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Localization of Deformations in Finite Elastoplasticity

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In this paper the guidelines for constructing a geometrical model for the localization of deformations during elastic-plastic deformations are given. A geometrical object, namely the physical metric, is introduced to take into account the internal disarrangement during the plastic flow. A number of very general thermomechanical relations are obtained. Constitutive relations giving the conditions for the absence of localization phenomena are also obtained for the two different cases of decomposition of the total deformation gradient into the elastic and the plastic part (Lee, 1969; Nemat-Nasser, 1979).

1 Introduction

We will first introduce an experimentally observed phenomenon: the formation of shear bands in geotechnical structures due to ground motion (Peck and Terzaghi, 1984). It has been observed (Mühlhaus, 1991) that inside these bands the strain can become very large eventually leading to fracture. Then, for this kind of phenomenon the stability analysis and the search for the critical stress level at which the bands first appear, is of evident importance for many engineering purposes. The analysis of the band geometry appears to be significant too, also because it can be shown (Mühlhaus, 1991) that the onset of localization is often the point of inception of rupture.
Next we consider another very well known, at least from a practical viewpoint, problem, i.e. the formation of landslips due to the creation of capillar disarrangement on the backside (Peck and Terzaghi, 1984).
These two phenomena give examples of two rather different physical problems having a basic common feature: the presence of local inhomogeneities in the material distribution and deformations. Moreover, they both need a more refined geometrical setting than the usual one of classical continuum mechanics, aimed to better connect the phenomena acting at the microscale to the visible effects at the macroscopic level.[1] The presence of discontinuities generate in fact difficulties in applying the classical theory of finite elastoplasticity. In the next section we will introduce some problems connected with the application of the basic features of the classical theory to these kinds of phenomena.

2 The Intermediate Configuration in the Classical Theory of Finite Elastoplasticity

We are considering, as usual in classical continuum mechanics, a body $\mathbb{B}$ as a regular manifold, the elements of which are called *particles.* A configuration C of the body $\mathbb{B}$ is an embedding of $\mathbb{B}$ into the Euclidean 3-dimensional space $\mathbb{R}^3$. The motion of the continuum is defined as a 1-parameter family $\{C_t\}_{t\in\mathbb{R}}$ of successive configurations. One can construct a diffeomorphism $\chi_t \in Diff(C_0, C_t)$ which gives the deformation map of the body, assuming C_0 as initial *reference configuration.* We will consider as initial reference configuration C_0 the one assumed by the body in its undisturbed state (free of loads and strains) at uniform temperature. The general deformation of the body is given in Euclidean coordinates in $\mathbb{R}^3$ by the diffeomorphism χ_t, locally expressed as $\mathbf{x} = \mathbf{f}(\mathbf{X}, t)$. The deformation gradient is then $\nabla\mathbf{f}(\mathbf{X}, t) = \frac{\partial \mathbf{f}}{\partial \mathbf{X}}$.

Lee and Liu (Lee, 1969) introduced the notion of *released intermediate configuration,* by using the explicit decomposition

$$\nabla\mathbf{f} = \mathbf{F}^e\mathbf{F}^p \tag{1}$$

where $\mathbf{F}^e$ and $\mathbf{F}^p$ denote the elastic part of the gradient of deformation and the plastic one, respectively. This decomposition is unique up to a rotation of the intermediate configuration. The decomposition (1) introduces the idea of an intermediate configuration C_* which can be considered as the one associated to the body when the loads are removed and the temperature is reduced to the initial one, thus releasing

