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Evolution Criterion in the Coupled Fields Theory

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Dedicated to Prof. Wolfgang Muschik on his 65-th Anniversary.

The evolution criterion, as formulated by Glansdorff and Prigogine, allows to find the kinetic potential describing a dissipative process. This paper deals with the construction of the kinetic potential for micropolar media embedded in the dielectric fluid and subjected to the coupled field interactions. We extend the results derived by Muschik and Papenfuß in Muschik and Papenfuss (1993) for the liquid crystals immersed in the perfect fluid, playing there the role of the thermostat. Generalization deals with the dissipative coupled mechanical, thermal, and electromagnetic processes occurring in the micropolar medium. The description of the micropolar medium is based on the approach of Eringen (Eringen, 1999). The formulated criterion is applied to obtain the equilibrium conditions for micropolar elastic solids, micropolar electromagnetic fluids, and liquid crystals.

1 Introduction

The tendency to unify the description of various important processes in one physically, mathematically, methodologically, and logically correct system has been observed in science for many years. One of such branches of physics, where the attempts toward the unification are systematically done, is thermodynamics of irreversible processes.

Big efforts are done to formulate the theories describing dissipation in frames of a variational principle. The most important physical version of a variational principle is the Hamilton principle, because there the full information about the system is included in one function (Anthony, 1986). One can find a short survey of variational principles e.g., in Muschik and Trostel (1983).

A weaker form, nevertheless also very difficult in practice, is the evolution criterion formulated by Glansdorff and Prigogine (1971). Since that time the possibility of the occurrence of a given physical process is a problem which has been studied by many authors. A more modern approach, taking into account the recent achievements of the non-equilibrium thermodynamics of the irreversible processes, was given in Muschik and Papenfuss (1993).

We start from the energy conservation law and construct a functional, which models the physical process. The extremum of that functional gives the condition for the equilibrium state.

In a previous paper (Radzikowska et al., 2001) we have constructed the evolution criterion for the materials immersed in vacuum and interacting with the external electromagnetic fields emitted by the perfect blackbody. These interactions disturb the thermal, electromagnetic, and mechanic fields in the micropolar body. The evolution criterion formulated in the global form for the non-equilibrium processes occurring in the micropolar body, were applied to obtain the equilibrium conditions for liquid crystals.

In the present paper we develop the approach of Muschik and Papenfuss (1993) in order to consider the electromagnetic material environment with constant temperature T^*. As examples of the application of the proposed methodology, we show how it works for the micropolar elastic solids, micropolar fluids, and liquid crystals immersed in the dielectric liquid and interacting with the electromagnetic field.

2 Basic Thermodynamic Principles

Let $\mathcal{V}$ be an open region of the Euclidean three-dimensional space E^3 occupied by the material micropolar body at time t and sweeping by the discontinuity surface Σ. The energy conservation law (the first principle of thermodynamics) for the considered body subjected to the electromagnetic interactions can be formulated in the following global form (Eringen and Maugin, 1990a,b; Eringen, 1999)

$$\begin{aligned}&\frac{d}{dt}\int\limits_{\mathcal{V}-\Sigma}[\rho\,(\epsilon+{}_{\mathrm{K}}\epsilon)+{}_{\mathrm{M}}\epsilon]\,dv=\\&\int\limits_{\partial\mathcal{V}-\Sigma}\mathbf{n}\cdot[(\mathbf{t}+{}_{\mathrm{M}}\mathbf{t}+\mathbf{v}\otimes\mathbf{G})\cdot\mathbf{v}+\mathbf{m}\cdot\boldsymbol{\omega}+\mathbf{q}-\mathcal{S}]\,da+\\&\int\limits_{\mathcal{V}-\Sigma}\rho(\tilde{\mathbf{f}}\cdot\mathbf{v}+\tilde{\mathbf{l}}\cdot\boldsymbol{\omega}+\tilde{h})\,dv\,.\end{aligned}\tag{1}$$

We have here: the mass density $\rho(\mathbf{x},t)$, the internal energy density $\epsilon(\mathbf{x},t)$, the kinetic energy density ${}_{\mathrm{K}}\epsilon$ per the unit of mass,

$${}_{\mathrm{K}}\epsilon\doteq\frac{1}{2}(\mathbf{v}\cdot\mathbf{v}+\boldsymbol{\omega}\cdot\mathbf{j}\cdot\boldsymbol{\omega})\,,\tag{2}$$

the material velocity $\mathbf{v}(\mathbf{x},t)$, the angular velocity of the director $\boldsymbol{\omega}(\mathbf{x},t)$, the microinertia tensor $\mathbf{j}(\mathbf{x},t)$, the energy of the "free" electromagnetic field ${}_{\mathrm{M}}\epsilon$ (see Eringen and Maugin, 1990a,b),

$${}_{\mathrm{M}}\epsilon\doteq\frac{1}{2}(\varepsilon_o\,\mathbf{E}\cdot\mathbf{E}+\frac{1}{\mu_o}\mathbf{B}\cdot\mathbf{B}),\tag{3}$$

the electric field strength $\mathbf{E}(\mathbf{x},t)$, the magnetic induction $\mathbf{B}(\mathbf{x},t)$, the exterior to the surface $\partial\mathcal{V}-\Sigma$ unit normal vector $\mathbf{n}$, the stress tensor $\mathbf{t}(\mathbf{x},t)$, the electromagnetic stress tensor ${}_{\mathrm{M}}\mathbf{t}$ (Eringen and Maugin, 1990a,b),

