

# Viscomagnetoelastic Interactions in a Vortex Array in the Type-II Superconductor<sup>1</sup>

A. Drzewiecki, J.A. Kołodziej, B.T. Maruszewski

*The paper develops considerations on viscomagnetoelastic interactions in a vortex array in a type-II superconductor. It is well known that a magnetic field penetrates such a material along lines called vortices of a special structure. Each of them consists of a core of material in the normal state, i.e. a material in which Ohm's law works and a surrounding where the supercurrent flows. The mean diameter of a core is called the coherence length. The penetration of the supercurrent outside the core exists in the London penetration depth. Since interactions among the vortices run with the help of the Lorenz force, the vortex field has elastic properties. Moreover, because of the normal state inside the vortex core also the viscosity of that field has been observed. The above situation occurs only between upper and lower magnetic field limits below the critical temperature regarding the phase diagram. The vortex field has a very interesting feature. In the vicinity of the lower magnetic field curve it possesses an ordered (quadratic or triangular) structure. Then going to the upper magnetic field limit that structure is losing its configuration behaving as a fluid. We assume smooth transition from ordered to disordered state although it is much more complicated in nature. Following the above statements all the "material" coefficients characteristic for the vortex field are also dependent on the magnetic field and temperature. The main aim of the paper is a formulation of the stress – strain constitutive law consisting of the following features:*

- *a coexistence of the ordered and disordered states,*
- *the viscosity of the vortex field,*
- *the dependence of the "material" coefficients related to the vortex field on the magnetic field.*

*An application for YBCO ceramics that deals with the use of the proposed constitutive law in vortex field equations and results coming from that are presented. Numerical calculations concern wave propagation in depinned parallel vortex line field versus magnitude of the applied magnetic field.*

## 1 Introduction

The paper develops the mechanics of the vortex lattice as a certain state and geometry in a medium (Tilley and Tilley, 1974; Tinkham, 1975; Orlando and Delin, 1991; Cyrot and Pavuna, 1992; Blatter et al., 1994; Brandt, 1998).

Magnetic flux can penetrate a type-II superconductor in the form of Abrikosov vortices (also called flux lines, flux tubes, or fluxons) each carrying a quantum of magnetic flux. These tiny vortices of supercurrent tend to arrange themselves in a triangular and/or quadratic flux-line lattice, which is more or less perturbed by material inhomogeneities that pin the flux lines. Pinning is caused by imperfections of the crystal lattice, such as dislocations, point defects, grain boundaries, etc. Hence, a honeycomb pattern of the vortex array presents some mechanical properties. The latter ones come mainly from the elastic properties of the superconducting body. However, since the vortices are formed from the applied magnetic field and around of each of them the supercurrent flows, there exist also some Lorentz force interactions among them. These interactions form an origin of an additional mechanical (stress) field occurring in the type-II superconductor. This field near the lower critical magnetic intensity limit  $H_{C1}$  is also of elastic character. However, if the density of the supercurrent is above its critical value and/or the temperature is sufficiently high, there occurs a flow of vortex lines in the superconducting body. Within such situation vortices behave rather as a fluid than as an elastic lattice. The "fluidity" of the vortex array is also observed when the applied magnetic field tends to its upper critical limit  $H_{C2}$ . So, there coexist two mechanical fields: one of them of a pure elastic character coming from the crystal lattice and the Lorentz interactions between vortices, and the second one that transfers smoothly into "fluid" form towards the upper magnetic field strength limit  $H_{C2}$ . However, if the Lorentz force of interactions between

