for the DC sources connected to the input of the power converter, it can be written

$$\frac{C}{2} \frac{dU_C^2}{dt} = -P_c + \sum_{x=1}^m U_x i_{L,x}, \quad \text{for } m \in \{1, 2, \dots, n\}, \quad (4.21)$$

where the product $U_x i_{L,x}$ is the power absorbed from the m primary sources.

4.2.1 DC bus capacitor with electronic load

Assuming $u_q = 0$, and considering an ELC connected to the DC bus, the capacitor C power dynamics in the continuous time domain can be written as

$$\frac{C}{2} \frac{dU_C^2}{dt} = -\frac{(U_C d_{\text{ELC}})^2}{R_{\text{ELC}}} + u_d i_d + \sum_{x=1}^m U_x i_{L,x}, \quad (4.22)$$

where d_{ELC} is the duty cycle of the ELC power switch, and R_{ELC} is the resistive load related to this equipment.

By assuming the last term of (4.21) as a disturbance, a proportional-integral controller can be used to provide control of the DC bus voltage in the discrete-time domain [48], for instance, as:

$$\begin{aligned} x_{\text{PI},k+1} &= x_{\text{PI},k} + T_s e_{C,k}, \\ P_{c,\text{ref},k} &= K_I x_{\text{PI},k} + K_P e_{C,k}, \end{aligned} \quad (4.23)$$

where K_P and K_I are respectively the proportional and integral gain of the PI controller, and

$$e_{C,k} = U_{C,\text{ref},k}^2 - U_{C,k}^2. \quad (4.24)$$

According to [151], the active and reactive power references can be related to the reference current by

$$\begin{bmatrix} i_{\alpha,\text{ref},k} \\ i_{\beta,\text{ref},k} \end{bmatrix} = -\frac{1}{u_{\alpha,k}^2 + u_{\beta,k}^2} \begin{bmatrix} u_{\alpha,k} & u_{\beta,k} \\ u_{\beta,k} & -u_{\alpha,k} \end{bmatrix} \begin{bmatrix} P_{c,\text{ref},k} \\ Q_{c,\text{ref},k} \end{bmatrix}. \quad (4.25)$$

With the dynamic models established, the following chapter builds on this foundation by introducing control strategies for both grid-tied and off-grid operation. It presents methods to regulate voltage and frequency at the converter terminals, also contributing to power quality. Moreover, the chapter explores operation in GFL and GFM modes, and the integration of grid-supporting functionalities. The scaling procedure for large-scale system assessments is also addressed.

5 System Operation and Control

This chapter presents the control strategies developed for the MISSI in both grid-tied and off-grid operation modes. These strategies are designed to regulate the voltage amplitude and frequency at the SEIG terminals, corresponding to the PCC, and to reduce harmonic distortion. The chapter details the application of MPC for internal control loops, discusses digital implementation aspects, and outlines the design considerations for inner and outer loop controllers. Additionally, it addresses ancillary control functions and the use of model aggregation for large-scale system evaluations.

An MPC strategy can be developed for the converter to investigate its operation, since this control easily explores the discrete nature of the inverter states as depicted in Table 2, showcasing the feasibility for the application, as well as being a suitable technique with good dynamic performance and low complexity hardware requirements, not requiring field programmable gate arrays (FPGA) or modulators. On the other hand, it demands high processing capacity.

The system configuration for grid-tied operation is presented in Figure 5.1. MPC aims to predict the future behavior of the variables over a known period. Therefore, an optimal vector must be defined in order to drive the variables of interest as close as possible to the desired reference values. For isolated operation, the system configuration is presented in Figure 5.2. The GFM control technique is employed, where the output voltages and its frequency are the references.

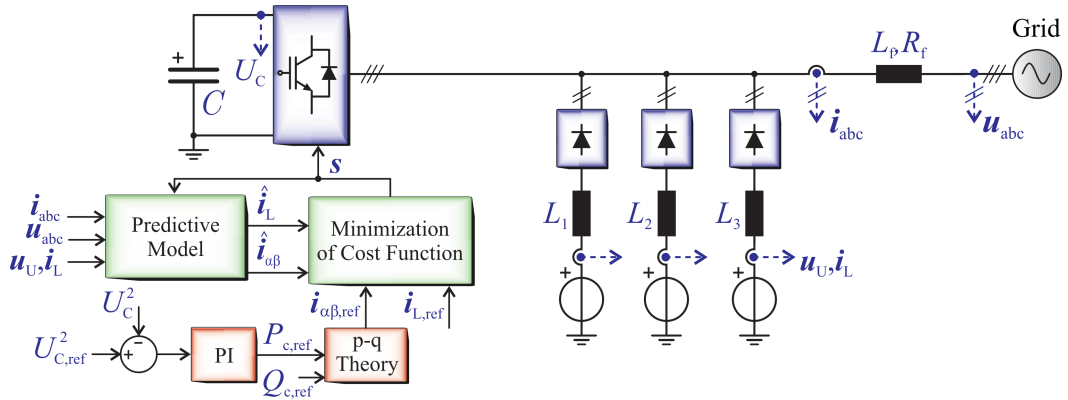


Figure 5.1: Block diagram of the control scheme and the grid-connected MISSI using an L filter

according to standards [177], [178]. MPC aims to predict the future behavior of the variables over a known period. Therefore, an optimal vector must be defined to drive the controlled variables as close as possible to desired reference values. In addition, the control system uses a Kalman filter synchronization method, well-suited for the off-grid application [56].

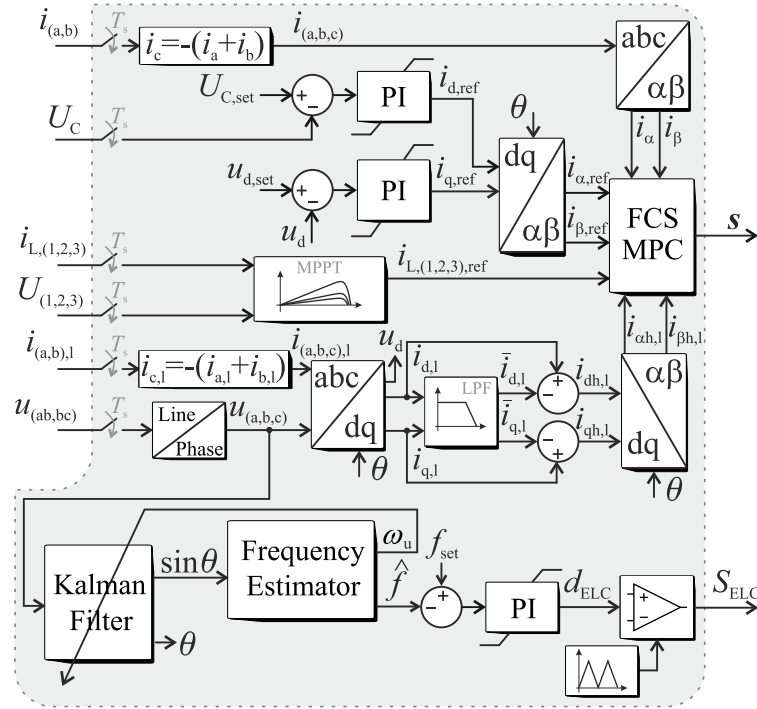


Figure 5.4: Block diagram of control system

This system has an outer control loop for the DC bus voltage, where the control action is the direct axis reference current. In addition, there is a control loop of the magnitude of the AC bus voltage u_d . The output of this loop is the quadrature axis reference current. The MISSI inputs stage control loop can be designed for the inductor currents $i_{L,(1,2,3)}$ as much as for the voltages of the capacitors on the PV side $U_{(1,2,3)}$.

The harmonics and power imbalances caused by some types of load can be identified through the alternating components of the load currents on dq synchronous axes [151]. Therefore, for harmonic mitigation, the alternate portions of the load current signals should be compensated [143]. For that, it is necessary to separate the alternate and the continuous portions. It can be done using a first-order low-pass filter (LPF) given by:

$$F_{\text{LPF}}(s) = \frac{\omega_{\text{LP}}}{s + \omega_{\text{LP}}}. \quad (5.1)$$

5.2 Control system for ancillary services ⁴

The test system for evaluating ancillary services is depicted in Figure 5.5. Load disturbances, generator disconnection, and different equipment arrangements are performed on a low-power radial microgrid employing SEIG generators, which operate with fixed rotor speed, above synchronous speed. The frequency control is performed by electronic load controllers as smart nonlinear loads, which can represent non-dispatchable energy storage such as hydrogen electrolyzers and heating systems. In addition, the positions of the test elements are highlighted in Figure 5.5 regarding the ELC 1 and 2, the secondary generator 3, and fault occurrence 4.

In this system, PCC voltages are regulated through reactive power control. Load currents are measured and filtered for harmonic component extraction and compensation. The SEIG should assume the fundamental component and active portion of the load current. The location of the proposed controller is highlighted in the control block diagram depicted in Figure 5.6 and will be discussed in Section 5.7.

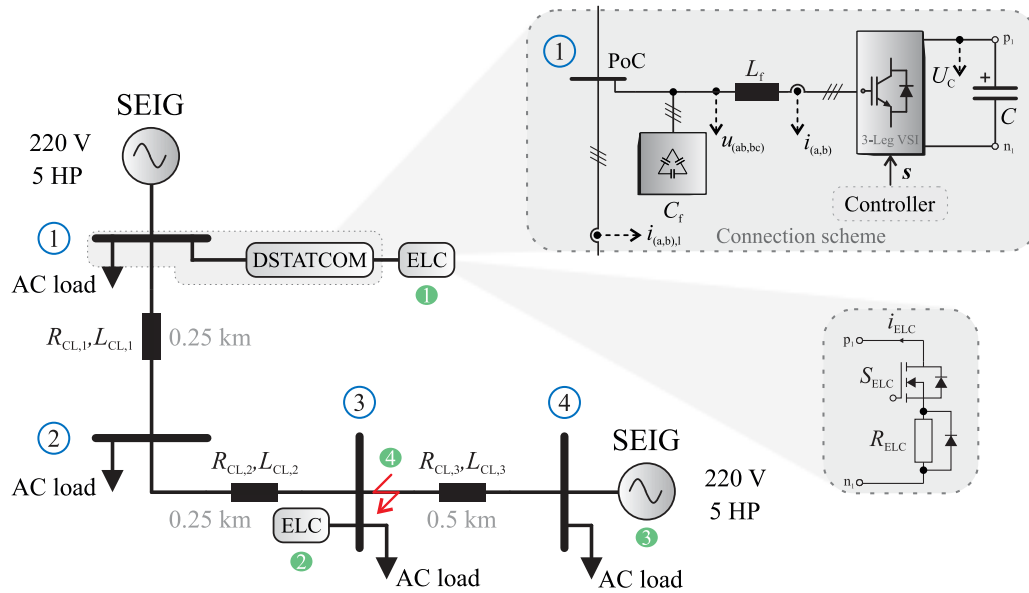


Figure 5.5: Single-line diagram of the SEIG-based tested microgrid

The DC bus capacitor voltage control loop is related to the active power through the inverter. The voltage reference has a fixed value related to the point of operation of the power converter. On the other hand, the DSTATCOM can allow a suitable range of capacitor voltage without compromising the operation or integrity of the VSI. The energy available can then be written as (5.2). In this sense, the DSTATCOM can improve

⁴This section has been partially published in [179].

the system stability through the PCC as an ancillary service using this degree of freedom. DC bus and PCC voltage control can be achieved by producing current references in a dq synchronous frame, as well as by the active and reactive power references, according to [151].

$$E_{\text{FFR}} = \frac{C}{2} \left(U_{C,\text{set}}^2 - U_{C,\text{min}}^2 \right). \quad (5.2)$$

Moreover, for a symmetry in the available energy, the maximum voltage can be defined as

$$U_{C,\text{max}} = \sqrt{2 U_{C,\text{set}}^2 - U_{C,\text{min}}^2}. \quad (5.3)$$

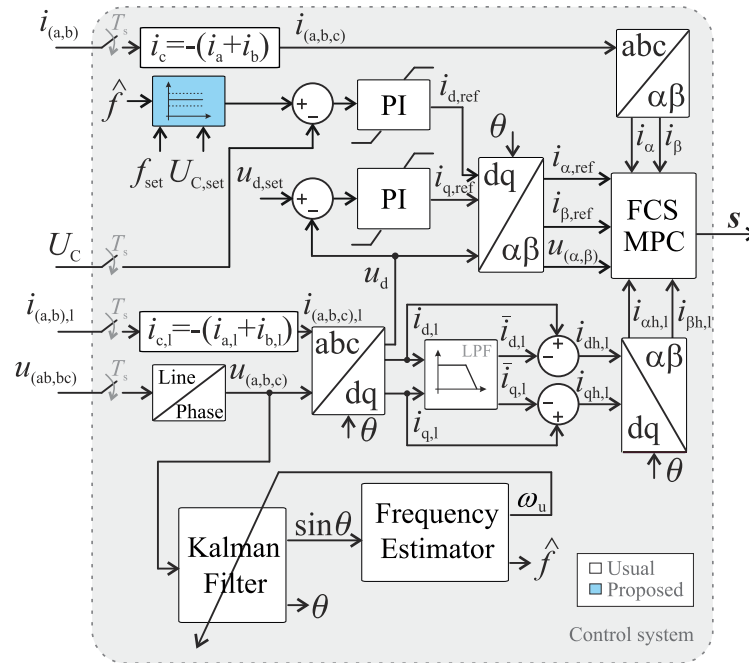


Figure 5.6: DSTATCOM control block diagram

Proportional-integral controllers can be used with good performance to provide control of continuous signals of the outer loops as the DC bus voltage, in the discrete-time domain. The design of the controller gains follows a well-known method discussed in [50]. The controller bandwidth and the gain and phase margins ensure the stability of an upstream-placed cascaded controller. The inner loop should have a crossover frequency at least one decade below the switching frequency of the converter, while the outer loop should have a crossover frequency at least one decade below the inner loop and a phase margin of at least 40° .

Figure 5.8 shows the block diagram of the control applied to the inverters involved in the

case study for coordination of the ancillary controllers with EES, as depicted in Figure 5.7. Three main parts can be noted: the generic control of the VSI; the ELC controller for the IBR with the ELC connected to the DC bus, which has a proportional-integral control that seeks to regulate the system frequency; and the control of the BESS. The inverter containing the BESS has an input current control, corresponding to its active power injection or absorption. The proposed ancillary controllers are highlighted in cyan. It can be seen that the bus voltage reference is an input parameter for both blocks, and its operation will also be discussed in Section 5.7.

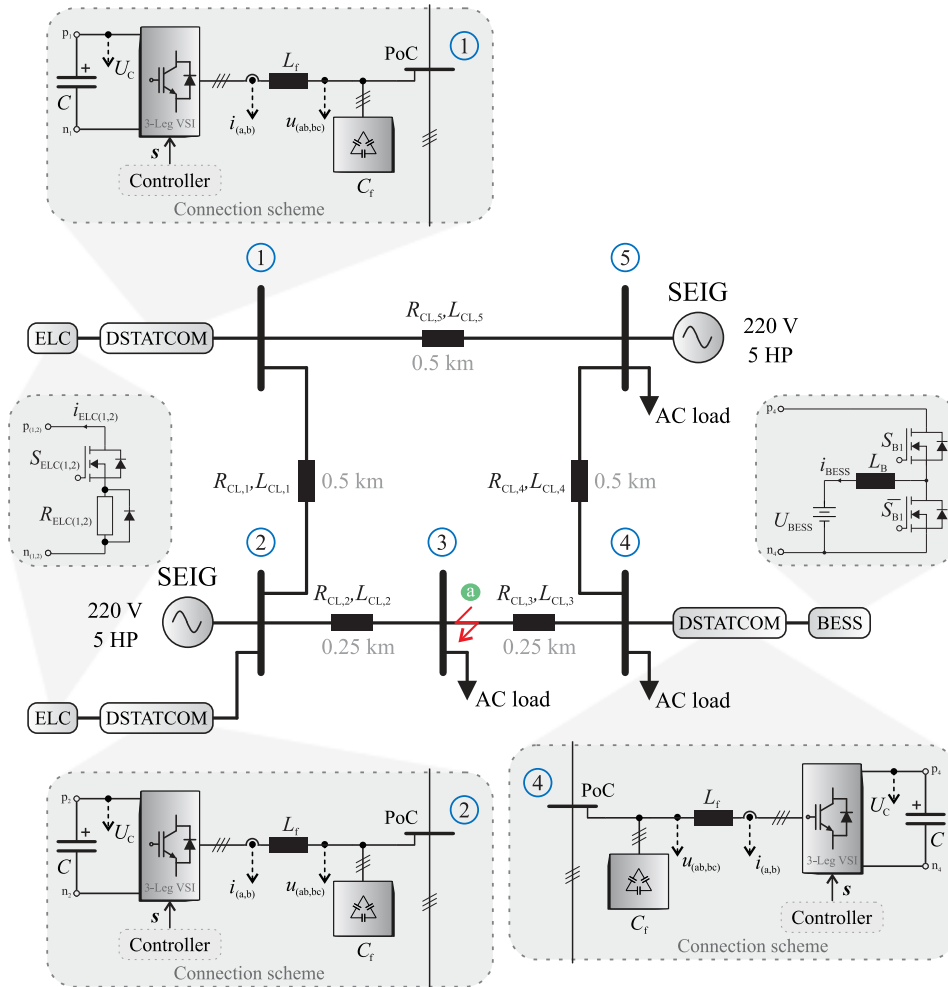


Figure 5.7: Renewable microgrid with two generators and three inverters (two as loads and one as BESS)

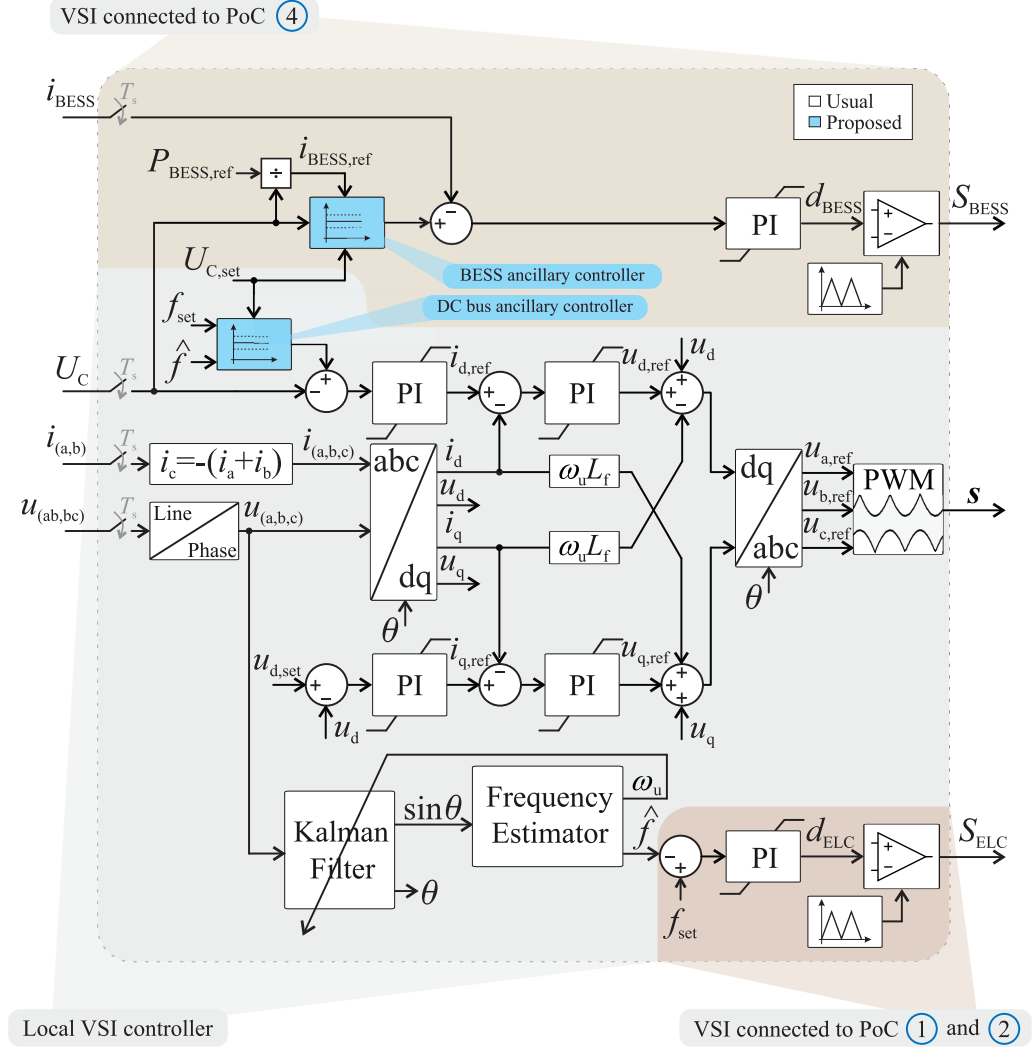


Figure 5.8: Generic control block diagram for the IBRs

5.3 Outer voltage loops

Outer loops are employed for the DC bus voltage control and PCC or PoC voltage control as shown in Figures 5.4, 5.6, and 5.8. Moreover, these controllers can be implemented in the discrete-time domain, where the k th sampling instant is expressed as:

$$\begin{bmatrix} i_{d,ref,k} \\ i_{q,ref,k} \end{bmatrix} = \begin{bmatrix} i_{d,ref,k-1} \\ i_{q,ref,k-1} \end{bmatrix} + \begin{bmatrix} K_{P,d} + K_{I,d} & 0 \\ 0 & K_{P,q} + K_{I,q} \end{bmatrix} \begin{bmatrix} e_{d,k} \\ e_{q,k} \end{bmatrix} + \begin{bmatrix} K_{P,d} \\ K_{P,q} \end{bmatrix} \begin{bmatrix} e_{d,k-1} \\ e_{q,k-1} \end{bmatrix}, \quad (5.4)$$

or even

$$i_{dq,ref,k} = i_{dq,ref,k-1} + k_P (e_{dq,k} + e_{dq,k-1}) + k_I e_{dq,k}, \quad (5.5)$$

where $\mathbf{k}_p$ and $\mathbf{k}_i$ are respectively the proportional and integral gain of the PI controller, arranged as column vectors as $\mathbf{k}_p = [K_{p,d} \ K_{p,q}]^\top$ and so on, and $\mathbf{e}_{dq}$ represents the error:

$$\mathbf{e}_{dq,k} = [U_{C,ref,k} - U_{C,k} \quad u_{d,ref,k} - u_{d,k}]^\top. \quad (5.6)$$

Another implementation of the discrete controller is also possible using

$$\begin{cases} \dot{\mathbf{x}}_{I,k} = \mathbf{e}_{dq,k} \\ \mathbf{i}_{dq,ref,k} = \mathbf{k}_i \mathbf{x}_{I,k} + \mathbf{k}_p \mathbf{e}_{dq,k}, \end{cases} \quad (5.7)$$

with

$$\mathbf{x}_{I,k+1} = \mathbf{x}_{I,k} + T_s \mathbf{e}_{dq,k}, \quad (5.8)$$

where $\mathbf{x}_{I,k} = [x_{I,d,k} \ x_{I,q,k}]^\top$ are the accumulation variables for the error integration. This implementation is particularly interesting for the application of the conditional integration anti-windup technique.

5.4 FCS-MPC technique ⁵

The measured variables $\mathbf{x}_k$ are used to compensate for the control implementation time through a predictive function y_p that relates the sampled values and the present inverter state $\mathbf{s}_k$, getting $\hat{\mathbf{x}}_{k+1}$. Thus, in order to predict the future values of the system, the predictive function is used to obtain $\hat{\mathbf{x}}_j$ for the possible states $\mathbf{s}_j$, described by

$$\hat{\mathbf{x}}_j = y_p \{ \hat{\mathbf{x}}_{k+1}, \mathbf{s}_j \}, \quad \forall j \in \{1, 2, \dots, 8\}, \quad (5.9)$$

where, y_p are the discrete model for the predicted output currents $\mathbf{i}_{\alpha\beta}$ described in (4.16) as well as the predicted input currents $\mathbf{i}_{L,x}$ described in (3.5) and (3.6).

Hereupon, a cost function g_j can be defined to find the optimal state, relating the eight resultant values to the eight possible predictions. Thus, the cost function y_G will relate the references of the variables and their predicted values by

$$g_j = y_G \{ \mathbf{x}_{ref}, \hat{\mathbf{x}}_j \}, \quad \forall j \in \{1, 2, \dots, 8\}. \quad (5.10)$$

A cost function can be defined to find the optimal state, relating the eight resultant values to the eight possible predictions. Thus, the cost function y_G will relate the references of

⁵This section has been partially published in [52], [60], and [179].

the variables and their predicted values, taking the inverter currents, for instance,

$$g_j = \left(i_{\alpha,\text{ref}} - \hat{i}_{\alpha,j,k+2}\right)^2 + \left(i_{\beta,\text{ref}} - \hat{i}_{\beta,j,k+2}\right)^2. \quad (5.11)$$

There are several ways to define a cost function depending on the nature of the different terms involved in the problem formulation [136]. For the control problem of a grid-connected MISSI through an inductive filter, a suitable cost function can be defined as

$$g_j = \left(i_{\alpha,\text{ref}} - \hat{i}_{\alpha,j,k+2}\right)^2 + \left(i_{\beta,\text{ref}} - \hat{i}_{\beta,j,k+2}\right)^2 + \lambda \sum_{x=1}^3 \left(i_{L,x,\text{ref}} - \hat{i}_{L,x,j,k+2}\right)^2, \quad (5.12)$$

where λ is a weighting factor that is used to impose importance on input currents in the decision of the control system, thus improving the control performance of these variables.

Therefore, the control action s_{k+1} that drives $\mathbf{x}$ as close as possible to the desired reference $\mathbf{x}_{\text{ref}}$ must minimize the cost function y_G by

$$s_{k+1} = \underset{s_j}{\operatorname{argmin}} y_G \{\mathbf{x}_{\text{ref}}, \hat{\mathbf{x}}_j(s_j)\}, \quad \forall j \in \{1, 2, \dots, 8\}. \quad (5.13)$$

The control algorithms to generically control the output currents of an L filter (1) and applied to a grid-tied MISSI (2) are detailed below as pseudocodes. The minimization of the cost function can be implemented as a *for loop* predicting for each voltage vector, evaluating the cost function, and storing the minimum value and the index value of the corresponding switching state. In addition, if the amplitude and the frequency of the grid are constant, then the time derivative of the voltage can be expressed as:

$$\begin{aligned} \frac{du_{\alpha}}{dt} &= -\omega_u u_{\beta}, \\ \frac{du_{\beta}}{dt} &= \omega_u u_{\alpha}. \end{aligned} \quad (5.14)$$

The grid voltages at the $(k+1)$ th sampling interval can be estimated by the discretization of (5.14), that is:

$$\begin{aligned} \hat{u}_{\alpha,k+1} &= u_{\alpha,k} - \omega_u T_s u_{\beta,k}, \\ \hat{u}_{\beta,k+1} &= u_{\beta,k} + \omega_u T_s u_{\alpha,k}. \end{aligned} \quad (5.15)$$

Moreover, a step ahead is included to compensate for the implementation time of

the microprocessor, according to [180]. In addition, as the frequency of $\mathbf{u}_{\alpha\beta}$ and $\mathbf{i}_{\alpha\beta}$ are much smaller than the sampling frequency, it can be assumed that $\mathbf{u}_{\alpha\beta,\text{ref},k} \approx \mathbf{u}_{\alpha\beta,\text{ref},k+1}$ and $\mathbf{i}_{\alpha\beta,\text{ref},k} \approx \mathbf{i}_{\alpha\beta,\text{ref},k+1}$ [48].

Algorithm 1: Pseudocode of MPC for output currents

```

Sampling  $\mathbf{i}_{\alpha\beta,k}, \mathbf{u}_{\alpha\beta,k}, U_{C,k}$ ;
 $\hat{\mathbf{i}}_{\alpha\beta,k+1} \leftarrow y_p \{\mathbf{x}_k, \mathbf{s}_k\}$ ;
 $j_{\min} \leftarrow \infty$ ;
for each  $j$  from 1 to 8 do
     $\hat{\mathbf{i}}_{\alpha\beta,j,k+2} \leftarrow y_p \{\hat{\mathbf{i}}_{\alpha\beta,k+1}, \mathbf{s}_j\}$ ;
     $\mathbf{g}_j = y_G \{\mathbf{x}_{\text{ref}}, \hat{\mathbf{x}}_j\}$ ;
    if  $\mathbf{g}_j < j_{\min}$  then
         $j_{\min} \leftarrow \mathbf{g}_j$ ;
         $j_{\text{opt}} \leftarrow j$ ;
    end
end
 $\mathbf{s}_{k+1} \leftarrow \mathbf{s}_{j_{\text{opt}}}$ ;
  
```

Algorithm 2: Pseudocode of Proposed FCS-MPC for the grid-tied MISSI

```

Sampling  $\mathbf{i}_{\alpha\beta,k}, \mathbf{u}_{\alpha\beta,k}, U_{C,k}, \mathbf{i}_{L,k}, \mathbf{u}_{U,k}$ ;
 $\hat{\mathbf{i}}_{\alpha\beta,k+1}, \hat{\mathbf{i}}_{L,k+1} \leftarrow y_p \{\mathbf{x}_k, \mathbf{s}_k\}$ ;
 $g_{\min} \leftarrow \infty$ ;
for each  $j$  from 1 to 8 do
     $\hat{\mathbf{i}}_{\alpha\beta,k+2,j} \leftarrow y_p \{\hat{\mathbf{i}}_{\alpha\beta,k+1}, \mathbf{s}_j\}$ ;
    if  $S_{(1,2,3),j} == 1$  and  $S_{(2,3,1),j} == 1$  then
         $\hat{\mathbf{i}}_{L,(1,2,3),k+2,j} \leftarrow y_{\text{dem}} \{\hat{\mathbf{i}}_{L,(1,2,3),k+1}\}$ ;
    else
         $\hat{\mathbf{i}}_{L,(1,2,3),k+2,j} \leftarrow y_{\text{mag}} \{\hat{\mathbf{i}}_{L,(1,2,3),k+1}\}$ ;
    end
    end
     $\mathbf{g}_j = y_G \{\mathbf{x}_{\text{ref}}, \hat{\mathbf{x}}_j\}$ ;
    if  $\mathbf{g}_j < g_{\min}$  then
         $g_{\min} \leftarrow \mathbf{g}_j$ ;
         $j_{\text{opt}} \leftarrow j$ ;
    end
end
 $\mathbf{s}_{k+1} \leftarrow \mathbf{s}_{j_{\text{opt}}}$ ;
  
```

According to previous work, as the MPC is based on a model of the system, the variation of parameters can influence the control performance. In some applications, performance may deteriorate if the declared inductance is less than the actual value. Also, performance may not be affected by an overestimated inductance value [65].

5.5 Frequency control and synchronization method

The synchronization method is based on the Kalman filter-phase-locked loop. This method features single-phase and three-phase synchronization based on optimum filtering theory. These explicitly include the presence of harmonics in the problem formulation, voltage unbalance, measurement noise, transients, and frequency deviation. The Kalman filter provides a way of obtaining the best compromise between transient response and measurement noise rejection for the synchronization signals [56].

For the off-grid application with induction generators, in this work, a rotating shaft coupled to the SEIG, operating above the synchronous speed (1840 rpm), is considered the prime mover, and the frequency control can be performed by the ELC. The voltage frequency depends on the balance between the input mechanical active power and the output active power of the SEIG. Therefore, the power consumed by the system must be maintained constant irrespective of the load variations.

The ELC control is based on the estimated frequency of the voltage at the point of connection (PoC), which is performed by a Kalman-based synchronization method followed by a frequency identification algorithm [143]. The Kalman filter is well known due to its ability to deal with linear systems corrupted by uncertainties in the states of the plant as well as measurement noise [56]. The estimated frequency ($\hat{f}$) is then compared with the fixed set point, which generates a frequency error

$$e_{\omega} = f_{\text{set}} - \hat{f}. \quad (5.16)$$

The error feeds a PI controller, which defines the power to be consumed by the ELC (P_{ELC}). The control action d_{ELC} is modulated to generate a PWM signal to be applied to the power switch, which drives the R_{ELC} .

5.6 Incremental conductance MPPT technique

For the MPPT implementation in the systems of this work, the incremental conductance algorithm (Inc-Cond) algorithm was chosen. This technique is considered a hill climbing method [53]. The MPP can be tracked by comparing the instantaneous conductance to the incremental conductance $\Delta i_{L,x,k} / \Delta U_{x,k}$. The current $i_{L,(1,2,3),\text{ref}}$ can be the reference value for the PV arrays. In other words, the solution of the Inc-Cond equation (5.17) is zero at the MPP, positive on the left side of the MPP, and negative on the right side of the MPP. The incremental conductance algorithm is efficient and simple, like the perturb and observe (P&O) algorithm. It exhibits good performance under rapid changes in

atmospheric conditions.

$$\frac{dP_{PV,k}}{dU_x} = \frac{d(i_{L,x,k}U_{x,k})}{dU_{x,k}} = i_{L,x,k} + U_{x,k} \frac{di_{L,x,k}}{dU_{x,k}} \cong i_{L,x,k} + U_{x,k} \frac{\Delta i_{L,x,k}}{\Delta U_{x,k}}. \quad (5.17)$$

5.7 Ancillary controller description⁶

Considering the appropriate stable control operation limit, and also that inverters are usually designed with a spare voltage capacity, the control strategy can be drawn. For reference, this thesis approaches the system conditions which are depicted in Figure 5.9.

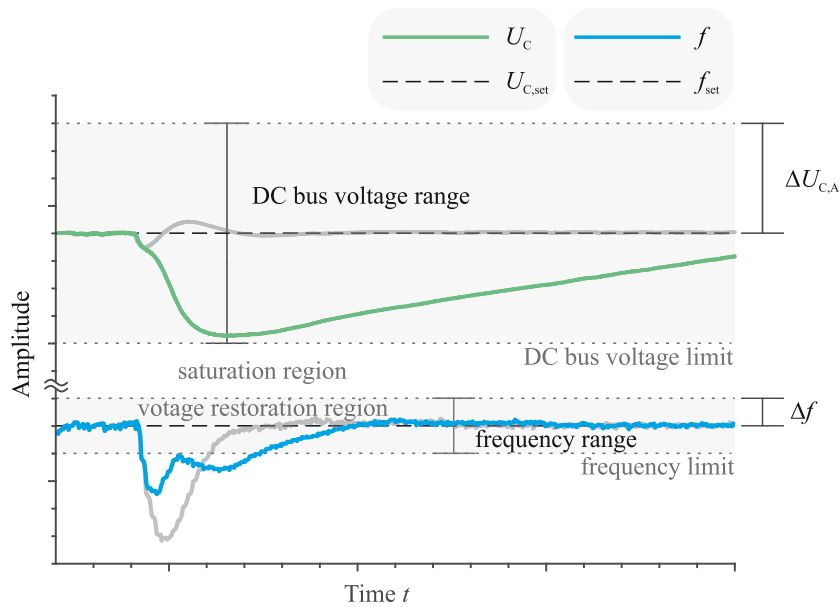


Figure 5.9: Outline of the control strategy pattern for VSI regarding the DC bus capacitor voltage and PoC frequency

Based on these definitions, the proposed method explores specific characteristics desired for off-grid systems with shunt compensation. The controller arrangement prioritizes the system frequency restoration over the voltage recovery rate, seeking to avoid premature contingencies such as equipment shutdown when the frequency exceeds the threshold. It can be achieved through an FFR strategy [131], [132].

A controller should be designed to provide active power support only if the system frequency is out of the determined range, since the task of the ancillary control is not to control the system frequency. The controller operation is flexible and can be based

⁶This section has been published in a similar form in [179] and [181].

on system operation standards or other desired limits [132]. The system frequency restoration is considered dependent on external contingency management, frequency monitoring, and security algorithms of external equipment.