$${}_{\mathrm{M}}\mathbf{t}\doteq\mathbf{P}\otimes\mathcal{E}-\mathbf{B}\otimes\mathcal{M}+\varepsilon_o\,\mathbf{E}\otimes\mathbf{E}+\frac{1}{\mu_o}\mathbf{B}\otimes\mathbf{B}-({}_{\mathrm{M}}\epsilon-\mathcal{M}\cdot\mathbf{B})\,\mathbf{I}\,,\tag{4}$$

the polarization $\mathbf{P}$, the magnetization $\mathbf{M}$,

$$\begin{aligned}&\mathbf{P}\doteq\mathbf{D}-\varepsilon_o\,\mathbf{E}\,,\qquad\mathbf{M}\doteq\frac{1}{\mu_o}\mathbf{B}-\mathbf{H}\,,\\&\mathcal{E}\doteq\mathbf{E}+\mathbf{v}\times\mathbf{B}\,,\quad\mathcal{M}\doteq\mathbf{M}+\mathbf{v}\times\mathbf{P}\,,\end{aligned}\tag{5}$$

the electromagnetic momentum $\mathbf{G}$, the couple stress tensor $\mathbf{m}(\mathbf{x},t)$, the heat vector $\mathbf{h}$, the Poynting vector $\mathcal{S}$,

$$\mathbf{G}\doteq\varepsilon_o\,\mathbf{E}\times\mathbf{B}\,,\qquad\mathcal{S}\doteq\mathcal{E}\times\mathcal{H}\,,\qquad\mathcal{H}\doteq\mathbf{H}-\mathbf{v}\times\mathbf{D}\,,\tag{6}$$

the mechanical body force density $\tilde{\mathbf{f}}$, the body couple density $\tilde{\mathbf{l}}$ and the heat energy source $\tilde{h}$. Let us note that $\mathcal{E}$, $\mathcal{H}$, $\mathcal{M}$, $\mathcal{S}$ are the electric field strength, the magnetic field strength, the magnetization, and the Poynting vector, respectively, written in the coordinate system co-moving with the particle.

The volume and the surface integrals in (1) do not include the surface and the line intersections of a discontinuity surface Σ, which may sweep the body with its own velocity $\mathbf{u}$, within the bulk of the body, where

$$\mathcal{V}-\Sigma\doteq\mathcal{V}-\mathcal{V}\cap\Sigma\,,\qquad\partial\mathcal{V}-\Sigma\doteq\partial\mathcal{V}-\partial\mathcal{V}\cap\Sigma\,.\tag{7}$$

Usually the first law of thermodynamics states, that the sum of the time rates of the internal and the kinetic energies equals to the work done by the internal and the external forces acting on the bulk and/or on the surface of the body. In our formulation we include in the first term the contribution from the

electromagnetic energy, because we need that for the final formulation of our evolution criterion. That energy can be introduced by the source terms on the r.h.s. of the expression (1) (Eringen and Maugin, 1990a,b).

The second law of thermodynamics can be written in the classical global form

$$\frac{d}{dt}\int\limits_{\mathcal{V}-\Sigma}\rho\eta\,dv-\oint\limits_{\partial\mathcal{V}-\Sigma}\mathbf{n}\cdot\mathbf{s}\,da-\int\limits_{\mathcal{V}-\Sigma}\rho\tilde{b}\,dv\geq 0\,, \tag{8}$$

where η is the entropy density, $\mathbf{s}$ - entropy flux vector and $\tilde{b}$ - entropy supply.

The thermodynamic laws (1) and (8) have the following local forms

- local energy conservation law

$$\rho\dot{\epsilon}-\mathbf{t}^T:\mathbf{a}-\mathbf{m}:\mathbf{b}-\nabla\cdot\mathbf{q}-{}_{\mathrm{M}}h-\rho\tilde{h}=0\,, \tag{9}$$

with the jump condition on Σ

$$\mathbf{n}\cdot[\![\rho(\epsilon+{}_{\mathrm{K}}\epsilon+{}_{\mathrm{M}}\epsilon)(\mathbf{v}-\mathbf{u})-(\mathbf{t}+{}_{\mathrm{M}}\mathbf{t}+\mathbf{u}\otimes\mathbf{G})\cdot\mathbf{v}-\mathbf{m}\cdot\boldsymbol{\omega}-(\mathbf{q}-\mathcal{S})]\!]=0\,, \tag{10}$$

and where the electromagnetic energy source ${}_{\mathrm{M}}h$ is by definition

$${}_{\mathrm{M}}h\doteq\mathcal{E}\cdot[\mathcal{J}+\dot{\mathbf{P}}+\mathbf{P}(\nabla\cdot\mathbf{v})]-\mathcal{M}\cdot\dot{\mathbf{B}}-(\mathbf{P}\times\mathcal{E}+\mathcal{M}\times\mathbf{B})\cdot\boldsymbol{\omega}\,, \tag{11}$$

- local entropy inequality

$$\rho\dot{\eta}-\boldsymbol{\nabla}\cdot\mathbf{s}-\rho\tilde{b}\geq 0\,, \tag{12}$$

with the jump condition on Σ

$$\mathbf{n}\cdot[\![\rho\eta(\mathbf{v}-\mathbf{u})-\mathbf{s}]\!]\geq 0\,. \tag{13}$$

Here $\mathbf{a}$ and $\mathbf{b}$ are the tensorial deformation rates,

$$\boldsymbol{a}\doteq\boldsymbol{\nabla}\otimes\mathbf{v}-\boldsymbol{\varepsilon}\cdot\boldsymbol{\omega}\,,\qquad\mathbf{b}\doteq(\boldsymbol{\nabla}\otimes\boldsymbol{\omega})^T\,, \tag{14}$$

$\boldsymbol{\varepsilon}$ is the Ricci pseudotensor, $\mathcal{J}$ is the electric current density vector written in the co-moving coordinate system

$$\mathcal{J}\doteq\mathbf{J}-\tilde{q}_f\,\mathbf{v}\,, \tag{15}$$

where $\mathbf{J}$ is the electric current density vector and $\tilde{q}_f$ is the free electric charge distribution.