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the vortices is much bigger than the pinning force, the vortex lattice behaves elastically (Cyrot and Pavuna, 1992; Blatter et al., 1994). Such a situation occurs if an external current density is applied to the vortex and is bigger than its critical value. So we assume following (Cyrot and Pavuna, 1992) that the pinning force is negligible in the sequel and we deal with soft vortices. The vortex motion (creep) is accompanied by an energy dissipation. Their motion is damped by a force proportional to the vortex velocity. Hence, except for the elastic properties, the vortex field is also of a viscous character. The resistivity in area of vortex creep is the same as the resistivity of the current, which would flow inside the vortex core. Hence the viscosity coefficient reads (Cyrot and Pavuna, 1992)

$$\eta = \frac{\Phi_o \mu_o H_{C2}}{\rho_n}, \quad (1)$$

where  $\Phi_o$  is the magnetic flux,  $\mu_o$  denotes the permeability of vacuum, and  $\rho_n$  is the resistivity in the normal state.

Assuming, for the sake of simplicity, that the vortex lines are parallel to each other we have considered waves that can propagate across such defined vortex field in a continuous model. Both dispersion and damping have been calculated.

## 2 Thermodynamical Foundations

Following the above properties the extended thermodynamical model for the viscoelastic field of vortices in the type-II superconductor is presented below. We have assumed that the mass density  $\rho$  of the vortex field concerns the density of the material in the normal state as the counterpart in the mixed type-II superconductor (i.e. the mass of the normal part of the body related to the total volume of the material), and the energy dissipation occurs only because of the viscosity of the vortex field caused by the ohmic-like resistivity (normal-state resistivity) inside the vortex core (Blatter et al., 1994). Hence the general form of the state vector (the set of independent variables) reads (cf. Maruszewski, 1997, 1998)

$$C = \left\{ \varepsilon_{ij}, \dot{\varepsilon}_{ij}, \varphi, A_i, T, T_{,i}, \psi, \psi^*, \psi_{,i}, \psi_{,i}^*, q_i, j_i^S \right\}, \quad (2)$$

where  $\varepsilon_{ij}$  denotes the strain tensor, its time derivative in (2) indicates the viscoelasticity of the vortex field,  $\varphi$  and  $A_i$  are the scalar and vector potentials, respectively,  $T$  is the absolute temperature,  $\psi$  is the order parameter (the wave function of a Cooper pair) and  $\psi^*$  is its complex conjugate,  $j_i^S$  is the supercurrent density. The fundamental laws, which govern the set (2) are the follows

$$\dot{\rho} + \rho v_{k,k} = 0 \quad (3)$$

$$\rho \dot{v}_k - \sigma_{ik,i} - e_{ijk} (j_i^N + j_i^S) B_j - f_k = 0 \quad (4)$$

$$\rho \dot{e} - \sigma_{ik} v_{k,i} - q_{k,k} - (j_i^N + j_i^S) \mathcal{E}_i - \rho r = 0 \quad (5)$$

$$\frac{1}{\mu_o} A_{i,kk} - j_i^N - j_i^S = 0 \quad (6)$$

$$\dot{q}_k - Q_k(C) = 0 \quad (7)$$

$$\dot{\psi} - \Psi(C) = 0 \quad (8)$$

$$\dot{\psi}^* - \Psi^*(C) = 0 \quad (9)$$

$$\dot{j}_k^S - J_k^S(C) = 0. \quad (10)$$

$v_k$  denotes the velocity of the vortex field point,  $\sigma_{ik}$  is the viscoelastic stress tensor,  $j_k^N$  is the normal current,  $B_j$  is the magnetic induction,  $f_k$  is the body force,  $e$  is the internal energy density,  $\mathcal{E}_i$  is the electromotive intensity,  $r$  is the heat source distribution. The set (3) to (10) consists of:

- the equation for vortex field whose form ensures the conservation of the vortex mass in the sense indicated above,
- the momentum balance of the vortex field where elastic interactions are due to the Lorentz force,
- the internal energy balance of the vortex field where the only dissipation occurs because of the Joule-like heat produced by the total current,
- the electromagnetic vector potential equation,
- the evolution equations for heat flux and supercurrent because of the extended thermodynamical model (2),
- the evolution equations for the Cooper pairs wave function as the order parameter (internal variable) evolution equations.