Therefore, differing from the classical methods, it is interesting that the ancillary controller presents different dynamics under different conditions, such as fast active power supply or demand in the face of surges and smooth or no active power dynamics when the system frequency is critical. It demands a flexible arrangement of control laws. The intended behavior is achieved by means of the conditional integration of the frequency error and the smooth emptying of this information, as presented in Figure 5.10.

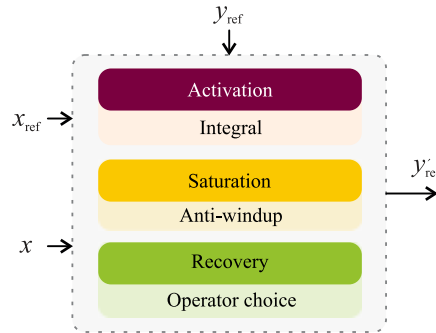


Figure 5.10: Proposed strategy

The ancillary controller uses a conditional integration anti-windup strategy, also presenting a variable structure, enabling a preloading feature that is straightforward in discrete-time implementation, known as manually resetting the integrator value to an offline predetermined value [182], [183]. This system, which is depicted in Figure 5.11, can characterize a discontinuous adaptive generator emulation control [114], [124], [127] and is performed through a backward Euler digital implementation. From the system model, the frequency variation is directly proportional to the active power variation.

Therefore, the discrete-time controller for the operation out of the frequency range can be written as

$$\begin{aligned} u_{C,\omega,\text{ref},k} &= K_I u_{C,I,k}, \\ u_{C,I,k+1} &= u_{C,I,k} + T_s e_{\omega,k} - \text{sgn}(e_{\omega,k}) \Delta f T_s, \end{aligned} \quad (5.18)$$

where

$$e_{\omega,k} = f_{\text{set},k} - \hat{f}_k \quad (5.19)$$

represents the error. The gain K_I influences how fast the active power compensation is done, and its design is dependent on the sampling frequency T_s , also presenting a negative value, since for a drop in voltage reference, active power is injected into the

system. In addition, the last term of (5.18) ensures a non-abrupt dynamic since the error starts from the defined frequency limit.

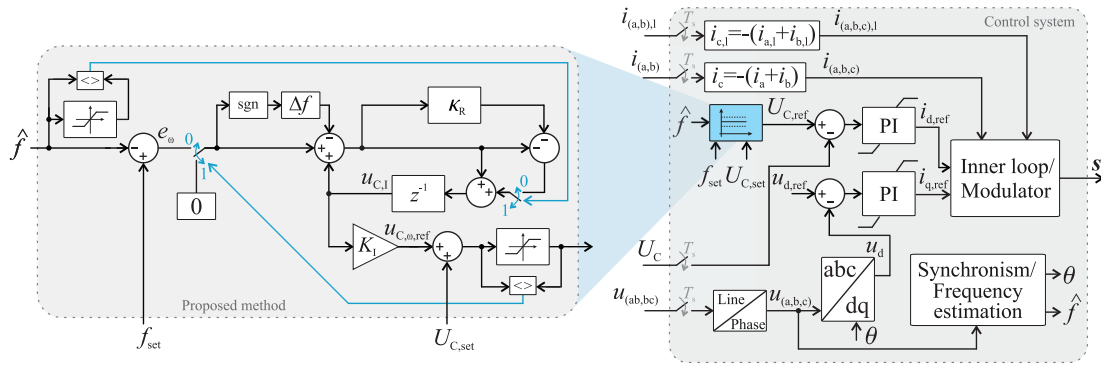


Figure 5.11: Control block diagram of the proposed strategy (blue highlighted section) located in a generic control system

If the voltage reaches saturation, the integration condition sets the error to zero, then

$$u_{C,I,k+1} = u_{C,I,k}. \quad (5.20)$$

If the frequency is within the limits, adequate active power demand may occur. The integrator reset dynamics at the voltage recovery region can be chosen to achieve soft active power demand for the system, so the dynamic function (5.21) or a ramp function (5.22) can be employed.

$$u_{C,I,k+1} = u_{C,I,k} - u_{C,I,k} \kappa_R T_s, \quad (5.21)$$

$$u_{C,I,k+1} = u_{C,I,k} - \text{sgn}(u_{C,I,k}) \kappa_R T_s, \quad (5.22)$$

where the gain function κ_R influences the recovery active power rate, and therefore the voltage recovery time. It can be assumed as a constant $\kappa_R = K_R$, for instance.

Regarding achieving fast power support and maintaining the system frequency within the acceptable steady-state limits, it can be stated that the active power disturbance when the frequency is close to the established border is undesirable; it is preferable that the voltage recovery rate is lower in this region and can be higher near the frequency reference. As pointed out in [130], an unsuitable recovery strategy may cause a second frequency drop during the recovery time.

In the lack of inertia and the need for an adequately fast voltage recovery time, the mitigation of undesirable active power oscillations during the voltage recovery time for sustained operation in the frequency limits, such as overload or surplus energy conditions, a function of frequency error can be used. An interesting decreasing or

asymptotic adaptive coefficient candidate can be chosen as (5.23); the voltage recovery rate will be lower around established frequency limits and higher around the frequency reference.

$$\kappa_R(e_\omega) = K_R \left(1 - \frac{|e_{\omega,k}|}{\Delta f} \right), \text{ or } e^{-\frac{K_R |e_{\omega,k}|}{\Delta f}}, \quad (5.23)$$

where Δf is half of the frequency range defined in Figure 5.9.

In summary, some key features of the developed method can be pointed out:

- Fast and adequate response to undesirable frequency deviation, prioritizing to lead the system frequency into the operating region
- Among the others, the approach can benefit the lifetime of the capacitor since the fixed operating point of the voltage is prioritized under normal conditions
- Active recovery power will only be requested if the frequency is considered adequate

For coordination with BESS, the control can follow the same method. The control action can be given by monitoring the DC bus voltage through the error

$$e_{C,k} = U_{C,\text{set},k} - U_{C,k}, \quad (5.24)$$

and a preset voltage range ΔU_C , as depicted in Figure 5.12. It is possible to consider the employment of proportional control action to the voltage deviation. However, the information stored in the integrator for smooth recovery and regulation would not be available, and the FFR would follow the variable behavior proportionally.

Therefore, for the operation outside the frequency range, the controller description will follow (5.18), which leads to:

$$i_{\text{BESS},U,\text{ref},k} = K_{U,I} i_{B,I,k}, \quad (5.25)$$

where $i_{\text{BESS},U,\text{ref},k}$ will be summed with the original current reference $i_{\text{BESS},\text{ref},k}$ of the BESS.

The controller operation in the recovery range will reset the integral control action with a term for smooth recovery, and can be written as

$$i_{B,I,k+1} = i_{B,I,k} - i_{B,I,k} K_{B,R} \left(1 - \frac{|e_{U,k}|}{\Delta U_C} \right) T_s. \quad (5.26)$$

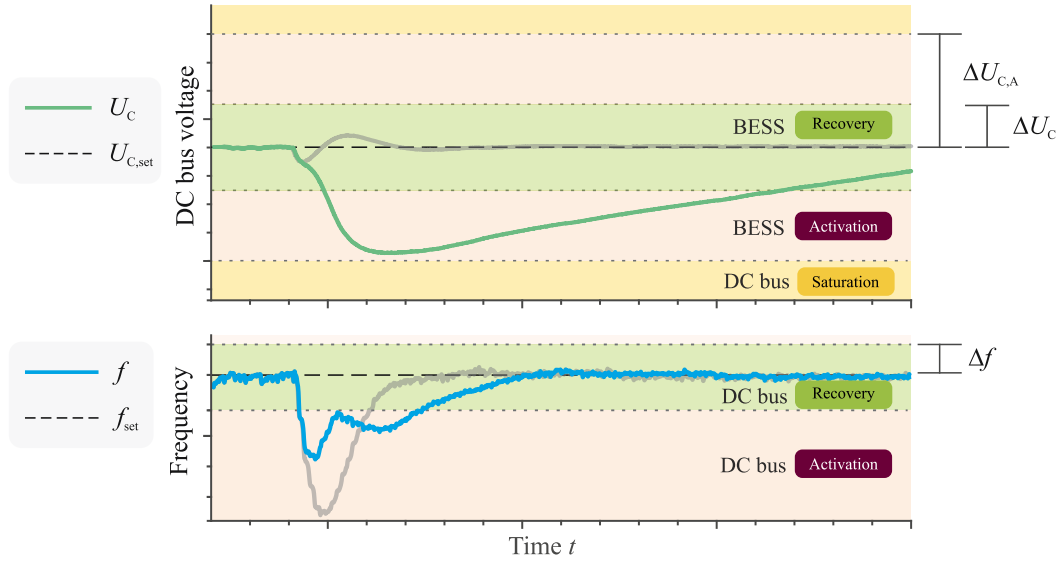


Figure 5.12: Outline of the control strategy pattern for VSI regarding the DC bus capacitor voltage, PoC frequency, and BESS coordination

Another suitable FFR may be proportional to the frequency deviation given by

$$P_{\text{BESS,FFR,ref},k} = K_{\text{FFR,P}} e_{\omega,k}. \quad (5.27)$$

However, the BESS will continuously compensate for disturbances, and the FFR performance can be degraded by interaction with the DC bus controller when no ancillary controller is considered. In addition, by considering frequency thresholds, the final contribution of the controller regarding active power injection to the system will be reduced.

5.7.1 Complementarity of FFR methods

Various types of FFR controllers can be implemented, including step-based, proportional, and proportional with RoCoF schemes. Each configuration presents specific limitations that must be evaluated for different applications. Integral control has been less explored due to challenges in practical implementation and smooth recovery; however, it holds potential for mitigating excessive responses and improving performance under slow frequency deviations, such as subsynchronous oscillations.

Typically, a dead-band is employed, which limits the improvement of the initial RoCoF, which is a limitation addressed by GFM inverters. To enable optimized ancillary services, the deployment of FFR-supporting functionalities is essential, as increasing

system inertia alone has proven insufficient. The Effective FFR implementation requires consideration of the technological constraints of each asset (e.g., BESS, SCES), informing the development of coordinated control strategies that range from fast to slow response. Moreover, the spatial distribution of FFR resources, particularly their proximity to the disturbance, is more critical than the total quantity deployed [184]–[186].

To ensure that frequency nadirs do not fall below the required thresholds, a solution is to utilize the inherent capabilities of existing IBRs to deliver FFR, focusing on elevating the frequency nadir by injecting the active power. Basically, IBRs can provide three types of FFR, as pointed out in section 2.

For the step response, when a certain frequency or RoCoF is reached, a constant amount of active power is injected, or the load is managed and decreased by a constant amount [121], [187]. The IBR provides a step response after a time delay, injecting the active power until it reaches the predetermined saturation value. Even presenting an interesting behavior, this method, on the other hand, can expend unnecessary amounts of energy, depending on the nature of the disturbance.

For the derivative response or synthetic inertia, the injection of active power or decrease of load is directly proportional to the RoCoF. It enables IBRs to supply synthetic inertia to the power system with:

$$P_{\text{FFR}} = K_P \frac{d\hat{f}}{dt}, \quad (5.28)$$

where K_P can be related with $-2H_c$.

For the proportional response, the amount of active power injection or decrease in load is directly proportional to the frequency fluctuations. In response to variations in grid frequency, the IBR can provide a proportional power adjustment. This approach mimics the P/f droop control or primary frequency regulation, where the change in the IBR output is directly related to the magnitude of the frequency deviation:

$$P_{\text{FFR}} = K_P e_{\omega,k}. \quad (5.29)$$

Therefore, the following control strategy can be implemented to calculate the reference current for a DC-GFM IBR connected to a BESS:

$$i_{\text{BESS,FFR,ref}} = \begin{cases} K_{P,\min}(\hat{f} - f_{\text{th},1}), & \text{if } \hat{f} \leq f_{\text{th},1} \\ K_{P,\max}(\hat{f} - f_{\text{th},2}), & \text{if } \hat{f} \geq f_{\text{th},2} \\ 0, & \text{otherwise,} \end{cases} \quad (5.30)$$

where, $i_{\text{BESS,FFR,ref}}$ is the battery current reference related to this grid-supporting

functionality, $\hat{f}$ is the estimated grid frequency, $f_{th,1}$ and $f_{th,2}$ are the threshold frequencies for triggering response, and $K_{P,min}$ and $K_{P,max}$ are the droop constants, which play a critical role in determining how aggressively the system responds to frequency deviations. It defines the slope of the frequency to the current reference curve and is calculated based on the maximum allowable battery current and the operational frequency threshold range, as

$$K_{P,min} = \frac{I_{BESS,max}}{f_{th,1} - f_{min}}, \quad (5.31)$$

$$K_{P,max} = \frac{I_{BESS,max}}{f_{max} - f_{th,2}}, \quad (5.32)$$

where $I_{BESS,max}$ is the BESS maximum current.

5.7.2 Grid-forming control approach for grid-tied operation

Grid-following converter controls, which rely on explicit synchronization mechanisms, are widely implemented in modern power systems. However, their performance can be compromised in converter-dominated systems, particularly those characterized by low rotational inertia, where they exhibit notable robustness and stability challenges. To address these issues, the concept of GFM converters has been introduced, offering fast and resilient voltage and frequency regulation suitable for low-inertia environments [10], [108], [188], [189].

Recent advancements in control strategies for GFM converters have demonstrated significant improvements in transient stability when compared to conventional GFL approaches. Core GFM methods, such as droop control and virtual synchronous machine (VSM) control, have been shown to exhibit equivalent dynamic behavior [190]–[192]. To further enhance system stability, state-of-the-art implementations incorporate droop control in combination with virtual admittance and virtual power concepts, achieving a broader stability margin [193]. In addition, current-limited GFM converters can be assessed using area-based stability criteria, as illustrated in Figure 5.13. For optimal performance in current regulation, advanced strategies such as model predictive control have been proposed for the inner control loops.

The transient stability of GFM inverters during grid voltage sag depends on the existence and nature of stable equilibrium points during the fault. If such points exist, which means that the power reference intersects the fault angle curve, as shown in Figure 5.13(a), the system may retain transient stability throughout the disturbance [194]. For converters with predominantly first-order angle-forming control, such as droop control, stability can generally be maintained as long as an equilibrium point is present. In the case of

second-order angle-forming control, such as virtual synchronous machines, transient stability is determined by the balance between the accelerating and decelerating energy areas, as evaluated by the equal-area criterion.

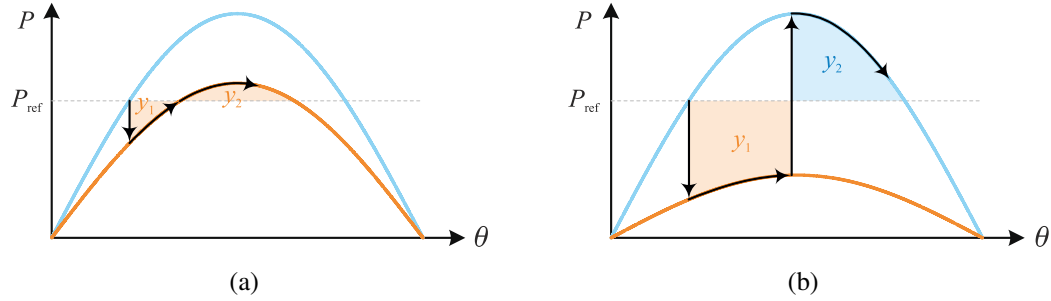


Figure 5.13: Area-based stability criteria (adapted from [192], [195])

If no equilibrium point exists during the voltage sag, as illustrated in Figure 5.13(b), synchronization cannot be achieved during the fault. However, post-fault synchronization may still occur, provided the decelerating area is sufficient to counter the acceleration. The equal-area criterion, a classical method from synchronous machine analysis, remains widely used for evaluating transient stability in GFM inverters. An alternative approach is the energy function method, which provides a more detailed stability assessment. For VSMs, the kinetic energy term represents the virtual energy stored in the virtual inertial mass rotating at the forming frequency, while the potential energy term reflects the virtual energy associated with angular displacement from the equilibrium point. These virtual energy exchanges are driven by internal power imbalances and mirror the mechanical and electromagnetic interactions in synchronous machines [189].

Energy-based methods can be used to approximate the region of attraction and estimate critical fault clearance times. However, both the equal-area criterion and standard energy functions neglect damping effects and may thus be conservative. More refined methods that account for damping are available in the literature. In practical applications, such as when GFM inverters connect renewable energy sources to the grid, the active power reference of the inverter and the primary energy source output should be reduced during voltage sags [195]. This ensures compliance with transmission line limits and prioritizes fault ride-through services, such as the provision of reactive current.

In inverter-dominated systems using AC-GFM control, frequency dynamics occur more rapidly and require less inertia. Although the maximum RoCoF increases with total system inertia, the average RoCoF often improves due to the fast response of GFM converters. Consequently, in converter-dominated systems, large RoCoF values may not indicate faults, and their use as protection signals should be reconsidered. Emulating virtual inertia is neither necessary nor well-suited to the characteristics of VSCs, as

explicitly stated in [108]. The recommendation is that future work should focus on exploiting the flexibility and fast control response of VSCs to address key challenges in integrating IBRs into large-scale power systems. Therefore, GFM control must explicitly consider the dynamic behavior and operational constraints of renewable energy sources, energy storage systems, and power electronic converters.

Grid-supporting control strategies for VSCs are commonly designed to replicate the behavior of the classical third-order synchronous generator model [108]. Dispatchable virtual oscillator control (dVOC) is the only control method that has been developed from first principles based on explicit design specifications by exploiting a universal physical principle: transmission lines inherently propagate synchronization information through voltage/current differences. By measuring these locally, inverters synchronize without communication or emulation layers. It formalizes this insight into tunable control laws with global stability certificates. On the other hand, in the context of inductive networks such as transmission systems, and under the assumption of near-nominal voltage magnitudes, dVOC exhibits behavior similar to that of droop control [196].

The AC-GFM droop control strategy can be derived from the one-axis SG model with an exciter [108]. Linearizing around the nominal terminal voltage magnitude and introducing low-pass filtered power measurements yields the droop control law:

$$\frac{d\theta}{dt} = \omega_{\text{set}} + K_{P,P} (P_{\text{ref}} - P_g), \quad (5.33)$$

$$U_{\text{ref}} = U_{\text{set}} + K_{P,Q} (Q_{\text{ref}} - Q_g), \quad (5.34)$$

where $K_{P,P}$ and $K_{P,Q}$ are the droop gains, and P_{ref} and Q_{ref} are the active and reactive power references, respectively. The droop gains are typically specified by grid codes or market mechanisms, and usually range between 1% and 5%. The droop coefficients governs the proportional behavior and can be calculated as (5.35) and (5.36) [191]. This approach allows decentralized load sharing without the need for communication links and is considered a baseline solution [94].

$$K_{P,P} = \frac{\Delta\omega}{\Delta P}, \quad (5.35)$$

$$K_{P,Q} = \frac{\Delta U}{\Delta Q}. \quad (5.36)$$

A related control strategy is the VSM, which emulates the dynamic behavior of a synchronous generator. However, due to the fast actuation capability of VSCs and typical renewable energy sources, emulating slow dynamics would unnecessarily limit

performance [108]. The VSM control law is

$$\frac{2H}{\omega_{\text{set}}} \frac{d\omega}{dt} = -B\omega + P_{\text{ref}} - P_g, \quad (5.37)$$

$$U_{\text{ref}} = U_{\text{set}} + K_{P,Q} (Q_{\text{ref}} - Q_g), \quad (5.38)$$

where H is the virtual inertia constant and B denotes the damping coefficient.

It is important to note that the limited internal energy storage capacity of VSCs restricts their ability to emulate significant inertia values. Finally, it is worth emphasizing that the AC-GFM droop control law and the VSM model are mathematically equivalent up to a change of coordinates [108]. The equivalent inertia constant of AC-GFM droop control can be tuned by the low-pass filter, which is typically small in practice.

Matching control has emerged as a promising GFM strategy by directly incorporating DC-link dynamics into frequency regulation. However, its performance is highly sensitive to the selection of the control coefficient, where suboptimal tuning can degrade frequency response. To address this, [197] proposes an enhanced matching control approach that improves frequency regulation through refined tuning, thereby achieving better alignment with DC bus dynamics.

Hybrid Adaptive Control HAC offers another promising solution, combining the advantages of both AC-based droop control and DC-based matching control [196]. Like droop control, HAC improves small-signal frequency stability in low-inertia systems. Simultaneously, it maintains robustness under practical converter limitations, similar to matching control, making it a versatile candidate for future low-inertia grids.

The equivalence between droop control, VSMs, and the SG swing equation provides a basis for interoperability among GFM-controlled VSCs and SGs. This correspondence allows established transient stability methods for reduced-order SG models to be applied to droop and VSM-controlled converters [108]. However, general global stability results for interconnected SG networks, based on either dynamic or quasi-steady-state models, remain an open research challenge.

Current-limiting strategies in IBR control are generally classified into voltage-based and current-based methods. These strategies typically employ power feedback in the outer control loops, while applying saturated current limits in the inner loops [198]. Such strategies effectively reduce the grid impedance seen by the PLL, thereby modifying the voltage-power-angle characteristic [194]. Under high current saturation, the active power exchanged is predominantly limited by the filter and virtual impedance. Depending on inverter capability and the severity of voltage sags, a stable operating point can still be achieved. These methods have also demonstrated effectiveness against a range of

$$\begin{bmatrix} i_{\alpha,\text{ref},k+1} \\ i_{\beta,\text{ref},k+1} \end{bmatrix} = \left(1 - T_s \frac{R_{\text{vr}}}{L_{\text{vr}}}\right) \begin{bmatrix} i_{\alpha,\text{ref},k} \\ i_{\beta,\text{ref},k} \end{bmatrix} + T_s \frac{1}{L_{\text{vr}}} \begin{bmatrix} e_{\text{U},\alpha,k} \\ e_{\text{U},\beta,k} \end{bmatrix}. \quad (5.40)$$

Considering the case studies in this work, the circular saturation can be adopted and implemented as follows:

$$\begin{bmatrix} i'_{\alpha,\text{ref},k+1} \\ i'_{\beta,\text{ref},k+1} \end{bmatrix} = \min \left(1, \frac{I_{\text{c},\alpha\beta,\text{max}}}{\sqrt{i_{\alpha,\text{ref},k+1}^2 + i_{\beta,\text{ref},k+1}^2}} \right) \begin{bmatrix} i_{\alpha,\text{ref},k+1} \\ i_{\beta,\text{ref},k+1} \end{bmatrix}. \quad (5.41)$$

The virtual power feedback utilizes the unsaturated current reference for the calculation of power in the droop control [189]. From the same formulation demonstrated in [195], it follows that

$$P_{\text{ref}} = \text{Re} \left\{ \underline{u}_{\alpha\beta,\text{ref}} \cdot i_{\alpha\beta,\text{ref}}^* \right\} = \frac{U_{\text{ref}} U_{\text{g}}}{X_{\text{vr}} + \frac{\|i'_{\alpha\beta,\text{ref}}\|}{\|i_{\alpha\beta,\text{ref}}\|} X_{\text{g}}} \sin(\theta - \theta_{\text{g}}), \quad (5.42)$$

showcasing that power angle deviations during disturbances are reduced and transient stability is potentially improved.

5.7.3 GFM mode for DSTATCOM and E-STATCOM

The DSTATCOM or the energy-STATCOM (E-STATCOM) operating in GFM mode can represent various inverter-based systems with a zero active power set point [200]. This includes applications such as BESS, charging stations, redundant photovoltaic systems during periods of no generation (e.g., nighttime operation), and other distribution-level compensators [201], [202]. These systems can also employ AC-GFM and DC-GFM control methods, providing voltage support and dynamic grid stabilization, even when not actively injecting power into the grid [203]–[206]. Figure 5.15 presents the electrical schematic and control diagram of an inverter operating as a DSTATCOM in GFM mode, including the ancillary controller, which characterizes a grid-supporting functionality.

The slow response of the synchronous machine, primarily due to its large turbine time constant, extends the duration of DC source saturation in GFM converters [196], [207]. If the AC current of the converters is not properly limited, this condition may lead to depletion of the DC link. To ensure the stability of GFM converters in the presence of synchronous machines, it is essential to consider DC-side limitations, either through AC current limiting or DC voltage monitoring. In contrast, in systems composed entirely

of GFM converters (whether by design or as a result of the absence of synchronous machines), DC-side limitations are typically engaged only for brief periods, and the overall system remains stable. This underscores that interactions between the fast dynamics of GFM converters and the slower dynamics of synchronous machines can potentially lead to system instability [196].

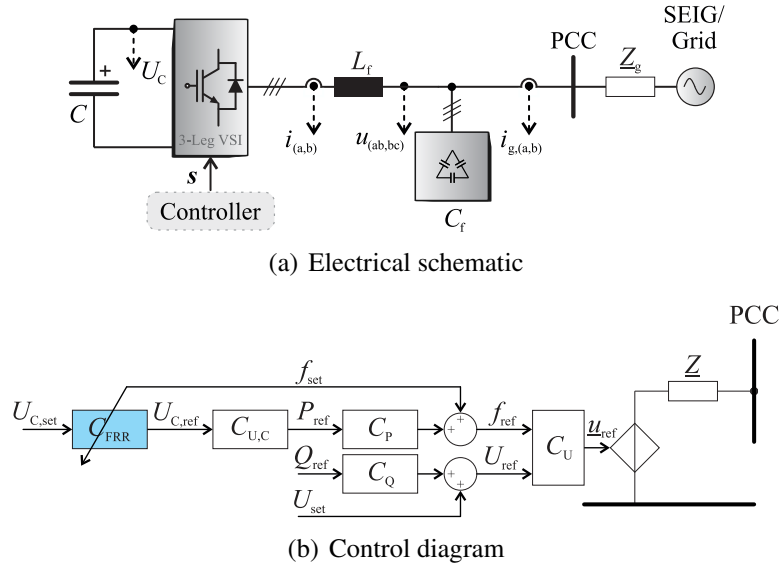


Figure 5.15: DSTATCOM operating in grid-forming mode

Considering that, in the AC/DC-GFM control mode, P_{ref} is the control action of the DC-bus voltage PI controller, the voltage regulation will be directly affected by frequency events. Moreover, the protection and anti-windup strategies associated with PI control should account for the maximum power of the inverter. This implies that, under a sufficiently large frequency disturbance, the PI controller will reach saturation, and the DC-bus voltage will no longer be able to prevent a violation of the voltage limits, as illustrated in Figure 5.16(a) for a critical frequency drop. The GFM mode demonstrates improved performance in utilizing DC-bus energy during frequency events when compared to the GFL mode. However, in a low-inertia system, the DC-bus recovery can lead to an undesired large frequency drop. To mitigate this, the ancillary controller ensures a smooth recovery by considering the actual frequency conditions. An adaptive coefficient κ_1 with a decreasing or asymptotic profile can be employed to smooth the integral action in (5.43), as shown in Figure 5.16(b).

$$u_{C,I,k+1} = u_{C,I,k} + \kappa_1 T_s e_{\omega,k} - \text{sgn}(e_{\omega,k}) \Delta f T_s, \quad (5.43)$$

with

$$\kappa_1(u_{C,\omega,\text{ref}}, e_\omega) = \begin{pmatrix} 1 - \frac{|u_{C,\omega,\text{ref},k}| \frac{1}{1 + |\psi e_{\omega,k}|}}{\Delta U_C \frac{1}{1 + |\psi e_{\omega,k}|}} \end{pmatrix}, \text{ or } \begin{pmatrix} 1 - \frac{|u_{C,\omega,\text{ref},k}| \frac{1}{\psi}}{\Delta U_C \frac{1}{\psi}} \end{pmatrix}, \quad (5.44)$$

where $\psi > 0$ for the latter is a tuning factor used to smooth the overshoot and maintain the DC bus voltage controller speed in the presence of known disturbances.

In typical power system events such as faults, load steps, or generator trips, the energy required to maintain system stability is relatively small compared to the nominal power of the system. These transient disturbances, often lasting from a few hundred milliseconds to a few seconds, demand fast and controlled energy support to preserve voltage and frequency stability. The required energy, while modest in absolute terms, must be delivered promptly, which emphasizes the importance of fast-response elements such as DC-link capacitors, BESS, or virtual inertia emulation. Understanding this dynamic behavior is essential for appropriately sizing such components in modern power electronic converters and grid-supporting devices. It has been demonstrated in literature that the power and energy capacity requirements of ESS are influenced by the location of the FFR reserve [185].

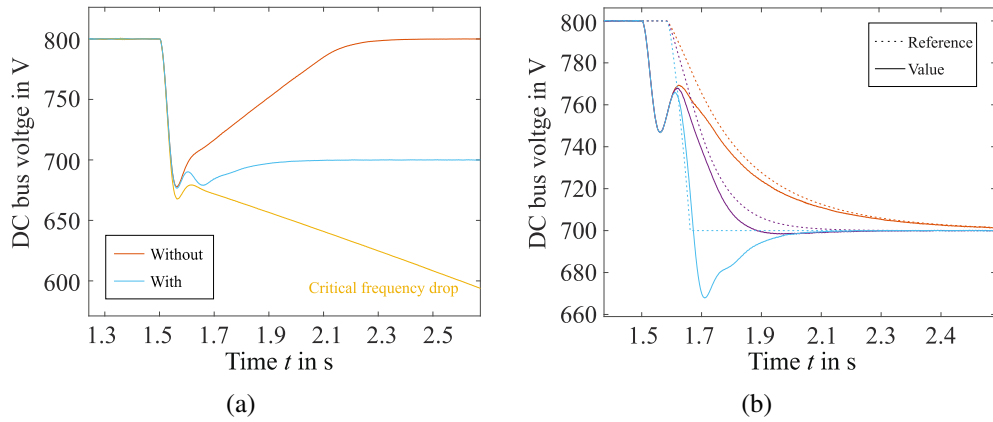


Figure 5.16: DC bus voltage behavior for critical frequency step

Beyond their primary function of voltage stabilization and ripple filtering, these capacitors offer a valuable source of fast-response energy that can be utilized as a form of early-term reserve during transient disturbances. Due to their ability to release or absorb energy almost instantaneously, they are well-suited to support the grid in the critical initial moments following a fault or other dynamic event. This characteristic positions them as a naturally embedded, low-latency energy buffer within distributed IBRs, contributing

to grid stability without requiring additional infrastructure. Leveraging this inherent energy potential is particularly relevant in systems with high penetration of renewables, where traditional inertia and reserves are diminished.

5.8 Scaling and analysis for transmission-level power systems

Power flow equations provide the foundation for steady-state analysis and optimization in power systems and are typically applied in conjunction with synchronous generator models for dynamic simulation, analysis, and control. These equations are based on the quasi-steady-state assumption, which relies on a time-scale separation between transmission line dynamics and synchronous generator dynamics, as justified by singular perturbation methods. However, this assumption becomes invalid in systems with a high penetration of power electronic converters, where the dynamics of the converters and their control systems occur on similar time scales as the line dynamics. This can lead to resonance phenomena and, ultimately, instability. Consequently, it becomes necessary either to consider the full dynamic network model or to remain critically aware of the limitations associated with quasi-steady-state models [108]. To prevent instability, converter controllers are often intentionally slowed down or designed with low-pass filters, as demonstrated in various recent studies [208]–[211].

The IEEE 9-bus system, representing a transmission-level network, serves as a well-established benchmark for evaluating controller scalability, model aggregation techniques, and the broader dynamics of system oscillations. It further facilitates performance assessment under scenarios involving both parallel service providers and synchronous machine-based generation, as well as estimation of power and energy storage requirements during frequency events.

The implementation used in this study includes two thermal and one hydro-based synchronous generators, modeled according to [211], [212]. Transmission lines are represented by nominal π -sections, while transformers are modeled using three-phase linear equivalents. Loads are assumed to be constant impedance. Each generator is modeled as an 8th-order synchronous machine, equipped with an ST1A excitation system that includes an automatic voltage regulator and a power system stabilizer. The governor dynamics are represented by a proportional speed-droop controller with a first-order delay, and the steam turbine parameters are adopted from [208].

Figure 5.17 presents the single-line diagram of the IEEE 9-bus system. The system is composed of 3 synchronous generators, named as 3 two-winding transformers, 6 transmission lines, 3 loads, and a total of 9 buses. A comprehensive model of the IEEE 9-bus system has been developed in MATLAB/Simulink for a detailed EMT

simulation, incorporating key system parameters such as transmission line data, generator specifications, and exciter characteristics. The complete list of parameters utilized in the model is provided in the Appendix D.

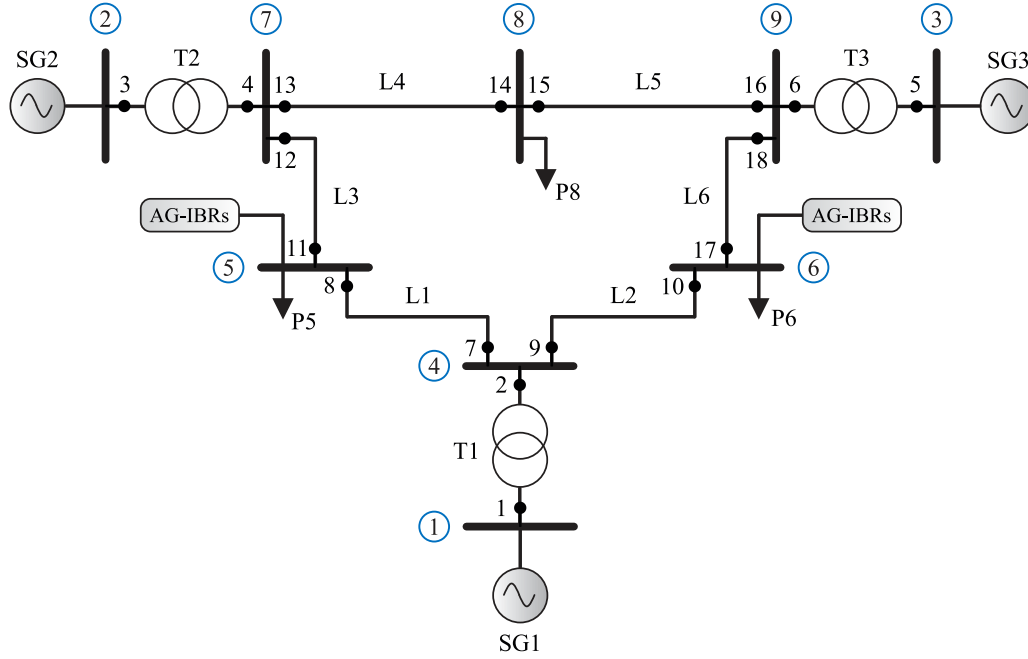


Figure 5.17: IEEE 9-bus system

It is important to note that line dynamics cannot be neglected in the presence of GFM converters due to potential adverse interactions between their fast control responses and the line dynamics, according to [208]. In this case study, each block referred to as aggregate (AG)-IBRs represents an aggregation of 8,000 commercial converter modules employing AC/DC GFM control. Each module has a rated power of 12 kVA, resulting in a total aggregated rating of 96 MVA, which is comparable to that of a synchronous machine. Each group of 320 modules is connected to the medium-voltage network through a 300 kVA low-to-medium voltage transformer, as illustrated in Figure 5.18, and in accordance with the parameters provided in Appendix E. The parameters of the aggregated transformer are obtained by considering 60 commercial 1.6 MVA transformers connected in parallel, as described in [211]. Although a detailed derivation is beyond the scope of this work, the aggregation methodology follows approaches similar to those presented in [213]–[215].