3 Description of the Problem

The two thermodynamic principles (1) and (8) are satisfied for an arbitrary material volume $\mathcal{V}$ sweeping by the discontinuity surface Σ. Let us consider the discrete system $\mathcal{G}(t)$ at the instant t, separated from the environment $\mathcal{G}^*(t)$ by the border surface $\partial\mathcal{G}(t)$ (Muschik and Papenfuss, 1993) (now $\mathcal{G}(t)$ is used instead of $\mathcal{V}$). The surface $\partial\mathcal{G}(t)$ with its positive unite normal vector $\mathbf{n}(\mathbf{x},t)$ moves with the velocity $\mathbf{u}(\mathbf{x},t)$. On the surface $\partial\mathcal{G}(t)$ the surface free electric charge distribution $\tilde{\omega}_f(\mathbf{x},t)$ and the surface electric current density $\tilde{\mathcal{K}}(\mathbf{x},t)$ are present,

$$\tilde{\mathcal{K}}\doteq\tilde{\mathbf{K}}-\tilde{\omega}_f\,\mathbf{u}\,, \tag{16}$$

where $\tilde{\mathbf{K}}$ is the surface electric current density vector. From now, the jump conditions (10) and (13) become the boundary conditions for our problem on $\partial\mathcal{G}(t)$. The vicinity $\mathcal{G}^*(t)$ of the system $\mathcal{G}(t)$ is presupposed to be a dielectric perfect fluid reservoir of constant thermostatic temperature T^* controlling the system during its evolution to equilibrium. The system is immersed in the electromagnetic fields $\mathbf{E}^*$ and $\mathbf{B}^*$ interacting with internal fields of the body. The equilibrium state of vicinity is determined by the following constitutive relations (Eringen and Maugin, 1990b)

$$\begin{aligned}
&\mathbf{t}^* = -\pi^* \mathbf{I}\,,\\
&\pi^* = \pi_o^* + \tfrac{1}{2}\mathbf{P}^* \cdot \mathbf{E}^*\,,\\
&\mathbf{P}^* = \chi^* \mathbf{E}^*\,,\\
&\mathbf{D}^* = \varepsilon^* \mathbf{E}^*\,, \quad \mathbf{H}^* = \tfrac{1}{\mu_o}\mathbf{B}^*\,,\\
&{}_{\mathrm{M}}\mathbf{t}^* = \mathbf{D}^* \otimes \mathbf{E}^* + \mathbf{H}^* \otimes \mathbf{B}^* - {}_{\mathrm{M}}\epsilon^* \mathbf{I}\,,\\
&{}_{\mathrm{M}}\epsilon^* \doteq \tfrac{1}{2}\left(\varepsilon_o \mathbf{E}^* \cdot \mathbf{E}^* + \tfrac{1}{\mu_o}\mathbf{B}^* \cdot \mathbf{B}^*\right),
\end{aligned} \tag{17}$$

where $\mathbf{t}^*$ is the Cauchy stress tensor, π^* is the thermodynamic pressure of the dielectric fluid, π_o^* is the constant pressure of the homogeneous dielectric fluid at rest when no electromagnetic fields are present, $\mathbf{P}^*$ is the polarization vector, χ^* is the constant dielectric susceptibility in $\mathcal{G}^*$, $\mathbf{D}^*$ is the dielectric displacement vector, $\varepsilon^* \doteq \varepsilon_o + \chi^*$ is the dielectric constant, $\mathbf{H}^*$ is the magnetic field, ${}_{\mathrm{M}}\mathbf{t}^*$ is the electromagnetic stress tensor and ${}_{\mathrm{M}}\epsilon^*$ is the free electromagnetic field energy density.
Let us note that according to the mass conservation law, the following boundary condition is satisfied on $\partial\mathcal{G}$

$$\mathbf{n} \cdot [\![\rho(\mathbf{v} - \mathbf{u})]\!] = 0\,. \tag{18}$$

Since $\mathbf{u} = \mathbf{v}|_{\partial\mathcal{G}(t)}$, it follows that

$$\mathbf{n} \cdot \mathbf{v}|_{\partial\mathcal{G}(t)} = \mathbf{n} \cdot \mathbf{v}^*|_{\partial\mathcal{G}(t)}\,. \tag{19}$$

It follows from the jump conditions on the boundary $\partial\mathcal{G}(t)$ for the first thermodynamic law (10) that

$$\mathbf{n} \cdot [(\mathbf{t} + {}_{\mathrm{M}}\mathbf{t} + \mathbf{v} \otimes \mathbf{G}) \cdot \mathbf{v} + \mathbf{m} \cdot \boldsymbol{\omega} + \mathbf{q} - \mathcal{S}] = \mathbf{n} \cdot [(-\pi_o^* + {}_{\mathrm{M}}\epsilon^* + \mathbf{E}^* \cdot \mathbf{P}^*)\mathbf{v} - \mathbf{E}^* \times \mathbf{H}^* + \mathbf{q}^*]\,. \tag{20}$$