The extended thermodynamical description has been chosen here since all the interactions run within low temperatures. Moreover, the electromagnetic field quantities satisfy the Maxwell equations, and the following relations hold

$$\mathcal{E}_i = E_i + e_{ijk} v_j B_k \quad (11)$$

$$E_i = -\varphi_{,i} - \frac{\partial A_i}{\partial t} \quad (12)$$

$$B_i = e_{ijk} A_{k,j} \quad (13)$$

$$B_i = \mu_o H_i , \quad (14)$$

where  $H_k$  is the magnetic field strength. In the sequel we follow the assumption that  $\varphi$  vanishes by gauging (Orlando and Delin, 1991).

The use of the second law of thermodynamics in the form of the entropy inequality

$$\rho \dot{s} + \Phi_{k,k} - \frac{\rho r}{T} \geq 0 , \quad (15)$$

where  $s$  is the entropy density and  $\Phi_k$  denotes the entropy flux, gives us a possibility to determine all the constitutive functions which, in our case form the set

$$Z = \{ \sigma_{ik}, e, s, \Phi_k, j_k^N, Q_k, \Psi, \Psi^*, J_i^S \} . \quad (16)$$

As we have mentioned before, we are mostly interested in the mechanical properties of the vortex field vs. magnetic field  $H_{C1} < H < H_{C2}$ . Having several possibilities to create a proper constitutive law for the stress tensor  $\sigma_{ij}$  whose form should consists both of terms related to the solid viscoelastic state of the vortex field and the viscous fluidity of the vortices vs. the magnitude of the applied magnetic field following (Maruszewski and Restuccia, 1999), its required form ( $p$  denotes the pressure of the vortex fluid) reads

$$\sigma_{ij} = \left[ \left( \frac{1}{3} \alpha K - \frac{2}{3} \alpha G \right) \varepsilon_{kk} - \frac{2}{3} \alpha \eta \varepsilon_{kk} - \beta p \right] \delta_{ij} + 2 \alpha G \varepsilon_{ij} + 2 (\alpha + \beta) \eta \varepsilon_{ij} , \quad (17)$$

where

$$\alpha + \beta = \begin{cases} 0 & \text{if } H = H_{C1} \text{ or } H = H_{C2} \\ f(H) & \text{if } H_{C1} < H < H_{C2} \end{cases} \quad (18)$$

$$\alpha = \left( \frac{H_{C2} - H}{H_{C2} - H_{C1}} \right)^2, \quad \alpha = \begin{cases} 0 & \text{if } H = H_{C2} \\ 1 & \text{if } H = H_{C1} \end{cases} \quad (19)$$

$$\beta = \left( \frac{H - H_{C1}}{H_{C2} - H_{C1}} \right)^2, \quad \beta = \begin{cases} 0 & \text{if } H = H_{C1} \\ 1 & \text{if } H = H_{C2} \end{cases} \quad (20)$$

$$3K = 2\mu + 3\lambda, \quad G = \mu. \quad (21)$$

$\lambda, \mu$  are the Lamé constants.

### 3 Field Equations

On using now (17) in the momentum balance (4) and modelling the first London equation (10) with the help of the relation (11) and confining only to magnetoelastic interactions in the considered superconductor, we arrive at the following nonlinear field equations

$$\begin{aligned} & \left[ \alpha G + (\alpha + \beta) \eta \frac{\partial}{\partial t} \right] u_{i,jj} + \frac{1}{3} \left[ (K + G) \alpha + (\alpha + 3\beta) \eta \frac{\partial}{\partial t} \right] u_{j,ij} + \\ & + \left[ \alpha_{,j} G + (\alpha + \beta)_{,j} \eta \frac{\partial}{\partial t} \right] (u_{i,j} + u_{j,i}) + \frac{1}{3} \alpha_{,j} \left( K - 2G - 2\eta \frac{\partial}{\partial t} \right) u_{k,k} - \\ & - \beta_{p,i} - \beta_{,i} p - \mu_o (H_{r,i} - H_{i,r}) H_r = \rho \ddot{u}_i \end{aligned} \quad (22)$$

$$\lambda_o^2 \dot{H}_{i,kk} - \dot{H}_i + \dot{u}_{i,k} H_k - \dot{u}_{k,k} H_i = 0. \quad (23)$$