[1] For the problem of an adequate geometry needed to describe the underlying mechanisms, which might be responsible for the dissipative character of plastic deformations, see Aifantis (1999)

the thermoelastic strains. The final and the intermediate configuration are associated with the state of the body at each instant of time during the plastic flow, so that the transformations depend on time as well as on space variables. As one can see, as far as kinematics is concerned, the idea is that the plastic deformation considered from C_0 to C_* simply introduces a change in shape for the unstressed state. Then, the total deformation is considered as being obtained through thermoelastic strain from the plastically deformed configuration C_* (Lee, 1969).
In their work Lee and Liu point out a fundamental remark: "*For a body subject to a nonhomogeneous stress distribution which has caused plastic flow, removal of the loads and the temperature differences in general leaves a distribution of residual stress so that the unstressed state is not achieved at each element.*" This means that the intermediate configuration is not usually a possible one in a continuous body (destressing may require the body to be cut into infinitesimal elements) so that the mapping $\mathbf{F}^p$ may be discontinuous and even not one-to-one. Accordingly the tensor fields $\mathbf{F}^p$ and $\mathbf{F}^e$ are in general not given in terms of displacement derivatives, although the total deformation $\nabla\mathbf{f}$ is. In other words we can say that neither $\mathbf{F}^p$ nor $\mathbf{F}^e$ is separately integrable into a displacement function. We want to stress that in any case, due to the migration of dislocations associated to the plastic flow, discontinuous displacement fields occur in the study of the crystal lattice according to the physical theory of plasticity. Thus it happens that when a homogeneous stress distribution causes a macroscopically homogeneous plastic flow, the removal of the surface tractions leaves the body without residual stresses, so that the unstressed configuration appears to be a continuous one. However, at the crystal lattice level, lack of continuity occurs owing to the migration of dislocations due to plastic flow. This is an experimentally observed problem: macroscopically uniform material domains which develop localized, i.e. not uniform, deformations although subject to uniform surface tractions. This is one of the reasons why anelastic phenomena can be considered to be dissipative. Moreover, in the case of large deformations of inelastic solids, the inner coupling between the various microscopic processes involved becomes very influential.

3 A New Geometric Internal Variable

Our model is essentially based on the introduction of a non-Euclidean metric tensor characterizing the incompatibility of the intermediate configuration in the classical multiplicative decomposition of finite elastoplasticity. Then, this non-Euclidean metric tensor is introduced by means of its gradient as an internal variable in the state space of non equilibrium processes with a material-dependent rate equation. In our case, as a standard method of the thermodynamical theory with internal variables, the state space is enlarged to describe microscopic instabilities inducing plastic behaviour, so obtaining constitutive equations of dissipative character (Muschik and Maugin, 1994). The physical meaning of this new variable $\mathbf{g}$, which appears among the internal variables, has been extensively discussed in previous literature (Valanis, 1995; Valanis et al., 2001) and in a forthcoming paper (Francaviglia and Ciancio, 2001). It has to do with an average tensor characterization of local deviations from the Euclidean structure associated with thermomechanical effects on the mesoscopic scale (atomic, molecular or grain level).
According to the classical theory, in the initial configuration C_0, free of loads and strains at uniform temperature, the squared mutual distance between points is given by the Euclidean expression

$$ds_0^2 = (d\mathbf{X})^T d\mathbf{X} = \delta_{LM} dX^L dX^{,} \tag{2}$$

After the deformation $\mathbf{f}$, the distance is expressed by

$$ds_0^2 = (d\mathbf{x})^T \mathbf{C}_E d\mathbf{x} = C_{ij} dx^i dx^j \tag{3}$$

where the Euclidean right Cauchy-Green tensor given by

$$\mathbf{C}_E = (\nabla\mathbf{f})^T \nabla\mathbf{f} \tag{4}$$

has been introduced. In our model a purely microscopic deformation characterized by a suitably thermodynamically induced variation of the metric from Euclidean into non-Euclidean (Valanis, 1995; Valanis et al., 2001) is superposed upon the macroscopic elastic-plastic one. The deformation is macroscopically given by the identity map $\mathbf{i}$ so that the positions of points do not change. Accordingly, in our model the mutual distance between points is given by an expression formally analogous to (4) where the new expression for the right Cauchy-Green tensor involves now the non-Euclidean metric $\mathbf{g}$, namely

$$\mathbf{C} = (\nabla\mathbf{f})^T \mathbf{g} \nabla\mathbf{f} \tag{5}$$

By using the classical multiplicative decomposition for the macroscopic deformation gradient

$$\nabla \mathbf{f} = \mathbf{F}^e \mathbf{F}^p \tag{6}$$

one has then

$$\mathbf{C} = (\mathbf{F}^p)^T (\mathbf{F}^e)^T \mathbf{g} \mathbf{F}^e \mathbf{F}^p. \tag{7}$$