By the derivation of the formula (20) we have used the identity (Eringen and Maugin, 1990a,b)

$$\mathcal{S} = \mathbf{E} \times \mathbf{H} + [{}_{\mathrm{M}}\mathbf{t} + \mathbf{v} \otimes \mathbf{G} - ({}_{\mathrm{M}}\epsilon + \mathbf{E} \cdot \mathbf{P})\mathbf{I}] \cdot \mathbf{v}\,, \tag{21}$$

and the boundary condition (19).

Let us extend the definitions of the fields $\mathbf{E}^*(\mathbf{x}, t)$, $\mathbf{B}^*(\mathbf{x}, t)$, $\mathbf{H}^*(\mathbf{x}, t)$ and $\mathbf{D}^*(\mathbf{x}, t)$ on the domain $\mathcal{G}(t) \cup \mathcal{G}^*(t)$ as the solutions of the Maxwell equations

$$\begin{aligned}
&\nabla \times \mathbf{E}^* + \mathbf{B}^*{}_{,t} = \mathbf{0}\,, \qquad \nabla \cdot \mathbf{D}^* = 0\,,\\
&\nabla \times \mathbf{H}^* - \mathbf{D}^*{}_{,t} = \mathbf{0}\,, \qquad \nabla \cdot \mathbf{B}^* = 0\,,
\end{aligned} \tag{22}$$

with the continuity conditions on $\partial\mathcal{G}(t)$

$$\begin{aligned}
&\mathbf{n} \times [\![\mathbf{E}^* + \mathbf{u} \times \mathbf{B}^*]\!] = \mathbf{0}\,, \quad \mathbf{n} \cdot [\![\mathbf{D}^*]\!] = 0\,,\\
&\mathbf{n} \times [\![\mathbf{H}^* - \mathbf{u} \times \mathbf{D}^*]\!] = \mathbf{0}\,, \quad \mathbf{n} \cdot [\![\mathbf{B}^*]\!] = 0\,.
\end{aligned} \tag{23}$$

If we additionally assume that in $\mathcal{G}(t)$

$$\mathbf{H}^* = \frac{1}{\mu_o}\mathbf{B}^*\,, \quad \mathbf{D}^* = \varepsilon^* \mathbf{E}^*\,, \tag{24}$$

where $\varepsilon^* \doteq \varepsilon_o + \chi^*$ is the dielectric constant in $\mathcal{G}^*(t)$, then we can write that

$$\oint_{\partial\mathcal{G}(t)} \mathbf{n}\cdot[(-\pi^* + {}_{\mathrm{M}}\epsilon^* + \mathbf{E}^*\cdot\mathbf{P}^*)\mathbf{v} - \mathbf{E}^*\times\mathbf{H}^*]\,dA = \frac{d}{dt}\int_{\mathcal{G}(t)} ({}_{\mathrm{M}}E^* - \pi_o^*)\,dV\,, \tag{25}$$

where

$${}_{\mathrm{M}}E^* \doteq \frac{1}{2}(\varepsilon^*\,\mathbf{E}^*\cdot\mathbf{E}^* + \frac{1}{\mu_o}\mathbf{B}^*\cdot\mathbf{B}^*)\,. \tag{26}$$

We assume here, similarly as in our previous paper (Radzikowska et al., 2001), that the body force $\tilde{\mathbf{f}}$ and the couple $\tilde{\mathbf{l}}$ have the following properties

$$\begin{aligned} &\tilde{\mathbf{f}} = -\nabla\tilde{g}\,, \quad \tilde{g} = \tilde{g}(\mathbf{x})\,, \quad \frac{\partial\tilde{g}}{\partial t} = 0\,,\\ &\tilde{\mathbf{l}} = \frac{d\tilde{\lambda}}{d\boldsymbol{\xi}}\times\boldsymbol{\xi}\,, \quad \tilde{\lambda} = \tilde{\lambda}(\boldsymbol{\xi})\,, \end{aligned} \tag{27}$$

where $\boldsymbol{\xi}$ is the director field, (Eringen, 1999).

Taking into account equations (20), (25) and (27) it is easy to write the energy conservation law (1) in the form which is useful for our purpose

$$\frac{d}{dt}\int_{\mathcal{G}(t)} [\rho(\epsilon + {}_{\mathrm{K}}\epsilon + \tilde{g} + \tilde{\lambda}) + {}_{\mathrm{M}}\epsilon - {}_{\mathrm{M}}E^* + \pi_o^*]\,dV = \oint_{\partial\mathcal{G}(t)} \mathbf{n}\cdot\mathbf{q}^*\,dA\,. \tag{28}$$

We have additionally assumed, that the heat energy source vanishes ($\tilde{h} = 0$).