The base values for voltage, power, and frequency can be scaled by

$$U_{r,s} = N_U U_r, \quad (5.45)$$

$$S_{r,s} = k_S S_r, \quad (5.46)$$

$$\omega_{g,s} = N_{\omega} \omega_g, \quad (5.47)$$

where $U_{r,s}$ is the new voltage at the PoC, PCC, or bus of connection. $S_{r,s}$ is the total power of the aggregated IBRs, and $\omega_{g,s}$ is the system frequency. The system parameters can, therefore, be rewritten as

$$\frac{Z_s}{Z} = \frac{U_{r,s}^2 S_r}{S_{r,s} U_r^2}, \quad (5.48)$$

$$\frac{L_s}{L} = \frac{Z_s}{\omega_{g,s}} \frac{\omega_g}{Z}, \quad (5.49)$$

$$\frac{C_s}{C} = \frac{Z \omega_g}{Z_s \omega_{g,s}}. \quad (5.50)$$

The aggregate connection impedance seen from the grid is given by

$$\underline{Z}_{T,LV,s} = N_U^2 \underline{Z}_{T,LV}. \quad (5.51)$$

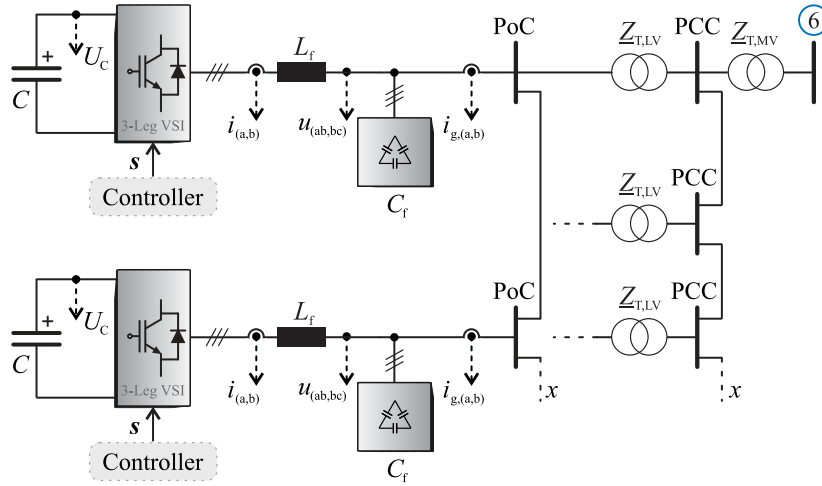


Figure 5.18: Aggregated DER (AG-IBRs) model

For the case studies in this work, the frequency ratio N_{ω} is neglected (equal to 1), as converters operating in a 60 Hz system are generally able to operate in a 50 Hz power system. Therefore, the system impedances and controllers are not scaled in this dimension; it is equivalent to connecting the IBR to a feeder with a different frequency, seen from the inverter.

The PI controller gains are calculated according to their respective controlled elements,

as in [216], and can be scaled together with the scaled parameter values as

$$K_{P,C,s} = C_s \frac{\zeta_C \omega_{C,LP,s}}{\sqrt{2\zeta_C^2 + 1} + \sqrt{(1 + 2\zeta_C^2)^2 + 1}}, \quad (5.52)$$

$$K_{I,C,s} = C_s \frac{\omega_{C,LP,s}^2}{2 \left(2\zeta_C^2 + 1 + \sqrt{(1 + 2\zeta_C^2)^2 + 1} \right)}, \quad (5.53)$$

$$K_{P,BESS,s} = L_{1,s} \frac{2\zeta_{BESS} \omega_{BESS,LP,s}}{\sqrt{2\zeta_{BESS}^2 + 1} + \sqrt{(1 + 2\zeta_{BESS}^2)^2 + 1}}, \quad (5.54)$$

$$K_{I,BESS,s} = L_{1,s} \frac{\omega_{BESS,LP,s}^2}{2\zeta_{BESS}^2 + 1 + \sqrt{(1 + 2\zeta_{BESS}^2)^2 + 1}}. \quad (5.55)$$

For the ancillary controllers, the aggregated gains are written as

$$K_{I,s} = N_U K_I, \quad (5.56)$$

$$K_{U,I,s} = \frac{N_U^2}{k_S} K_{U,I}. \quad (5.57)$$

As the controllers based on MPC are model-based, the aggregation procedure is straightforward, as the scaled parameters are incorporated in the control law.

Figure 5.19 shows the comparison of the relative angle of the inverter with respect to the grid, considering the system with BESS and ancillary controllers enabled. The model is initialized by applying a large disturbance, introduced through an angle difference, followed by a grid voltage step of -0.9 p.u at 2.1 s. The results highlight the contribution to transient stability of the IBR, reducing the angle deviation.

Figure 5.20 shows the comparison of the relative angle of the AG-IBR model with respect to the grid, considering the same system. The results indicate that the aggregated model satisfactorily follows the angle behavior of three individual inverters operating in parallel.

Taking into account the nonidealities introduced by the detailed models of the VSI semiconductors, the aggregated model can provide sufficient accuracy for a wide range of disturbances. However, it is worth noting that when aggregating a large number of inverters, events such as current saturation may lead to reduced accuracy for

specific studies. This highlights the potential value of semiconductor-level aggregation techniques as a useful tool for modeling distributed power plants.

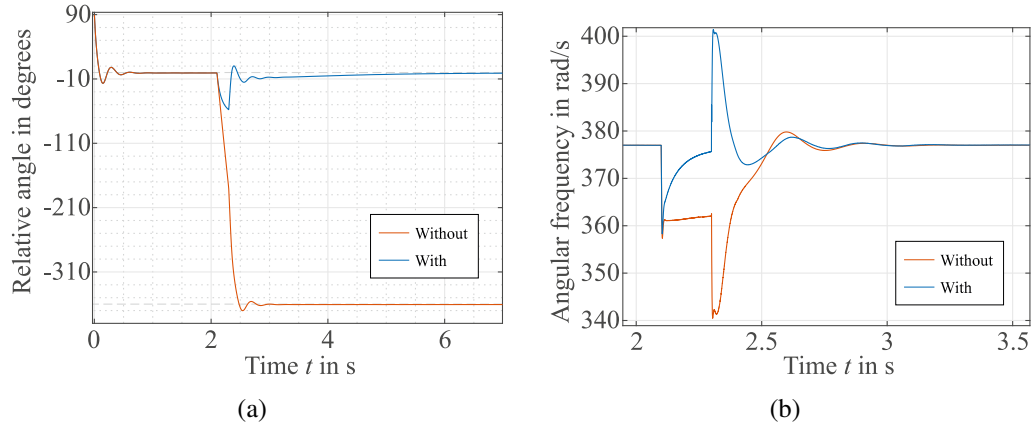


Figure 5.19: Simulation results for the relative angle and internal frequency for ancillary controllers enabled and disabled

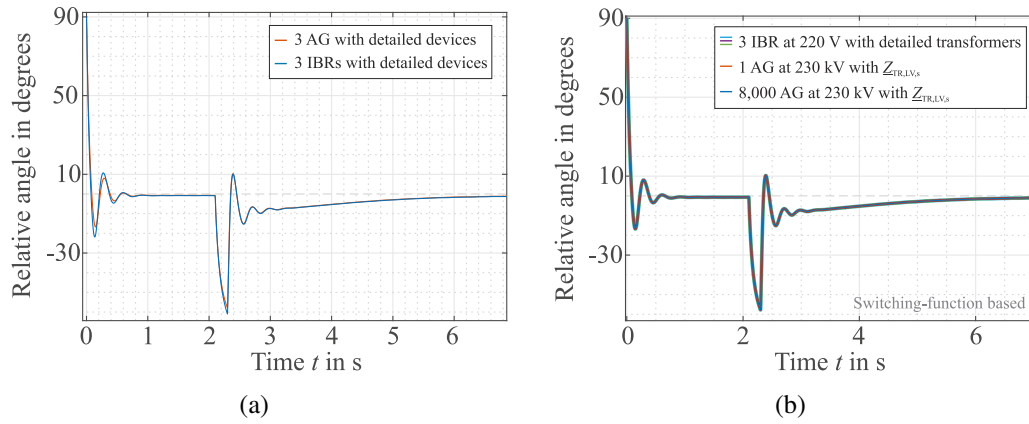


Figure 5.20: Simulation results for the aggregated model

The slight difference is depicted in the results of Figure 5.21(a). On the other hand, Figure 5.21(b) shows the high fidelity representation of the system behavior for considering ideal switches, where the complex oscillations after a critical event are successfully scaled considering all the cascaded devices and controllers.

Considering the same voltage event, Figure 5.22 shows the output currents, the DC-bus voltage, and BESS current for the AG-IBR present the same dynamics as the simulation for multiple inverters. The current saturation is also achieved by the MPC technique as depicted in Figure 5.22(d).

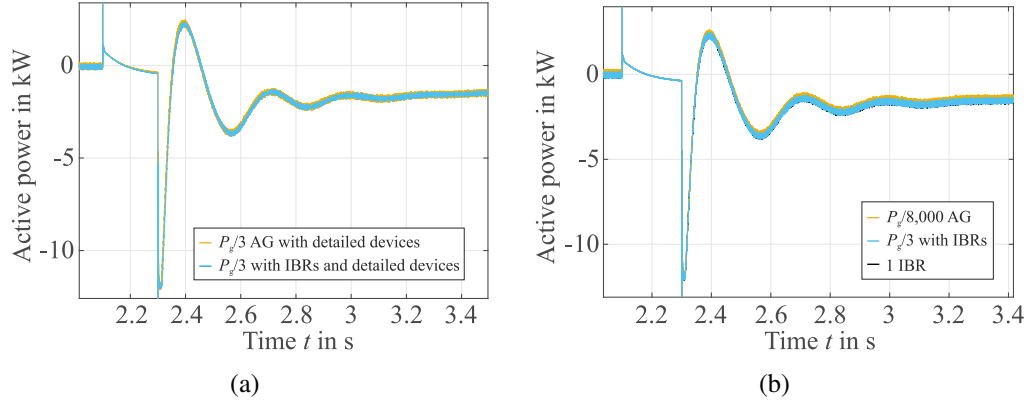


Figure 5.21: Simulation results for the active power of the inverter

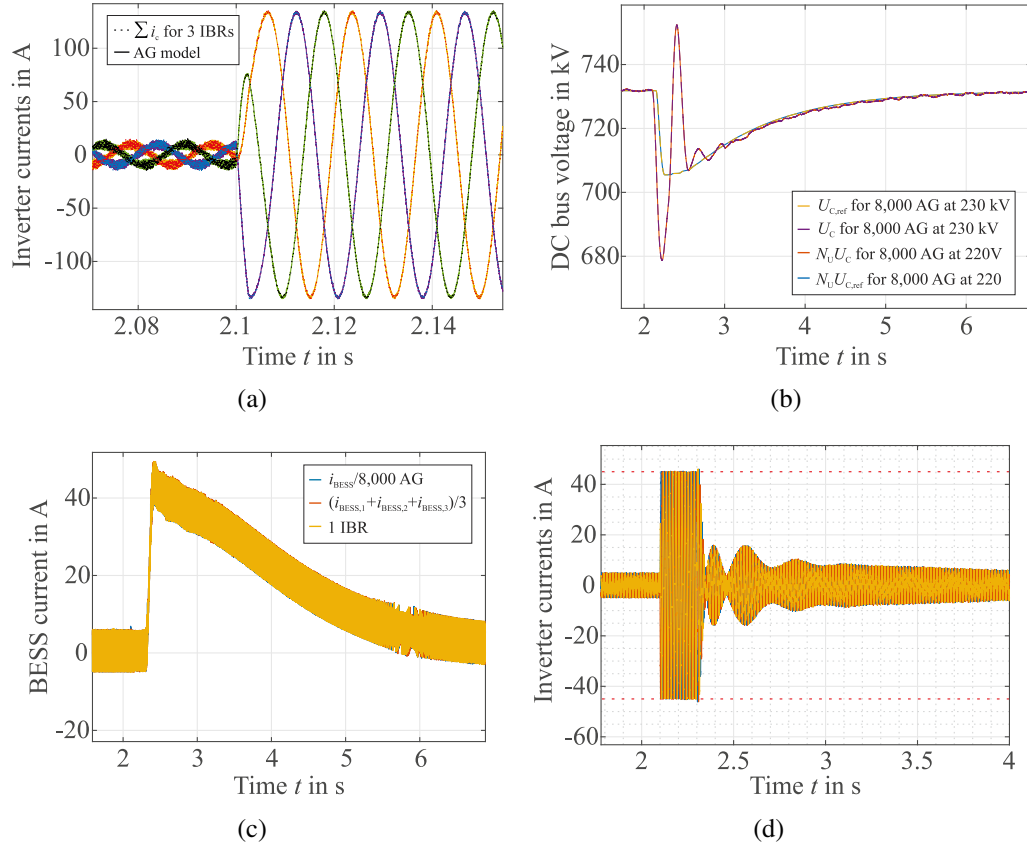


Figure 5.22: Simulation results for the inverter currents, BESS current, and DC-bus voltage of the aggregated model

5.8.1 Performance metrics

The standard power system frequency performance metrics are adopted to assess the performance in the case studies, including the maximum frequency deviation $\|\Delta\hat{f}\|_\infty$, called frequency nadir when negative and zenith when positive, and the RoCoF $|\dot{\hat{f}}|$, are defined, respectively, by [208] as:

$$\|\Delta\hat{f}\|_\infty := \max_{t \geq t_0} |\hat{f}(t_0) - \hat{f}(t)|, \quad (5.58)$$

$$|\dot{\hat{f}}| := \left| \frac{\hat{f}(t_0 + T) - \hat{f}(t_0)}{T} \right|, \quad (5.59)$$

where $t_0 > 0$ is the time when the disturbance is applied, and $T > 0$ is the RoCoF calculation window, which, according to [208], the use of $T = 250$ ms is in accordance with the settings of the recommended protection scheme. Therefore, these metrics provide a quantitative assessment of the dynamic frequency response of the system.

5.8.2 Price mechanism for FFR ⁷

According to [218], potential instantaneous reserve (IR) providers must define their positive and negative limits. If the average active power provided by inverter-based units within a 15-minute interval exceeds the calculated threshold $P_{\text{lim, neg}}$ for negative instantaneous reserve or falls below $P_{\text{lim, pos}}$ for positive IR, the unit is considered available during that interval. The thresholds for positive and negative IR are determined by (5.60):

$$P_{\text{lim, (neg, pos)}} = \mp |P_{\text{c, max}}| \pm \left(M T_A P_r \frac{0.04}{s} \right) \quad (5.60)$$

where M is the proportion of the rated active power of the unit that is marketed for IR, P_r represents the active power, and $0.04/s$ reflects the RoCoF of 2 Hz/s and nominal frequency.

The marketable energy amount of IR is calculated according to [218] via the grid start-up time constant T_A of the respective technology and includes the ratio M of IR and nominal active power:

$$E_{\text{IR}} = \frac{1}{2} M T_A P_r \quad (5.61)$$

As shown in [219], T_A for battery storage sources is freely selectable and therefore an optimization parameter, with respect to market strategies.

⁷This section has been partially published in [217].

The technical requirements for IR are specified in [220], detailing the capabilities of Type-1 and Type-2 storage systems capable of delivering IR. These guidelines define essential GFM characteristics, including the provision of IR and PR, and establish fixed technical requirements that generation facilities, storage systems, and controllable loads must meet to ensure grid stability in a converter-dominated energy landscape.

All units must also provide Primary Reserve during frequency deviations of ± 200 mHz. Type-2 units must operate as controlled voltage sources with impedance limits of 0.35 p.u. (medium voltage) and 0.5 p.u. (high voltage). They must damp oscillations in the 100 Hz to 1 kHz range with a minimum damping factor $\zeta \geq 0.3$. Units must maintain stability at a short-circuit ratio (SCR) ≥ 1 , regulate voltage within ± 10 % of nominal with recovery times under 1 s, and withstand voltage disturbances. They must also handle cumulative fault energy of $P_{\max}$ 2 s within 30 minutes and resume normal operation after current limitations. These specifications ensure IR compatibility with grid stability requirements.

As an example of power and energy requirements for IR, specific results from [221], [222] show that the infinite norm over the required FFR for all scenarios is 33 MW, and the energy consumption during FFR support is very small compared to the power due to the low supply duration, with an average consumption of 0.18 MWh. Relating the energy and power to these values, it represents rated power during 19.6 s, considering the energy for a frequency event as

$$E_{\omega} = H_{\omega} S_r, \quad (5.62)$$

$$T_{\omega} = 2 H_{\omega}. \quad (5.63)$$

For the proposed control strategy for the IBRs analyzed in this work, the percentage of the required energy for a determined frequency event can be defined as

$$h_{C,\%} = \frac{C \left(U_{C,\text{set}}^2 - U_{C,\text{min}}^2 \right) 50}{S_r H_{\omega}}. \quad (5.64)$$

For using p.u. values, the expression becomes

$$h_{C,\%} = \frac{C \left(U_{C,\text{set}}^2 - U_{C,\text{min}}^2 \right) 50}{\omega_g H_{\omega}}. \quad (5.65)$$

For the BESS, it can be written as follows

$$h_{\text{BESS},\%} = \frac{E_{\text{BESS}}}{S_r H_\omega} 100 = \frac{H_{\text{BESS}}}{H_\omega} 100. \quad (5.66)$$

Therefore, the total energy available for FFR for an inverter module is

$$H_c = \frac{E_C + E_{\text{BESS}}}{S_r}. \quad (5.67)$$

Although the first energy term is negligible for IBRs equipped with large BESS, which are designed for long-duration events, it is worth noting that it can be more representative for very short-frequency events and may be dispatched first or in parallel, depending on the control tuning, thereby reducing the utilization of the BESS.

For a practical example, a capacitor rated at 7 p.u. can be considered, along with a DC bus voltage of 3 p.u., with a lower limit of 0.72 times the nominal value. In this scenario, the IBR would be capable of fully supplying very short-duration events lasting less than 41 ms. Considering the lower energy requirement mandated by the grid code, this corresponds to 2%. However, when accounting for fast frequency events lasting up to 300 ms, the contribution increases to 13.6%.

It is worth noting that the capacitor design for a specific IBR can now also consider the most suitable and cost-effective energy storage technology to support grid events. Changes in the overall inverter lifespan and potential revenue from participation in instantaneous reserve markets, or even avoided resiliency costs for the system, can be taken into account when compared to the increased size and initial investment cost of the IBR.

Having defined the control framework and implementation strategies for the system, Chapter 6 focuses on validating these approaches through high-fidelity simulations and experimental results. It examines the performance of the system under various scenarios, assessing both transient and steady-state responses.

6 Simulation and Experimental Results

This chapter presents the results obtained from simulation models and experimental prototypes demonstrating the performance of the topologies along with the designed control methods for power injection, voltage regulation, frequency support, and rejection of harmonics and load disturbances. The system is initially characterized based on its relevant parameters, followed by showcasing simulation results obtained in the Matlab® Simulink and Altair® PSIM environments to illustrate the behavior of the system variables under specific conditions. Also, experimental results are presented to confirm the expected performance of the proposed topology and control system for the multi-input split-source inverter in both steady-state and transient conditions.

Furthermore, the harmonic spectrum analysis and total harmonic distortion of the obtained waveforms are presented to verify compliance with IEEE Standard 1547 [177] concerning power quality. The SEIG used in the simulations matches the experimental testbed of the microgeneration system, featuring a nominal power of 3.73 kW, line voltages of 220 V, and a frequency of 60 Hz.

6.1 Case studies with MPC

The case studies were selected to evaluate the operation of the proposed topologies and control strategies in different applications. The three-phase MISSI is employed in grid-tied and off-grid operations, and its multilevel and bidirectional topology derivations are evaluated for grid-tied operation as well.

6.1.1 Grid-tied MISSI ⁸

To demonstrate the performance of the proposed topology driven by the MPC control technique for grid connection, Matlab® Simulink model simulation results of the three-phase MISSI employing one and three DC power sources are presented. The simulated system parameters are shown in Table 3.

Figure 6.1 shows the reference steps of the variables for the converter with a single DC power source. The input current reference steps up and down by 40 A during the interval from 0.3 to 0.5 s. The DC-bus voltage reference steps up and down by 30 V during the interval from 0.7 to 0.82 s. During the interval from 1 to 1.1 s, the reactive power

⁸This section has been published in a similar form in [52].

reference steps up and down by 2 kVA, and a fast dynamic response can be observed in the output currents.

Table 3: Simulation Parameters for the Grid-Connected MISSI

Description	Variable value
Sampling period	$T_s = 20 \mu s$
Rated power	$S_r = 12 \text{ kVA}$
DC bus voltage	$U_C = 850 \text{ V}$
DC voltage source 1	$U_1 = 100 \text{ V}$
DC voltage source 2	$U_2 = 50 \text{ V}$
DC voltage source 3	$U_3 = 80 \text{ V}$
Line voltages (RMS)	$U_{g,r} = 220 \text{ V}$
DC-link capacitor	$C = 4.7 \text{ mF}$
Input inductors	$L_{(1,2,3)} = 2 \text{ mH}, R_{(1,2,3)} = 0.1 \Omega$
Operating frequency of inductors	$f_{L,(1,2,3)} \approx 5.6, 2.6, 4.2 \text{ kHz}$
Output inductors	$L_f = 3 \text{ mH}, R_f = 0.2 \Omega$

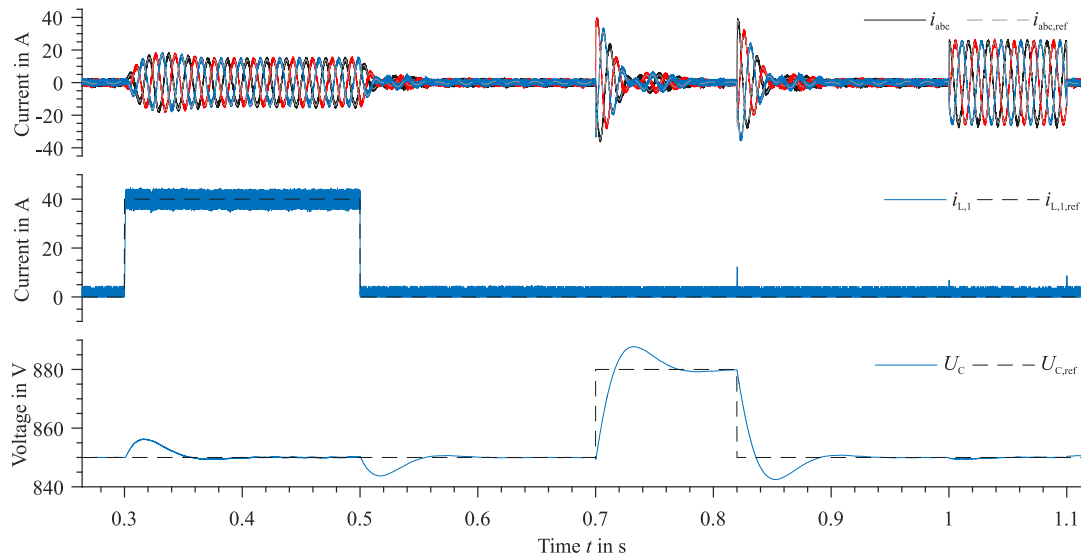


Figure 6.1: Simulation results for three-phase MISSI with one DC source: output currents (upper), input current (middle), and capacitor voltage (lower)

Figure 6.2 presents the reference steps of the variables for the converter with three DC power sources. The input currents reference steps up and down with different values and times at the interval 0.3 to 0.7 s. The DC bus voltage reference steps up and down by 30 V during the interval from 0.7 to 0.82 s. Finally, at the interval 1 to 1.1 s, the reactive power reference steps up and down by 2 kVA, and it can be seen the same dynamic in the output currents.

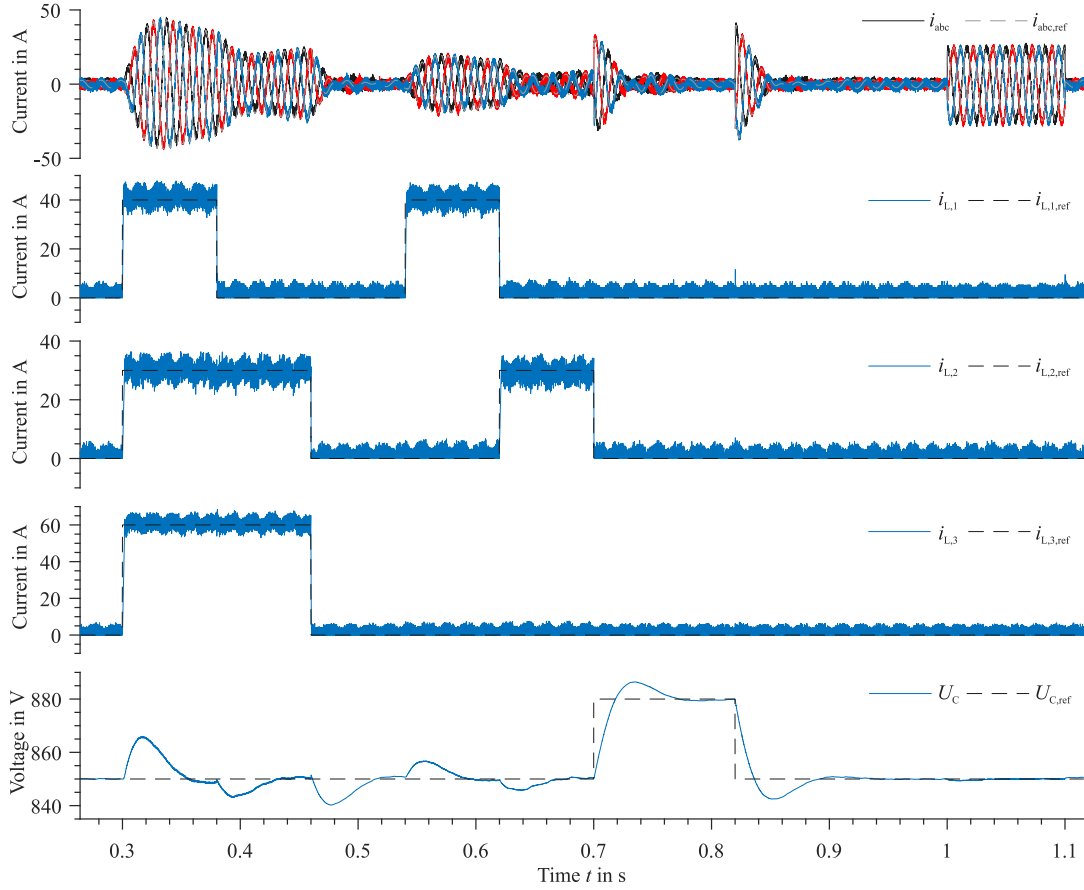


Figure 6.2: Simulation results for three-phase MISSI with three DC sources: output currents (upper), three individual input currents, and capacitor voltage (lower)

It can be seen that the converter was able to inject power from three independent sources into the grid. System modeling and operation were presented. The simulation results presented the capabilities of the system and the good performance of the control technique to manage the MISSI variables. Regarding the system parameters, according to [65], performance may not be affected by an overestimated inductance value.

6.1.2 Standalone MISSI

Supplying an isolated load is a general role for a power converter. Thus, the MISSI is evaluated considering a system of around 10 kW, designed to supply a load with a line voltage of 220 V. The DC bus voltage is 800 V, and $\mathbf{u}_U^T = [50 \ -50]$ V. Moreover, a simulation using the well-established electrical circuit simulation software, Altair® PSIM, is conducted, also presenting verification using different simulation software.

Using the MPC strategy with the LC model [55], coded in a C language block, the

control references are the output voltages, as well as the DC bus voltage and the input sources. The power injection by the input sources is, therefore, governed by the value of the load power. Figure 6.3 illustrates the operation of the inverter with two input sources.

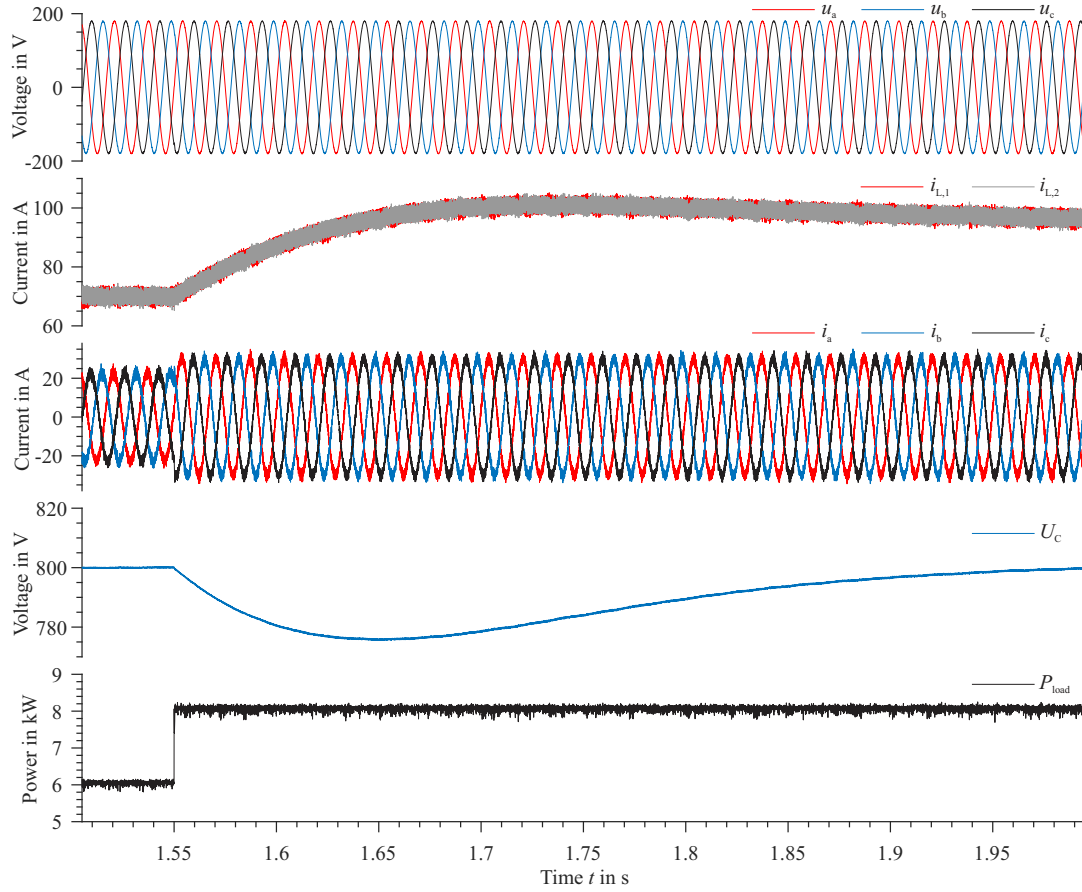


Figure 6.3: Simulation results for three-phase isolated MISSI with two DC sources: load voltages, input currents, output currents, DC bus voltage, and active power consumed by the load

From the results, it is possible to observe a good dynamic performance where, in response to an abrupt power step, the inverter currents instantaneously increase and the reference power of the input sources is also increased. The regulation of the output voltages demonstrated good performance.

6.1.3 Multilevel and bidirectional inverters

This section presents the simulation results for the proposed converters based on the generalization of the MISSI topology with multiple input sources. Results for the

grid-tied application of the multilevel and bidirectional converters are presented, where the applied control is the previously described MPC. The output filter parameters are the same presented in Table 3. Once the output filter model is the same, the main parameters of change are the switching table for each converter and the characteristics of the cost function. For multilevel converters that require voltage regulation of multiple capacitors, a term in the cost function, weighting the voltage on the capacitors, should be added.

The main objective of these results is to demonstrate the feasibility of the proposed converters in injecting power into the system to which they are connected from different DC sources, independently, with balanced or unbalanced voltages, employing different renewable energy generation technologies, in this case study. Input current reference steps are performed to evaluate the system dynamics.

Figure 6.4 presents the simulation results for the FC-MISSI with three input sources connected to the switching nodes of the inverter, composing a system connected to a grid with line voltages of 220 V, and $\mathbf{u}_U^\top = [100 \ 80 \ 50]$ V. The results indicate successful power injection from the input sources and voltage regulation of the system capacitors. This is achieved using a specific cost function, in the natural reference frame, as expressed in (6.1).

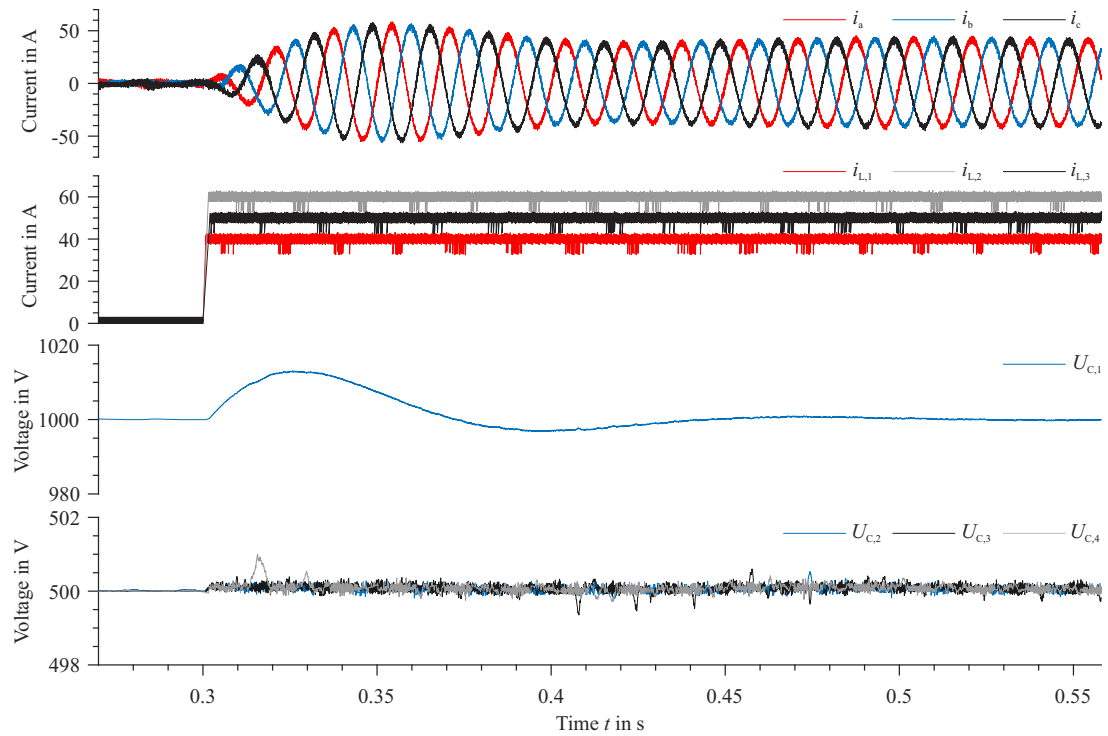


Figure 6.4: Simulation results for three-phase FC-MISSI with three DC sources: output currents, input currents, DC bus voltage, and leg capacitor voltages

$$\begin{cases} g_i = \left(i_{a,\text{ref}} - \hat{i}_{a,j,k+2}\right)^2 + \left(i_{b,\text{ref}} - \hat{i}_{b,j,k+2}\right)^2 + \left(i_{c,\text{ref}} - \hat{i}_{c,j,k+2}\right)^2, \\ g_C = \left(\frac{U_{C,\text{ref}}}{2} - \hat{U}_{C,2,j,k+2}\right)^2 + \left(\frac{U_{C,\text{ref}}}{2} - \hat{U}_{C,3,j,k+2}\right)^2 + \left(\frac{U_{C,\text{ref}}}{2} - \hat{U}_{C,4,j,k+2}\right)^2, \\ g_{i,L} = \left(i_{L,1,\text{ref}} - \hat{i}_{L,1,j,k+2}\right)^2 + \left(i_{L,2,\text{ref}} - \hat{i}_{L,2,j,k+2}\right)^2 + \left(i_{L,3,\text{ref}} - \hat{i}_{L,3,j,k+2}\right)^2, \\ g_c = g_i + \lambda_1 g_{i,L} + \lambda_2 g_C. \end{cases} \quad (6.1)$$

with $\lambda_1 = 10^6$ and $\lambda_2 = 10^{-2}$.

Figure 6.5 presents the simulation results for the DC-MISSI connected to a grid with line voltages of 220 V, with input sources also connected to the midpoint of each leg, and $\mathbf{u}_U^\top = [50 \ 50 \ 50] \text{ V}$. The results demonstrate effective power injection from the input sources and voltage regulation of the capacitors, utilizing the MPC technique.