Now we set

$$(\mathbf{F}^e)^T \mathbf{g} \mathbf{F}^e = \mathbf{H}^T \mathbf{H} = \tilde{\mathbf{C}}, \tag{8}$$

where $\mathbf{H}$ is any regular matrix such that $\mathbf{F}^e \mathbf{H}^{-1}$ diagonalizes $\mathbf{g}$, i.e.

$$(\mathbf{F}^e \mathbf{H}^{-1})^T \mathbf{g} (\mathbf{F}^e \mathbf{H}^{-1}) = \mathbf{1};$$

this defines a new tensor $\tilde{\mathbf{C}}$ which will be later used as an independent variable. The expression for the right Cauchy-Green tensor takes now the form

$$\mathbf{C} = (\mathbf{H}\mathbf{F}^p)^T (\mathbf{H}\mathbf{F}^p) = (\mathbf{F}^p)^T \tilde{\mathbf{C}} \mathbf{F}^p, \tag{9}$$

which resembles the Euclidean expression (4) provided $\nabla \mathbf{f}$ is replaced by the now *total deformation gradient* (the microscopic plus the macroscopic one) given by $\mathbf{H}\mathbf{F}^p$.

We remark that the above decomposition is introduced only with the purpose of distinguishing among different contributions to the anelastic phenomena, and, of course, the intermediate configurations introduced are not really attained by the body during the motion. Moreover in particular we want to distinguish between microscopic phenomena (more geometrical) and macroscopic ones (more phenomenological). The definition (8), together with (33), has three basic features:

- A physical background for the introduction of the variation of the non-Euclidean metric as an internal variable in the state space of non-equilibrium processes, which is not in contrast to the usual features of the macroscopic theory of continuum mechanics.

- A mathematical justification of the physically based deductions of Lee and Liu (Lee, 1969) on the non-integrability of the elastic and plastic part of the total gradient of deformation.

- The agreeement with the theory of structured deformations of Owen and Del Piero (1993) (where our quantity $\mathbf{H}$ offers a possible example of structured deformation).

4 Energetic Considerations for the Model Applied to the Decomposition $\nabla \mathbf{f} = \mathbf{F}^e \mathbf{F}^p$

To describe the process we adopt now as state variables the "elastic" Cauchy-Green tensor $\tilde{\mathbf{C}}$, the plastic part $\mathbf{F}^p$ of the classical deformation gradient, the gradient of the non-Euclidean metric tensor $\mathbf{g}$ considered as an internal variable, and the absolute temperature θ. Then the free energy Ψ is given as a functional of the following type

$$\Psi = \Psi(\tilde{\mathbf{C}}, \mathbf{F}^p, \nabla \mathbf{g}, \theta). \tag{10}$$

Notice that Ψ depends on $\mathbf{g}$ through $\tilde{\mathbf{C}}$. By derivation with respect to time we obtain

$$\dot{\Psi} = \tilde{\mathbf{T}} \cdot \dot{\tilde{\mathbf{C}}} + \mathbf{A} \cdot \dot{\mathbf{F}^p} + \mathbf{B} \cdot \dot{(\nabla \mathbf{g})} - \tilde{s}\dot{\theta} \tag{11}$$

where we set

$$\tilde{\mathbf{T}} = \frac{\partial \Psi}{\partial \tilde{\mathbf{C}}}, \mathbf{A} = \frac{\partial \Psi}{\partial \mathbf{F}^p}, \mathbf{B} = \frac{\partial \Psi}{\partial (\nabla \mathbf{g})}, \tilde{s} = -\frac{\partial \Psi}{\partial \theta}. \tag{12}$$