Being, however, conscious of the fact that the amplitude of the magnetic field inside the material can not exceed the limiting values  $H_{C1}$  and  $H_{C2}$  to keep the superconductor within the vortex state, we assume that amplitude of the form

$$H_r = H_{or} + h_r, \quad |h_r| \ll |H_{or}|, \quad (24)$$

where  $H_{or}$  is constant and  $h_r$  is a small perturbation of the magnetic field  $H_{or}$ . That idea allows us to linearize the equations (22) and (23).

### 4 Waves in the Vortex Lattice

Let us assume now, that the applied magnetic field is taken as  $\mathbf{H}_o = [0, 0, H_{O3}]$ .

Supposing that the superconducting body occupies the whole space and that vortices are parallel one to another being in  $x_3$  direction we consider a propagation of magnetomechanical waves along the  $x_1$ -direction. If we use (24) in (22), (23), the linear field equations can be rewritten to the form

$$\begin{aligned} & \mu u_{i,jj} + \eta \dot{u}_{i,jj} + (\lambda + \mu) u_{j,ij} + \frac{1}{3} \eta \dot{u}_{j,ij} - \mu_o (h_{r,i} - h_{i,r}) H_{or} - \rho \ddot{u}_i = 0, \\ & u_{i,k} H_{ok} - u_{k,k} H_{oi} + \lambda_o^2 h_{i,kk} - h_i = 0. \end{aligned} \quad (25)$$

We see that only a ‘‘magnetic’’ displacement  $u_k$  can propagate as a wave. So, it is responsible both for dynamics of the magnetic field and the perturbation of its geometry. Following the above assumption concerning the geometry of the applied magnetic field, the solutions of (25) are looked in the form

$$\begin{aligned} u_1 &= u_{o1} \exp [i k (x_1 - v t)] \\ u_2 &= u_{o2} \exp [i k (x_1 - v t)] \\ h_3 &= h_{o3} \exp [i k (x_1 - v t)]. \end{aligned} \quad (26)$$

Hence the final form of the field equations is the following

$$\begin{aligned}
c_L^2 u_{1,11} + \frac{4\eta}{3\rho} \dot{u}_{1,11} - \frac{\mu_o}{\rho} h_{3,1} H_{o3} - \ddot{u}_1 &= 0 \\
c_T^2 u_{2,11} + \frac{\eta}{\rho} \dot{u}_{2,11} - \ddot{u}_2 &= 0 \\
-u_{1,1} H_{o3} + \lambda_o^2 h_{3,11} - h_3 &= 0
\end{aligned} \tag{27}$$

where  $c_L^2 = \frac{\lambda + 2\mu}{\rho}$ ,  $c_T^2 = \frac{\mu}{\rho}$ .

So we can observe two waves: the longitudinal magnetomechanical  $u_1$  and the viscoelastic transverse wave  $u_2$  which is independent of the magnetic perturbation  $h_3$ . The dispersion for the magnetomechanical  $u_1$  wave is the following

$$v^2 + \frac{4\mu}{3\rho} i k v - \left( c_L^2 + \frac{\mu_o H_{o3}^2}{\rho (\lambda_o^2 k^2 + 1)} \right) = 0. \tag{28}$$