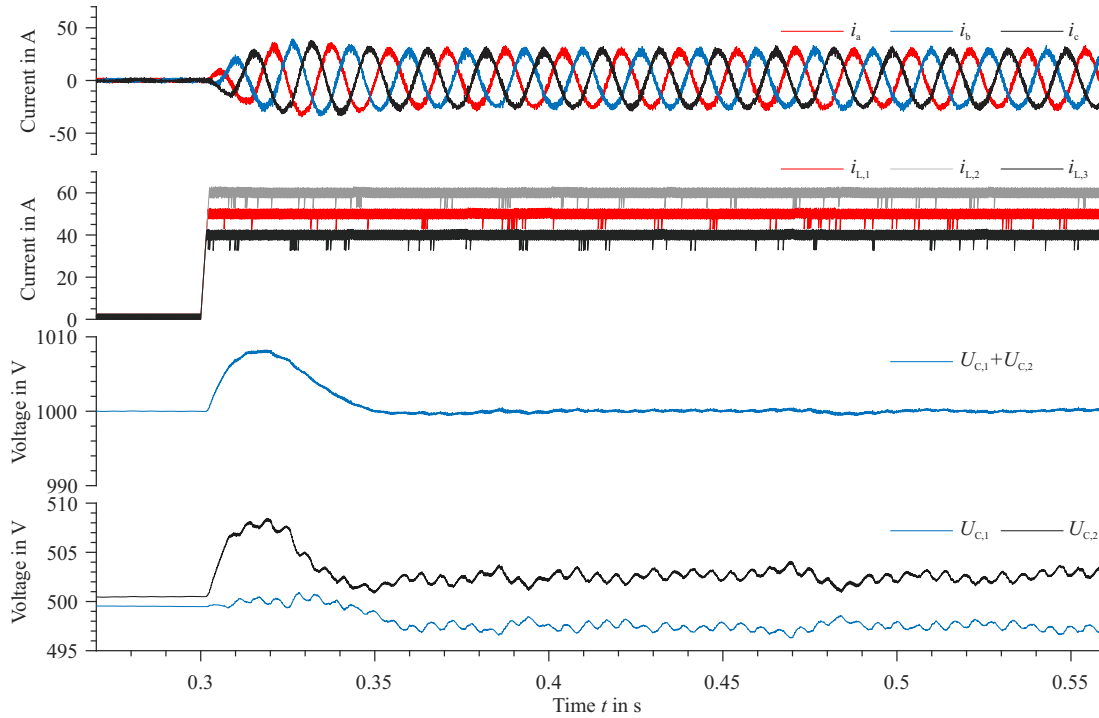


Figure 6.5: Simulation results for three-phase DC-MISSI with three DC sources: output currents, input currents, and DC bus voltages

Figure 6.6 presents the results for the DC-MISSI with four independent input sources, $\mathbf{u}_U^\top = [50 \ 50 \ 50 \ -700] \text{ V}$, where three sources have a lower voltage and the fourth has a higher voltage. The results show effective power injection into the grid and voltage regulation of the system capacitors. This topology may be more suitable for powering

small electric vehicles due to its restricted linear voltage region when employing all six possible sources. The specific cost function in the natural reference frame can be written as

$$\begin{cases} g_i = \left(i_{a,\text{ref}} - \hat{i}_{a,j,k+2}\right)^2 + \left(i_{b,\text{ref}} - \hat{i}_{b,j,k+2}\right)^2 + \left(i_{c,\text{ref}} - \hat{i}_{c,j,k+2}\right)^2, \\ g_C = \left(\hat{U}_{C,1,j,k+2} - \hat{U}_{C,2,j,k+2}\right)^2, \\ g_{i,L} = \sum_{x=1}^4 \left(i_{L,x,\text{ref}} - \hat{i}_{L,x,j,k+2}\right)^2, \\ g_c = g_i + \lambda_1 g_{i,L} + \lambda_2 g_C. \end{cases} \quad (6.2)$$

with $\lambda_1 = 10^6$ and $\lambda_2 = 10^1$.

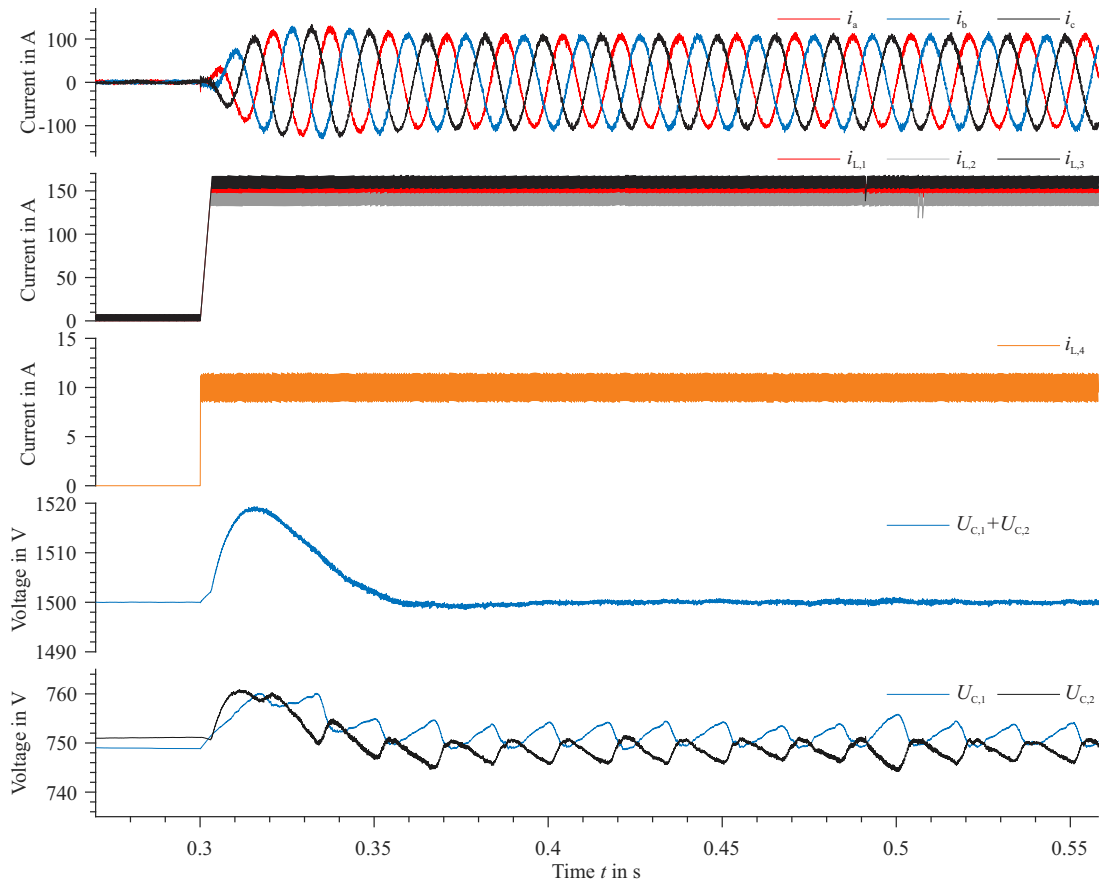


Figure 6.6: Simulation results for three-phase DC-MISSI with four DC sources

Figure 6.7 presents the simulation results for the 9S-MISSI with six sources with common ground, and $\mathbf{u}_U^T = [35 \ 35 \ 35 \ 35 \ 35 \ 35] \text{ V}$, composing a system connected to a grid with a line voltage of 220 V. The results demonstrate successful power injection from the input sources and voltage regulation of the system capacitors. In addition, it is

worth noting that from instant 0.5 to 0.7 s, internal energy transfer can be observed from unidirectional to bidirectional sources, which could be batteries.

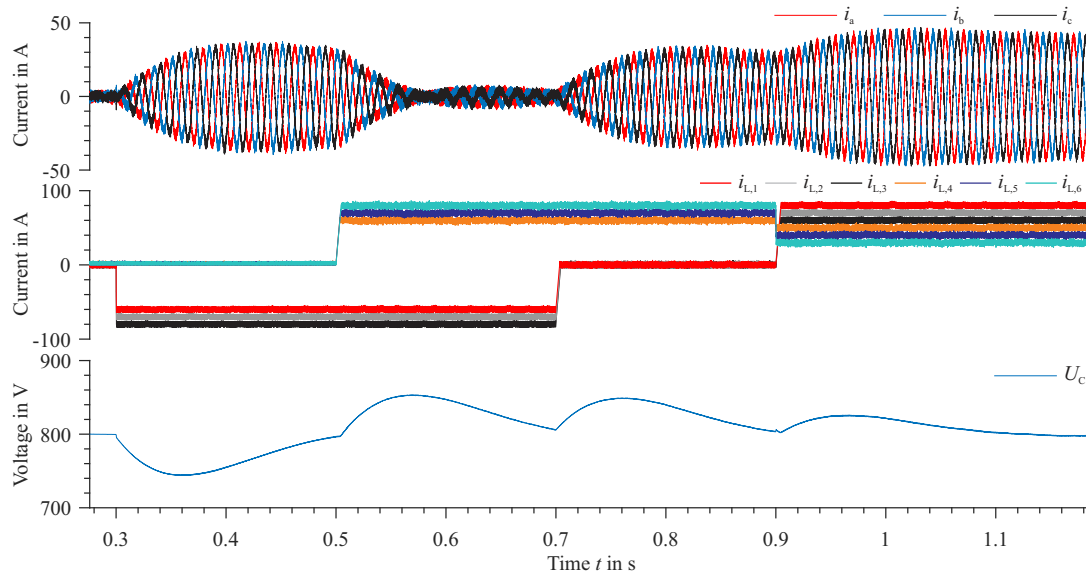


Figure 6.7: Simulation results for three-phase 9S-MISSI with six DC sources

Grid-supporting functionalities can be added to a GFL or GFM strategy. It provides ancillary services such as transient support due to sudden frequency dips and reactive support during voltage dips, as presented in Figure 2.6(d). In this sense, Figure 6.8(a) shows the behavior of the active power injected and absorbed by the inverter in response to variations in grid frequency. Over a timescale of 2 s, the frequency oscillated between the upper and lower limits based on those recommended by relevant standards. Despite these variations, the frequency-active power curve remained compliant with the expected behavior. Considering the test was performed with a frequency ramp at a rate of -5 Hz per second, this dynamic is sufficiently fast to interact with the linear controller of the DC bus voltage, leading to observable overshoots around 57 Hz and 61 Hz. Under steady-state conditions or during slow frequency variations, these dynamics can be neglected. The operating points, including the maximum and minimum points, are determined by the maximum power capacity of the inverter and the grid operation characteristics. In this case, 58.8 Hz and 61.2 Hz were adopted as frequency thresholds, simulating a scenario where the support operation of this low-power equipment aims to assist the power system in returning to ideal operating conditions. Various methods and frequency envelopes can also be employed, as discussed in [125].

Moreover, Figure 6.8(b) depicts the voltage-reactive power curve. The simulation results demonstrate that the converter was able to inject reactive power into the grid when the voltage dropped below the established limit and absorb reactive power when the voltage

exceeded the limit. The timescale for this result is 1 s, and consequently, the behavior of the control loop can be observed as exhibiting slight overshoots. The voltage thresholds were set to 10% of the ideal operating value. It is noteworthy that converters exhibit a rapid response capability to dynamic power variations, depending on the control design. This highlights the significant potential of converters to enhance the dynamic characteristics of the grid if provided for in the control strategy.

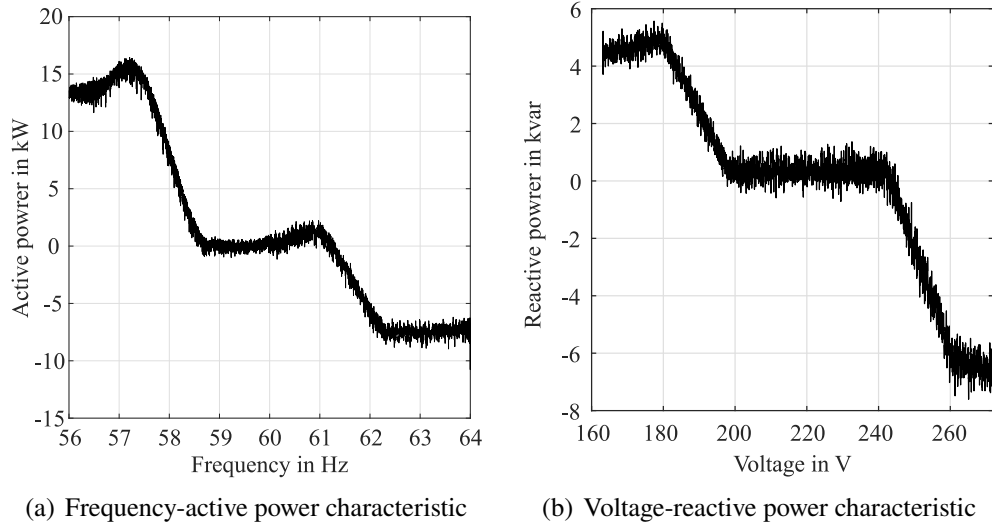


Figure 6.8: Simulation results for the grid-supporting response by the inverter

It is worth noting that for multilevel converters, which have more switching states, the employed predictive control technique, being an exhaustive search method, becomes difficult to implement in commercial hardware as the processing time increases. This contrasts with the two-level converter, which has fewer switching states and thus shorter processing times. Therefore, new control techniques can and should be explored. For instance, using Space Vector Modulation, as previously described, to develop a control strategy. Additionally, predictive control methods with shorter processing times, such as convex optimization techniques employing the Karush-Kuhn-Tucker conditions [137], the Frank-Wolfe algorithm, Nesterov's accelerated gradient, and other methods [223], [224], can be utilized.

6.1.4 SEIG-based off-grid system ⁹

The performance of the MISSI driven by the MPC control technique is presented in this section. Model simulation results of the three-phase MISSI employing three DC power

⁹This section has been published in a similar form in [60].

sources connected to independent PV arrays are presented next. The simulated system parameters are shown in Table 4. The system diagram was presented in Figure 5.3.

Table 4: Simulation Parameters for the Off-Grid System

Description	Value
SEIG	
Nominal power	$P_N = 3.7 \text{ kW (5 cv)}$
Line voltages (RMS)	$U_{g,r} = 220 \text{ V}$
Frequency	$f = 60 \text{ Hz}$
Excitation capacitor	$C_f = 40 \text{ }\mu\text{F}$
Thévenin equivalent	$L_g = 2.5 \text{ mH}, R_g = 0.02 \text{ }\Omega$
PV Array	
Rated power (25°C and 1 kW/m ²)	$P_{PV,MPP} = 3 \times 1000 \text{ W}$
Voltage at MPP	$U_{MPP,x} = 34.4 \text{ V}$
Current at MPP	$i_{L,MPP,x} = 29.08 \text{ A}$
Inverter	
Sampling period	$T_s = 20 \text{ }\mu\text{s}$
DC bus voltage	$U_C = 650 \text{ V}$
DC-link capacitor	$C = 4.7 \text{ mF}$
Capacitor associated with PV	$C_{PV} = 1 \text{ mF}$
Input inductors	$L_{(1,2,3)} = 2 \text{ mH}, R_{(1,2,3)} = 0.1 \text{ }\Omega$
Operating frequency of inductors	$f_{L,(1,2,3)} \approx 2.6 \text{ kHz}$
Output inductors	$L_f = 3 \text{ mH}, R_f = 0.2 \text{ }\Omega$
ELC resistor	$R_{ELC} = 48 \text{ }\Omega$

Figure 6.9 presents the simulation results for irradiance steps on the PV arrays of the system under no load conditions. The fast reference tracking can be seen in the dynamics of the input currents. The system variables present a good regulation and PCC voltages remain within power quality limits.

The power generation of the system depends on multiple power sources. In this sense, Figure 6.10 shows the current characteristics of an unbalanced nonlinear load that exceeds the power generation capacity of the SEIG. Figures 6.11 and 6.12 present the system performance for the 5 kVA load step at instant 2 s. An adequate performance can be seen from the controlled variables. From the power waveforms, the converter supplies the unbalanced power for the system.

It is in the control targets to minimize the harmonics caused by nonlinear loads; for that purpose, the PCC voltages total harmonic distortion (THD), SEIG total rated-current distortion (TRD), and their harmonic content are measured and shown in Figure 6.13. It is possible to note that even with nonlinear loads connected to PCC, the system is able to maintain the individual harmonics lower than the maximum level recommended by the IEEE 1547 standard [177]. On the other hand, in terms of total distortion, the

system remains at values close to the limits of THD (8%) and TRD (5%).

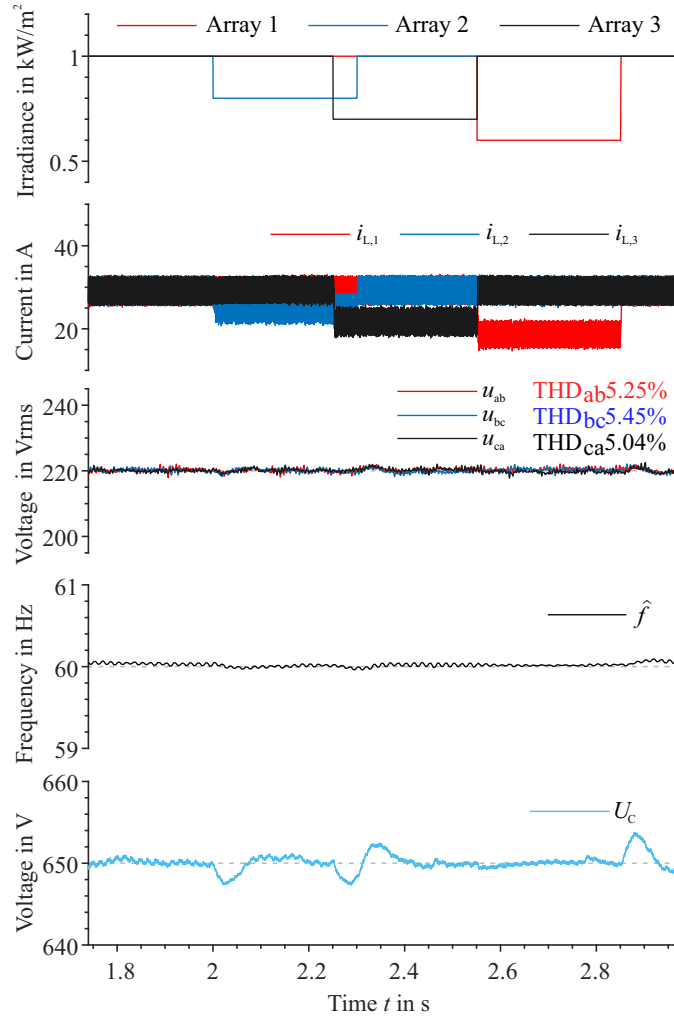


Figure 6.9: Simulation results for the system under irradiance steps

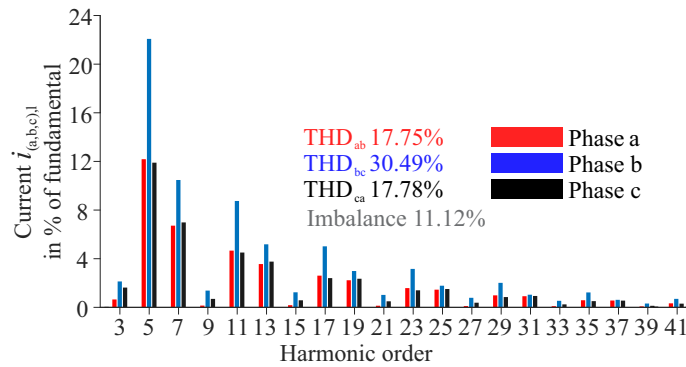


Figure 6.10: Characterization of the 5 kVA load and its current harmonic content

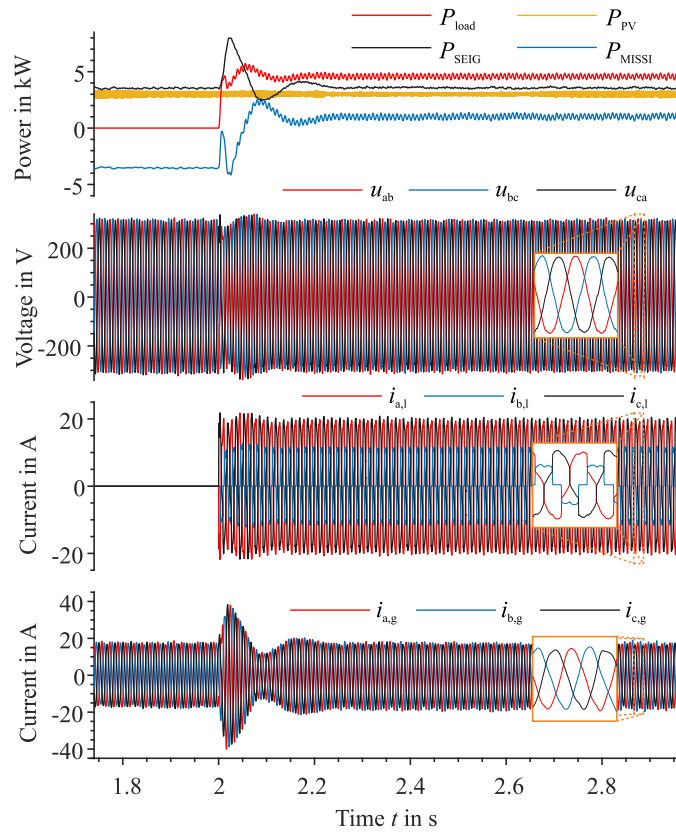


Figure 6.11: Simulation results for the system under unbalanced nonlinear load - part 1

Regarding the presented results, a discontinuous control strategy was applied and evaluated, showing that the converter can inject power into the system from three independent MPPTs of the PV arrays. In addition, the converter is able to regulate the PCC voltages in terms of amplitude, imbalance, and harmonic distortion.

6.1.4.1 Experimental analysis

Following the configuration of the aforementioned system, an experimental analysis is performed to verify the simulated behavior and the converter operation. Due to hardware restrictions, a MISSI with a maximum of two input sources is employed, and the PV system is replaced by DC voltage sources with a maximum output current of 10 A. The primary machine (PM) consists of a three-phase induction motor, which is driven by a WEG-CFW700 frequency inverter. The PM is mechanically coupled to another three-phase asynchronous machine, the SEIG, whose parameters are shown in Table 20 in Appendix G.

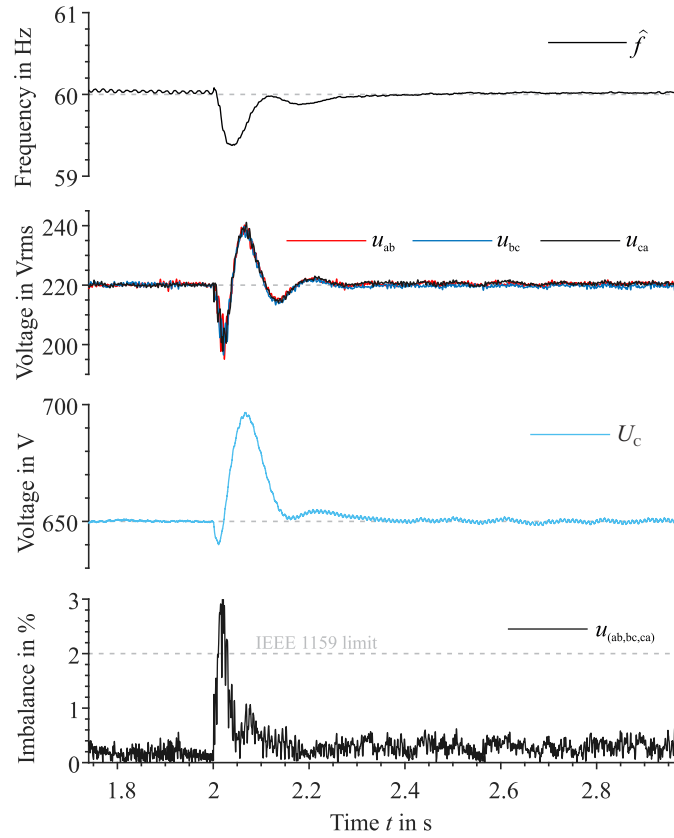


Figure 6.12: Simulation results for the system under unbalanced nonlinear load - part 2

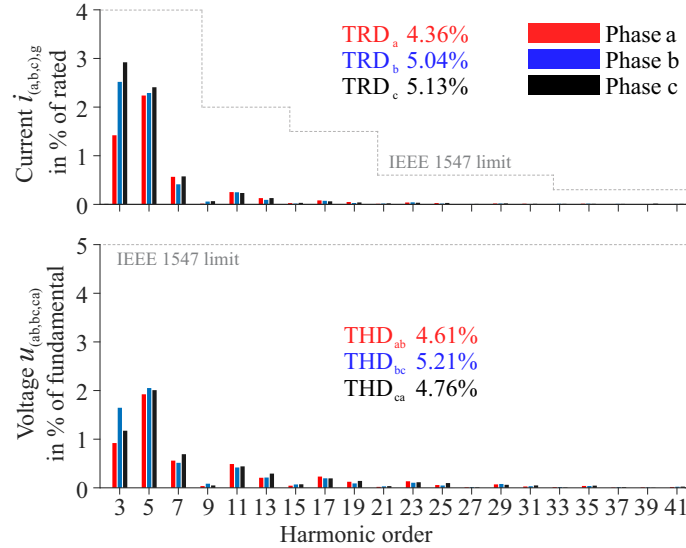


Figure 6.13: Simulation results of harmonic spectrum of SEIG currents and PCC line voltages under unbalanced nonlinear load condition

The frequency inverter is programmed to maintain the speed of the primary machine constant above the synchronous speed of the SEIG, ensuring that the electrical frequency of the SEIG with nominal load does not deviate significantly from the 60 Hz nominal frequency of the system. To assist the excitation of the SEIG, a three-phase excitation capacitor bank of 40 μF arranged in a delta connection is connected. This value is obtained from the magnetization curve of the machine, in order to obtain the nominal line voltage of 220 V with the SEIG at no-load conditions.

Voltage regulation, frequency control, harmonic compensation, and power injection from the PV source are performed by the MISSI. Experimentally, a converter manufactured by Semikron is used, containing four half-bridges composed of IGBTs, drive units, and capacitors for the DC bus. The components of the inverter are described in Table 5.

Table 5: Hardware Specifications of the MISSI

Component	Model
Diode Modules	SKKD42F
IGBT Modules	SKM75GB128D
IGBT Drivers	SKHI22A

As the studied system deals with a three-wire topology, only three legs of the VSI are used. The connection of the MISSI to the PCC is made through an inductive filter, which, along with the excitation capacitors, forms a second-order low-pass filter, attenuating high-frequency harmonic components.

The designed controllers are implemented digitally through a DSP, model TMS32028335 from Texas Instruments. In this DSP, analog inputs corresponding to the measurement of system variables are connected. Additionally, digital outputs are used for the frequency inverter drive signals and also for connecting and disconnecting loads in the system.

Figure 6.14 presents, initially, the experimental results for the three-phase SSI connected to the PCC. A resistive load of 1.6 kW is connected to the system. In this configuration, the inverter injects active power into the system from the input source and compensates for reactive power for the voltage regulation at PCC.

Figure 6.15, shows the behavior of the DC bus voltage for the connection of a load at the initial instant and, at the second instant, the connection of the input source, injecting power into the system, as evidenced by the behavior of the DC bus voltage.

In Figure 6.16, the MISSI topology with only one input source is employed, representing the removal of one diode from the previous configuration. It can be noted that with the same bus voltage, input power can be injected, and the PCC voltage can be regulated. However, due to the reduction of the linear region, a ripple in the input current appears,

also due to the reduced weighting in the cost function ($\lambda = 2$).

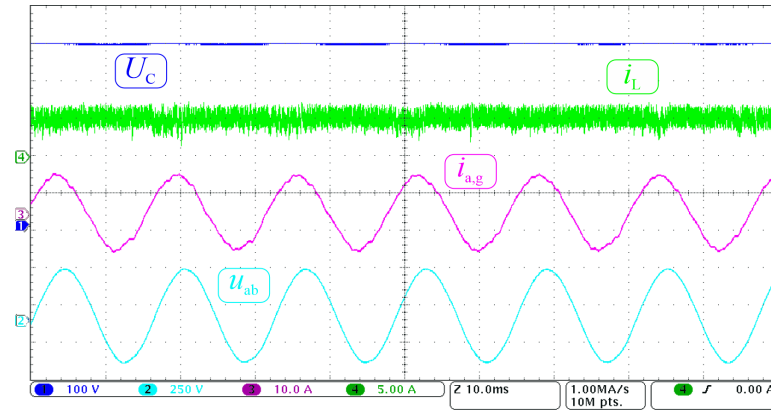


Figure 6.14: Experimental results for the three-phase SSI with a DC input voltage of 100 V and a DC bus voltage of 500 V

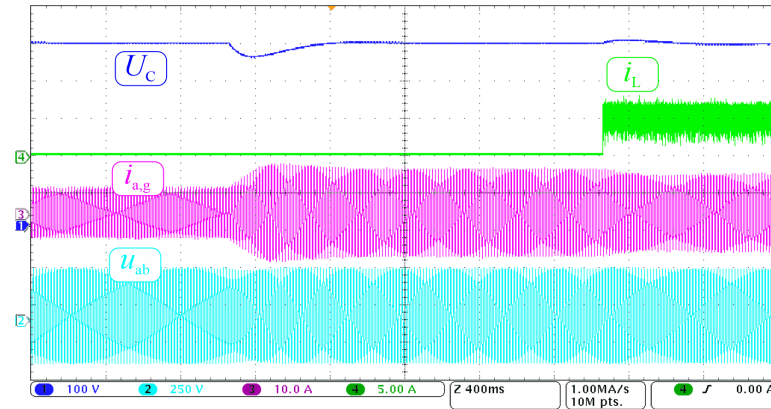


Figure 6.15: Load connection and input source connection for the SSI

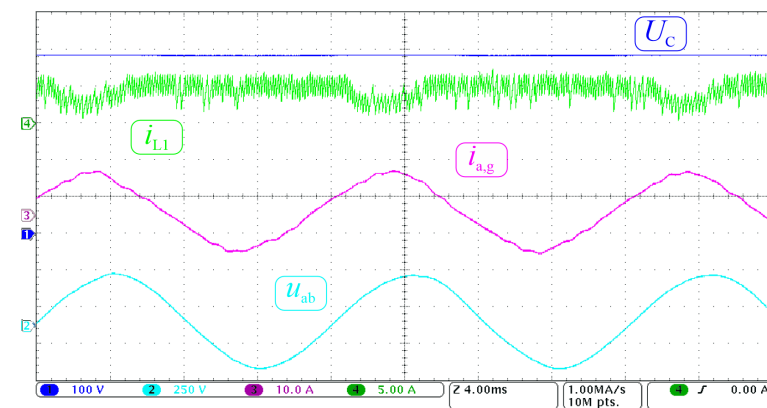


Figure 6.16: MISSI with one 70 V input source and DC bus voltage of 500 V

Figure 6.17 highlights the behavior of the input inductor current, presenting its switching cycle. In this condition, the estimated maximum operation frequency is 7.5 kHz.

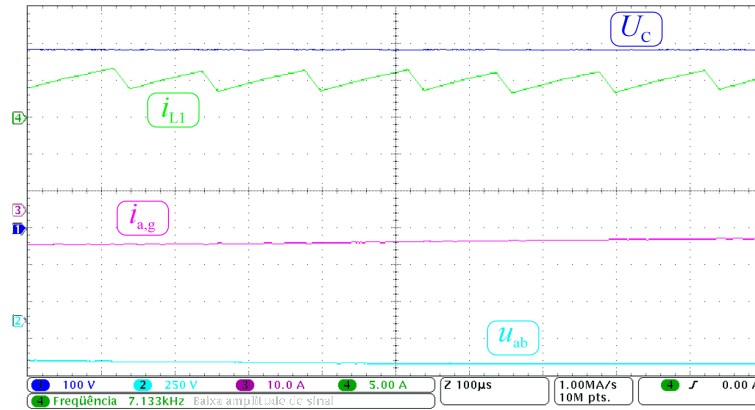


Figure 6.17: Input current waveform and frequency for the MISSI with one input source

Figure 6.18 presents the input currents, output current, and the line voltage synthesized by the inverter. The DC bus voltage is set to 650 V and the input voltages are $\mathbf{u}_U^T = [30 \text{ } -40] \text{ V}$. Based on the analysis derived in Section 3.2, the linear region limit is $U_a = 141.42 \text{ V}$. Since the inverter provides reactive power for the SEIG, the reference step in the input currents produces small changes in the output currents of the inverter.

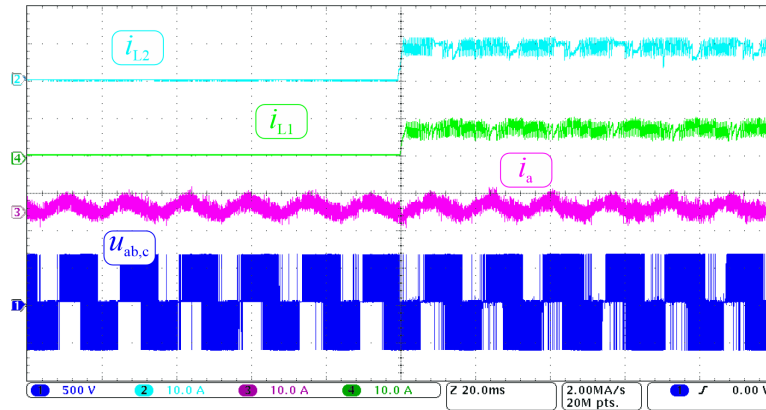


Figure 6.18: Input current step for the MISSI with two input sources

In Figure 6.19, the two input currents, the output current, and the line voltage synthesized by the converter can be seen. The variable switching frequency of the voltage can be noted. The average operating frequency of the input inductor, which can be approximated by the equation described in Section 3.1.1, can be verified, where $\mathbf{u}_U^T = [35 \text{ } -35] \text{ V}$, $U_C = 650 \text{ V}$ and $T_s = 20 \text{ } \mu\text{s}$, and therefore $f_{L,\max,(1,2)} \approx 2.69 \text{ kHz}$. In Figure 6.20, reference steps in the input currents were applied, demonstrating the rapid dynamic response to power changes by the input sources. Figure 6.21 shows the instantaneous

power injected by source 1. The DC input voltage sources are set to 40 V, while the current references are 7 A. Similarly, the power for source 2 can also be verified.