By using a simple vectorial relation we can write

$$\dot{\Psi} = \tilde{\mathbf{T}} \cdot \dot{\tilde{\mathbf{C}}} + \mathbf{A} \cdot (\dot{\mathbf{F}^p}) - \tilde{s}\dot{\theta} + \nabla \cdot (\mathbf{B}\dot{\mathbf{g}}) - \nabla \mathbf{B} \cdot \dot{\mathbf{g}}. \tag{13}$$

For the velocity gradient $\mathbf{L}$ one has

$$\mathbf{L} = (\dot{\nabla \mathbf{f}})\nabla \mathbf{f}^{-1} \tag{14}$$

which, together with the multiplicative decomposition of the deformation gradient (1), gives the following relation for the stress power

$$\begin{aligned} \mathbf{T}\cdot\mathbf{L} &= \mathbf{T}\cdot[(\dot{\mathbf{F}}^{e})\mathbf{F}^{p}\nabla\mathbf{f}^{-1}] + \mathbf{T}\cdot[\mathbf{F}^{e}(\dot{\mathbf{F}}^{p})\nabla\mathbf{f}^{-1}] = \\ &= [\mathbf{T}(\mathbf{F}^{e})^{-T}]\cdot(\dot{\mathbf{F}}^{e}) + [(\mathbf{F}^{e})^{T}\mathbf{T}(\nabla\mathbf{f})^{-T}]\cdot(\dot{\mathbf{F}}^{p}), \end{aligned} \tag{15}$$

where the usual rules for the tensor product have been used. By multiplying the whole relation by the Jacobian $J = det(\nabla\mathbf{f}) \geq 0$ one obtains

$$J\mathbf{T}\cdot\mathbf{L} = \mathbf{T}^{e}_{R}\cdot(\dot{\mathbf{F}}^{e}) + [(\mathbf{F}^{e})^{T}\mathbf{T}_{R}]\cdot(\dot{\mathbf{F}}^{p}) \tag{16}$$

where $\mathbf{T}_R = J\mathbf{T}(\nabla\mathbf{f})^{-T}$ is the first Piola-Kirchoff stress tensor, and the definition

$$\mathbf{T}^{e}_{R} = J\mathbf{T}(\mathbf{F}^{e})^{-T} \tag{17}$$

for what will be called in the following the "elastic" first Piola-Kirchoff stress tensor is made. The entropy function s satisfies the dissipation inequality

$$\theta\dot{s} + \nabla\cdot(\theta\mathbf{J}_s) - \mathbf{J}_s\cdot\nabla\theta \geq 0 \tag{18}$$

where $\mathbf{J}_s$ is the entropy flux density. By using the Legendre transformation as in [?] $\Psi = e - \theta s$ together with relation (18) and the balance law for the internal energy e

$$\dot{e} = \mathbf{T}\cdot\mathbf{L} + h, \tag{19}$$

with $h = -\nabla\cdot\mathbf{q}$, we obtain the Clausius-Duhem inequality in the form

$$-(\dot{\Psi} + s\dot{\theta}) + \mathbf{T}\cdot\mathbf{L} - \mathbf{J}_s\cdot\nabla\theta + \nabla\cdot(\theta\mathbf{J}_s) + h \geq 0. \tag{20}$$

With respect to the classical prescription for the entropy flux density $\mathbf{J}_s$, a different relation with the heat flux density $\mathbf{q}$ is now given following Müller and Maugin, by setting

$$\mathbf{J}_s = \frac{\mathbf{q}}{\theta} + \mathbf{k} \tag{21}$$

which includes a phenomenological coefficient $\mathbf{k}$ to be determined in the sequel (see Müller (1985) and Maugin (1992)). By replacing (21) in (20), the following relation is obtained

$$-(\dot{\Psi} + s\dot{\theta}) + \mathbf{T}\cdot\mathbf{L} - \mathbf{J}_s\cdot\nabla\theta + \nabla\cdot(\theta\mathbf{k}) \geq 0. \tag{22}$$