4 The Evolution Criterion

The second law of thermodynamics (8), in case of vanishing entropy supply $\tilde{b}$, takes the form

$$\frac{d}{dt}\int_{\mathcal{G}(t)} \rho\eta\,dV \geq \oint_{\partial\mathcal{G}(t)} \mathbf{n}\cdot\mathbf{s}\,dA\,. \tag{29}$$

We use here the conventional methodology of obtaining evolution criteria (Muschik and Papenfuss, 1993), (Radzikowska et al., 2001). Accordingly, we multiply the inequality (29) by the constant equilibrium temperature of the environment T^*, and next we subtract the result from the equation (28), and we obtain the following inequality

$$\frac{d}{dt}\int_{\mathcal{G}(t)} [\rho(T^*\eta - \epsilon - {}_{\mathrm{K}}\epsilon - \tilde{g} - \tilde{\lambda}) - {}_{\mathrm{M}}\epsilon + {}_{\mathrm{M}}E^* - \pi_o^*]\,dV \geq \oint_{\partial\mathcal{G}(t)} \mathbf{n}\cdot(T^*\mathbf{s} - \mathbf{q}^*)\,dA\,. \tag{30}$$

It follows from the boundary conditions for the second law (13) that on the boundary material surface $\partial\mathcal{G}(t)$

$$\mathbf{n}\cdot\mathbf{s} \geq \mathbf{n}\cdot\mathbf{s}^*\,, \tag{31}$$

is valid.

Let us assume that in the environment $\mathcal{G}^*(t)$ the classical expression for the entropy flux is valid (Eringen and Maugin, 1990a,b)

$$\mathbf{s}^* = \frac{1}{T^*}\,\mathbf{q}^*\,. \tag{32}$$

It is easily seen that we can now write the evolution criterion in the following form

$$\frac{d}{dt} L(\mathbf{x}, t) \geq 0 , \tag{33}$$

where the kinetic potential $L(\mathbf{x}, t)$ is defined as

$$L(\mathbf{x}, t) \doteq \int_{\mathcal{G}(t)} [\rho(T^*\eta - \epsilon - {}_{\kappa}\epsilon - \tilde{g} - \tilde{\lambda}) - {}_{M}\epsilon + {}_{M}E^* - \pi_o^*] \, dV . \tag{34}$$

Let us summarize the assumptions we have made in order to obtain the above evolution criterion:

1. the vicinity of the micropolar medium has constant temperature T^*;
2. the entropy flux density of the vicinity of the micropolar medium is proportional to the heat vector density (32);
3. the acting mechanical force density is conservative and the torque has the form given by (27);
4. there are no energy and entropy supplies;
5. the vicinity of the micropolar medium is a homogeneous dielectric fluid.

If the above assumptions are valid, the evolution criterion (33) is satisfied even for the processes running far away from equilibrium. In the next section we apply the obtained criterion (33) to investigate the equilibrium conditions for micropolar continua.

5 The Equilibrium Conditions

According to the definition of the equilibrium state of the discrete system $\mathcal{G}(t)$ there exists such a global frame $(\boldsymbol{x}', t)$ that all time derivatives and fluxes vanish and the material is at rest (Muschik and Papenfuss, 1993). In the equilibrium state the system of equations describing the evolution of the micropolar electromagnetic body degenerates to the following system of equilibrium equations in $\mathcal{G}'$:

$$\begin{aligned}
&\nabla' \cdot (\mathbf{t} + {}_{M}\mathbf{t}) + \rho \tilde{\mathbf{f}} = \mathbf{0} , \\
&\nabla' \cdot \mathbf{m} + \boldsymbol{\varepsilon} : (\mathbf{t} + {}_{M}\mathbf{t}) + \rho \tilde{\mathbf{l}} = \mathbf{0} , \\
&\nabla' \times \mathbf{E} = \mathbf{0} , \quad \nabla' \times \mathbf{E}^* = \mathbf{0} , \\
&\nabla' \times \mathbf{H} = \mathbf{0} , \quad \nabla' \times \mathbf{H}^* = \mathbf{0} , \\
&\nabla' \cdot \mathbf{B} = 0 , \quad \nabla' \cdot \mathbf{B}^* = 0 , \\
&\nabla' \cdot \mathbf{D} = \tilde{q}_f , \quad \nabla' \cdot \mathbf{D}^* = 0 ,
\end{aligned} \tag{35}$$

with the boundary conditions on $\partial\mathcal{G}'$

$$\begin{aligned}
&\mathbf{n} \cdot (\mathbf{t} + {}_{M}\mathbf{t}) = \mathbf{n} \cdot (\pi^* \mathbf{I} + {}_{M}\mathbf{t}^*) , \quad \mathbf{n} \cdot \mathbf{m} = \mathbf{0} , \\
&\mathbf{n} \times (\mathbf{E}^* - \mathbf{E}) = \mathbf{0} , \quad \mathbf{n} \times [\![\mathbf{E}^*]\!] = \mathbf{0} , \\
&\mathbf{n} \times (\mathbf{H}^* - \mathbf{H}) = \mathbf{0} , \quad \mathbf{n} \times [\![\mathbf{H}^*]\!] = \mathbf{0} , \\
&\mathbf{n} \cdot (\mathbf{B}^* - \mathbf{B}) = 0 , \quad \mathbf{n} \cdot [\![\mathbf{B}^*]\!] = 0 , \\
&\mathbf{n} \cdot (\mathbf{D}^* - \mathbf{D}) = \tilde{\omega}_f , \quad \mathbf{n} \cdot [\![\mathbf{D}^*]\!] = 0 ,
\end{aligned} \tag{36}$$

where $\mathbf{t}$ is the mechanical stress tensor, $\mathbf{m}$ is the mechanical couple stress tensor, and