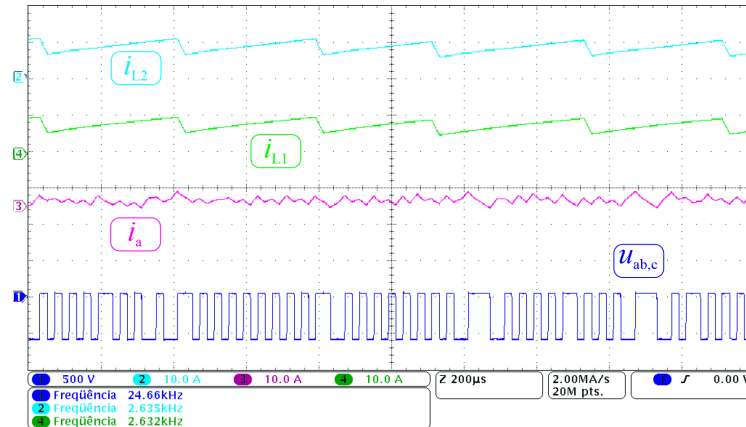


Figure 6.19: Operation of the input currents and line voltages synthesized by the inverter

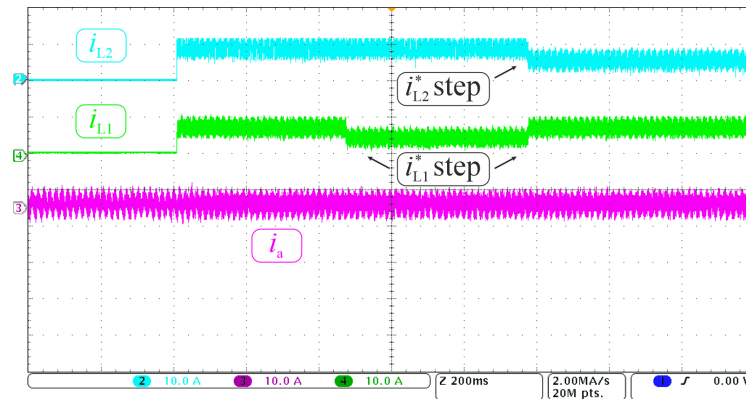


Figure 6.20: Reference steps for the input currents

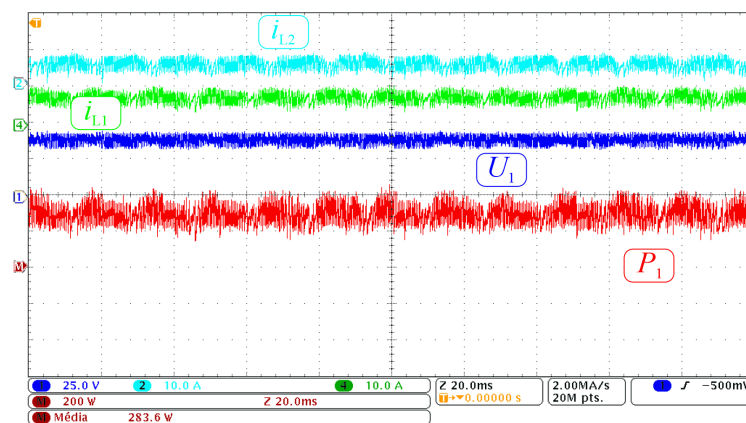


Figure 6.21: Active power injection

The experimental results confirm that the split-source converter employing multiple input sources with a reduced number of active switches can be successfully implemented, allowing power injection into the system. The MPC control, implemented on commercial hardware, demonstrated the expected performance. However, at the point of operation, due to hardware limitations, the linear region was not sufficiently adequate to minimize the level of harmonics in the converter output currents. Nonetheless, these experimental results validate the feasibility of the converter implementation. Furthermore, C-HIL testing can provide a high-fidelity and flexible means of validating control strategies.

6.2 Case studies for ancillary services

For case studies, the DC voltage ancillary controller is evaluated considering small-size low-power islanded power systems, applied to VSI, since it is a generic control block that can be employed in different DER spread throughout the system. The evaluation of coordination with BESS is also presented. The feasibility and performance of the proposed method are verified through software-in-the-loop simulations.

6.2.1 SEIG based microgrid ¹⁰

In order to demonstrate the performance of the proposed control method driving the off-grid system, simulation results of the three-phase VSI as a shunt DSTATCOM are presented. The simulated system parameters are shown in Table 6.

Table 6: Simulation Parameters for the Microgrid

Description	Variable value
Sampling period	$T_s = 20 \mu\text{s}$
DC bus voltage	$U_C = 500 \text{ V}$
Line voltages (RMS)	$U_{g,r} = 220 \text{ V}$
DC capacitor	$C = 4.7 \text{ mF}$
Output inductor	$L_f = 3 \text{ mH}, R_f = 0.2 \Omega$
Excitation capacitor	$C_f = 40 \mu\text{F}$
Distribution line impedance	$L_{CL,(1,2)} = 86 \mu\text{H}, R_{CL,(1,2)} = 0.36 \Omega$ $L_{CL,3} = 172 \mu\text{H}, R_{CL,3} = 0.725 \Omega$
Ancillary discrete controller gains	$K_I = -0.2/T_s, K_r = 1$

Figures 6.22(a), 6.22(c), and 6.22(e) show the results for sudden load connection and disconnection with ELC located in position 2, pointed out in Figure 5.5.

¹⁰This section has been published in a similar form in [179].

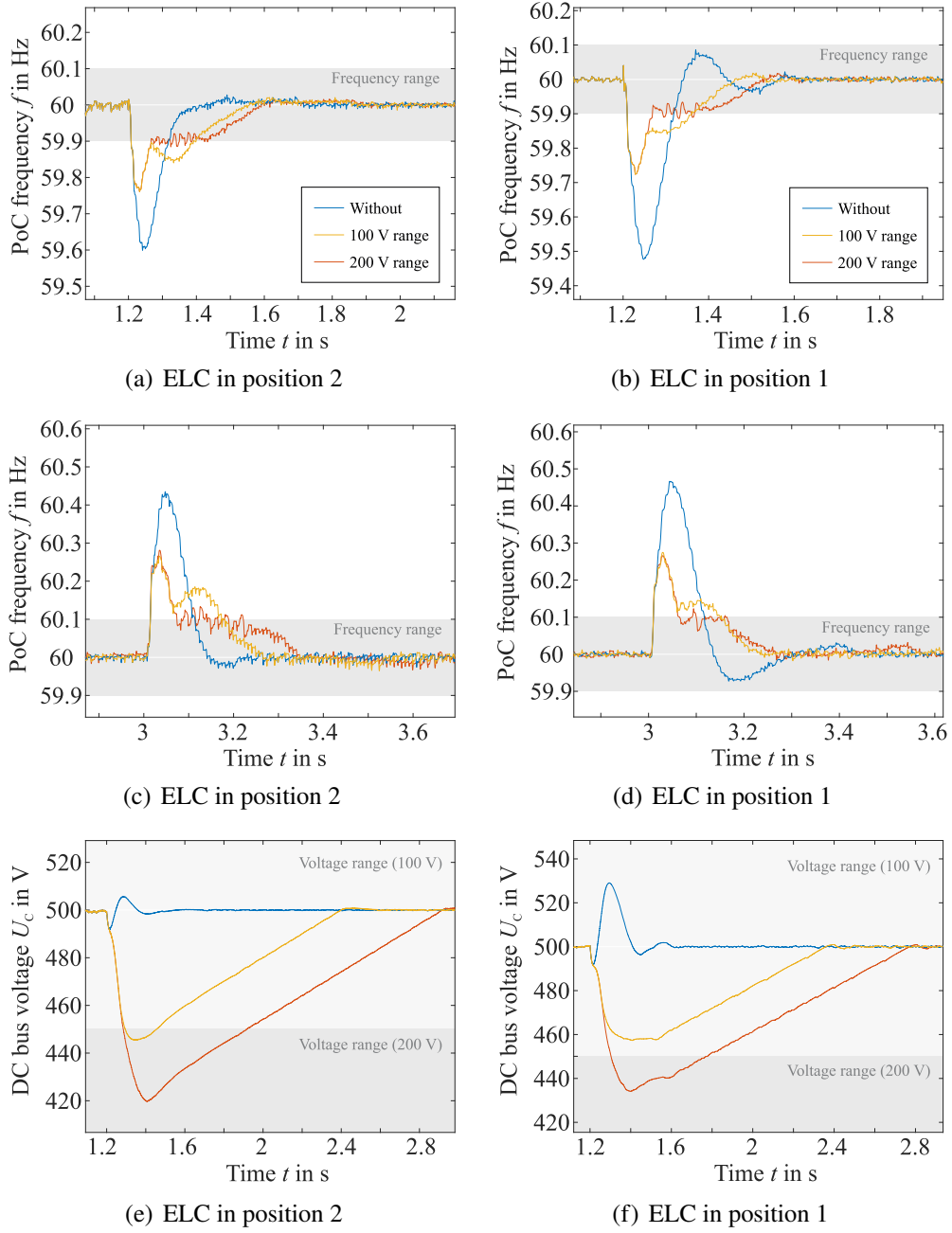


Figure 6.22: Simulation results with the ancillary controller for different DC bus voltage ranges and for the ELC located in different positions, (a) and (b) PCC voltage frequency for load connection (92%), (c) and (d) load disconnection, (e) and (f) DC bus voltage for load connection

The acceptable operating frequency range is set to 59.9 to 60.1 Hz. The frequency deviation was reduced by a fast actuation of the designed controller, and the DC voltage

recovery was performed properly by a ramp function. Figures 6.22(b), 6.22(d), and 6.22(f) show the results for sudden load connection and disconnection with ELC located in position 1. The same compensation features shown previously can be seen, even facing more interaction with the DC bus voltage controller due to the ELC connection to the DC bus.

Figures 6.23(a), 6.23(b), and 6.23(c) show the control system performance over the disconnection and connection of a generator. More interaction with the ELC controller dynamics when the ELC is connected to the DC bus is observed, pointing to the need for a compatibility-driven design.

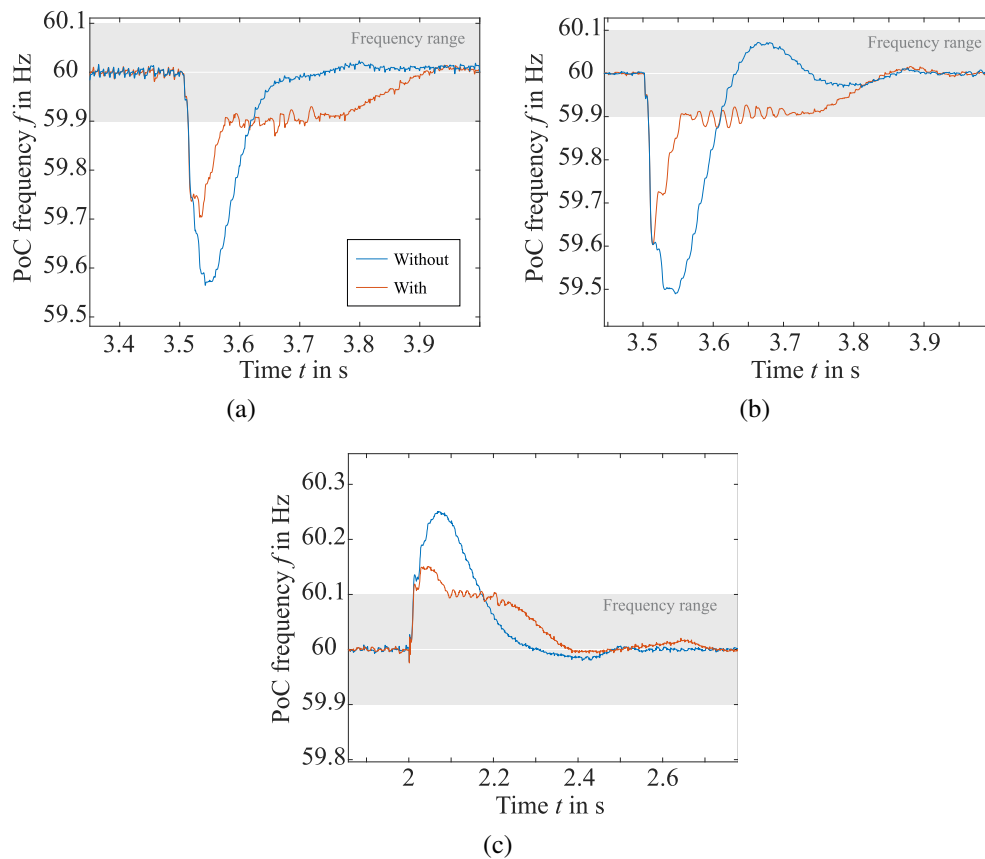


Figure 6.23: Simulaton results for (a) SEIG 3 sudden disconnection with ELC in position 2, (b) SEIG 3 sudden disconnection with ELC in position 1, and (c) SEIG 3 sudden connection with ELC in position 1

The system response to the overload condition is depicted in Figures 6.24(a) and 6.24(c), for the ELC in position 2. The frequency deviation is reduced, and the frequency worsening, due to active power demand, is avoided by the controller since the frequency is out of the range. Figures 6.24(b) and 6.24(d) show the same results for the ELC in position 1.

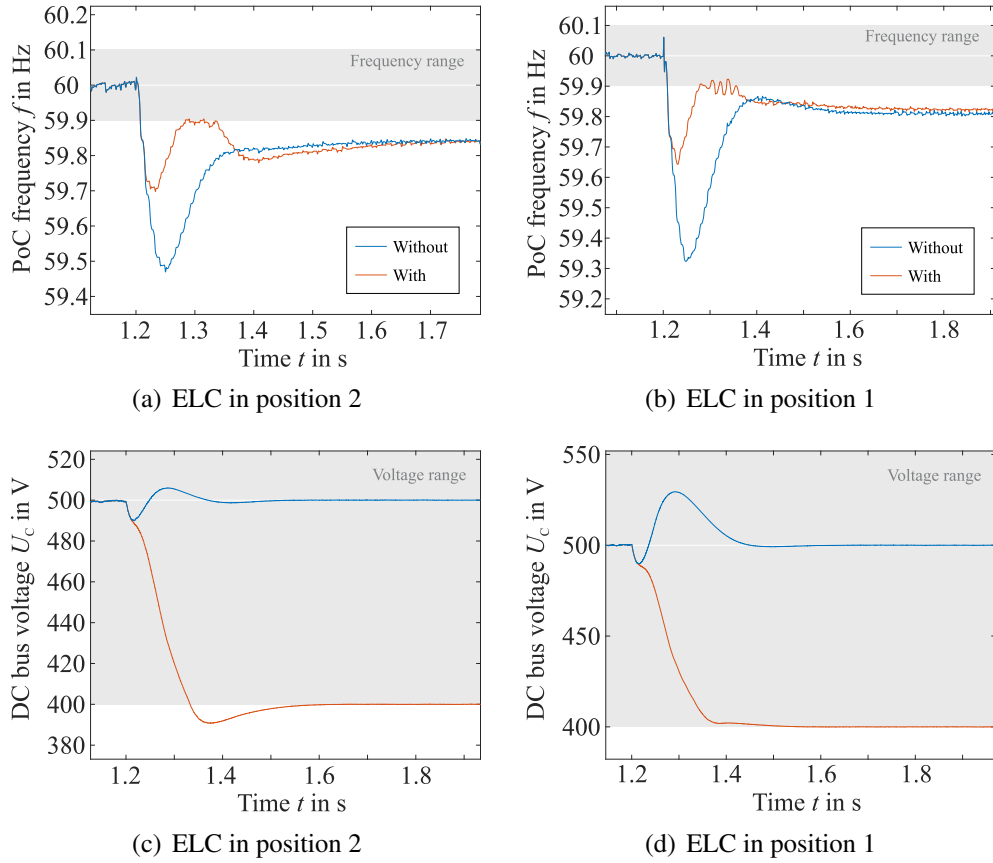


Figure 6.24: Simulation results with the ancillary controller for the ELC located in different positions: PCC voltage frequency and DC bus voltage for load connection (137%)

The design of the controller gains and the recovery dynamics is an important issue. Figures 6.25(a) and 6.25(b) show the effect of the choice of an inappropriate recovery gain and function for a required recovery time. For operation around the frequency threshold, the choice for an asymptotic recovery function $\kappa_R(e_\omega)$ enables the proper voltage recovery; meanwhile, the oscillation produced by a constant gain K_R can produce a voltage stagnation.

Regarding the gain K_I , Figure 6.25(c) illustrates the effect of selecting a high design value. This choice enhances the ability to compensate for fast transients and improves the capacity to maintain the frequency within acceptable limits at higher capacitance levels, as more energy is available. Nonetheless, significant power oscillations can be observed due to the dynamic interactions with the system. Therefore, an appropriate gain value should be selected.

The fault occurrence was performed and can be seen in Figures 6.26(a) and 6.26(b). Without the presence of the proposed controller, the system collapses, facing a 40 ms

three-phase short-circuit. The DC bus voltage controller naturally demands active power for voltage regulation. On the other hand, with the ancillary controller, a more grid-friendly response was achieved, maintaining system operation.

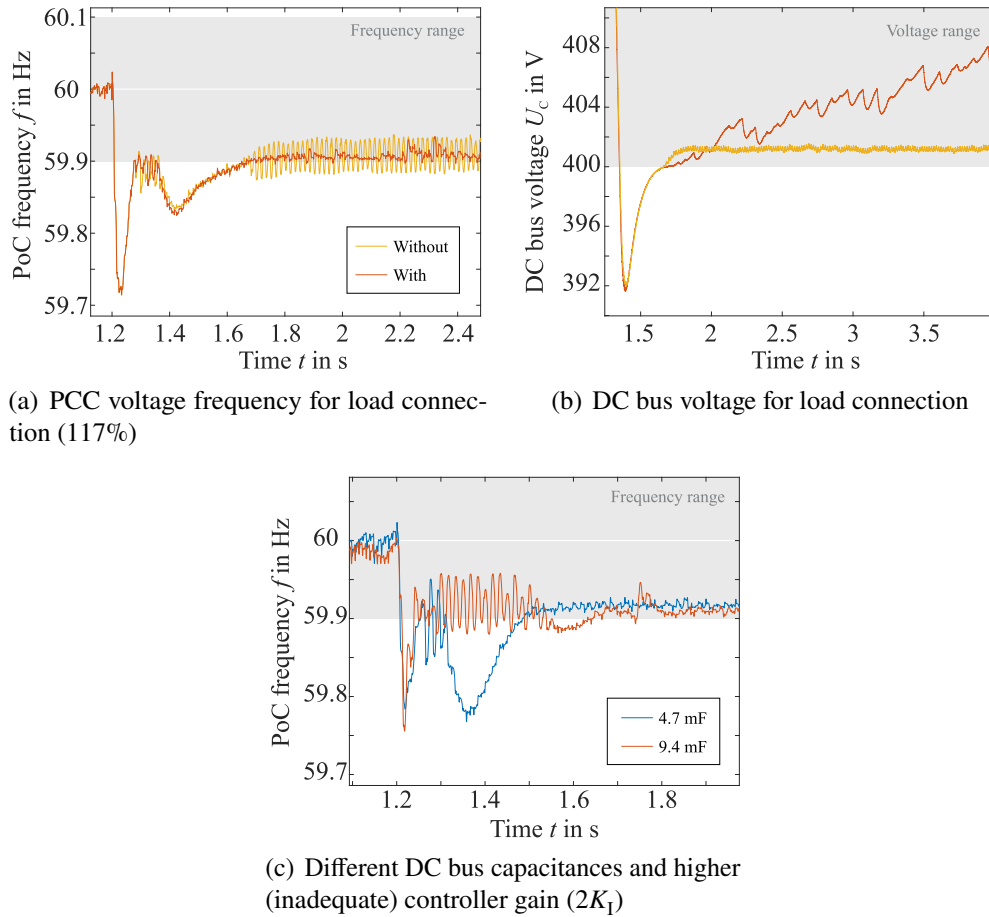


Figure 6.25: Simulation results with different ancillary controller recovery functions

The simulation verification shows the helpful contribution of the controller to the system, achieving less frequency deviation during substantial perturbations and avoiding worsening frequency dynamics during unacceptable operation regions. For small DC bus capacitance, the energy stored in the capacitor will contribute poorly to the system frequency. Furthermore, as the method prioritizes the DC voltage operating point, the contribution to the system frequency is not persistent and can be seen as a drawback in some applications. This can be overcome by combining with other controller structures covered in the literature.

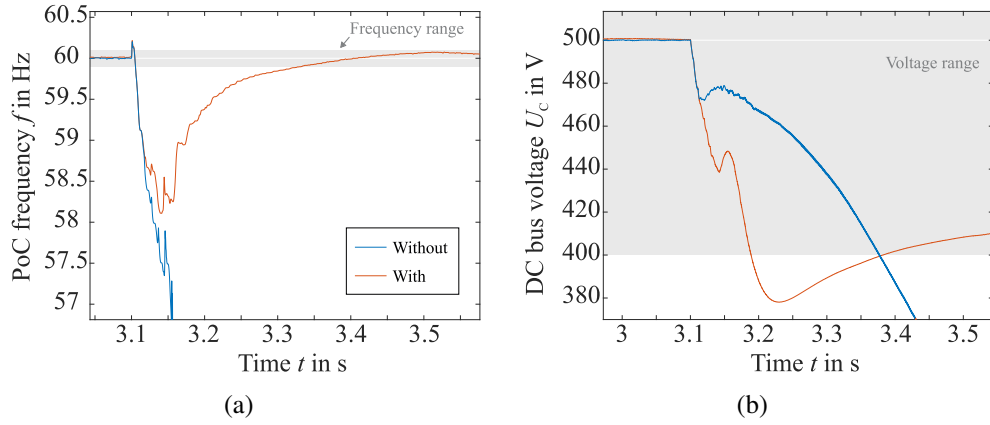


Figure 6.26: Simulation results for a 40 ms three-phase fault occurrence at position 4 (a) PCC voltage frequency, (b) DC bus voltage

6.2.2 Multi-device system with energy storage

Table 7 shows some of the relevant parameters used in the simulation for the case study microgrid. The ELCs can be considered, for instance, as heat-storage systems, water-pumping systems, or electrolyzers for hydrogen conversion. It is noteworthy that this is a low-power system with low active power generation, representing a system with very low inertia that is easily and highly vulnerable to disturbances.

Table 7: Simulation Parameters for the Microgrid with Bidirectional ESS

Description	Value
BESS rated power	960 W
System rated power	8417 W
VSI DC-bus voltage	500 V
Frequency range Δf	0.2 Hz
Voltage range ΔU_C	20 V
Sampling period T_s	100 μ s
$K_{\omega,I}, K_{\omega,R}$	$-1/T_s, 10$
$K_{B,I}, K_{B,R}$	$-0.001/T_s, 1$
$K_{FFR,P}$	500
Distribution line impedance	$L_{CL,(1,4,5)} = 172 \mu\text{H}, R_{CL,(1,4,5)} = 0.725 \Omega$ $L_{CL,(2,3)} = 86 \mu\text{H}, R_{CL,(2,3)} = 0.36 \Omega$

Figure 6.27 presents the system performance for short circuits at point “a” and load connections to the system. The system presents a clear improvement in its stability when the developed auxiliary controllers are used. The frequency presents faster recovery

and less deviation, while the PCC voltages remain outside the ideal operating range for less time. It can also be noted that the system becomes capable of sustaining a fault for longer. For the load connection results, a lower frequency drop can be noted for the proposed method, as well as a better use of the total instantaneous active power available in the IBR.

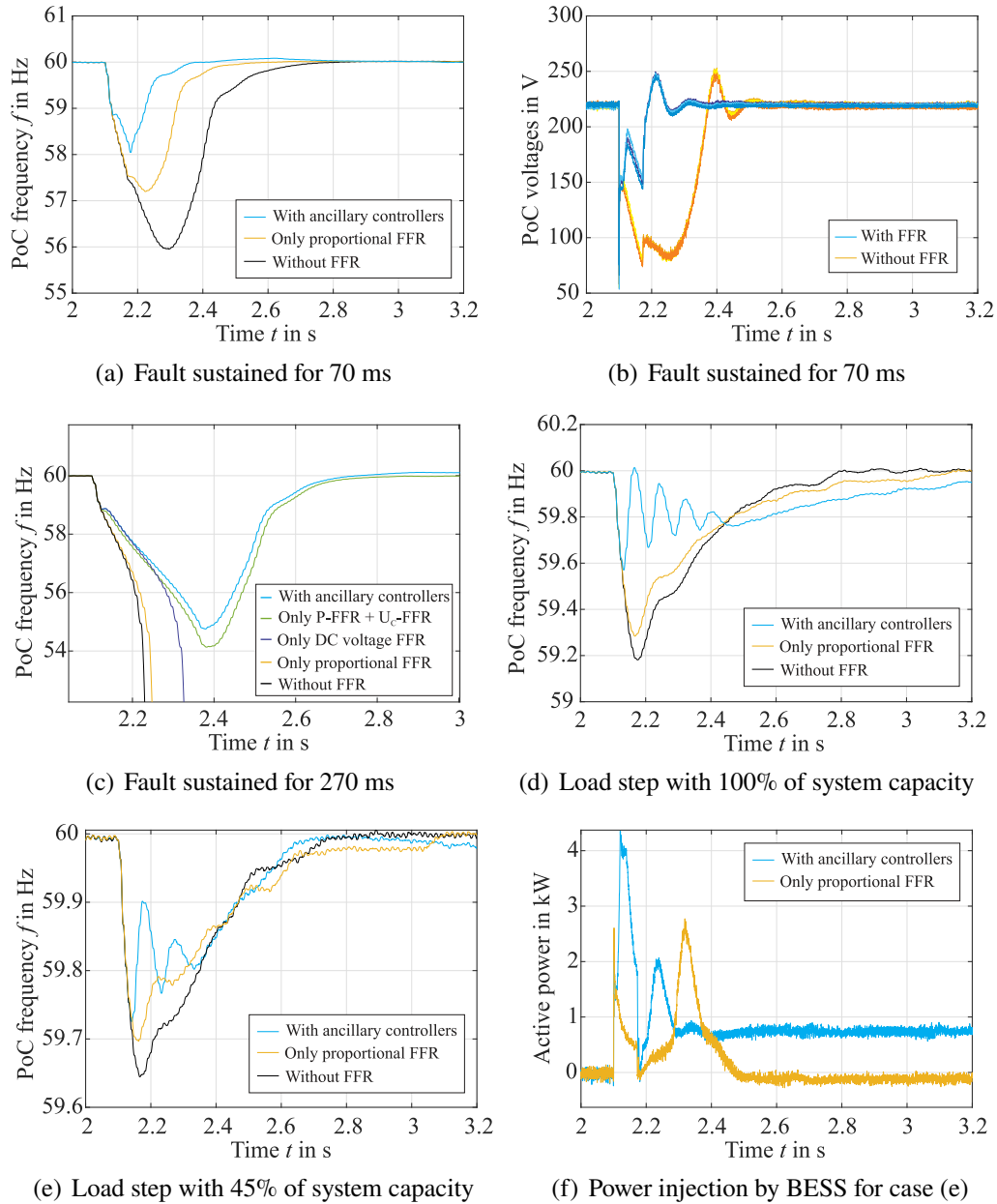


Figure 6.27: FFR performance for three-phase symmetrical fault at point “a” and load connection

Figure 6.28 presents the BESS currents and the PCC frequency for a severe load step.

It can be noted that even with similar frequency support when compared with the proportional FFR, the proposed strategy presents a naturally smoother recovery, an important feature for an FFR response in several power systems and standards as pointed out in [225].

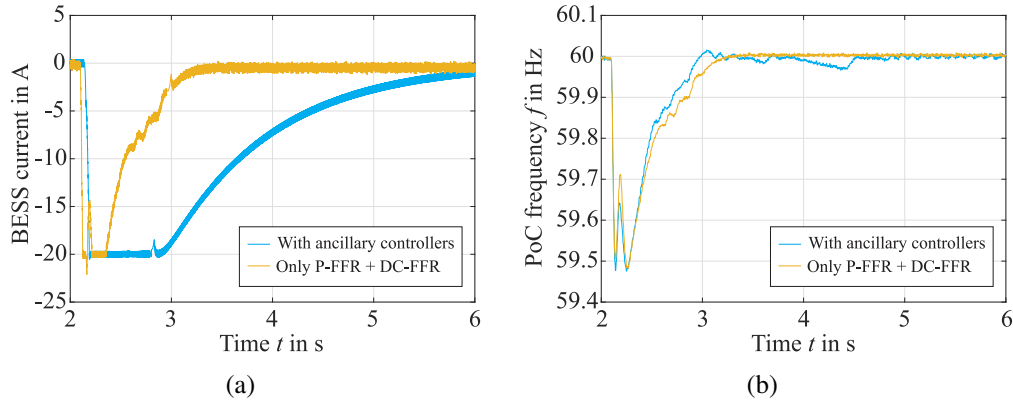


Figure 6.28: BESS current with the ancillary controller and the proportional FFR under severe load steps

Figure 6.29 shows the BESS currents and the PCC frequency for a light load step. The connection of this kind of load can be considered a frequent event in the system. From the results, it can be seen that the BESS currents are not affected due to its threshold limits, even providing frequency support for the power system, while using a proportional FFR produces current demand for the BESS in that case. It is worth noting that the proportional FFR could also be activated with thresholds; on the other hand, the frequency support would be poor.

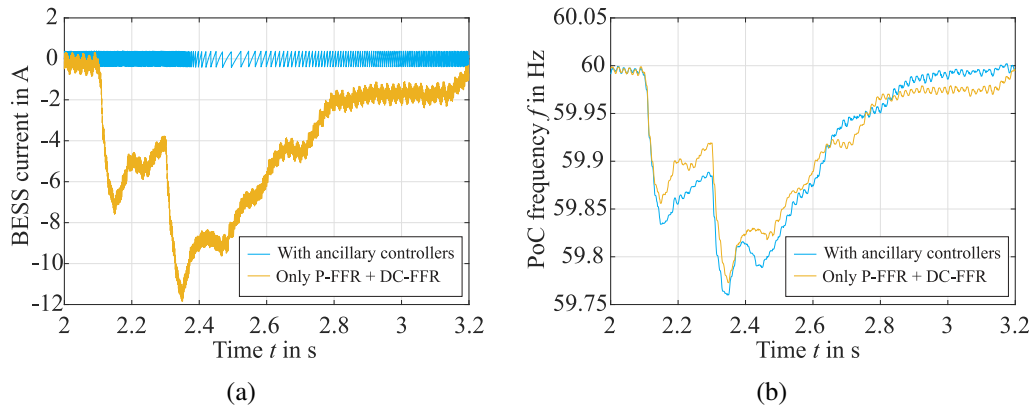


Figure 6.29: BESS currents with the ancillary controller and the proportional FFR under light load steps

The control method has shown good performance in improving the dynamic security of the system. The ancillary controllers and coordination can also be employed in several applications considering inverter-based resources.

Applying the GFM control technique presented in Figure 5.14 in the same system, and modifying the reactive power control loop, including the PCC voltage in the structure with two integrators [226], for ensuring better voltage regulation in steady state and comparability, Figure 6.30(a) presents the results for a three-phase fault lasting 270 ms, comparing the GFM with and without the proposed ancillary controller, following the results from Figure 6.27(c). It can be observed that the GFM controller inherently exhibits grid-supporting capabilities, since the system frequency is directly linked to the DC-bus voltage control. Therefore, even without the ancillary controller, the grid-forming scheme is capable of providing both low-frequency and low-voltage ride-through, as can be seen in 6.30(b). Nevertheless, when the ancillary controller is considered, an improvement in both frequency and voltage characteristics is achieved. For instance, the shall trip zone with a maximum time of 0.16 s defined by the IEEE Standard 1547 is highlighted, where, in this specific scenario, for the GFL, the time during which the frequency remains outside the permissible limits is 0.1813 s. For the GFM without the ancillary controller, this duration is 0.1704 s. In contrast, when the GFM operates with the ancillary controller, the time outside the frequency limits is reduced to 0.043 s, avoiding possible equipment disconnection from the system.

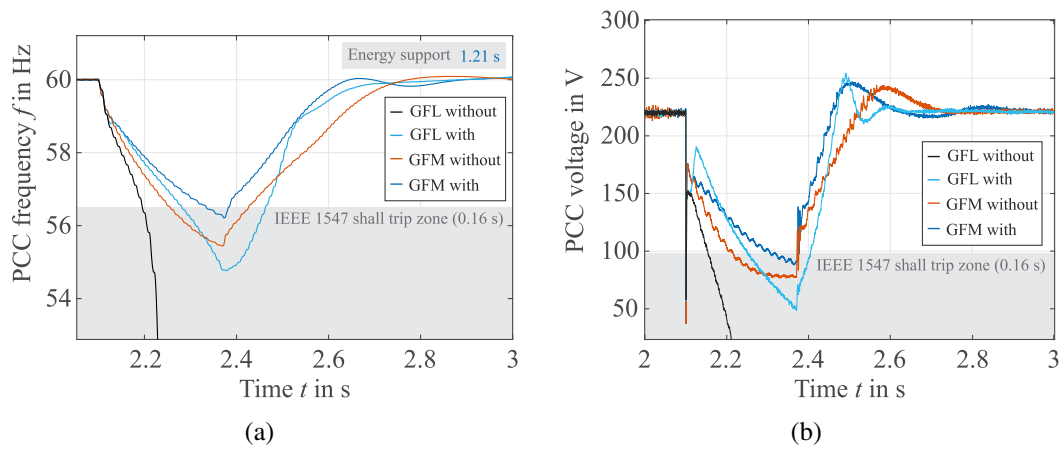


Figure 6.30: Performance for GFM and GFL for three-phase symmetrical fault at point “a”

In this scenario, for all cases with the ancillary controller, the BESS current behavior is the same as shown in Figure 6.28(a). Calculating the energy support for this frequency event using the MATLAB function *trapz* on the inverter output power results in 1161.6 Ws (or 0.0003267 kWh). When normalized by the rated power of the BESS converter, this corresponds to 1.21 s; when compared with the rated power of the converter, 4 kW, it

represents 0.29 s. These values highlight the minimal energy requirement needed to enhance grid-side resiliency during transient frequency events.

6.2.3 Large-scale power system

To evaluate the potential contribution to FFR in utility-scale applications and assess the scalability of the proposed method operating in parallel to synchronous machines and other FFR suppliers in high-inertia grids, some scenarios are presented in this section considering the IEEE 9-bus test system depicted in Figure 5.17, and discussed in Section 5.8.

Figure 6.31(a) presents the sudden disconnection of SG3. For this case study, only the energy available in the DC-bus capacitors is injected, considering the proposed control framework. Due to the high inertia of the system, a larger amount of energy is required to significantly improve the frequency nadir. Moreover, the fast dynamics of the FFR can cause oscillations in the frequency profile, even though it slightly improves the frequency behavior during this low-frequency event. In this scenario, each AG-IBR contributes with 47 ms of energy support, considering its rated power of 96 MW.

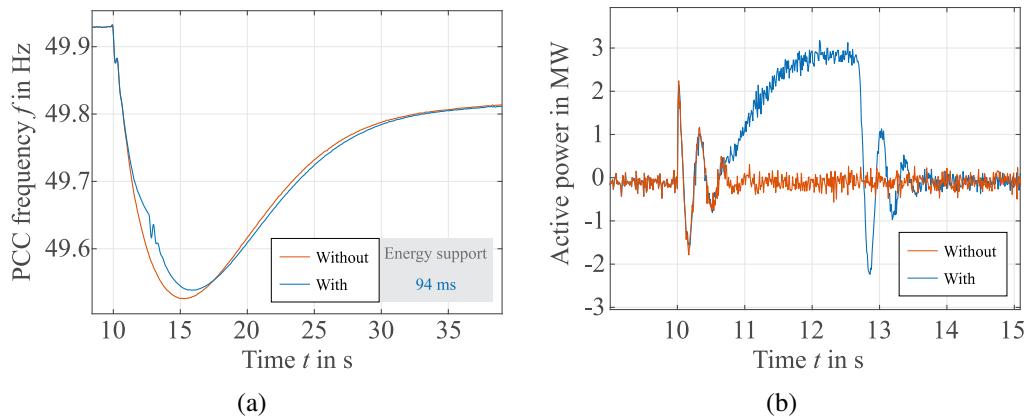


Figure 6.31: Simulation results at bus 6 for measured frequency and active power injection with and without DC-bus energy ancillary support

Figure 6.32 illustrates the system frequency response during a 90 MW load increase at Bus 6 (P6). In this scenario, coordination with a BESS is considered, and different combinations of FFR are evaluated. The energy delivered is calculated over a 90 s window, from 10 to 100 s. The proposed control strategy demonstrates the capability to provide both instantaneous and primary reserves automatically, leveraging equipment that may otherwise remain in standby mode. Once again, the small amount of energy relative to the rated power of the IBR is highlighted, as reported in the literature.