By substituting relation (13) together with (16) in the Clausius-Duhem inequality (20), one obtains

$$\begin{aligned} &[(\mathbf{F}^{e})^{T}\mathbf{T}_{R} - J\mathbf{A}]\cdot\dot{\mathbf{F}}^{p} + \mathbf{T}^{e}_{R}\cdot\dot{\mathbf{F}}^{e} - J\tilde{\mathbf{T}}\cdot\dot{\tilde{\mathbf{C}}} + \\ &+J(\tilde{s} - s)\dot{\theta} - J\mathbf{J}_s\cdot\nabla\theta + J\nabla\mathbf{B}\cdot\dot{\mathbf{g}} + J\nabla\cdot(\theta\mathbf{k} - \mathbf{B}\dot{\mathbf{g}}) \geq 0. \end{aligned} \tag{23}$$

After differentiating the "elastic" right Cauchy-Green tensor $\tilde{\mathbf{C}}$, the quantity $\tilde{\mathbf{T}}\cdot\dot{\tilde{\mathbf{C}}}$ appearing in (23) takes the form

$$\tilde{\mathbf{T}}\cdot\dot{\tilde{\mathbf{C}}} = [\tilde{\mathbf{T}}(\mathbf{F}^{e})^{T}\mathbf{g} + \mathbf{g}^{T}\mathbf{F}^{e}\tilde{\mathbf{T}}]\cdot(\dot{\mathbf{F}}^{e}) + [\mathbf{F}^{e}\tilde{\mathbf{T}}(\mathbf{F}^{e})^{T}]\cdot\dot{\mathbf{g}} \tag{24}$$

where the identity $\mathbf{A}^{T}\cdot\mathbf{B} = \mathbf{A}\cdot\mathbf{B}^{T}$ together with the usual algebraical rules are used. Inequality (23) together with (24) gives finally

$$\begin{aligned} &[(\mathbf{F}^{e})^{T}\mathbf{T}_{R} - J\mathbf{A}]\cdot(\dot{\mathbf{F}}^{p}) + \{\mathbf{T}^{e}_{R} - J[\tilde{\mathbf{T}}(\mathbf{F}^{e})^{T}\mathbf{g} + \mathbf{g}^{T}\mathbf{F}^{e}\tilde{\mathbf{T}}]\}\cdot(\dot{\mathbf{F}}^{e}) + \\ &+J[\nabla\mathbf{B} - \mathbf{F}^{e}\tilde{\mathbf{T}}(\mathbf{F}^{e})^{T}]\cdot\dot{\mathbf{g}} + J\nabla\cdot(\theta\mathbf{k} - \mathbf{B}\dot{\mathbf{g}}) + \\ &+J(\tilde{s} - s)\dot{\theta} - J\mathbf{J}_{S}\cdot\nabla\theta \geq 0. \end{aligned} \tag{25}$$

We assume now that the entropy s does not depend on $\dot{\theta}$, the coefficient of $(\dot{\mathbf{F}^e})$ in (25) is assumed to not depend on $(\dot{\mathbf{F}^e})$ itself while the remaining coefficients in (25) may in general depend on the corresponding variables. Since (25) cannot change sign, the argument of Maugin (Maugin, 1987) also applies to this case and we find

$$s = \tilde{s} = -\frac{\partial \Psi}{\partial \theta} \tag{26}$$

and

$$\mathbf{T}^e_R = J[(\tilde{\mathbf{T}}(\mathbf{F}^e)^T\mathbf{g} - \mathbf{g}^T\mathbf{F}^e\tilde{\mathbf{T}})]. \tag{27}$$