Solving (28) we obtain

$$\text{Re } v = \frac{\sqrt{\rho (9 H_{o3}^2 \mu_o + 9 c_L^2 \rho \Theta - 4 \eta^2 k^2 \rho \Theta)}}{3 \rho \sqrt{\Theta}} \tag{29}$$

where  $\Theta = k^2 \lambda_o^2 + 1$ ,

$$\text{Im } v = -\frac{2\eta k}{3}. \tag{30}$$

Then for the  $u_2$  mode the dispersion problem is the following

$$v^2 + \frac{i\eta}{\rho k} v - c_T^2 = 0 \tag{31}$$

$$\text{Re } v = \frac{\sqrt{4 c_T^2 k^2 \rho^2 - \eta^2}}{2 k \rho} \quad \text{Im } v = -\frac{\eta}{2 k \rho}. \tag{32}$$

The analysis of the relations (28) to (32) shows that for the case of a long wave approximation the  $u_1$  mode is not damped and dispersed, and the shortest possible  $u_1$  wave can propagate only if  $k = 2\pi / \lambda_o$ . Then the mode  $u_2$  behaves differently. There is no case of long wave form for that. For the short wave approximation, the  $u_2$  - wave propagates with the velocity  $c_T$  and is not damped.

The numerical analysis for the dispersion and damping related to  $u_1$  and  $u_2$  modes have been done for YBCO ceramics. The proper data are collected in the Table 1.

Quantity	Value	References
$\mu_o H_{o3}$	120 T	Cyrot and Pavuna (1992)
$\lambda_o$	$4 \cdot 10^{-7}$ m	Cyrot and Pavuna (1992)
$\xi$	$10^{-9}$ m	Cyrot and Pavuna (1992)
$T_o$	$\leq 92$ K	Cyrot and Pavuna (1992)
$\rho_n$	$6 \cdot 10^{-5} \Omega \cdot \text{m}$	Cyrot and Pavuna (1992)
$\rho_o$	$5 \cdot 10^3 \text{ kg} / \text{m}^3$	Cyrot and Pavuna (1992)
$\mu_o$	$4 \pi \cdot 10^{-7} \text{ T} \cdot \text{m} / \text{A}$	
$\Phi_o$	$2.07 \cdot 10^{-15} \text{ T} \cdot \text{m}^2$	Cyrot and Pavuna (1992)
$c_{11}$	$\mu_o H_{o3}^2$	Brandt (1998)
$H_{o3}$	$0.955 \cdot 10^8 \text{ A} / \text{m}$	

Table 1. The Data for YBCO Ceramics

We remark that the density  $\rho_o$  of the vortex field must be calculated as follows:  $\rho_o = \rho_{\text{YBCO}} \cdot w$ , where

$$w = \frac{\text{volume of the material in the normal state}}{\text{total volume of the material}} .$$

For the triangular structure  $w = \frac{\pi}{2\sqrt{3}}$  .

The results are shown on Figs. 1 to 4.

(I -  $H_{o3} = 3.17 \cdot 10^7$  A/m, II -  $H_{o3} = 4.76 \cdot 10^7$  A/m, III -  $H_{o3} = 6.35 \cdot 10^7$  A/m, IV -  $H_{o3} = 7.94 \cdot 10^7$  A/m)