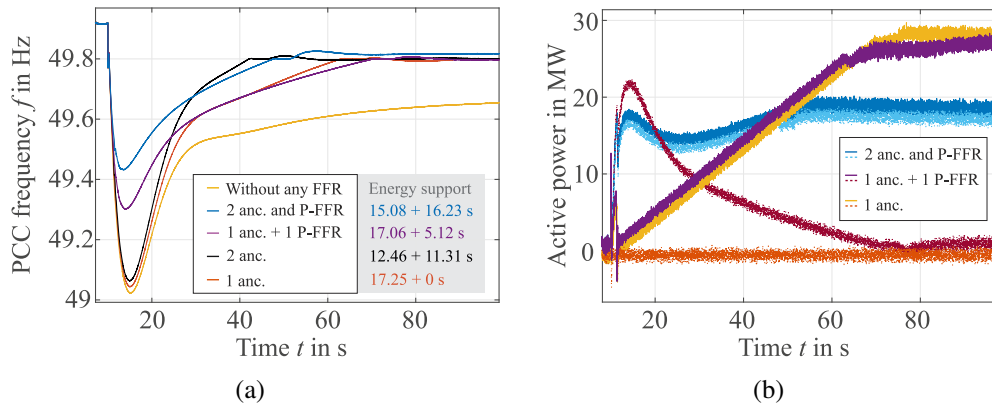


Figure 6.32: Simulation results at bus 6 under different combinations of FFR and AG-IBRs enabled

Although the primary function of the considered IBRs is not frequency regulation, they are capable of driving the system frequency close to the defined threshold. This is a valuable feature for enhancing system resilience in distributed energy systems, particularly where dynamic security becomes more critical due to the stochastic nature of loads and the increased failure probability of equipment.

The control technique enables the prioritized dispatch of DC-bus energy within a safe operating range, as shown in Figure 6.33. This is especially advantageous for rapid frequency events and may also improve the response speed of IBR active power injection, accounting for the inherent technological limitations of the BESS.

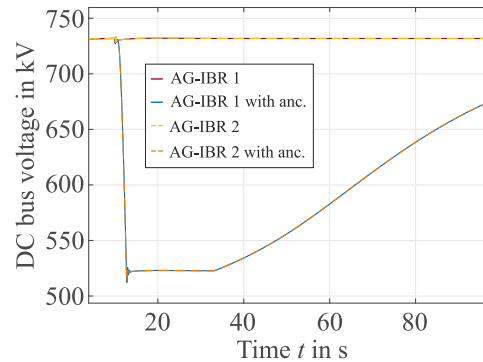


Figure 6.33: Simulation results for DC-bus voltage behavior of each AG-IBR with ancillary controller

Figure 6.34 presents the case of IBR disconnection from bus 6. Considering the power system operating at 50 Hz, the frequency limit defined by the standard [227] is indicated.

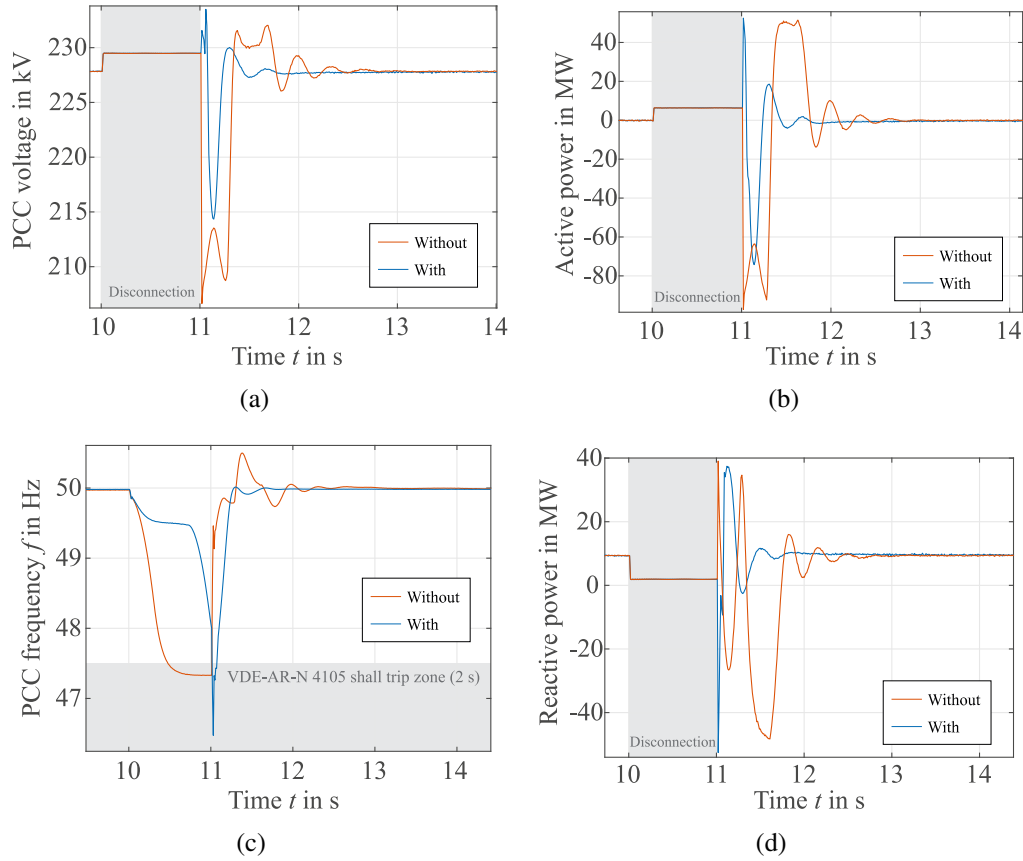


Figure 6.34: Simulation results for a sudden disconnection with a local load of 6 MW

Following the formulation presented in Section 5.8.2 for the energy available in the DC-link capacitors over a voltage range, and based on the parameters of the considered inverter, it is possible to estimate the capability to supply a 6 MW load (6.25% of the AG-IBR rated power) for approximately 0.75 s. From the results, it can be observed that the IBR is capable of supplying isolated loads while maintaining adequate voltage and frequency levels. Furthermore, with the ancillary controller active, a smaller frequency deviation is observed while the capacitor energy remains sufficient to supply the load within the defined voltage range.

After 1 s of interruption, the AG-IBR is suddenly reconnected to the grid. A transient response followed by successful resynchronization with the system can be observed. At this point, a resynchronization algorithm would typically be applied [94], [228]. However, the development of additional control loops related to microgrid operation is beyond the scope of this thesis.

6.2.3.1 Experimental analysis of the GFM controller

To validate the implementation feasibility of the proposed control framework, the controller is programmed in a DSP connected to a hardware-in-the-loop test bench using Typhoon HIL 404, as described in Appendix G. The simulated system is the same as depicted in Figure 5.15, with an additional resistor connected to the DC-bus, representing a load with a power of 0.5 p.u.

The experiment of a grid voltage amplitude step, as presented in Figure 6.35, shows the current-limiting characteristics implemented by the FW-M²PC technique, as well as the low-voltage ride-through characteristic. The connection parameters are chosen to present both weak and strong grid behavior regarding the X/R ratio, where the results show successful synchronization and stable operation with resistive and inductive grids.

When comparing the results from Figures 6.35(a) and 6.35(b), a more stable behavior can be noted, as well as when comparing Figures 6.35(c) and 6.35(d). The active power of the IBR is directly related to the DC-bus voltage, and the proposed ancillary controller acts in the way to improve the transient stability, once it automatically changes the active power reference.

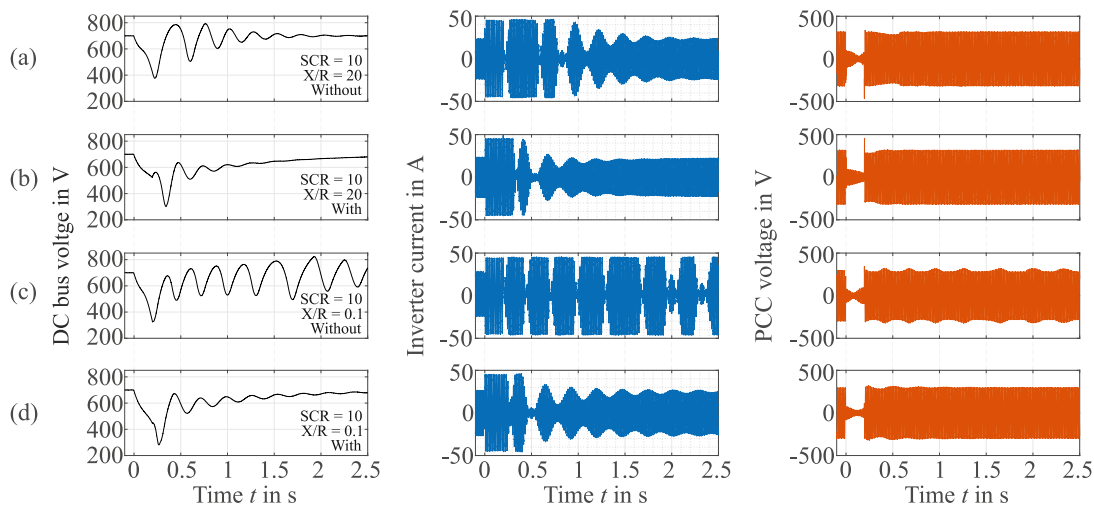


Figure 6.35: Experimental control hardware results using hardware-in-the-loop for voltage amplitude step of -0.9 p.u. for 200 ms

The results also show that when employing a GFM control strategy, violations of the DC-bus voltage limits can occur, highlighting the importance of also considering strategies to avoid undesirable negative overshoot, as presented in Section 5.7.3.

Throughout this chapter, the transient and steady-state responses of the systems employing the proposed converters and the control techniques were analyzed under various

conditions, including changes in linear and nonlinear loads, frequency and voltage events. Power quality and reliability were assessed in accordance with the relevant standards. Based on these results, the following chapter presents the final considerations of the thesis.

7 Conclusion and Future Work

This chapter provides the concluding remarks, summarizing the key findings of this work and highlighting its limitations and contributions. It also outlines gaps that require further research in the future, as well as the related publications.

7.1 Summary and conclusion

As discussed throughout this work, electrical energy is now a fundamental resource for the development of society. It is crucial that electrical energy is widely distributed and delivered reliably to support this development. However, many countries face significant infrastructure challenges, as the necessary resources are often limited. To address this challenge, hybrid generation systems have become increasingly popular in recent years. These systems offer a wide range of implementation possibilities in various situations, thanks to their ability to integrate renewable sources and address environmental issues.

To enable the integration of renewables, this thesis work presented the development, analysis, and control of split-source-type impedance source converters. The control strategies successfully achieved voltage regulation and frequency support, improving the power quality and stability of the power system to which they are connected.

By employing state-space continuous and discrete-time analysis of the proposed topologies, it was possible to develop switching control strategies using MPC simply and effectively, obtaining initial findings about the feasibility, operation, and capabilities of such power converters. Moreover, a selected space vector analysis was developed to evaluate the converter operation limits with fixed switching frequency, paving the way for converter design, loss, and thermal analysis, as well as the application of other classical and emergent control and modulation techniques.

In the literature, the need for compensating nonlinear currents connected to isolated systems with SEIG was identified, as these comprise a weak network that is greatly influenced in the presence of such load types. From this, contributions were developed for the control applied to an isolated hybrid microgeneration system, which uses a self-excited induction generator for partial base power supply, representing hydro generation, kinetically powered by a constant-speed primary machine. The simulation has considered a three-phase three-wire multi-input split-source inverter with three input sources, while the experimental results considered the same system with two input sources. The implemented MPC strategy provided DC bus and PCC voltage regulation, using PI controllers applied to the external voltage loops.

The experimental results demonstrated the proper operation of the designed controllers and a good match with the simulation results, where the control system adequately compensated for load adversities in transients and steady-state conditions, with both current TRD and voltage THD parameters staying within the recommended requirements of the IEEE Standard 1547. In addition, software-in-the-loop simulations were performed on different grid-connected and isolated operations of new inverter topologies, which showed successful collective power injection from different and independent sources.

Impedance source converters present advantages and disadvantages in some applications, considering other topologies such as the VSI plus boost converters and galvanically isolated topologies. The split-source converter and its multi-input versions are not an exception. The first evident disadvantage is the reduction of the DC bus voltage utilization, as well as increasing and unbalancing the current stress in the few components present in the inverter circuit. These topologies can be advantageous in specific applications, presenting low-cost and easy-maintenance solutions. The multi-input converters presented a modular and easy-to-implement construction aspect, and it is important to consider advancements in power electronic semiconductor technologies such as silicon carbide and gallium nitride, which can reduce losses and increase stress capabilities. Moreover, this thesis proposes and evaluates multilevel topologies that reduce voltage stress on the semiconductors.

Following the grid-supporting control concept, this work presented control strategies using an early-term reserve FFR from the DC-bus capacitors. The cascaded coordination with the larger storage systems was also verified, demonstrating an improvement in overall system stability.

Even though using the energy stored in capacitors as a fast frequency response can improve the dynamic security of the system, it will represent additional operation for the capacitor bank, potentially reducing the life cycle of such elements. On the one hand, this strategy goes against recent efforts to reduce the DC bus capacitance in inverters. On the other hand, larger capacitance can improve the operation of the system in terms of power quality and dynamic security. Furthermore, investments and tax incentives can be considered when evaluating the overall advantages they can bring to the electrical power system.

Therefore, the following contributions from this dissertation can be highlighted:

- (i) Analysis, modeling, and control of grid-connected and off-grid low-power generation systems considering stationary and synchronous reference frames, addressing voltage regulation, frequency support, and harmonics mitigation.
- (ii) Converter topologies that enable multiple, independent sources, along with low

component counts

- (iii) Development of predictive controllers for the proposed topologies
- (iv) Initial space vector modulation analysis for achieving fixed switching frequency and enabling other control techniques
- (v) Proposal of a hybrid power generation system with PV and SEIG, through a topology that offers power processing with a reduced number of conversion stages and active components, also enabling multiple independent MPPT controls
- (vi) Fast frequency response control strategy to address system stability, reliability, and ancillary services, with easy implementation and generalization
- (vii) Comprehensive testing of the proposed ancillary controllers in a theoretical low-inertia, low-power grid with parallel operation of inverters, considering state-of-the-art control modes (GFM and GFL).
- (viii) Analysis of the potential contributions to IR in a transmission-level, high-inertia power system by implementing a high-fidelity EMT aggregated model.

7.2 Suggestions for future work

- (i) Further generalization of the methods employed in this work, such as the use of predictive models and selected space vectors in multi-source, multi-objective, multi-phase, and multi-level converters and systems.
- (ii) Analysis of losses and efficiency of the proposed systems, including the analysis of the design of magnetic elements, switches, and diodes. Including device technology and increasing the switching frequency, in order to reduce losses and increase efficiency
- (iii) Investigate more specific applications to define gain design guidelines for the FFR control strategy. The establishment of stability related to parameter design for FFR control methods remains an open topic due to the high complexity and heterogeneous control strategies, and the nature of devices connected to the power system.
- (iv) Develop supply systems with the proposed topologies to further investigate their potential applications with fuel cells, batteries, and other DC power sources
- (v) Investigate further linear control strategies and modulation techniques applied to such systems

- (vi) Analyzing CCS-MPC based on the Frank-Wolfe algorithm, as it forms a foundation for developing a fixed switching frequency control together with a modulation scheme for the proposed topologies, enabling traditional filter design processes, and establishing fair comparisons with other topologies.
- (vii) Approach other control techniques, such as adaptive control, observers, and neural networks, which can be employed to give robustness to the model-based controllers
- (viii) Explore aspects of the GFM mode, identifying opportunities for stability improvements by considering the specific characteristics of the system for microgrid connection, and investigating power sharing, DC-bus sharing, as well as other grid-supporting functionalities and communication-free coordination capabilities.
- (ix) Assess the performance of FFR when IBRs are integrated into a market or a virtual power plant, and consider combining the proposed ancillary controller with other FFR techniques.
- (x) Extend the topologies and control strategies to 4-wire systems, considering designs that adequately supply unbalanced and nonlinear loads.

7.3 Own scientific publications

Since the beginning of doctoral studies to date, the following publications have been made as scientific contributions:

- (i) **G. M. Cocco**, G. A. Salvatti, J. P. S. Cipriani, H. Pinheiro, M. Wolter, C. Rech, and M. Sperandio. "DC Bus Ancillary Controller Enabling Fast Frequency Response from Fast Charging Stations." *Power Systems Computation Conference 2026*, Limassol, Cyprus, 2026 (Accepted).
- (ii) **G. M. Cocco**, G. A. Salvatti, J. P. S. Cipriani, H. Pinheiro, M. Wolter, C. Rech, and M. Sperandio. "Grid-Friendly Control Strategy for IBRs in Islanded Operation." 25. *Dresdener Kreis 2025*, Dresden, Germany, 2025.
- (iii) **G. M. Cocco**, J. P. S. Cipriani, C. Sauer, H. Pinheiro, and M. Wolter. "FFR Coordination with Energy Storage Systems for Grid-Supporting Control of Inverter-Based Resources." *NEIS 2024; Conference on Sustainable Energy Supply and Energy Storage Systems*. VDE, 2024.
- (iv) **G. M. Cocco**, J. P. S. Cipriani, C. Sauer, M. Wolter, F. E. Bisogno, and H. Pinheiro. "Employing a Reduced Component Count Inverter Fed by Multiple Sources for Grid Support." *PESS 2024; IEEE Power and Energy Student Summit 2024*. VDE, 2024.

- (v) **G. M. Cocco**, F. E. Bisogno, R. F. de Camargo, E. Glende, and M. Wolter, "Dc voltage ancillary controller for dynamic security of shunt-compensated microgrids." *at-Automatisierungstechnik*, vol. 71, no. 12, pp. 1040–1050, 2023.
- (vi) **G. M. Cocco**, F. E. Bisogno, R. F. De Camargo, T. A. Woldu, C. Ziegler, and M. Wolter, "Analysis and control of an isolated multi-source renewable microgrid employing the multi-input split-source inverter." In *2023 IEEE Power & Energy Society General Meeting (PESGM)*, pp. 1–5, IEEE 2023.
- (vii) **G. M. Cocco**, F. E. Bisogno, R. F. de Camargo, and F. B. Grigoletto, "Multi-input split-source inverter (missi)." In *2022 14th Seminar on Power Electronics and Control (SEPOC)*, pp. 1–6, IEEE 2022 (Best Paper Award).
- (viii) **G. M. Cocco**, C. A. de Souza, F. B. Grigoletto, R. F. de Camargo, and F. E. Bisogno, "Model predictive control for voltage regulation of standalone self-excited induction generator." *Simpósio Brasileiro de Sistemas Elétricos-SBSE*, vol. 2, no. 1, 2022.
- (ix) **G. M. Cocco**, F. B. Grigoletto, L. G. Scherer, and R. F. de Camargo, "Modeling and control of hydro-pv hybrid power system with three-phase three-leg split- source inverter." *Journal of Control, Automation and Electrical Systems*, pp. 1–13, 2022.
- (x) J. P. S. Cipriani, **G. M. Cocco**, A. Kashtanov, C. Sauer, H. Pinheiro, M. Sperandio, and M. Wolter. "Distributed Co-Simulation of IBRs: Extending Latency Tolerance by Extending Line Models." 2026 (Draft).
- (xi) J. P. S. Cipriani, **G. M. Cocco**, R. Viaro, H Pinheiro, V. F. Montagner, and M. Wolter. "FW-M²PC: Projection-Free Modulated Model Predictive Control of Grid-Tied Converters." *Control Engineering Practice*, 2026 (Submitted).
- (xii) J. P. S. Cipriani, C Sauer, **G. M. Cocco**, H Pinheiro, M Wolter, and M. Sperandio. "Potential Market for the Simultaneous Provision of Instantaneous and Primary Reserves in Germany." *IEEE PowerTech 2025 Conference*. IEEE, 2025.
- (xiii) G. A. Salvatti, **G. M. Cocco**, M. Sperandio, M Wolter. H Pinheiro, and C. Rech. "Evaluation Guidelines for Grid Impedance of Fast Charging Stations Connected in Medium Voltage." *2025 IEEE PES Innovative Smart Grid Technologies Europe (ISGT EUROPE)*. IEEE, 2025.
- (xiv) J. V. L. Rosa, J. P. S. Cipriani, **G. M. Cocco**, M. Wolter, F. Carnielutti, and H. Pinheiro. "Iterative Solvers for Model Predictive Control of Grid-Tied Inverters." *2025 Brazilian Power Electronics Conference (COBEP)*. IEEE, 2025. (Submitted).
- (xv) C. A. Souza, **G. M. Cocco**, R. F. de Camargo, F. E. Bisogno, and M. Wolter. "OSV-MPC for Harmonic and Zero-Sequence Compensation in Four-Wire Off-Grid Microgeneration Systems Based on SEIG." *Eletrônica de Potência* 29 (2024): e202458-e202458.

- (xvi) C. Sauer, J. P. S. Cipriani, **G. M. Cocco**, H. Pinheiro, and M. Wolter. "Batteries for fast frequency containment response: market impacts on outage dynamics." *at-Automatisierungstechnik*, vol. 72, no. 11, pp. 1007–1016, 2024.
- (xvii) C. Sauer, J. P. S. Cipriani, **G. M. Cocco**, H. Pinheiro, and M. Wolter. "Instantaneous Reserve Implementations: Method for Testing Worst-Case Resilience Scenarios." *NEIS 2024; Conference on Sustainable Energy Supply and Energy Storage Systems*. VDE, 2024.
- (xviii) C. A. de Souza, **G. M. Cocco**, R. F. De Camargo, M. L. Flach, and F. E. Bisogno, "FS-MPC Approach for Zero-Sequence Compensation in SEIG-Based Off-Grid Systems." *17th Brazilian Power Electronics Conference and 8th IEEE Southern Power Electronics Conference (COBEP/SPEC)*, pp. 1-6, IEEE 2023.
- (xix) C. A. de Souza, **G. M. Cocco**, L. G. Scherer, and R. F. de Camargo, "Proportional integral resonant and proportional derivative controllers applied to zero sequence harmonic compensation in four-wire microgeneration systems." *Journal of Control, Automation and Electrical Systems*, vol. 34, no. 2, pp. 443–454, 2023.
- (xx) R. F. De Camargo, D. H. Chrischon, **G. M. Cocco**, C. A. de Souza, A. R. Cukla, and F. R. Gentile, "Compensación de Armónicos y Desequilibrio Usando Control Predictivo y Teoría p-q Aplicado al DSTATCOM." *XX Reunión de Trabajo en Procesamiento de la Información y Control (RPIC)*, pp. 1–6, 2023.
- (xxi) D. H. Chrischon, R. F. De Camargo, **G. M. Cocco**, and C. A. de Souza, "Finite-Set Model Predictive Control and p-q Theory Applied on Shunt Active Filter in an Off-Grid Microgeneration System." *2023 15th Seminar on Power Electronics and Control (SEPOC)*, pp. 1-6, IEEE 2023.
- (xxii) M. V. S. Gonçalves, **G. M. Cocco**, and R. F. De Camargo, "p-q theory based predictive control for voltage regulation and harmonic compensation of standalone seig." In *2022 14th Seminar on Power Electronics and Control (SEPOC)*, pp. 1–6, IEEE 2022.

Moreover, the following Master's theses were supervised during the doctoral research:

- (xxiii) J. Kulkarni, "Transient Stability Analysis of IEEE 9-Bus System with Grid-Forming Inverters." 2025.
- (xxiv) I. A. Sani, "Real-Time Simulation and Assessment of Wind Power Systems Providing Fast Frequency Response." 2025.
- (xxv) N. A. Italiya, "Modeling Bidirectional Chargers for Vehicle-to-Grid Impact Analysis on Power Systems." 2025.

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A Dynamic state-space model for stationary and synchronous reference frame

A.1 Introduction

This appendix details the procedure for obtaining the state equations, or dynamic model, of the AC plant in the $\alpha\beta 0$ and dq0 coordinates. Starting from the well-established Clarke (power invariant) and Park transformations, state space matrices are obtained to adequately represent the system. This appendix refers to and is based on the equations deduced and presented in Chapter 4 of this work.

A.2 Defining circuit model variables

Considering the system already presented, the compact form equation for state space representation in stationary coordinates is given by:

$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t) + \mathbf{F}\mathbf{w}(t), \quad (\text{A.1})$$

where:

$$\mathbf{x}(t) = \begin{bmatrix} i_a \\ i_b \\ i_c \\ u_{a,c} \\ u_{b,c} \\ u_{c,c} \\ i_{a,g} \\ i_{b,g} \\ i_{c,g} \end{bmatrix}, \quad \mathbf{u}(t) = \begin{bmatrix} u_a \\ u_b \\ u_c \end{bmatrix}, \quad \mathbf{w}(t) = \begin{bmatrix} u_{a,g} \\ u_{b,g} \\ u_{c,g} \end{bmatrix}; \quad (\text{A.2})$$

$$\mathbf{A} = \begin{bmatrix} \mathbf{I}_{3 \times 3}(-R_f/L_f) & \mathbf{\Xi}(1/L_f) & \mathbf{0}_{3 \times 3} \\ \mathbf{I}_{3 \times 3}(1/C_f) & \mathbf{I}_{3 \times 3}(-1/C_f Z_l) & \mathbf{I}_{3 \times 3}(1/C_f) \\ \mathbf{0}_{3 \times 3} & \mathbf{\Xi}(1/L_g) & \mathbf{I}_{3 \times 3}(-R_g/L_g) \end{bmatrix}; \quad (\text{A.3})$$

$$\mathbf{B} = \begin{bmatrix} \mathbf{\Xi}(-1/L_f) \\ \mathbf{0}_{3 \times 3} \\ \mathbf{0}_{3 \times 3} \end{bmatrix}, \quad \mathbf{F} = \begin{bmatrix} \mathbf{0}_{3 \times 3} \\ \mathbf{0}_{3 \times 3} \\ \mathbf{\Xi}(-1/L_g) \end{bmatrix}, \quad (\text{A.4})$$

where:

$$\mathbf{\Xi} = \begin{bmatrix} -2/3 & 1/3 & 1/3 \\ 1/3 & -2/3 & 1/3 \\ 1/3 & 1/3 & -2/3 \end{bmatrix}. \quad (\text{A.5})$$

A.3 State equation of the plant in $\alpha\beta 0$ coordinates

The transformation of the three-phase system from stationary abc coordinates to a stationary $\alpha\beta 0$ coordinate system is performed by applying the following transformation matrix:

$$\mathbf{T}_{\alpha\beta 0} = \sqrt{\frac{2}{3}} \begin{bmatrix} 1 & -1/2 & -1/2 \\ 0 & \sqrt{3}/2 & -\sqrt{3}/2 \\ 1/\sqrt{2} & 1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix}. \quad (\text{A.6})$$

Hence, it follows that:

$$\begin{aligned} \dot{\mathbf{x}}_{\alpha\beta 0}(t) &= \mathbf{T}_{\alpha\beta 0} \dot{\mathbf{x}}(t); \\ \mathbf{x}_{\alpha\beta 0}(t) &= \mathbf{T}_{\alpha\beta 0} \mathbf{x}(t); \\ \mathbf{u}_{\alpha\beta 0}(t) &= \mathbf{T}_{\alpha\beta 0} \mathbf{u}(t); \\ \mathbf{w}_{\alpha\beta 0}(t) &= \mathbf{T}_{\alpha\beta 0} \mathbf{w}(t). \end{aligned} \quad (\text{A.7})$$

Upon isolating the terms, the expression becomes:

$$\begin{aligned} \dot{\mathbf{x}}(t) &= \mathbf{T}_{\alpha\beta 0}^{-1} \dot{\mathbf{x}}_{\alpha\beta 0}(t); \\ \mathbf{x}(t) &= \mathbf{T}_{\alpha\beta 0}^{-1} \mathbf{x}_{\alpha\beta 0}(t); \\ \mathbf{u}(t) &= \mathbf{T}_{\alpha\beta 0}^{-1} \mathbf{u}_{\alpha\beta 0}(t); \\ \mathbf{w}(t) &= \mathbf{T}_{\alpha\beta 0}^{-1} \mathbf{w}_{\alpha\beta 0}(t). \end{aligned} \quad (\text{A.8})$$

Applying (A.7) to (A.1), leads to:

$$\mathbf{T}_{\alpha\beta 0}^{-1} \dot{\mathbf{x}}_{\alpha\beta 0}(t) = \mathbf{A} \mathbf{T}_{\alpha\beta 0}^{-1} \mathbf{x}_{\alpha\beta 0}(t) + \mathbf{B} \mathbf{T}_{\alpha\beta 0}^{-1} \mathbf{u}_{\alpha\beta 0}(t) + \mathbf{F} \mathbf{T}_{\alpha\beta 0}^{-1} \mathbf{w}_{\alpha\beta 0}(t). \quad (\text{A.9})$$

Pre-multiplying both sides of (A.8) by the matrix $\mathbf{T}_{\alpha\beta 0}$ yields:

$$\dot{\mathbf{x}}_{\alpha\beta 0}(t) = \mathbf{T}_{\alpha\beta 0} \mathbf{A} \mathbf{T}_{\alpha\beta 0}^{-1} \mathbf{x}_{\alpha\beta 0}(t) + \mathbf{T}_{\alpha\beta 0} \mathbf{B} \mathbf{T}_{\alpha\beta 0}^{-1} \mathbf{u}_{\alpha\beta 0}(t) + \mathbf{T}_{\alpha\beta 0} \mathbf{F} \mathbf{T}_{\alpha\beta 0}^{-1} \mathbf{w}_{\alpha\beta 0}(t), \quad (\text{A.10})$$

where:

$$\mathbf{A}_{\alpha\beta 0} = \mathbf{T}_{\alpha\beta 0} \mathbf{A} \mathbf{T}_{\alpha\beta 0}^{-1}, \quad \mathbf{B}_{\alpha\beta 0} = \mathbf{T}_{\alpha\beta 0} \mathbf{B} \mathbf{T}_{\alpha\beta 0}^{-1}, \quad \mathbf{F}_{\alpha\beta 0} = \mathbf{T}_{\alpha\beta 0} \mathbf{F} \mathbf{T}_{\alpha\beta 0}^{-1}. \quad (\text{A.11})$$

As a result of (A.9), the state space representation in stationary $\alpha\beta 0$ coordinates is given by:

$$\dot{\mathbf{x}}_{\alpha\beta 0}(t) = \mathbf{A}_{\alpha\beta 0} \mathbf{x}_{\alpha\beta 0}(t) + \mathbf{B}_{\alpha\beta 0} \mathbf{u}_{\alpha\beta 0}(t) + \mathbf{F}_{\alpha\beta 0} \mathbf{w}_{\alpha\beta 0}(t), \quad (\text{A.12})$$

where:

$$\mathbf{x}_{\alpha\beta 0}(t) = \begin{bmatrix} i_{\alpha} \\ i_{\beta} \\ i_0 \\ u_{\alpha,c} \\ u_{\beta,c} \\ u_{0,c} \\ i_{\alpha,g} \\ i_{\beta,g} \\ i_{0,g} \end{bmatrix}, \quad \mathbf{u}_{\alpha\beta 0}(t) = \begin{bmatrix} u_{\alpha} \\ u_{\beta} \\ u_0 \end{bmatrix}, \quad \mathbf{w}_{\alpha\beta 0}(t) = \begin{bmatrix} u_{\alpha,g} \\ u_{\beta,g} \\ u_{0,g} \end{bmatrix}; \quad (\text{A.13})$$

$$\mathbf{A}_{\alpha\beta 0} = \begin{bmatrix} \mathbf{I}_{3 \times 3}(-R_f/L_f) & \mathbf{\Lambda}(1/L_f) & \mathbf{0}_{3 \times 3} \\ \mathbf{I}_{3 \times 3}(1/C_f) & \mathbf{I}_{3 \times 3}(-1/C_f Z_l) & \mathbf{I}_{3 \times 3}(1/C_f) \\ \mathbf{0}_{3 \times 3} & \mathbf{\Lambda}(1/L_g) & \mathbf{I}_{3 \times 3}(-R_g/L_g) \end{bmatrix}; \quad (\text{A.14})$$

$$\mathbf{B}_{\alpha\beta 0} = \begin{bmatrix} \mathbf{\Lambda} L_f \\ \mathbf{0}_{3 \times 3} \\ \mathbf{0}_{3 \times 3} \end{bmatrix}; \quad (\text{A.15})$$

$$\mathbf{F}_{\alpha\beta 0} = \begin{bmatrix} \mathbf{0}_{3 \times 3} \\ \mathbf{0}_{3 \times 3} \\ \mathbf{\Lambda} L_g \end{bmatrix}, \quad (\text{A.16})$$

with:

$$\mathbf{\Lambda} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}. \quad (\text{A.17})$$

A.4 State equation of the plant in dq0 coordinates

The transformation of the three-phase system from $\alpha\beta 0$ coordinates to a synchronous dq0 coordinate system is carried out through the transformation matrix:

$$\mathbf{T}_{dq0}(\omega_u t) = \begin{bmatrix} \cos(\omega_u t) & \sin(\omega_u t) & 0 \\ -\sin(\omega_u t) & \cos(\omega_u t) & 0 \\ 0 & 0 & 1 \end{bmatrix}. \quad (\text{A.18})$$

Accordingly, the following can be expressed:

$$\begin{aligned} \mathbf{x}_{dq0}(t) &= \mathbf{T}_{dq0}(\omega_u t) \mathbf{x}_{\alpha\beta 0}(t); \\ \mathbf{u}_{dq0}(t) &= \mathbf{T}_{dq0}(\omega_u t) \mathbf{u}_{\alpha\beta 0}(t); \\ \mathbf{w}_{dq0}(t) &= \mathbf{T}_{dq0}(\omega_u t) \mathbf{w}_{\alpha\beta 0}(t). \end{aligned} \quad (\text{A.19})$$

Isolating the terms yields:

$$\begin{aligned} \mathbf{x}_{\alpha\beta 0}(t) &= \mathbf{T}_{dq0}^{-1}(\omega_u t) \mathbf{x}_{dq0}(t); \\ \mathbf{u}_{\alpha\beta 0}(t) &= \mathbf{T}_{dq0}^{-1}(\omega_u t) \mathbf{u}_{dq0}(t); \\ \mathbf{w}_{\alpha\beta 0}(t) &= \mathbf{T}_{dq0}^{-1}(\omega_u t) \mathbf{w}_{dq0}(t). \end{aligned} \quad (\text{A.20})$$

Applying (A.18) to (A.11) results in:

$$\begin{aligned} \left(\mathbf{T}_{dq0}^{-1}(\omega_u t) \dot{\mathbf{x}}_{dq0}(t) \right) &= \mathbf{A}_{\alpha\beta 0} \mathbf{T}_{dq0}^{-1}(\omega_u t) \mathbf{x}_{dq0}(t) + \mathbf{B}_{\alpha\beta 0} \mathbf{T}_{dq0}^{-1}(\omega_u t) \mathbf{u}_{dq0}(t) \\ &\quad + \mathbf{F}_{\alpha\beta 0} \mathbf{T}_{dq0}^{-1}(\omega_u t) \mathbf{w}_{dq0}(t). \end{aligned} \quad (\text{A.21})$$

Expanding the right-hand side term in (A.19) in terms of its partial derivative leads to:

$$\begin{aligned} \dot{\mathbf{T}}_{dq0}^{-1}(\omega_u t) \mathbf{x}_{dq0}(t) + \mathbf{T}_{dq0}^{-1}(\omega_u t) \dot{\mathbf{x}}_{dq0}(t) &= \mathbf{A}_{\alpha\beta 0} \mathbf{T}_{dq0}^{-1}(\omega_u t) \mathbf{x}_{dq0}(t) \\ &\quad + \mathbf{B}_{\alpha\beta 0} \mathbf{T}_{dq0}^{-1}(\omega_u t) \mathbf{u}_{dq0}(t) + \mathbf{F}_{\alpha\beta 0} \mathbf{T}_{dq0}^{-1}(\omega_u t) \mathbf{w}_{dq0}(t). \end{aligned} \quad (\text{A.22})$$

Isolating the term from the left-hand side of (A.20) yields:

$$\begin{aligned} \dot{\mathbf{x}}_{dq0}(t) &= \left(-\dot{\mathbf{T}}_{dq0}^{-1}(\omega_u t) + \mathbf{A}_{\alpha\beta 0} \mathbf{T}_{dq0}^{-1}(\omega_u t) \right) \mathbf{T}_{dq0}(\omega_u t) \mathbf{x}_{dq0}(t) \\ &\quad + \mathbf{T}_{dq0}(\omega_u t) \mathbf{B}_{\alpha\beta 0} \mathbf{T}_{dq0}^{-1}(\omega_u t) \mathbf{u}_{dq0}(t) + \mathbf{T}_{dq0}(\omega_u t) \mathbf{F}_{\alpha\beta 0} \mathbf{T}_{dq0}^{-1}(\omega_u t) \mathbf{w}_{dq0}(t), \end{aligned} \quad (\text{A.23})$$

where:

$$\begin{aligned} \mathbf{A}_{dq0} &= \mathbf{T}_{dq0}(\omega_u t) \left(-\dot{\mathbf{T}}_{dq0}^{-1}(\omega_u t) + \mathbf{A}_{\alpha\beta 0} \mathbf{T}_{dq0}^{-1}(\omega_u t) \right); \\ \mathbf{B}_{dq0} &= \mathbf{T}_{dq0}(\omega_u t) \mathbf{B}_{\alpha\beta} \mathbf{T}_{dq0}^{-1}(\omega_u t); \\ \mathbf{F}_{dq0} &= \mathbf{T}_{dq0}(\omega_u t) \mathbf{F}_{\alpha\beta} \mathbf{T}_{dq0}^{-1}(\omega_u t). \end{aligned} \quad (\text{A.24})$$

Consequently, from (A.21), the following is obtained:

$$\dot{\mathbf{x}}_{dq0}(t) = \mathbf{A}_{dq0} \mathbf{x}_{dq0}(t) + \mathbf{B}_{dq0} \mathbf{u}_{dq0}(t) + \mathbf{F}_{dq0} \mathbf{w}_{dq0}(t), \quad (\text{A.25})$$

with:

$$\mathbf{x}_{dq0}(t) = \begin{bmatrix} i_d \\ i_q \\ i_0 \\ u_{d,c} \\ u_{q,c} \\ u_{0,c} \\ i_{d,g} \\ i_{q,g} \\ i_{0,g} \end{bmatrix}, \quad \mathbf{u}_{dq0}(t) = \begin{bmatrix} u_d \\ u_q \\ u_0 \end{bmatrix}, \quad \mathbf{w}_{dq0}(t) = \begin{bmatrix} u_{d,g} \\ u_{q,g} \\ u_{0,g} \end{bmatrix}; \quad (\text{A.26})$$

$$\mathbf{A}_{dq0} = \begin{bmatrix} \boldsymbol{\Phi}_1 & \Lambda(1/L_f) & \mathbf{0}_{3 \times 3} \\ \mathbf{I}_{3 \times 3}(1/C_f) & \boldsymbol{\Phi}_2 & \mathbf{I}_{3 \times 3}(1/C_f) \\ \mathbf{0}_{3 \times 3} & \Lambda(1/L_g) & \boldsymbol{\Phi}_3 \end{bmatrix}; \quad (\text{A.27})$$

$$\mathbf{B}_{dq0} = \begin{bmatrix} \Lambda L_f \\ \mathbf{0}_{3 \times 3} \\ \mathbf{0}_{3 \times 3} \end{bmatrix}; \quad (\text{A.28})$$

$$\mathbf{F}_{dq0} = \begin{bmatrix} \mathbf{0}_{3 \times 3} \\ \mathbf{0}_{3 \times 3} \\ \Lambda L_g \end{bmatrix}, \quad (\text{A.29})$$

where:

$$\begin{aligned}
 \boldsymbol{\Phi}_1 &= \begin{bmatrix} -R_f/L_f & \omega_u & 0 \\ -\omega_u & -R_f/L_f & 0 \\ 0 & 0 & -R_f/L_f \end{bmatrix}; \\
 \boldsymbol{\Phi}_2 &= \begin{bmatrix} -1/C_f Z_1 & \omega_u & 0 \\ -\omega_u & -1/C_f Z_1 & 0 \\ 0 & 0 & -1/C_f Z_1 \end{bmatrix}; \\
 \boldsymbol{\Phi}_3 &= \begin{bmatrix} -R_g/L_g & \omega_u & 0 \\ -\omega_u & -R_g/L_g & 0 \\ 0 & 0 & -R_g/L_g \end{bmatrix}.
 \end{aligned} \tag{A.30}$$

B Small-signal model of split-source inverter

B.1 Introduction

For the stable controller design using the Split-Source Inverter (SSI) or the Multi-Input Split-Source Inverter (MISSI), it is necessary to model the system in both the time domain (state-space) and the frequency domain (transfer functions). For this purpose, the deduction of small-signal models can be employed, as already analyzed and developed in [48] for this converter, based on a classical methodology well established in modern control theory [229], [230].