Moreover, following again an earlier argument of Maugin (1987), it is reasonable to choose the following value for the phenomenological coefficient $\mathbf{k}$

$$\mathbf{k} = \frac{\mathbf{B}}{\theta}\dot{\mathbf{g}} \tag{28}$$

so that at the end we have the following dissipation inequality

$$\begin{aligned}&[(\mathbf{F}^e)^T\mathbf{T}_R - J\mathbf{A}]\cdot(\dot{\mathbf{F}^p}) + \\ &+J[\nabla\mathbf{B} - \mathbf{F}^e\tilde{\mathbf{T}}(\mathbf{F}^e)^T]\cdot\dot{\mathbf{g}} - J\mathbf{J}_s\cdot\nabla\theta \geq 0\end{aligned} \tag{29}$$

where the three contributions due to the plastic part of the classical deformation $\mathbf{F}^p$, the non-Euclidean metric tensor $\mathbf{g}$ and the heat flow are recognizable. We remark that the localization of deformation depends on the gradient $\nabla\mathbf{F}^p$ (with this we mean that a spatially homogeneous $\mathbf{F}^p$ does not produce effects of localization). Using the standard rules for total time derivatives $\dot{\mathbf{F}}^p = \frac{\partial \mathbf{F}^p}{\partial t} + \frac{\partial \mathbf{F}^p}{\partial x^i}\dot{x}^i$ we see that $(\mathbf{F}^e)^T\mathbf{T}_R - J\mathbf{A}$ is the coefficient of $\nabla\mathbf{F}^p$ in Eq. (29). Then the non localization condition gives the following constitutive equation

$$\mathbf{T} = (\mathbf{F}^e)^{-T}\frac{\partial \Psi}{\partial \mathbf{F}^p}(\nabla\mathbf{f})^T \tag{30}$$

where the definition of the first Piola-Kirchoff stress tensor and the assuption $(12)_2$ have been used, together with $\mathbf{T}_R = J\mathbf{T}(\nabla\mathbf{f})^{-T}$.

5 The Model Applied to the Decomposition $\nabla\mathbf{f} = \mathbf{F}^p\mathbf{F}^e$

By using the alternative decomposition

$$\nabla\mathbf{f} = \mathbf{F}^p\mathbf{F}^e \tag{31}$$

(see Nemat-Nasser (1979), Francaviglia and Dolfin (2000)) relation (4) is replaced by

$$\mathbf{C} = (\mathbf{F}^e)^T(\mathbf{F}^p)^T\mathbf{g}\mathbf{F}^p\mathbf{F}^e. \tag{32}$$

As for (8) we choose a regular matrix $\mathbf{K}$ such that

$$(\mathbf{F}^p)^T\mathbf{g}\mathbf{F}^p = \mathbf{K}^T\mathbf{K} = \hat{\mathbf{C}} \tag{33}$$

so that the corresponding expression for the right Cauchy-Green tensor takes now the form

$$\mathbf{C} = (\mathbf{K}\mathbf{F}^e)^T(\mathbf{K}\mathbf{F}^e) = (\mathbf{F}^e)^T\hat{\mathbf{C}}\mathbf{F}^e. \tag{34}$$

With the position (33) the metric in the actual configuration is again in the Euclidean form with the total deformation gradient given by $\mathbf{K}\mathbf{F}^e$. With respect to the energetic considerations, in this second case we will not deal explicitly with all the passages as in the previous section but we will give only the basic results. The free energy Ψ is supposed, in this case, to be a functional of the following type

$$\Psi = \Psi(\hat{\mathbf{C}}, \mathbf{F}^e, \nabla\mathbf{g}, \theta) \tag{35}$$

and the stress power, multiplied by the Jacobian, is given by

$$J\mathbf{T}\cdot\mathbf{L} = \mathbf{T}^p_R\cdot(\dot{\mathbf{F}^p}) + [(\mathbf{F}^p)^T\mathbf{T}_R]\cdot(\dot{\mathbf{F}^e}), \tag{36}$$