$$
\begin{aligned}
&{}_{\mathrm{M}}\mathbf{t} \doteq \mathbf{D}\otimes\mathbf{E} + \mathbf{H}\otimes\mathbf{B} - ({}_{\mathrm{M}}\epsilon - \mathbf{M}\cdot\mathbf{B})\mathbf{I}\,,\\
&{}_{\mathrm{M}}\mathbf{t}^* \doteq \mathbf{D}^*\otimes\mathbf{E}^* + \mathbf{H}^*\otimes\mathbf{B}^* - {}_{\mathrm{M}}\epsilon^*\,\mathbf{I}\,,\\
&\mathbf{D} \doteq \varepsilon_o\,\mathbf{E} + \mathbf{P}\,, \quad \mathbf{H} \doteq \tfrac{1}{\mu_o}\mathbf{B} - \mathbf{M}\,,\\
&\mathbf{D}^* \doteq \varepsilon^*\,\mathbf{E}^*\,, \quad \mathbf{H}^* \doteq \tfrac{1}{\mu_o}\mathbf{B}^*\,.
\end{aligned}
\tag{37}
$$

The kinetic potential (34) in the equilibrium coordinates system $(\boldsymbol{x}', t)$ takes the form

$$
L(\mathbf{x}', t) = \int_{\mathcal{G}'} [\rho(T^*\eta - \epsilon - \tilde{g} - \tilde{\lambda}) - ({}_{\mathrm{M}}\epsilon - {}_{\mathrm{M}}E^* + \pi_o^*)]\, dV'\,. \tag{38}
$$

The electromagnetic potential functions for the static electromagnetic fields are

$$
\begin{aligned}
&\mathbf{E} = \nabla'\varphi\,, \quad \mathbf{B} = \nabla'\times\mathbf{A}\,,\\
&\mathbf{E}^* = \nabla'\varphi^*\,, \quad \mathbf{B}^* = \nabla'\times\mathbf{A}^*\,.
\end{aligned}
\tag{39}
$$

The potentials $\mathbf{A}$ and φ are the differentiable functions in $\mathcal{G}'$ and the potentials $\mathbf{A}^*$ and φ^* are the differentiable functions in $\mathcal{G}'\cup\mathcal{G}^*$ and on the boundary surface $\partial\mathcal{G}'$ they are equal to each other

$$
\mathbf{A} = \mathbf{A}^*\,, \quad \varphi = \varphi^*\,. \tag{40}
$$

It follows (Ingarden et al., 1980) from the electric Gauss laws $(35)_6$ and $(36)_5$ that

$$
\int_{\mathcal{G}'} \tilde{q}_f\, dV' + \oint_{\partial\mathcal{G}'} \tilde{\omega}_f\, dA' = 0\,. \tag{41}
$$

and

$$
\int_{\mathcal{G}'} (\mathbf{D}\cdot\mathbf{E} - \mathbf{D}^*\cdot\mathbf{E}^*)\, dV' = -\int_{\mathcal{G}'} \tilde{q}_f\,\varphi\, dV' - \oint_{\partial\mathcal{G}'} \omega_f\,\varphi\, dA'\,, \tag{42}
$$

Now we apply the last result (42) into the expression for the kinetic potential (38) and we obtain that

$$
L = -\int_{\mathcal{G}'} [\rho(\psi + \tilde{g} + \tilde{\lambda}) - ({}_{\mathrm{M}}e - {}_{\mathrm{M}}e^* - \pi^*) - \tilde{q}_f\,\varphi]\, dV' + \oint_{\partial\mathcal{G}'} \tilde{\omega}_f\,\varphi\, dA'\,, \tag{43}
$$

where

$$
\psi \doteq \epsilon - T^*\eta - \frac{1}{\rho}\mathbf{P}\cdot\mathbf{E}\,, \tag{44}
$$

is the generalized Helmholtz free energy function in the case of constant temperature T^*, while ${}_{\mathrm{M}}e$ and ${}_{\mathrm{M}}e^*$ are the Lagrange densities for the electromagnetic fields

$$
\begin{aligned}
&{}_{\mathrm{M}}e \doteq \tfrac{1}{2}(\varepsilon_o\,\mathbf{E}\cdot\mathbf{E} - \tfrac{1}{\mu_o}\mathbf{B}\cdot\mathbf{B})\,,\\
&{}_{\mathrm{M}}e^* \doteq \tfrac{1}{2}(\varepsilon_o\,\mathbf{E}^*\cdot\mathbf{E}^* - \tfrac{1}{\mu_o}\mathbf{B}^*\cdot\mathbf{B}^*)\,.
\end{aligned}
\tag{45}
$$

The fundamental assumption of the variational approach (Kotowski and Radzikowska, 1998, 1999) is that

$$
\delta\mathbf{X} = \mathbf{0}\,. \tag{46}
$$

Let us note that the above assumption means the consistence of variations with deformations of the body.