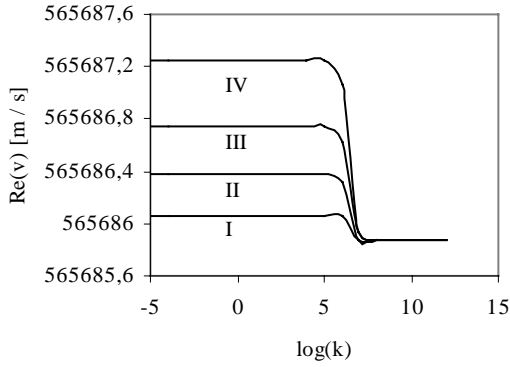


Figure 1. The Dispersion of the  $u_1$  Mode

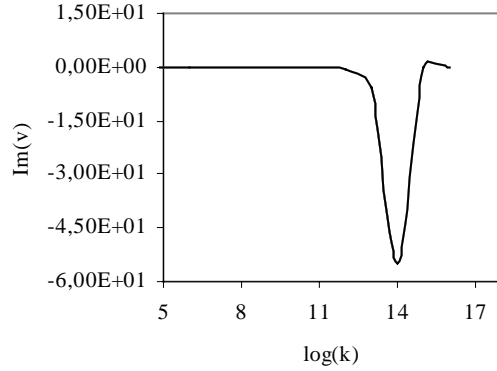


Figure 2. The Damping of the  $u_1$  Mode

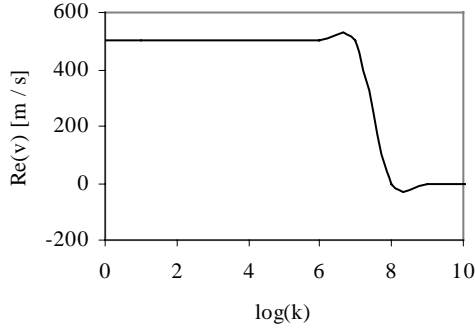


Figure 3. The Dispersion of the  $u_2$  Mode

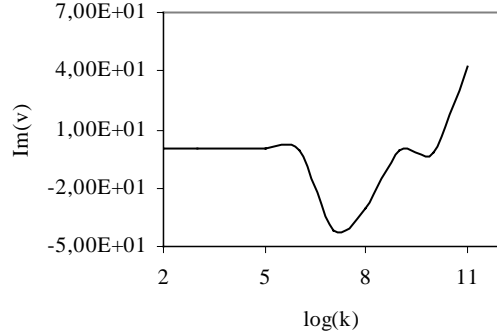


Figure 4. The Damping of the  $u_2$  Mode

A simple look at the above curves shows that for the value of  $k < 10^5 \text{ m}^{-1}$  ( $\zeta = 10^{-9} \text{ m}$ ,  $\lambda_o = 4 \cdot 10^{-7} \text{ m}$ )  $u_1$  and  $u_2$  modes are nondispersive and undamped, so they can carry any information on a long distance and at a long time. Moreover, the transverse  $u_2$  wave does not depend on the magnitude of the applied magnetic field.

## 5 Magnetoacoustic Waves in a Vortex Fluid

Confining now only to the fluid state of the vortex array and assuming the applied magnetic field also as  $\mathbf{H}_o = [0, 0, H_{o3}]$  the basic set of wave equations coming from (22) to (24) read

$$\begin{aligned} \dot{\rho} + \rho_o v_{k,k} &= 0 \\ \eta v_{i,jj} + \frac{1}{3} \eta v_{j,ij} - p_{,i} - \mu_o (h_{r,i} - h_{i,r}) H_{or} &= \rho_o \dot{v}_i \\ \lambda_o^2 \dot{h}_{i,kk} - \dot{h}_i + v_{i,k} H_{ok} - v_{k,k} H_{oi} &= 0 . \end{aligned} \quad (33)$$

Assuming now that the propagation direction is  $x_1$  and the magnetic field direction is  $x_3$ , we take that

$$\begin{aligned} v_1 &= v_{10} \exp[i k (x_1 - c t)] \\ \rho &= \rho_{00} \exp[i k (x_1 - c t)] \\ h_3 &= h_{30} \exp[i k (x_1 - c t)]. \end{aligned} \quad (34)$$

Since we know only a general form of the vortex fluid pressure with respect of the vortex density, i.e.

$$p = p(\rho) , \quad (35)$$

we find its gradient in the form

$$p_{,i} = \frac{d p}{d \rho} \rho_{,i} . \quad (36)$$

But the relation (36) makes equation (33)<sub>2</sub> nonlinear. So, to linearize (31) we expand  $\frac{d p}{d \rho}$  into the Taylor series

around the rest state of the fluid, and we confine only to the first term of the expansion  $\left(\frac{d p}{d \rho}\right)_o$  (index "0" indicates the state of the fluid at rest).