where

$$\mathbf{T}^p_R = J\mathbf{T}(\mathbf{F}^p)^{-T} \tag{37}$$

(in full analogy to the previous section) is the first "plastic" Piola-Kirchoff stress tensor. Clausius-Duhem inequality takes now the form

$$\begin{aligned}&[\mathbf{T}^p_R - J(\hat{\mathbf{T}}(\mathbf{F}^p)^T\mathbf{g} + \mathbf{g}^T\mathbf{F}^p\hat{\mathbf{T}})]\cdot(\dot{\mathbf{F}^p}) + [(\mathbf{F}^p)^T\mathbf{T}_R - J\hat{\mathbf{A}}]\cdot(\dot{\mathbf{F}^e}) + \\ &+J[\nabla\mathbf{B} - \mathbf{F}^p\hat{\mathbf{T}}(\mathbf{F}^p)^T]\cdot\dot{\mathbf{g}} - JJ_S\cdot\nabla\theta \geq 0\end{aligned} \tag{38}$$

where definitions analogous to those giving rise to (12) have been made, i.e.

$$\hat{\mathbf{T}} = \frac{\partial\Psi}{\partial\hat{\mathbf{C}}}, \hat{\mathbf{A}} = \frac{\partial\Psi}{\partial\mathbf{F}^e}, \mathbf{B} = \frac{\partial\Psi}{\partial(\nabla\mathbf{g})}, \tilde{s} = -\frac{\partial\Psi}{\partial\theta}. \tag{39}$$

In this case the dissipation inequality reads

$$\begin{aligned}&[\mathbf{T}^p_R - J(\hat{\mathbf{T}}(\mathbf{F}^p)^T\mathbf{g} + \mathbf{g}^T\mathbf{F}^p\hat{\mathbf{T}})]\cdot(\dot{\mathbf{F}^p}) + \\ &J[\nabla\mathbf{B} - \mathbf{F}^e\hat{\mathbf{T}}(\mathbf{F}^e)^T]\cdot\dot{\mathbf{g}} - J\mathbf{J}_S\cdot\nabla\theta \geq 0\end{aligned} \tag{40}$$

and the condition for which no localization occurs is given by

$$\mathbf{T} = \frac{\partial\Psi}{\partial\hat{C}}(\mathbf{F}^p)^T\mathbf{g} - \mathbf{g}^T\mathbf{F}^p\frac{\partial\Psi}{\partial\hat{C}}(\mathbf{F}^p)^T. \tag{41}$$

So, in this second case of decomposition $\nabla\mathbf{f} = \mathbf{F}^p\mathbf{F}^e$, the constitutive relation giving the condition of no localization involves only the plastic part of the deformation gradient together with the non-Euclidean metric (internal variable) $\mathbf{g}$.

6 Conclusions

In this paper we have investigated the role played by the new mesoscopic variable $\mathbf{g}$ (a non-Euclidean metric) assumed as an internal variable in finite elastoplasticity, both in Lee's hypothesis of decomposition $\nabla\mathbf{f} = \mathbf{F}^e\mathbf{F}^p$ and in Nemat-Nasser's alternative decomposition $\nabla\mathbf{f} = \mathbf{F}^p\mathbf{F}^e$. We were not interested in the evolution equations for the internal variables $\mathbf{g}, \mathbf{F}^e, \mathbf{F}^p$ (and their gradients) but simply in the contributions of these internal variables to Clausius-Duhem inequality and to the extra entropy flux. We have seen that in the two cases the effects of localization (i.e. those related to the inhomogeneities of the gradient of plastic deformation) depend on suitable constitutive equations, i.e. (30) and (41). The two cases are rather different, since in the case of Lee's decomposition the constitutive equation does not depend on the internal variable $\mathbf{g}$, while in the opposite case the constitutive equation is more complicated and depends on $\mathbf{g}$ and only on the plastic part of the deformation gradient. Moreover looking at the dissipation inequalities (29) and (40) for the two different decomposition we can see that in the second case the elastic deformation is not at all involved in the plastic one.

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