B.2 Small-signal model

Considering that the natural frequencies of the converter, as well as the signal variation frequencies of the inputs, are much lower than the switching frequency, the modeling of the Split-Source Inverter (SSI) can start by assuming that it is a function of the following parameters:

- State variables ($n = 2$ states): $U_C(t)$ and $i_{L,x}(t)$
- Output variables ($r = 2$ outputs): $U_C(t)$ and $i_{L,x}(t)$
- Input variables ($m = 2$ inputs): $U_x(t)$, $i_o(t)$

The system is represented in a generic form as:

$$\begin{cases} \mathbf{K} \frac{d\mathbf{x}(t)}{dt} = \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t) \\ \mathbf{y}(t) = \mathbf{C}\mathbf{x}(t) + \mathbf{E}\mathbf{u}(t) \end{cases} \quad (\text{B.1})$$

where the state vector $\mathbf{x}(t)$ is given by:

$$\mathbf{x}(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \\ \vdots \\ x_n(t) \end{bmatrix} \Rightarrow \frac{d\mathbf{x}(t)}{dt} = \dot{\mathbf{x}}(t) = \begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \\ \vdots \\ \dot{x}_n(t) \end{bmatrix}, \quad (\text{B.2})$$

with the dimensions of matrices and vectors as follows:

- $\mathbf{A}_{n \times n} = \mathbf{A}_{4 \times 4}$

- $B_{n \times m} = B_{4 \times 2}$
- $C_{r \times n} = C_{4 \times 4}$
- $E_{r \times m} = E_{4 \times 2}$

Assuming the split-source inverter modulated by pulse width modulation (PWM), operating in continuous conduction mode (DCM), meaning that the current in the inductor $i_{L,x}$ does not reach zero in the switching period in steady-state, it has two operating stages. Thus, there are two dynamic models that compose and represent these stages. The first configuration relates to the state of charge of the inductor L_x , and the second one is related to the discharge of the same inductor.

The average model (at the steady-state equilibrium point, i.e., with no variation in the state variables) is represented as:

$$\begin{cases} \mathbf{0} = A\bar{x} + B\bar{u} \\ \bar{y} = C\bar{x} + E\bar{u} \end{cases} \quad (\text{B.3})$$

The average matrices are:

$$\begin{cases} A = DA_1 + (1-D)A_2 \\ B = DB_1 + (1-D)B_2 \\ C = DC_1 + (1-D)C_2 \\ E = DE_1 + (1-D)E_2 \end{cases}, \quad (\text{B.4})$$

where D is the duty cycle.

The DC components at equilibrium are determined by the equilibrium point equation:

$$\begin{aligned} \mathbf{0} &= A\bar{x} + B\bar{u} \Rightarrow -A\bar{x} = B\bar{u}; \\ \bar{x} &= \frac{B\bar{u}}{-A}; \\ \bar{y} &= C\bar{x} + E\bar{u} \Rightarrow \bar{y} = C(-A^{-1}B\bar{u}) + E\bar{u}; \\ \bar{y} &= (-CA^{-1}B + E)\bar{u}. \end{aligned} \quad (\text{B.5})$$

Since the modeling is based on the average model, small disturbances around the operating point are included, where:

- Perturbed state variables $\tilde{x}(t)$;

- Perturbation in input variables $\tilde{\mathbf{u}}(t)$;
- Perturbed outputs $\tilde{\mathbf{y}}(t)$;
- Perturbation in the duty cycle $\tilde{d}(t)$.

To insert the disturbances into the model, it is considered that the resulting state vector consists of a DC term, which comes from the average model, plus an AC term, which arises from the disturbance around the operating point due to the parameters chosen for the model, as demonstrated in the equation:

$$\mathbf{x}(t) = \underbrace{\bar{\mathbf{x}}}_{\text{DC}} + \underbrace{\tilde{\mathbf{x}}(t)}_{\text{AC}} . \quad (\text{B.6})$$

By substituting the DC terms added by disturbances into the average model, and considering that the DC terms are much larger in magnitude than the AC terms, then using the small signal approximation, assuming that $\mathbf{x}(t)$ and $\mathbf{u}(t)$ do not present large variations in a switching period T_s , it holds that:

$$\langle \mathbf{x}(t) \rangle_{T_s} = \frac{1}{T_s} \int_t^{t+T_s} \mathbf{x}(\tau) d\tau; \quad (\text{B.7})$$

$$\begin{aligned} \langle \mathbf{x}(t) \rangle_{T_s} &= \bar{\mathbf{x}} + \tilde{\mathbf{x}}(t) \Rightarrow |\bar{\mathbf{x}}| \gg |\tilde{\mathbf{x}}(t)|; \\ \langle \mathbf{u}(t) \rangle_{T_s} &= \bar{\mathbf{u}} + \tilde{\mathbf{u}}(t) \Rightarrow |\bar{\mathbf{u}}| \gg |\tilde{\mathbf{u}}(t)|; \\ \langle \mathbf{y}(t) \rangle_{T_s} &= \bar{\mathbf{y}} + \tilde{\mathbf{y}}(t) \Rightarrow |\bar{\mathbf{y}}| \gg |\tilde{\mathbf{y}}(t)|. \end{aligned} \quad (\text{B.8})$$

Considering the second stage of the duty cycle as

$$D' = (1 - D), \quad (\text{B.9})$$

and being $D \gg |\tilde{d}(t)|$, it follows that:

$$\begin{aligned} d(t) &= D + \tilde{d}(t); \\ d'(t) &= D' - \tilde{d}(t). \end{aligned} \quad (\text{B.10})$$

Thus, it is assumed that $\tilde{\mathbf{u}}(t)$ and $\tilde{d}(t)$ are small alternating variations in the input vector and in the duty cycle. Also, the vectors $\tilde{\mathbf{x}}(t)$ and $\tilde{\mathbf{y}}(t)$ are small alternating variations resulting in the state vector and the output vector.

Substituting the variables with the perturbed terms, and considering that for the average model there is no variation in the states, hence, $\dot{\bar{\mathbf{x}}} = 0$, then,

$$\mathbf{K}\dot{\mathbf{x}}(t) = (\mathbf{D}\mathbf{A}_1 + \mathbf{D}'\mathbf{A}_2) (\bar{\mathbf{x}} + \tilde{\mathbf{x}}(t)) + (\mathbf{D}\mathbf{B}_1 + \mathbf{D}'\mathbf{B}_2) (\bar{\mathbf{u}} + \tilde{\mathbf{u}}(t)). \quad (\text{B.11})$$

Substituting (B.10) into (B.11) leads to:

$$\begin{aligned} \mathbf{K}\dot{\mathbf{x}}(t) = & \left[(\mathbf{D} + \tilde{d}(t)) \mathbf{A}_1 + (\mathbf{D}' - \tilde{d}(t)) \mathbf{A}_2 \right] (\bar{\mathbf{x}} + \tilde{\mathbf{x}}(t)) \\ & + \left[(\mathbf{D} + \tilde{d}(t)) \mathbf{B}_1 + (\mathbf{D}' - \tilde{d}(t)) \mathbf{B}_2 \right] (\bar{\mathbf{u}} + \tilde{\mathbf{u}}(t)). \end{aligned} \quad (\text{B.12})$$

Developing,

$$\begin{aligned} \mathbf{K}\dot{\mathbf{x}}(t) = & \underbrace{[\mathbf{D}\mathbf{A}_1 + (1 - \mathbf{D}) \mathbf{A}_2]}_{\mathbf{A}} \bar{\mathbf{x}} + \underbrace{[\mathbf{D}\mathbf{B}_1 + (1 - \mathbf{D}) \mathbf{B}_2]}_{\mathbf{B}} \bar{\mathbf{u}} \\ & + \underbrace{\mathbf{A}\tilde{\mathbf{x}}(t) + \mathbf{B}\tilde{\mathbf{u}}(t) + [(\mathbf{A}_1 - \mathbf{A}_2) \bar{\mathbf{x}} + (\mathbf{B}_1 - \mathbf{B}_2) \bar{\mathbf{u}}] \tilde{d}(t)}_{\text{AC1}} \\ & + \underbrace{(\mathbf{A}_1 - \mathbf{A}_2) \tilde{\mathbf{x}}(t)\tilde{d}(t) + (\mathbf{B}_1 - \mathbf{B}_2) \tilde{\mathbf{u}}(t)\tilde{d}(t)}_{\text{AC2}}, \end{aligned} \quad (\text{B.13})$$

where AC1 are the first-order AC terms and AC2 are the second-order nonlinear terms.

Similarly, the output $\mathbf{y}(t)$ is obtained as:

$$\begin{aligned} \bar{\mathbf{y}} + \tilde{\mathbf{y}}(t) = & \underbrace{[\mathbf{D}\mathbf{C}_1 + (1 - \mathbf{D}) \mathbf{C}_2]}_{\mathbf{C}} \bar{\mathbf{x}} + \underbrace{[\mathbf{D}\mathbf{E}_1 + (1 - \mathbf{D}) \mathbf{E}_2]}_{\mathbf{E}} \bar{\mathbf{u}} \\ & + \underbrace{\mathbf{C}\tilde{\mathbf{x}}(t) + \mathbf{E}\tilde{\mathbf{u}}(t) + [(\mathbf{C}_1 - \mathbf{C}_2) \bar{\mathbf{x}} + (\mathbf{E}_1 - \mathbf{E}_2) \bar{\mathbf{u}}] \tilde{d}(t)}_{\text{AC3}} \\ & + \underbrace{(\mathbf{C}_1 - \mathbf{C}_2) \tilde{\mathbf{x}}(t)\tilde{d}(t) + (\mathbf{E}_1 - \mathbf{E}_2) \tilde{\mathbf{u}}(t)\tilde{d}(t)}_{\text{AC4}}, \end{aligned} \quad (\text{B.14})$$

where AC3 are the first-order AC terms and AC4 are the second-order nonlinear terms.

Eliminating the DC terms from (B.13) and (B.14), and assuming the second-order (nonlinear) terms are negligible, yields the generic reference model for the linearized small-signal state equations, as presented in (B.15).

$$\begin{cases} \mathbf{K}\dot{\mathbf{x}}(t) = \mathbf{A}\tilde{\mathbf{x}}(t) + \mathbf{B}\tilde{\mathbf{u}}(t) + [(\mathbf{A}_1 - \mathbf{A}_2)\tilde{\mathbf{x}} + (\mathbf{B}_1 - \mathbf{B}_2)\tilde{\mathbf{u}}] \tilde{d}(t) \\ \tilde{\mathbf{y}}(t) = \mathbf{C}\tilde{\mathbf{x}}(t) + \mathbf{E}\tilde{\mathbf{u}}(t) + [(\mathbf{C}_1 - \mathbf{C}_2)\tilde{\mathbf{x}} + (\mathbf{E}_1 - \mathbf{E}_2)\tilde{\mathbf{u}}] \tilde{d}(t) \end{cases} \quad (\text{B.15})$$

For the model, the following generic form is obtained for the states, based on (B.15):

$$\begin{cases} \mathbf{K}\dot{\tilde{\mathbf{x}}}(t) = \mathbf{A}\tilde{\mathbf{x}}(t) + \mathbf{B}\tilde{\mathbf{u}}(t) + \mathbf{F}\tilde{d}(t) \\ \tilde{\mathbf{y}}(t) = \mathbf{C}\tilde{\mathbf{x}}(t) + \mathbf{E}\tilde{\mathbf{u}}(t) + \mathbf{G}\tilde{d}(t) \end{cases} \quad (\text{B.16})$$

From this, the following can be defined:

$$\begin{aligned} \mathbf{A}_p &= \mathbf{K}^{-1}\mathbf{A}; \\ \mathbf{B}_p &= \begin{bmatrix} \mathbf{K}^{-1}\mathbf{B} & \mathbf{K}^{-1}\{(\mathbf{A}_1 - \mathbf{A}_2)\tilde{\mathbf{x}} + (\mathbf{B}_1 - \mathbf{B}_2)\tilde{\mathbf{u}}\} \end{bmatrix}; \\ \mathbf{C}_p &= \mathbf{C}; \\ \mathbf{E}_p &= \begin{bmatrix} \mathbf{E} & \{(\mathbf{C}_1 - \mathbf{C}_2)\tilde{\mathbf{x}} + (\mathbf{E}_1 - \mathbf{E}_2)\tilde{\mathbf{u}}\} \end{bmatrix}; \\ \tilde{\mathbf{u}}(t) &= \begin{bmatrix} \tilde{i}_o(t) \\ \tilde{U}_x(t) \end{bmatrix}; \\ \tilde{\mathbf{u}}_p(t) &= \begin{bmatrix} \tilde{\mathbf{u}}(t) \\ \tilde{d}(t) \end{bmatrix}. \end{aligned} \quad (\text{B.17})$$

Thus, the system can be represented again in the known canonical form:

$$\begin{cases} \dot{\tilde{\mathbf{x}}}(t) = \mathbf{A}_p\tilde{\mathbf{x}}(t) + \mathbf{B}_p\tilde{\mathbf{u}}_p(t) \\ \tilde{\mathbf{y}}(t) = \mathbf{C}_p\tilde{\mathbf{x}}(t) + \mathbf{E}_p\tilde{\mathbf{u}}_p(t) \end{cases} \quad (\text{B.18})$$

B.3 Small-signal state-space model of SSI

As already demonstrated, the modeling of the energization and de-energization of the split-source inverter for its x th input can be represented by the following dynamic models:

Magnetization of L_x :

$$\begin{aligned} \frac{di_{L,x}}{dt} &= \frac{U_x}{L_x} - \frac{R_x}{L_x}i_{L,x}; \\ \frac{dU_C}{dt} &= \frac{i_o}{C}. \end{aligned} \quad (\text{B.19})$$

Demagnetization of L_x :

$$\begin{aligned}\frac{di_{L,x}}{dt} &= -\frac{R_x}{L_x}i_{L,x} + \frac{(U_x - U_C)}{L_x}; \\ \frac{dU_C}{dt} &= \frac{i_{L,x} + i_o}{C}.\end{aligned}\quad (\text{B.20})$$

The output current can be modeled as follows:

$$i_o = (1 - 2s^\top) i_{abc}. \quad (\text{B.21})$$

Performing the state-space modeling:

$$\dot{\mathbf{x}} = \begin{bmatrix} di_{L,x}/dt \\ dU_C/dt \end{bmatrix} = \begin{bmatrix} \dot{i}_{L,x} \\ \dot{U}_C \end{bmatrix}, \quad \mathbf{x} = \begin{bmatrix} i_{L,x} \\ U_C \end{bmatrix}, \quad \mathbf{y} = \begin{bmatrix} i_{L,x} \\ U_C \end{bmatrix}, \quad \mathbf{u} = \begin{bmatrix} U_x \\ i_o \end{bmatrix}. \quad (\text{B.22})$$

From this, the model representing the first stage can be obtained according to (B.23) and (B.24).

$$\underbrace{\begin{bmatrix} di_{L,x}/dt \\ dU_C/dt \end{bmatrix}}_{\dot{\mathbf{x}}} = \underbrace{\begin{bmatrix} -R_x/L_x & 0 \\ 0 & 0 \end{bmatrix}}_{\mathbf{A}_1} \underbrace{\begin{bmatrix} i_{L,x} \\ U_C \end{bmatrix}}_{\mathbf{x}} + \underbrace{\begin{bmatrix} 1/L_x & 0 \\ 0 & 1/C \end{bmatrix}}_{\mathbf{B}_1} \underbrace{\begin{bmatrix} U_x \\ i_o \end{bmatrix}}_{\mathbf{u}}, \quad (\text{B.23})$$

$$\underbrace{\begin{bmatrix} i_{L,x} \\ U_C \end{bmatrix}}_{\mathbf{y}} = \underbrace{\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}}_{\mathbf{C}_1} \underbrace{\begin{bmatrix} i_{L,x} \\ U_C \end{bmatrix}}_{\mathbf{x}} + \underbrace{\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}}_{\mathbf{E}_1} \underbrace{\begin{bmatrix} U_x \\ i_o \end{bmatrix}}_{\mathbf{u}}. \quad (\text{B.24})$$

For the second stage, the modeling results in (B.25) and (B.26).

$$\underbrace{\begin{bmatrix} di_{L,x}/dt \\ dU_C/dt \end{bmatrix}}_{\dot{\mathbf{x}}} = \underbrace{\begin{bmatrix} -R_x/L_x & -1/L_x \\ 1/C & 0 \end{bmatrix}}_{\mathbf{A}_2} \underbrace{\begin{bmatrix} i_{L,x} \\ U_C \end{bmatrix}}_{\mathbf{x}} + \underbrace{\begin{bmatrix} 1/L_x & 0 \\ 0 & 1/C \end{bmatrix}}_{\mathbf{B}_2} \underbrace{\begin{bmatrix} U_x \\ i_o \end{bmatrix}}_{\mathbf{u}}, \quad (\text{B.25})$$

$$\underbrace{\begin{bmatrix} i_{L,x} \\ U_C \end{bmatrix}}_y = \underbrace{\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}}_{C_2} \underbrace{\begin{bmatrix} i_{L,x} \\ U_C \end{bmatrix}}_x + \underbrace{\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}}_{E_2} \underbrace{\begin{bmatrix} U_x \\ i_o \end{bmatrix}}_u. \quad (\text{B.26})$$

Based on the models representing the operating modes of the split-source inverter, the state matrices can be applied to the steady-state state-space equations for small signals (B.15). Developing each of the terms:

$$\mathbf{A} = D_x \mathbf{A}_1 + (1 - D_x) \mathbf{A}_2 = D_x \begin{bmatrix} -R_x/L_x & 0 \\ 0 & 0 \end{bmatrix} + (1 - D_x) \begin{bmatrix} -R_x/L_x & -1/L_x \\ 1/C & 0 \end{bmatrix}, \quad (\text{B.27})$$

$$\mathbf{A} = \begin{bmatrix} -D_x R_x/L_x & 0 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} (1 - D_x)(-R_x/L_x) & (1 - D_x)(-1/L_x) \\ (1 - D_x)(1/C) & 0 \end{bmatrix}, \quad (\text{B.28})$$

$$\mathbf{A} = \begin{bmatrix} -R_x/L_x & (1 - D_x)(-1/L_x) \\ (1 - D_x)(1/C) & 0 \end{bmatrix}; \quad (\text{B.29})$$

$$\mathbf{B} = D_x \mathbf{B}_1 + (1 - D_x) \mathbf{B}_2 = D_x \begin{bmatrix} 1/L_x & 0 \\ 0 & 1/C \end{bmatrix} + (1 - D_x) \begin{bmatrix} 1/L_x & 0 \\ 0 & 1/C \end{bmatrix}, \quad (\text{B.30})$$

$$\mathbf{B} = \begin{bmatrix} D_x/L_x & 0 \\ 0 & D_x/C \end{bmatrix} + \begin{bmatrix} (1 - D_x)/L_x & 0 \\ 0 & (1 - D_x)(1/C) \end{bmatrix}, \quad (\text{B.31})$$

$$\mathbf{B} = \begin{bmatrix} 1/L_x & 0 \\ 0 & 1/C \end{bmatrix}; \quad (\text{B.32})$$

$$\mathbf{A}_1 - \mathbf{A}_2 = \begin{bmatrix} -R_x/L_x & 0 \\ 0 & 0 \end{bmatrix} - \begin{bmatrix} -R_x/L_x & -1/L_x \\ 1/C & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1/L_x \\ -1/C & 0 \end{bmatrix}, \quad (\text{B.33})$$

$$\mathbf{B}_1 - \mathbf{B}_2 = \begin{bmatrix} 1/L_x & 0 \\ 0 & 1/C \end{bmatrix} - \begin{bmatrix} 1/L_x & 0 \\ 0 & 1/C \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}. \quad (\text{B.34})$$

Substituting the terms into the steady-state small-signal model equation yields:

$$\begin{aligned} \dot{\tilde{\mathbf{x}}} = & \underbrace{\begin{bmatrix} -R_x/L_x & (1-D_x)(-1/L_x) \\ (1-D_x)(1/C) & 0 \end{bmatrix}}_A \tilde{\mathbf{x}} + \underbrace{\begin{bmatrix} 1/L_x & 0 \\ 0 & 1/C \end{bmatrix}}_B \tilde{\mathbf{u}} \\ & + \left\{ \underbrace{\begin{bmatrix} 0 & 1/L_x \\ -1/C & 0 \end{bmatrix}}_{A_1-A_2} \tilde{\mathbf{x}} + \underbrace{\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}}_{B_1-B_2} \tilde{\mathbf{u}} \right\} \tilde{d}_x, \end{aligned} \quad (\text{B.35})$$

where:

$$\dot{\tilde{\mathbf{x}}} = \begin{bmatrix} \dot{\tilde{i}}_{L,x} \\ \dot{\tilde{U}}_C \end{bmatrix}, \quad \tilde{\mathbf{x}} = \begin{bmatrix} \tilde{i}_{L,x} \\ \tilde{U}_C \end{bmatrix}, \quad \tilde{\mathbf{u}} = \begin{bmatrix} \tilde{U}_x \\ \tilde{i}_o \end{bmatrix}, \quad \tilde{\mathbf{x}} = \begin{bmatrix} i_{L,x} \\ U_C \end{bmatrix}, \quad \tilde{\mathbf{u}} = \begin{bmatrix} U_x \\ i_o \end{bmatrix}. \quad (\text{B.36})$$

Substituting also the input and output variables of the system results in:

$$\begin{aligned} \begin{bmatrix} \dot{\tilde{i}}_{L,x} \\ \dot{\tilde{U}}_C \end{bmatrix} = & \begin{bmatrix} -R_x/L_x & (1-D_x)(-1/L_x) \\ (1-D_x)(1/C) & 0 \end{bmatrix} \begin{bmatrix} \tilde{i}_{L,x} \\ \tilde{U}_C \end{bmatrix} + \begin{bmatrix} 1/L_x & 0 \\ 0 & 1/C \end{bmatrix} \begin{bmatrix} \tilde{U}_x \\ \tilde{i}_o \end{bmatrix} \\ & + \left\{ \begin{bmatrix} 0 & 1/L_x \\ -1/C & 0 \end{bmatrix} \begin{bmatrix} i_{L,x} \\ U_C \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} U_x \\ i_o \end{bmatrix} \right\} \tilde{d}_x. \end{aligned} \quad (\text{B.37})$$

Evidencing the equations representing the input current $i_{L,x}$ and the DC bus voltage U_C :

$$\begin{aligned} \dot{\tilde{i}}_{L,x} = & \left(-\frac{R_x}{L_x}\right) \tilde{i}_{L,x} + (1-D_x) \left(-\frac{1}{L_x}\right) \tilde{U}_C + \left(\frac{1}{L_x}\right) \tilde{U}_x + \left(\frac{1}{L_x}\right) U_C \tilde{d}_x; \\ \dot{\tilde{U}}_C = & (1-D_x) \left(\frac{1}{C}\right) \tilde{i}_{L,x} + \left(\frac{1}{C}\right) \tilde{i}_o + \left(-\frac{1}{C}\right) i_{L,x} \tilde{d}_x. \end{aligned} \quad (\text{B.38})$$

B.4 Representation of the SSI model in the frequency domain

Based on the property for converting the representation of a system in continuous time (state-space) to the frequency domain (the s domain), through the Laplace transform, assuming zero initial conditions:

$$\begin{cases} \dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t) \\ \mathbf{y}(t) = \mathbf{C}\mathbf{x}(t) + \mathbf{E}\mathbf{u}(t) \end{cases} \quad (\text{B.39})$$

$$\Downarrow$$

$$\mathcal{L}\{\dot{\mathbf{x}}(t)\} = \mathcal{L}\{\mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t)\} \Rightarrow s\mathbf{x}(s) = \mathbf{A}\mathbf{x}(s) + \mathbf{B}\mathbf{u}(s).$$

Applying the mentioned property initiates the process of converting the system to the s domain:

$$s\tilde{i}_{L,x}(s) = \left(-\frac{R_x}{L_x}\right)\tilde{i}_{L,x}(s) + (1 - D_x)\left(-\frac{1}{L_x}\right)\tilde{U}_C(s) + \left(\frac{1}{L_x}\right)\tilde{U}_x(s) + \left(\frac{1}{L_x}\right)U_C\tilde{d}_x(s). \quad (\text{B.40})$$

Given that the input voltage U_x is constant, this term can be disregarded for modeling purposes in the frequency domain.

$$s\tilde{i}_{L,x}(s) = \left(-\frac{R_x}{L_x}\right)\tilde{i}_{L,x}(s) + (1 - D_x)\left(-\frac{1}{L_x}\right)\tilde{U}_C(s) + \left(\frac{1}{L_x}\right)U_C\tilde{d}_x(s). \quad (\text{B.41})$$

Developing,

$$\tilde{i}_{L,x}(s) = \left(\frac{1}{sL_x + R_x}\right) \left[-(1 - D_x)\tilde{U}_C(s) + U_C\tilde{d}_x(s) \right]. \quad (\text{B.42})$$

By isolating the terms of the transfer function for $i_{L,x}$, the expression becomes:

$$\tilde{i}_{L,x}(s) = \left(\frac{U_C}{sL_x + R_x}\right)\tilde{d}_x(s) - \left(\frac{1 - D_x}{sL_x + R_x}\right)\tilde{U}_C(s). \quad (\text{B.43})$$

Now, performing the same process for the model of the DC bus voltage:

$$s\tilde{U}_C(s) = (1 - D_x)\left(\frac{1}{C}\right)\tilde{i}_{L,x}(s) + \left(\frac{1}{C}\right)\tilde{i}_o(s) + \left(-\frac{1}{C}\right)i_{L,x}\tilde{d}_x(s). \quad (\text{B.44})$$

Developing,

$$\tilde{U}_C(s) = \left(\frac{1}{sC} \right) \left[(1 - D_x) \tilde{i}_{L,x}(s) + \tilde{i}_o(s) - i_{L,x} \tilde{d}_x(s) \right]. \quad (\text{B.45})$$

By isolating the terms of the transfer function for U_C , the expression becomes:

$$\tilde{U}_C(s) = \left(\frac{1 - D_x}{sC} \right) \tilde{i}_{L,x}(s) + \left(\frac{1}{sC} \right) \tilde{i}_o(s) - \left(\frac{1}{sC} \right) i_{L,x} \tilde{d}_x(s). \quad (\text{B.46})$$

Based on (B.43) and (B.46), one can develop the transfer functions that form the basis for the multi-loop control developed in Chapter 5.

First, assuming that the DC bus voltage is constant, the transfer function of $\tilde{i}_{L,x}(s)$ (B.43), which relates the input current to the duty cycle, can be simplified as:

$$\frac{\tilde{i}_{L,x}(s)}{\tilde{d}_x(s)} = \frac{U_C}{sL_x + R_x}. \quad (\text{B.47})$$

Based on the transfer function of $\tilde{U}_C(s)$ (B.45), and assuming a constant duty cycle and output current, the transfer function relating the DC bus voltage to the input current is obtained as:

$$\frac{\tilde{U}_C(s)}{\tilde{i}_{L,x}(s)} = \left(\frac{1 - D_x}{sC} \right). \quad (\text{B.48})$$

C Bidirectional MISSI topologies

C.1 Topology description

Since the nine-switch inverter offers a high degree of freedom, it enables the possibility of using nine input sources. Figure C.1 shows an arrangement with nine DC sources and the three-phase AC terminals.

Some topological changes are promising alternatives to make the two-level three-phase MISSI bidirectional, such as the approach presented in [83], which was applied to the SSI. Trying to keep the low active component count and low-cost features, an alternative could be using solid-state circuit breaker technology together with a simple buck converter, as depicted in Figure C.2.

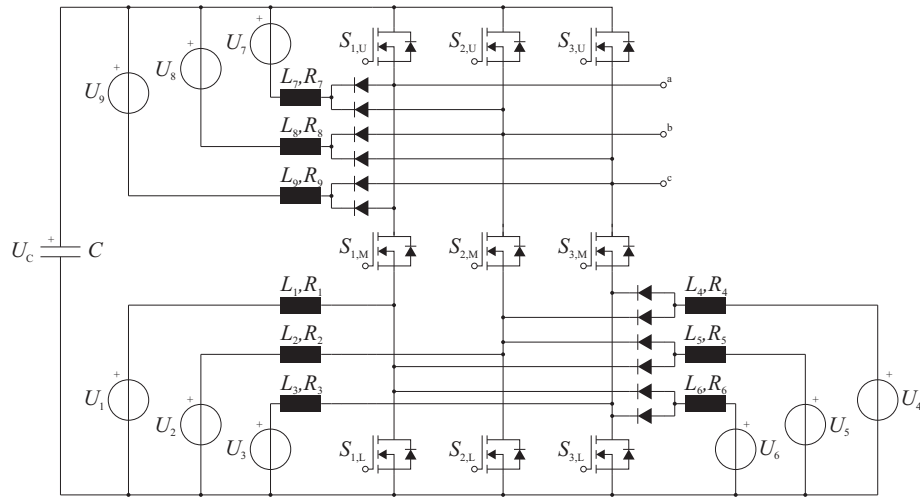


Figure C.1: Bidirectional 9S-MISSI with nine inputs

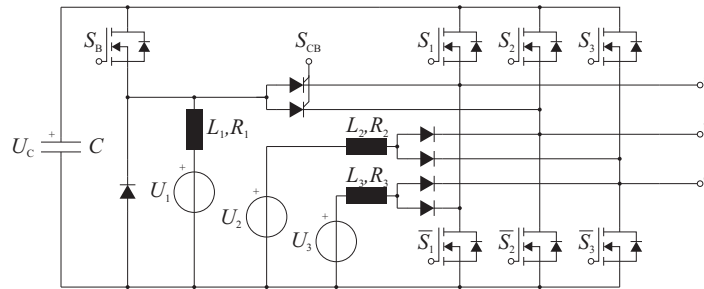


Figure C.2: MISSI with a bidirectional input

C.2 Simulation results

Following the same scheme from Section 6.1.3, Figure C.3 shows the results for the 9S-MISSI with nine input sources, where $\mathbf{u}_U^\top = [75 \ 75 \ 75 \ 50 \ 50 \ 50 \ 50 \ 50 \ 50]$ V, and 6 sources are unidirectional and 3 sources are bidirectional. The results show good dynamic performance, power injection into and absorption from the grid, and DC bus voltage regulation.