According to (46) we can calculate the variations of mass and the electromagnetic fields

$$\begin{aligned}
&\delta\rho = -\rho\,\nabla'\cdot\delta\mathbf{x}'\,,\\
&\delta\mathbf{E} = -\mathbf{E}\cdot(\nabla'\otimes\delta\mathbf{x}')^T + \nabla'(\delta\varphi)\,,\\
&\delta\mathbf{B} = (\mathbf{B}\cdot\nabla')\,\delta\mathbf{x}' - \mathbf{B}(\nabla'\cdot\delta\mathbf{x}') + \nabla'\times\hat{\delta}\,\mathbf{A}\,,
\end{aligned} \tag{47}$$

where the Weiss type gauge invariant variation $\hat{\delta}\,\mathbf{A}$ is defined as (Kotowski and Radzikowska, 1999)

$$\hat{\delta}\,\mathbf{A} \doteq \delta\,\mathbf{A} - (\nabla'\otimes\mathbf{A})^T\cdot\delta\mathbf{x}'\,. \tag{48}$$

The similar expression can be formulated for the "asterisk" quantities $\delta\,\mathbf{E}^*$ and $\delta\,\mathbf{B}^*$.

It follows from the equation (41) and from the conditions (27) that

$$\delta\int\limits_{\mathcal{G}'}\tilde{q}_f\,\varphi\,dV' + \delta\oint\limits_{\partial\mathcal{G}'}\tilde{\omega}_f\,\varphi\,dA' = \int\limits_{\mathcal{G}'}\tilde{q}_f\,\delta\varphi\,dV' + \oint\limits_{\partial\mathcal{G}'}\tilde{\omega}_f\,\delta\varphi\,dA'\,, \tag{49}$$

and

$$\delta\int\limits_{\mathcal{G}'}\rho(\tilde{g}+\tilde{\lambda})\,dV' = -\int\limits_{\mathcal{G}'}\rho(\tilde{\mathbf{f}}\cdot\delta\mathbf{x}' + \tilde{\mathbf{l}}\cdot\delta\boldsymbol{\kappa})\,dV'\,, \tag{50}$$

where the angular variation $\delta\boldsymbol{\kappa}$ is defined by

$$\delta\boldsymbol{\xi} = \delta\boldsymbol{\kappa}\times\boldsymbol{\xi}\,. \tag{51}$$

Taking into account the results (49) and (50) we can write

$$\delta L = -\int\limits_{\mathcal{G}'}\rho\,\delta[\psi - \frac{1}{\rho}({}_{\mathrm{M}}e - {}_{\mathrm{M}}e^* - \pi^*)]\,dV' + \int\limits_{\mathcal{G}'}[\rho(\tilde{\mathbf{f}}\cdot\delta\mathbf{x}' + \tilde{\mathbf{l}}\cdot\delta\boldsymbol{\kappa}) + \tilde{q}_f\,\delta\varphi]\,dV' + \oint\limits_{\partial\mathcal{G}'}\tilde{\omega}_f\,\delta\varphi\,dA'\,. \tag{52}$$

Let us notice that it follows from the Maxwell equations and from the formulae analogous to (47) for variations of the fields $\mathbf{E}^*$ and $\mathbf{B}^*$ and from the conditions (40) that

$$\int\limits_{\mathcal{G}'}\rho\,\delta[\frac{1}{\rho}({}_{\mathrm{M}}e^* + \pi^*)]\,dV' = \oint\limits_{\partial\mathcal{G}'}\mathbf{n}\cdot[(\pi^*\,\mathbf{I} + {}_{\mathrm{M}}\mathbf{t}^*)\cdot\delta\mathbf{x}' - \mathbf{D}^*\,\delta\varphi - \mathbf{H}^*\times\hat{\delta}\mathbf{A}]\,dA'\,, \tag{53}$$

where ${}_{\mathrm{M}}\mathbf{t}^*$ is defined by equation (37). We are ready now to write the variation of the kinetic potential in the more useful form

$$\begin{aligned}
\delta L = {}& \delta\int\limits_{\mathcal{G}'}({}_{\mathrm{M}}e - \rho\,\psi)\,dV' + \int\limits_{\mathcal{G}'}[\rho(\tilde{\mathbf{f}}\cdot\delta\mathbf{x}' + \tilde{\mathbf{l}}\cdot\delta\boldsymbol{\kappa}) + \tilde{q}_f\,\delta\varphi]\,dV' + \\
&\oint\limits_{\partial\mathcal{G}'}\{\mathbf{n}\cdot[(\pi^*\,\mathbf{I} + {}_{\mathrm{M}}\mathbf{t}^*)\cdot\delta\mathbf{x}' - \mathbf{D}^*\,\delta\varphi - \mathbf{H}^*\times\hat{\delta}\mathbf{A}] + \tilde{\omega}_f\,\delta\varphi\}\,dA'\,.
\end{aligned} \tag{54}$$

The generalized Helmholtz free energy function ψ defined by equation (44) is the thermodynamic potential for the electromagnetic micropolar body in the equilibrium state. This functional depends on the set of the independent constitutive variables different for different materials (Eringen, 1999). We discuss here the cases for the micropolar elastic solids, the micropolar fluids and the liquid crystals subjected to the electromagnetic interactions.