Hence, the final form of the field equations reads

$$\begin{aligned} \frac{4\eta}{3\rho_o} \dot{\rho}_{,11} + \mu_o h_{3,11} H_{o3} + \left(\frac{d p}{d \rho}\right)_o \rho_{,11} &= \ddot{p} \\ \lambda_o^2 h_{3,11} - h_3 + \frac{1}{\rho_o} \rho H_{o3} &= 0 . \end{aligned} \quad (37)$$

Calling now

$$\left(\frac{d p}{d \rho}\right)_o = c_o^2 = \frac{\lambda + 2 \mu}{\rho_o} = c_L^2 , \quad (38)$$

which means that the pure acoustic wave speed for the fluid is equal to the longitudinal elastic wave velocity (no shear phenomenon occurs). On using now (34) in (37) we arrive at the following dispersion relation for the vortex fluid

$$\left(\frac{4\mu}{3\rho_o} i k c - c_o^2 + c^2\right) (1 - \lambda_o^2 k^2) + \frac{\mu_o H_{o3}^2}{\rho_o} = 0 . \quad (39)$$

The numerical results coming from the relation (39) for YBCO ceramics are collected on Figs. 5, 6.

(I -  $H_{o3} = 3.17 \cdot 10^7$  A/m, II -  $H_{o3} = 4.76 \cdot 10^7$  A/m, III -  $H_{o3} = 6.35 \cdot 10^7$  A/m, IV -  $H_{o3} = 9.52 \cdot 10^7$  A/m)

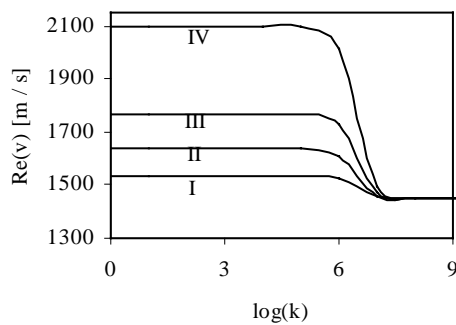


Figure 5. The Dispersion of the Magnetoacoustic Wave in the Fluid

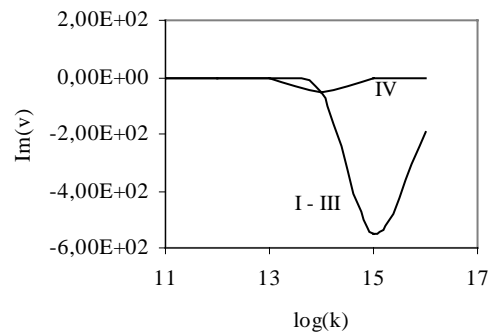


Figure 6. The Damping of the Magnetoacoustic Wave in the Fluid

We see that neither for the lattice, nor for the vortex fluid for  $k < 10^5 \text{ m}^{-1}$ , the magnetoacoustic wave is not dispersed or damped.

## 6 Conclusions

The complicated magnetomechanical structure of vortices in the type-II superconductor can be described within a continuum model. It is possible to show that both the magnetoelastic and the magnetoacoustic waves can propagate along such vortex field. The analysis of their dispersion and damping proves that propagation is possible up to  $k = 10^5 \text{ m}^{-1}$  (cf. Figs.1 to 6). It means that such waves can be modulated and carry signals superposed on them. This is a very interesting result. It shows that a vortex field might serve as a medium able to do a communication at low temperatures in a quite new way.

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*Address:* Dr. Andrzej Drzewiecki, Prof. Dr.-Ing. habil. Jan Adam Kołodziej and Prof. Dr. habil. Bogdan Tadeusz Maruszewski, Institute of Applied Mechanics, Poznan University of Technology, ul. Piotrowo 3, 60 – 965 Poznan, Poland, e-mail: Andrzej.Drzewiecki@put.poznan.pl