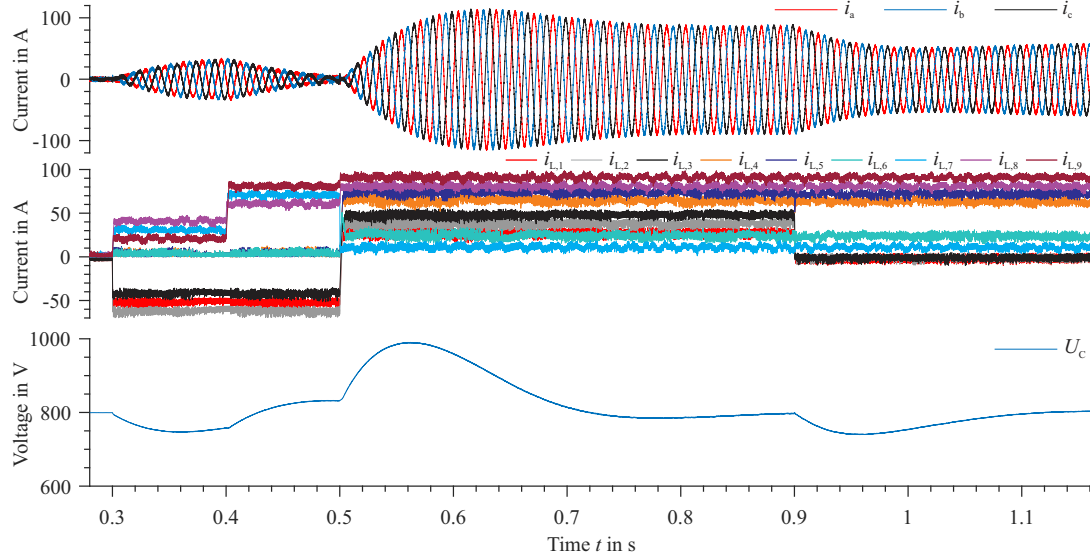


Figure C.3: Simulation results for three-phase 9S-MISSI with nine DC sources: output currents, input currents, and DC bus voltage

Figure C.4 presents the results for the two-level converter with a bidirectional source using a solid-state circuit breaker. The input voltages are $\mathbf{u}_U^\top = [35 \ 35 \ 35]$ V. The results demonstrate bus voltage regulation, power absorption from the grid, and good dynamic response. It can be highlighted that the approach is one among the potential alternatives using emerging technologies in power electronics, such as new bidirectional circuit breakers and low-loss semiconductors. From this arrangement, the input sources have an inherent disconnection device.

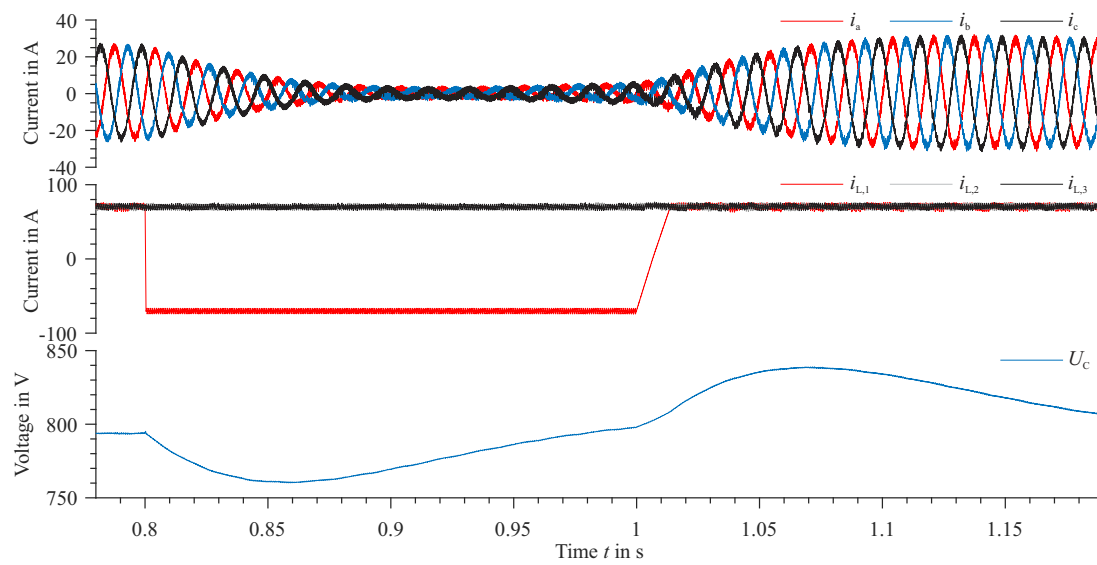


Figure C.4: Simulation results for three-phase MISSI with one bidirectional DC source: output currents, input currents, and DC bus voltage

D Data of IEEE 9-Bus test system

To evaluate the performance of IBRs and synchronous generators in a realistic power system environment, the IEEE 9-bus test system is adopted in this thesis. This widely recognized benchmark provides a simplified yet representative model of a transmission network that includes generators, loads, transmission lines, and transformers.

This section presents the key parameters and data used for simulation and analysis. These include transmission line impedances, synchronous generator ratings, system load demands, transformer specifications, and event scenarios. Additionally, detailed parameters for both steam and hydro governor-turbine models are provided, along with the inverter parameters used for GFM inverter design. The data is either derived from or based on established literature and simulation models [196], [212].

The following tables summarize the relevant system data used in this work.

Table 8: Transmission Line Parameter of IEEE 9-Bus System

Line	Resistance (p.u.)	Reactance (p.u.)	Susceptance (p.u.)
1-4	0.0000	0.0576	0.0000
4-5	0.0100	0.0850	0.1760
4-6	0.0170	0.0920	0.1580
6-9	0.0390	0.1700	0.3580
5-7	0.0320	0.1610	0.3060
9-3	0.0000	0.0586	0.0000
7-2	0.0000	0.0625	0.0000
9-8	0.0119	0.1008	0.2090
7-8	0.0085	0.0720	0.1490

Table 9: Ratings of Synchronous Generators

Parameter	SG1	SG2	SG3
Rating in p.u.	2.50	0.96	0.64
Rated Power in MVA	250	96	64
Inertia in s	6	13	13

Table 10: Total System Load Demand

Parameter	Value
Active power	300 MW
Reactive power	82.5 Mvar

Table 11: Events Applied During Simulation

Event	Value
Load Step at Bus 6	90 MW
Loss of SG3 at Bus 3	64 MW
IBR disconnection at Bus 6 with local load	6 MW

Table 12: Steam Governor and Reheat Turbine Parameters

Parameter	Value
Droop gain	0.03
Governor time constant	0.2
Steam chest time constant	0.5
Turbine fraction factor	0.3
Reheat time constant	7

Table 13: Hydro Govenor and Turbine Parameters

Parameter	Value
Droop gain	0.03
Governor time constant	0.2
Transient droop gain	0.38
Reset time constant	5
Waterways time constant	1

Table 14: Transformer Parameter of IEEE 9-Bus System

Transformer	LV	HV
T1	16.5 kV	230 kV
T2	18 kV	230 kV
T3	13.8 kV	230 kV

Table 15: Inverter Parameters

Symbol	Parameter	Value
S_r	Rated apparent power	12 kVA
$U_{g,r}$	Grid voltage	220 V
f_g	Grid frequency	60 Hz
U_C	DC bus voltage	700 V
R_f	Filter resistance	1 m Ω
L_f	Filter inductance	2 mH
C_f	Filter capacitance	40 μ F
R_ζ	Damping resistor	0.5 Ω
$K_{P,P}$	Active power droop coefficient	5%
$K_{P,Q}$	Reactive power droop coefficient	10%
R_{vr}	Virtual resistance	0.5 p.u.
L_{vr}	Virtual inductance	0.5 p.u.
f_s	Sampling frequency	20 kHz
f_{sw}	Switching frequency	10 kHz
U_{BESS}	BESS voltage	48 V
$i_{BESS,r}$	BESS rated current	250 A
R_B	BESS filter resistance	0.1 m Ω
L_B	BESS filter inductance	260 μ H
ζ_C	Damping ratio (voltage loop)	0.1
ζ_{BESS}	Damping ratio (BESS)	0.7
$\omega_{C,LP}$	Bandwidth of voltage loop	2 π 5 rad/s
$\omega_{BESS,LP}$	Bandwidth of current loop	2 π 100 rad/s
$K_{\omega,I}, K_{\omega,R}$	DC-bus ancillary controller gains	-0.05/ T_s , 70 T_s
$K_{B,I}, K_{B,R}$	BESS ancillary controller gains	-0.0005/ T_s , 14 T_s
k_S	Aggregation factor	8,000
$S_{r,s}$	Aggregated power	96 MVA
–	Aggregated connection impedance	SCR = 10, X/R = 6

E System connection parameters

E.1 Evaluation guidelines

Considering the typical scenario in which IBRs are connected to medium-voltage networks along highways via dedicated transformers. Here, R_{MV} , L_{MV} , R_{LV} , and L_{LV} represent the impedance of the medium-voltage (MV) and low-voltage (LV) segments, respectively. Additionally, each transformer is represented by an impedance $\underline{Z}_T$, composed of R_T and L_T .

E.1.1 Medium-voltage segment

In Brazil, two distribution line structures are commonly used for MV networks: the traditional, horizontally distributed configuration and the compact design [231]. Based on [232], the reactance per kilometer $X_{MV,km}$ can be calculated from the cable resistance using the following expression:

$$X_{MV,km} = 28.935325 \cdot 10^{-4} f \log \left(\frac{l_m}{l_s} \right) \Omega/\text{km}, \quad (\text{E.1})$$

in which f is the fundamental frequency, assumed to be 60 Hz, l_m is the geometric mean distance between phases, and l_s is the geometric mean radius of the cable.

Table 16 presents the resistance and reactance data compiled for the main types of cables. Here, 1 represents the traditional configuration, and 2 indicates the compact design.

Table 16: Resistance and Reactance Parameters for MV Cables [232]

Cable	Type	l_s in m	l_m in m	R_{MV} in Ω/km	X_{MV} in Ω/km	L_{MV} in mH/km
1 ACSR	1-Robin	0.00127	1.09027	0.8576	0.5093	1.3510
1 ACSR	2-Robin	0.00127	0.14000	0.8576	0.3546	0.9400
4/0 ACSR	1-Penguin	0.00248	1.09027	0.3679	0.4589	1.2172
4/0 ACSR	2-Penguin	0.00248	0.14000	0.3679	0.3041	0.8070
2/0 ACSR	1-Quail	0.00155	1.09027	0.5562	0.4943	1.3110
2/0 ACSR	2-Quail	0.00155	0.14000	0.5562	0.3395	0.9010

Since the analysis presented in this paper is conducted on the LV side, the impedance calculated for the MV segment is referred to the transformer secondary side using $U_{\text{prim}}/U_{\text{sec}}$.

E.1.2 Transformer

Considering overhead networks, the Brazilian standard NBR5440 covers the requirements for distribution transformers up to 300 kVA. Since the two main nominal voltages are 13.8 kV and 34.5 kV, Table 17 summarizes the main transformer parameters for ratings up to 300 kVA for both voltage levels.

Table 17: Main Transformer Parameters [233], [234]

U_{prim} in kV	S_T in kVA	$Z_{T,\%}$ in %	X/R	U_{prim} in kV	S_T in kVA	$Z_{T,\%}$ in %	X/R
13.8	45	3.5	3.5	34.5	45	4	3.5
	75	3.5	3.5		75	4	3.5
	112.5	3.5	3.5		112.5	4	3.5
	150	3.5	3.5		150	4	3.5
	225	4.5	3.5		225	5	3.5
	300	4.5	3.5		300	5	3.5

Based on the presented data, the base impedance of the transformer, reflected to the secondary side, can be calculated as:

$$Z_{T,\text{sec}} = \frac{Z_{T,\%}}{100} \frac{U_{r,\text{LV}}^2}{S_T}, \quad (\text{E.2})$$

in which $Z_{T,\%}$ is the typical impedance in percentage, $U_{r,\text{LV}}$ is the line-to-line voltage on the secondary side, and S_T is the rated power of the transformer.

The equivalent resistance of the transformer (R_T) can be determined by:

$$R_T = \frac{Z_{T,\text{sec}}}{\sqrt{1 + \frac{X}{R}}}. \quad (\text{E.3})$$

Through R_T and the X/R ratio, the transformer reactance X_T can be readily obtained. In this study, the magnetizing branch is not considered in the transformer model, and the reflected resistance and reactance values are used.

E.1.3 Low-voltage segment

Regarding the LV segment, the installation method is based on the Brazilian standard NBR5410, which determines the requirements for LV electrical systems. The resistance and reactance data for a set of LV cables are obtained from the tables provided in [235] and [236] and are summarized in Table 18.

Table 18: Resistance and Reactance Parameters for LV Cables [235], [236]

Sectional Area in mm ²	Current-carrying Capacity in A	R_{LV} in Ω/km	X_{LV} in Ω/km
16	68	1.45	0.13
25	89	0.93	0.13
35	110	0.66	0.12
50	134	0.46	0.12
70	171	0.33	0.12
95	207	0.25	0.12
120	239	0.20	0.11
150	275	0.16	0.11
185	314	0.13	0.11
240	370	0.10	0.11
300	426	0.08	0.11
400	510	0.06	0.11
500	587	0.05	0.11

After defining the connection method presented in NBR5410, the cable is established based on the current carrying capacity for the required demand.

E.1.4 Grid strenght

One way to assess the characteristics of the power grid is through the use of the short-circuit ratio N_k and the impedance ratio N_{XR} [237], [238]. These indices are defined as follows:

$$N_{XR} = \frac{X_g}{R_g}, \quad N_k = \frac{S_k''}{S_r} = \frac{1.1 U_{g,r}^2}{S_r \sqrt{R_g^2 + X_g^2}}, \quad (\text{E.4})$$

in which X_g and R_g are the reactance and resistance of the segment under evaluation, respectively, $U_{g,r}$ is the line-to-line voltage at the evaluation point, and S_r represents the rated apparent power. The variable S_k'' denotes the short-circuit power and is related to the maximum power that the grid can deliver at the PCC [237].

These indices provide insights into the amount of load the feeder can support, enabling the grid classification as either strong or weak. In this context, a weak grid indicates that the grid impedance is higher than the impedance seen at the PCC, thereby dominating the feeder's dynamic behavior [237].

F Continuous-control-set modulated MPC for current control loop

To mitigate the variable switching frequency issue, modulated model predictive control (M²PC) methods incorporate modulation directly into the optimization process [137], [239]–[245], effectively yielding a continuous control set MPC (CCS-MPC) with fixed switching frequency. In these schemes, at switching period T_s , a convex optimization problem is solved over the feasible region of the converter, usually via active-set methods that enforce Karush-Kuhn-Tucker (KKT) conditions [226], [245], [246], or interior-point methods [247] that maintain feasibility through barrier functions. Among the challenges is the real-time implementation of these algorithms within a few microseconds, considering, for instance, converters operating with switching frequencies above 50 kHz.

This appendix elaborates on the application of the Frank–Wolfe (FW) algorithm, also known as the conditional gradient method, to the optimization problem in M²PC for grid-tied power converters. The proposed approach minimizes the current reference tracking error over a finite prediction horizon while strictly enforcing control constraints. In contrast to conventional techniques that utilize KKT conditions and back-projection onto the feasible set, the FW method solves a sequence of linearized sub-problems, beginning from an initial feasible guess and iteratively converging to the constrained optimum. The algorithm exhibits robustness even when approximate solutions are employed, allowing a trade-off between computational burden and control accuracy by varying the number of iterations per sampling period.

F.1 Grid-tied VSI MPC problem statement

This chapter introduces the M²PC problem for regulating currents in a grid-tied VSI with a prediction horizon of $N_p = 1$. The objective is to determine the control action $\mathbf{u}$ that results in the smallest current tracking error $\mathbf{e}$ at the end of the next switching period while considering constraints on the maximum control action values — specifically, the limits defined by the two-dimensional $\alpha\beta$ space vector diagram.

F.1.1 MPC formulation

Let us consider a grid-connected VSI with an inductance L filter, as described in [138]. In $\alpha\beta$ coordinates, the ac-side currents $\mathbf{i}_{\alpha\beta} = [i_\alpha \ i_\beta]^\top$ are governed by the differential equations

$$\begin{bmatrix} \frac{di_\alpha}{dt} \\ \frac{di_\beta}{dt} \end{bmatrix} = \frac{1}{L} \begin{bmatrix} u_{\alpha,c} - u_\alpha \\ u_{\beta,c} - u_\beta \end{bmatrix} = \frac{1}{L} (\mathbf{u}_{\alpha\beta,c} - \mathbf{u}_{\alpha\beta}), \quad (\text{F.1})$$

where the vectors $\mathbf{u}_{\alpha\beta,c} = [u_{\alpha,c} \ u_{\beta,c}]^\top$ and $\mathbf{u}_{\alpha\beta} = [u_\alpha \ u_\beta]^\top$ contain in their entries the inverter-synthesized and grid voltages in $\alpha\beta$ coordinates, respectively. In the discrete time domain, the estimated current can be expressed as:

$$\begin{bmatrix} \hat{i}_{\alpha,k+1} \\ \hat{i}_{\beta,k+1} \end{bmatrix} = \mathbf{A}_p \begin{bmatrix} i_{\alpha,k} \\ i_{\beta,k} \end{bmatrix} + \mathbf{B}_p \begin{bmatrix} u_{\alpha,c,k} - u_{\alpha,k} \\ u_{\beta,c,k} - u_{\beta,k} \end{bmatrix}, \quad (\text{F.2})$$

where

$$\mathbf{A}_p = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \text{ and } \mathbf{B}_p = \frac{T_s}{L} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad (\text{F.3})$$

and T_s is the switching period. Note that the control action $\mathbf{u}_k$ is actually applied over the interval $[k, k+1)$. To account for the implementation delay, first (F.2) is used to predict $\hat{i}_{k+1}$, and then the MPC is formulated to find $\mathbf{u}_{k+1}$ so as to minimize the current error at time $k+2$. The two-step-ahead current prediction is

$$\begin{bmatrix} \hat{i}_{\alpha,k+2} \\ \hat{i}_{\beta,k+2} \end{bmatrix} = \begin{bmatrix} \hat{i}_{\alpha,k+1} \\ \hat{i}_{\beta,k+1} \end{bmatrix} + \frac{T_s}{L} \begin{bmatrix} \hat{u}_{\alpha,c,k+1} - \hat{u}_{\alpha,k+1} \\ \hat{u}_{\beta,c,k+1} - \hat{u}_{\beta,k+1} \end{bmatrix}, \quad (\text{F.4})$$

where $\hat{\mathbf{u}}_{\alpha\beta,k+1} = [\hat{u}_{\alpha,k+1} \ \hat{u}_{\beta,k+1}]^\top$ is the estimated grid voltage vector at $k+1$. Let us define the current error vector as:

$$\mathbf{e}_{k+2} = \begin{bmatrix} e_{\alpha,k+2} \\ e_{\beta,k+2} \end{bmatrix} = \begin{bmatrix} \hat{i}_{\alpha,\text{ref},k+2} - \hat{i}_{\alpha,k+2} \\ \hat{i}_{\beta,\text{ref},k+2} - \hat{i}_{\beta,k+2} \end{bmatrix} \quad (\text{F.5})$$

or, alternatively,

$$\mathbf{e}_{k+2} = \begin{bmatrix} \hat{i}_{\alpha,\text{ref},k+2} - \hat{i}_{\alpha,k+1} \\ \hat{i}_{\beta,\text{ref},k+2} - \hat{i}_{\beta,k+1} \end{bmatrix} - \frac{T_s}{L} \begin{bmatrix} \hat{u}_{\alpha,c,k+1} - \hat{u}_{\alpha,k+1} \\ \hat{u}_{\beta,c,k+1} - \hat{u}_{\beta,k+1} \end{bmatrix}. \quad (\text{F.6})$$

In order to simplify the notation, one can write the equation for the error $\mathbf{e}$ as:

$$\mathbf{e} = \mathbf{H}\mathbf{u} + \mathbf{h}. \quad (\text{F.7})$$

where the inverter phase voltages vector $\mathbf{u}_{\alpha\beta,c}$ is normalized by the dc bus voltage U_C resulting in the vector $\mathbf{u}$, that is:

$$\mathbf{u} = \mathbf{u}_{k+1} = \frac{1}{U_C} \begin{bmatrix} \hat{u}_{\alpha,c,k+1} \\ \hat{u}_{\beta,c,k+1} \end{bmatrix} = \frac{\hat{\mathbf{u}}_{\alpha\beta,c,k+1}}{U_C}. \quad (\text{F.8})$$

In addition, the matrices $\mathbf{H}$ and $\mathbf{h}$ are defined as follows:

$$\mathbf{H} = -\frac{U_C T_s}{L} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad (\text{F.9})$$

$$\mathbf{h}_{k+2} = \begin{bmatrix} \hat{i}_{\alpha,\text{ref},k+2} - \hat{i}_{\alpha,k+1} \\ \hat{i}_{\beta,\text{ref},k+2} - \hat{i}_{\beta,k+1} \end{bmatrix} + \frac{T_s}{L} \begin{bmatrix} \hat{u}_{\alpha,k+1} \\ \hat{u}_{\beta,k+1} \end{bmatrix}.$$

It is worth mentioning that, for fixed bus voltages, $\mathbf{H}$ can be pre-calculated and stored in the DSP memory.

In the case of the control of inverter currents, a strongly-convex function $J(\mathbf{u})$ can be defined as the squared norm of the error $\mathbf{e}$:

$$J(\mathbf{u}) = \|\mathbf{e}\|^2 = \|\mathbf{H}\mathbf{u} + \mathbf{h}\|^2 \quad (\text{F.10})$$

$$J(\mathbf{u}) = \mathbf{u}^\top \mathbf{H}^\top \mathbf{H} \mathbf{u} + 2\mathbf{h}^\top \mathbf{H} \mathbf{u} + \mathbf{h}^\top \mathbf{h}.$$

This is a standard quadratic programming MPC formulation, which naturally arises when formulating the current tracking error minimization as a constrained convex optimization problem [137], [245], that is, at each sampling cycle, find the control action $\mathbf{u}$ that minimizes the cost function $J(\mathbf{u})$, related to the error in the next period:

$$\begin{aligned} \min_{\mathbf{u} \in \mathbb{R}^2} J(\mathbf{u}) \\ \text{subject to } \mathbf{u} \in \mathcal{D}. \end{aligned} \quad (\text{F.11})$$

The solution is constrained to $\mathcal{D}$, which denotes the compact-convex hull that defines the boundaries for the control action $\mathbf{u}$. The convex set of vertices $\mathcal{V}$ represents the corners of the space vector modulation diagram, that is, the set of the switching vectors of the converter.

F.2 Frank-Wolfe algorithm applied to MPC

In this section, the projection-free FW algorithm is formulated for the efficient solution of the MPC problem defined in (F.11). The FW method is considered particularly suitable for real-time applications due to its low computational complexity and the avoidance of costly projection steps, especially when extended prediction horizons are employed.

First, the FW algorithm is analyzed in the context of quadratic cost functions and compact convex input sets, and a summary of standard offline convergence results is provided. Subsequently, the online implementation is addressed, and the behavior of warm-started FW iterations under time-varying objectives is characterized. Particular emphasis is placed on the use of exact line search, through which cost descent is ensured at each iteration and vanishing step sizes are produced as optimality is approached. These properties are utilized to establish the main result of this section: a closed-loop stability guarantee for the FW–MPC strategy, even when a single iteration is executed per control cycle.

F.2.1 Frank–Wolfe problem statement

Suppose $\mathcal{D} \subset \mathbb{R}^N$ is a convex and bounded set in a vector space and that $J(\mathbf{u}) : \mathcal{D} \rightarrow \mathbb{R}$ is a convex, differentiable and real-valued function. This is the case for the MPC of VSIs, as presented in the previous section. The optimization problem to be solved is given by (F.11).

The Frank–Wolfe Algorithm is a three-step iterative algorithm that solves this problem [248]. The algorithm starts from an initial guess $\mathbf{u}_i$ that is within the constrained region $\mathcal{D}$. At the end of each iteration i , it reaches a new approximation $\mathbf{u}_{i+1}$. The linear, supposedly simpler, sub-problem to be solved is defined in (F.12) [249]. This statement can be interpreted as: minimize the linearized form of the problem using the first-order Taylor approximation of J around $\mathbf{u}_i$ constrained to stay within $\mathcal{D}$.

$$\begin{aligned} \min_s \quad & s^\top \nabla J(\mathbf{u}_i) \\ \text{subject to } & s \in \mathcal{D}. \end{aligned} \tag{F.12}$$

With the proper formulation of the power converter control problem, the polytope $\mathcal{D} = \text{conv}(\mathcal{V})$ is defined as the convex hull of the space vector vertices, that is a convex compact set. For a convex compact N -dimensional polytope, the constraints can be expressed either in H-representation [250], [251], using a matrix inequality $\mathcal{D} : \mathbf{A}\mathbf{u} - \mathbf{b} \leq 0$, or in V-representation [251], as $\mathcal{V} = \{\mathbf{v}_1, \mathbf{v}_2, \dots, \mathbf{v}_n\}$ represents the set of vertices of $\mathcal{D}$.

This characterizes a compact convex linear problem. The fundamental theorem of linear programming (LP) guarantees that, at each step i , the solution $\mathbf{s}_i$ for the linearized problem $\mathbf{s}^\top \nabla J(\mathbf{u}_i)$ lies at some vertex $\mathbf{v}_j$ [252]. That is, the hyperplane which represents the first-order approximation of the function $J(\mathbf{u})$ around $\mathbf{u}_i$. This can be tested for each vertex $\mathbf{v}_j$ through exhaustive search, although some efficient linear minimization oracles (e.g. Simplex [248]) might speed up this search for higher number of vertices.

In the successive steps, the iterative solution $\mathbf{u}_{i+1}$ should walk towards $\mathbf{s}_i$ with the scalar step size $\alpha_i(\mathbf{s}_i - \mathbf{u}_i)$. Therefore, at the end of the iteration, it finds a new value $\mathbf{u}_{i+1} = \mathbf{u}_i + \alpha(\mathbf{s}_i - \mathbf{u}_i)$. Regarding the parameter α , there are a few alternatives for its selection, which all achieve a general convergence rate of $\mathcal{O}(1/i)$ [253], [254]. In offline settings, one can select α_i as $2/(i+2)$. Another possibility is to find the α value that minimizes J over the segment $(\mathbf{u}_i, \mathbf{s}_i)$:

$$\begin{aligned} \min_{\alpha} J(\mathbf{u}_i + \alpha(\mathbf{s}_i - \mathbf{u}_i)) \\ \text{subject to } \alpha \in [0, 1]. \end{aligned} \quad (\text{F.13})$$

That is, setting the gradient to zero, solving for α (F.14), and then limiting the solution to satisfy the bounds of α . This solution is preferred for online settings, where it leverages an improved convergence rate [255] and ensures that there will always be an improvement from a warm-start, which is an important assumption for system stability [256].

$$\nabla J(\mathbf{u}_i + \alpha(\mathbf{s}_i - \mathbf{u}_i))|_{\alpha} = 0. \quad (\text{F.14})$$

F.2.2 Frank–Wolfe for quadratic functions

Consider the following substitutions in (F.10):

$$\mathbf{Q} = \mathbf{H}^\top \mathbf{H} \quad \text{and} \quad \mathbf{C} = 2\mathbf{h}^\top \mathbf{H} \quad \text{and} \quad \mathbf{F} = \mathbf{h}^\top \mathbf{h}. \quad (\text{F.15})$$

Applying the FW algorithm, at each iteration i , the LP problem (F.16) is solved, where $s^\top \nabla J(\mathbf{u}_i)$ is the linearized first-order Taylor approximation of $J(\mathbf{u})$ around the initial point $\mathbf{u}_i$ using the auxiliary solution variable s . Here, s_i denotes the linear solution of the i th iteration, corresponding to a vertex $\mathbf{v}_j$ of the constraining polytope.

$$\begin{aligned} \min_s s^\top \nabla J(\mathbf{u}_i) &= (s_i - \mathbf{u}_i)^\top (2\mathbf{Q}^\top \mathbf{u}_i + \mathbf{C}^\top) \\ \text{subject to } s &\in \mathcal{D}. \end{aligned} \quad (\text{F.16})$$

Once again, the LP constrained sub-problem can then be solved by any linear optimization oracle or by simply evaluating the linearized function at each vertex $\mathbf{v}_j \in \mathcal{V}$ of the polytope and selecting the one that returns the smallest value. The initial guess of the next iteration $\mathbf{u}_{i+1}$ is then computed by moving from $\mathbf{u}_i$ to s_i with a step size proportional to α_i . The process is repeated until the maximum number of iterations $i_{\max}$ is reached. For quadratic functions, there exists a closed form for (F.13), so the exact line-search step-size α' is

$$\alpha'_i = \min \left\{ 1, \frac{-2\mathbf{q}_i^\top \mathbf{Q} \mathbf{u}_i - \mathbf{C} \mathbf{q}_i}{(2\mathbf{q}_i^\top \mathbf{Q} \mathbf{q}_i)} \right\}, \quad (\text{F.17})$$

where

$$\mathbf{q}_i = s_i - \mathbf{u}_i. \quad (\text{F.18})$$

The vanishing perturbation observed in the FW updates is consistent with recent studies on dynamic regret of online FW applications [255], which show that under exact line-search, the accumulated suboptimality decays under bounded variations on J_k . The asymptotic stability result also aligns with established findings in suboptimal MPC literature, where robust stability of the exact solution is maintained despite suboptimal control actions [256], [257].

The FW-M²PC (Algorithm 3) takes part in the innermost control loop of this setup.

Algorithm 3: FW-M²PC applied to a LCL-VSI

Pre-calculate matrices H and Q

for each switching period k :

Measure $u_{ab,k}, u_{bc,k}, i_{a,k}, i_{b,k}$

Transform to $u_{\alpha,k}, u_{\beta,k}, i_{\alpha,k}, i_{\beta,k}$

Synchronize to $\hat{i}_{\alpha,\text{ref},k+2}$ and $\hat{i}_{\beta,\text{ref},k+2}$ from outer loops

Estimate C_f **voltages** $\hat{u}_{\alpha,k+1}, \hat{u}_{\beta,k+1}$

Estimate L_1 **currents** $\hat{i}_{\alpha,k+1}, \hat{i}_{\beta,k+1}$

Mount vector h_{k+2}

Mount matrix C_{k+2}

 Let $i \leftarrow 0$

 Let u_0 be the last applied modulation index u_k .

$J_{\text{grad}} \leftarrow (2Q^T u_i + C^T)$

if ($J_{\text{grad},\alpha} \geq 0$), $\mathcal{V} \leftarrow \mathcal{V}^+$ **else**, $\mathcal{V} \leftarrow \mathcal{V}^-$

for each iteration $i = 0$ **to** $i_{\max}$:

$y_{\min} \leftarrow \infty$

for each vertex $v_j \in \mathcal{V}$:

$s^T \nabla f(u_i) \leftarrow (v_j - u_i)^T J_{\text{grad}}$

if ($s^T \nabla f(u_i) < y_{\min}$)

$s_i \leftarrow v_j$

end if

end for

$\alpha \leftarrow \frac{-2q^T Q u_i - C q}{(2q^T Q q)}$, constrained to $0 \leq \alpha \leq 1$.

$u_{i+1} \leftarrow u_i + \alpha(s_i - u_i)$.

end for

$u_k \leftarrow u_{k+1}$

$u_{k+1} \leftarrow u_{i_{\max}}$

end for

G Implementation setup

G.1 Introduction

Throughout this work, we have addressed energy generation from renewable sources, specifically solar and hydro power, considering an isolated system using photovoltaic panels and an induction generator. A control system was implemented to regulate the voltage and frequency of the SEIG through two approaches: computational simulation and experimental analysis.

For the simulation, the system was developed using Matlab®/Simulink, PSIM, and Typhoon HIL software, considering in some cases equivalent parameters identical to those of the experimental prototype. This appendix presents the characteristics and parameters of the prototype used to validate the theory developed in this work, along with additional information and relevant data.

G.2 Experimental testbed

The primary machine, with specifications shown in Table 19, is a three-phase induction motor powered by a WEG-CFW08 frequency inverter. The PM is mechanically coupled to another three-phase asynchronous machine, the SEIG, whose data are presented in Table 20.

Table 19: Nameplate Data of the Primary Machine.

Parameter	Value
Power	7.5 hp (5.5 kW)
Voltage	440/380/220 V
Frequency	60 Hz
Number of poles	4
Nominal speed	1745 rpm
Inrush current factor	7.7
Service factor	1.0
Efficiency	89.5%
Power factor	0.81
Operating speed	1840 rpm

Table 20: Additional Data of the Induction Generator.

Source	Parameter	Value
Nameplate data	Line voltage (RMS)	380/220 V
	Number of poles	4
	Inrush current factor	7.2
	Efficiency	87.5%
	Service factor	1.0
Test parameters	Rotor resistance	0.25 Ω
	Rotor inductive reactance	0.929 Ω
	Stator resistance	0.66 Ω
	Stator inductive reactance	0.929 Ω
	Mutual inductive reactance	24.2 Ω
	Inertia coefficient	0.034 kg·m ²

The frequency inverter is programmed to maintain the speed of the primary machine constant, above the synchronous speed of the SEIG, ensuring that the electrical frequency of the generator under nominal load does not deviate significantly from the nominal frequency (60 Hz) of the system. Moreover, to assist in the generator excitation, a delta-connected, three-phase excitation capacitor bank of 40 μF is used. This value was determined from the magnetization curve of the machine to obtain a nominal line voltage of 220 V when the SEIG is unloaded.

Voltage and frequency regulation, harmonic compensation, and PV power injection are performed by the proposed inverter, which experimentally uses a converter developed by Semikron. This converter features four half-bridges composed of IGBTs, gate drivers, and DC-link capacitors.

The designed control system is digitally implemented using a Texas Instruments Digital Signal Processor (DSP), model TMS32028335. This DSP interfaces with the analog inputs used for measuring system variables. Furthermore, digital outputs are used for controlling the frequency inverter and for connecting and disconnecting system loads.

Figure G.1 shows the experimental prototype used to obtain the test results. The key components of the generation system are identified in the images.



Figure G.1: Experimental setup

G.3 Hardware-in-the-loop testbench

All tests were performed on a Typhoon HIL 404 real-time simulator, Figure G.2, which executes the full LC-grid plant model with sub-microsecond precision. A physical DSP (TI TMS320F28379D) implements the FW-M²PC control law at $f_s = 10$ kHz. Lower f_s is also supported, provided the filter is redesigned accordingly to avoid excessive THD. At each T_s , line voltages $u_{g,ab}$, $u_{g,bc}$ in the filter capacitor C_f branch, as well as L_f currents i_a and i_b , are sampled by the analog-to-digital converters of the DSP and transformed to $\alpha\beta$ coordinates. The control input $u_{FW,k+1}$ is computed via $i_{\max}$ FW iterations and converted to PWM signals, which are sent back to the HIL via digital outputs.

The proposed C-HIL setup reproduces implementation delay, measurement noise, quantization, timing jitter, as well as synchronization dynamics and coordinate transformation that an actual inverter would present [258]. The real-time simulation results verify that the FW solution remains valid and convergent under realistic conditions and allows plant-robustness experiments, with parameter scaling, grid disturbances and impedance changes.

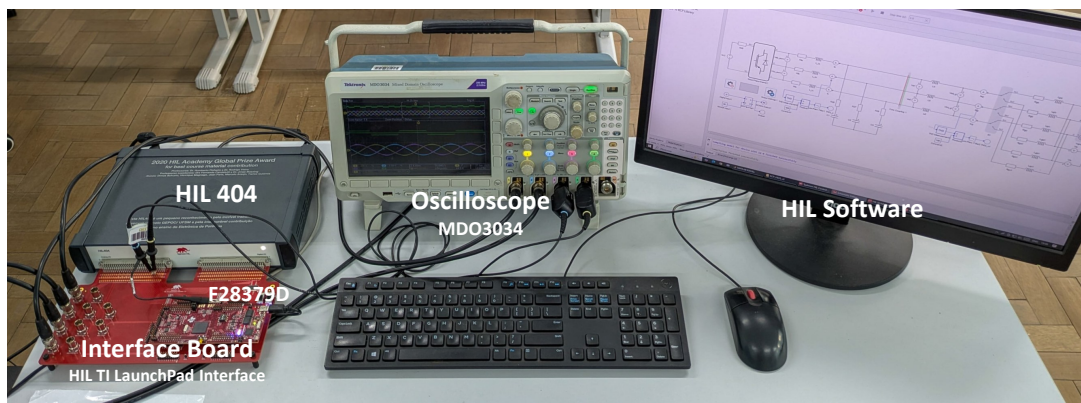


Figure G.2: Experimental setup photograph, including a TMS320F28379D DSP, a Typhoon HIL 404, and an official interface board

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