5.1 Micropolar Electromagnetic Elastic Solids

In the case of the micropolar electromagnetic elastic solids (Eringen, 1999)

$$\psi = \psi(\mathfrak{C}, \Gamma, \mathbf{E}, \mathbf{B}, \mathbf{X}), \tag{55}$$

where $\mathfrak{C}$ is the Cosserat deformation tensor defined by the following formula

$$\mathfrak{C}_{KL} \doteq x^k{}_{,K}\ \chi_{kL}, \tag{56}$$

and Γ it the material wryness deformation tensor

$$\Gamma_{KL} \doteq \frac{1}{2}\varepsilon_{KMN}\ \chi^{kM}{}_{,L}\ \chi_k{}^N. \tag{57}$$

Variations of the above deformation tensors are as follows (Kotowski and Radzikowska, 1998, 1999)

$$\begin{aligned} \delta\mathfrak{C}_{KL} &= x^k{}_{,K}\ [\chi_{lL}(\delta x^l)_{,k} + \varepsilon_{lnk}\,\chi^n{}_L\,\delta\kappa^l], \\ \delta\Gamma_{KL} &= x^k{}_{,L}\ \chi_{lK}(\delta\kappa^l)_{,k}. \end{aligned} \tag{58}$$

There hold the following equilibrium constitutive relations

$$t_{kl} \doteq \rho\frac{\partial\psi}{\partial\mathfrak{C}_{KL}}\,x_{k,K}\ \chi_{lL}, \tag{59}$$

$$m_{kl} \doteq \rho\frac{\partial\psi}{\partial\Gamma_{LK}}\,x_{k,K}\ \chi_{lL}, \tag{60}$$

$$\mathbf{P} \doteq -\rho\frac{\partial\psi}{\partial\mathbf{E}}, \quad \mathbf{M} \doteq -\rho\frac{\partial\psi}{\partial\mathbf{B}}. \tag{61}$$

5.2 Micropolar Electromagnetic Fluids

In the case of the micropolar electromagnetic fluids we have (Eringen, 1999)

$$\psi = \psi(\rho^{-1}, \mathbf{j}, \mathbf{E}, \mathbf{B}), \tag{62}$$

with the following additional constitutive relations

$$\mathbf{t} \doteq \frac{\partial\psi}{\partial\rho^{-1}}\mathbf{I}, \quad \mathbf{m} \doteq \mathbf{0}, \tag{63}$$

$$\mathbf{P} \doteq -\rho\frac{\partial\psi}{\partial\mathbf{E}}, \quad \mathbf{M} \doteq -\rho\frac{\partial\psi}{\partial\mathbf{B}}. \tag{64}$$

5.3 Liquid Crystals

For the liquid crystals functional ψ is of the form (Eringen, 1979)

$$\psi = \psi(\rho^{-1}, \mathbf{j}, \boldsymbol{\gamma}, \mathbf{E}, \mathbf{B}), \tag{65}$$

where $\boldsymbol{\gamma}$ is the spatial deformation tensor

$$\gamma_{lk} \doteq \frac{1}{2}\varepsilon_{lmn}\ \chi^{mK}\,\chi^n{}_{K,k}. \tag{66}$$

The variation of this tensor is given by Kotowski and Radzikoeska (1999)

$$\delta\boldsymbol{\gamma} = -\boldsymbol{\gamma}\cdot(\nabla\otimes\delta\mathbf{x})^T + \boldsymbol{\varepsilon} : \delta\boldsymbol{\kappa}\otimes\boldsymbol{\gamma} + (\nabla\otimes\delta\boldsymbol{\kappa})^T. \tag{67}$$

The following additional constitutive relations are satisfied:

$$\mathbf{t} \doteq \frac{\partial \psi}{\partial \rho^{-1}} \mathbf{I} - \mathbf{m} \cdot \boldsymbol{\gamma}, \quad \mathbf{m} \doteq \rho \frac{\partial \psi}{\partial \boldsymbol{\gamma}^T}, \tag{68}$$

$$\mathbf{P} \doteq -\rho \frac{\partial \psi}{\partial \mathbf{E}}, \quad \mathbf{M} \doteq -\rho \frac{\partial \psi}{\partial \mathbf{B}}. \tag{69}$$

Taking into account the axiom of the material objectivity (the material frame indifference) and putting the proper constitutive relations to the variation of the kinetic potential (54) we obtain from the equations (35) with the boundary conditions (36) the equilibrium condition in the final form

$$\delta L = 0. \tag{70}$$

It means that L satisfies the conditions of the kinetic potential because it is an increasing functional of its variables and reaches the extremum (here maximum) in the equilibrium state. Let us see that (70) states the classic variational principle describing the equilibrium processes in the electrodynamics of micropolar continua.

6 Conclusions

It was shown that an arbitrary dissipative process in a micropolar body being controlled by a reservoir of a dielectric fluid with the constant temperature T^* and tending to equilibrium, can be described by the obtained evolution criterion (33)

$$\frac{d}{dt} L(\mathbf{x}, t) \geq 0.$$

It is the generalization the evolution criterion derived by Muschik and Papenfuss (1993) and by Radzikowska et al. (2001). The description of the micropolar medium was based on the approach of Eringen (1999). Our evolution criterion was obtained with the following assumptions: the entropy flux density in the vicinity of the micropolar medium is proportional to the heat vector density (32), the acting mechanical force density is conservative and the torque has the form given by (27), there are no energy and entropy supplies and the vicinity of the micropolar medium is a homogeneous dielectric fluid. We applied our results to study the equilibrium conditions in micropolar bodies subjected to electromagnetic interactions. We stated, that in the discussed cases (micropolar electromagnetic elastic solids, micropolar electromagnetic fluids and liquid crystals) the functional (34)

$$L(\mathbf{x}, t) \doteq \int_{\mathcal{G}(t)} [\rho(T^*\eta - \epsilon - {}_{\mathrm{K}}\epsilon - \tilde{g} - \tilde{\lambda}) - {}_{\mathrm{M}}\epsilon + {}_{\mathrm{M}}E^* - \pi_o^*] \, dV.$$

is the kinetic potential of Glansdorff and Prigogine.

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