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Transverse Vibrations of a Continuous Beam on Rigid and Elastic Supports under the Action of Moving Bodies

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In the present paper the method of substructures is used to derive transverse vibration equations of a continuous beam on rigid and elastic supports under the action of moving bodies. An algorithm for calculating the solutions of vibration equations of a continuous beam is presented. From this algorithm, a computer program is created using C++ language.

1 Introduction

The transverse vibration of a simple beam and of a continuous beam on elastic supports under the action of moving bodies has been mentioned in works such as (Filippov et al., 1974; Popp and Schiehlen, 1993; Nguyen Van Khang et al., 1999; Nguyen Van Khang et al., 2000). Currently transverse vibrations of the beam on many rigid and elastic supports are attracting increased attention in cable stayed bridges. In this work, we use the method of substructures to derive transverse vibration equations of a continuous beam on rigid and elastic supports under the action of moving bodies. An algorithm is suggested to solve the received vibration equations. From this algorithm, a computer program is created using C++ language.

2 Derivation of Vibration Equations Using the Method of Substructures

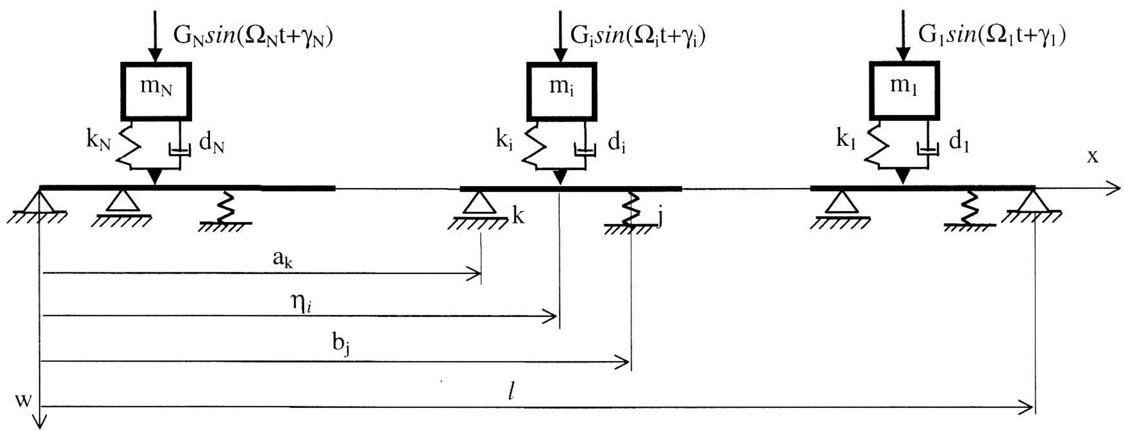

Figure 1. Vibration Model of a Continuous Beam on Elastic and Rigid Supports

Consider a continuous Euler-Bernoulli beam with length l on J elastic, and K rigid supports (Figure 1). Suppose that its mass per unit length is μ ($\mu = \rho A$), and the bending stiffness EI is constant along of its length where ρ is the mass density, A the cross sectional area, E the Young's modulus, I the centroidal moment of inertia. c_j and b_j ($j=1,...,J$) respectively represent the rigidity and the coordinate of the intermediate elastic support j and a_k ($k=1,...,K$) the coordinate of the intermediate rigid support k. The i–th body ($i=1,...,N$) consists of the mass m_i attached to the spring system with rigidity k_i and damping d_i directly proportional to the velocity. The i–th body moves with the velocity v_i and is subjected to the action of a force $G_i \sin(\Omega_i t+\gamma_i)$ caused by an unbalanced mass which rotates with angular velocity Ω_i. Here G_i is the amplitude of the force.

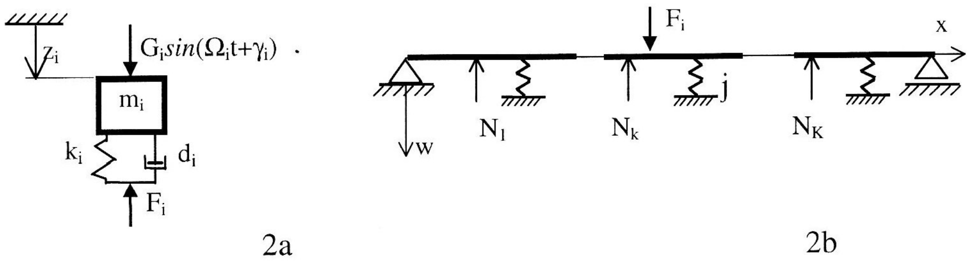

Figure 2. Substructures

Using the method of substructures in order to derive vibration equations of the beam and the bodies, we divide the system into N+1 substructures: beam and N bodies (Figure 2). Here the intermediate rigid supports are replaced by the reaction forces $N_k(t)$.
The position of the i–th body can be determined by (1)

$$\eta_i = v_i\,(t-\tau_i),\ \ t \geq \tau_i \tag{1}$$

where τ_i denotes the time when the i–th body starts moving along the beam with the constant velocity v_i.
Additionally it is supposed that during the motion, the i-th body is not separated from the beam and its velocity v_i satisfies the condition of non impact as following

$$\eta_i > \eta_j \quad (i<j)$$

The following loads are applied to the substructure beam:

- pressure load $p_1(x,z,t)$ of bodies on the beam

$$p_1(x,z,t) = \sum_{i=1}^{N} L_i(t)\left[m_i g + G_i \sin\varphi_i - m_i \ddot{z}_i\right]\delta(x-\eta_i) \tag{2}$$

with $\quad \varphi_i = \Omega_i\, t + \gamma_i$

- reaction load of elastic supports

$$p_2(x,z,t) = -\sum_{j=1}^{J} c_j w(b_j,t)\,\delta(x-b_j) \tag{3}$$

- reaction load of rigid supports

$$p_3(x,z,t) = -\sum_{k=1}^{K} N_k(t)\,\delta(x-a_k) \tag{4}$$

Within this we apply the Dirac-function $\delta(x\text{-}a)$ and the logic signal-function $L_i(t)$, which are determined by the following relations

$$L_i(t) = \begin{cases} 1 & when \quad \tau_i \leq t \leq T_i + \tau_i \\ 0 & when \quad t < \tau_i \ \ or\ \ t > T_i + \tau_i \end{cases}$$

with $\qquad T_i = \dfrac{l}{v_i}$

and $\qquad \delta\,(x\text{-}a) = \lim\limits_{\varepsilon\to 0} \delta_\varepsilon(x\text{-}a)$

with $$\delta_\varepsilon(x-a) = \begin{cases} \dfrac{1}{2\varepsilon} & when \quad |x-a| \leq \varepsilon \\ 0 & when \quad |x-a| > \varepsilon \end{cases}$$

Vibration differential equations of substructures can be obtained by applying the basic principles of dynamics. The equation describing transverse·vibration of a beam including internal friction is (Hagedorn, 1989; Petersen, 1996)

$$EI\left(\frac{\partial^4 w}{\partial x^4}+\alpha\frac{\partial^5 w}{\partial x^4 \partial t}\right)+\mu\left(\frac{\partial^2 w}{\partial t^2}+\beta\frac{\partial w}{\partial t}\right)=p(x,z,t) \tag{5}$$

$$p(x,z,t) = p_1(x,z,t) + p_2(x,z,t) + p_3(x,z,t) \tag{6}$$

in which α and β are damping constants.
The equation describing the vibration of the i-th body has the following form

$$L_i(t)(m_i\ddot{z}_i + d_i\dot{z}_i + k_i z_i) = L_i(t)(m_i g + G_i \sin\varphi_i + d_i \dot{w}_{\eta_i} + k_i w_{\eta_i}), \quad (i=1,...,N) \tag{7}$$

in which $w_{\eta_i} = w(\eta_i, t)$; $\quad \dot{w}_{\eta_i} = \dfrac{\partial w(\eta_i,t)}{\partial t}$ (8)

The constraints at the rigid supports are given in the form

$$w(a_k,t) = 0, \quad (k=1,...,K) \tag{9}$$

The equations of motion (5), (7) and the constraints (9) form a mixed system of partial differential equations, an ordinary differential equation and nonlinear algebraic equations. Four boundary conditions, two at x=0, two at x=l, and initial conditions must be specified for the solution of the equations.
The boundary conditions have the following form

$$x=0: \quad w(0,t) = \frac{\partial^2 w(0,t)}{\partial x^2} = 0 \tag{10}$$

$$x=l: \quad w(l,t) = \frac{\partial^2 w(l,t)}{\partial x^2} = 0 \tag{11}$$

The initial conditions are expressions of the form

$$t=\tau_1: \qquad w(x,\tau_1) = f^{(1)}(x), \quad \frac{\partial w(x,\tau_1)}{\partial t} = f^{(2)}(x) \tag{12}$$

$$t=\tau_i: \qquad z_i(\tau_i) = z_i^{(0)}(\tau_i), \quad \dot{z}_i(\tau_i) = \dot{z}_i^{(0)}(\tau_i), \quad (i=1,...,N) \tag{13}$$

From the constraints (9) we have

$$f^{(1)}(a_k) = 0, \quad f^{(2)}(a_k) = 0 \tag{14}$$

3 Transformation of the Mixed Equation System into Ordinary Differential Equations

Using the mode superposition principle a solution of equations (5) and (7) with the boundary conditions (10) and (11) is assumed in the form

$$w(x,t) = \sum_{r=1}^{n} q_r(t) \sin\frac{r\pi x}{l} \tag{15}$$

in which $q_r(t)$ (r=1,...,n) are generalized coordinates to be determined. From the relation (15) we can calculate the particular derivation

$$w_{\eta_i} = L_i(t)w(x,t)\big|_{x=\eta_i} = L_i(t)\sum_{r=1}^{n} q_r(t)\sin\frac{r\pi\eta_i}{l} \tag{16}$$

$$\dot{w}_{\eta_i} = L_i(t)\left[\sum_{r=1}^{n}\dot{q}_r \sin\frac{r\pi\eta_i}{l} + \sum_{r=1}^{n} q_r \frac{r\pi v_i}{l}\cos\frac{r\pi\eta_i}{l}\right] \tag{17}$$

By substituting the relation (15) into the equation (5) we obtain

$$EI\left[\sum_{r=1}^{n}\left(\frac{r\pi}{l}\right)^4 q_r \sin\frac{r\pi x}{l} + \alpha\sum_{r=1}^{n}\left(\frac{r\pi}{l}\right)^4 \dot{q}_r \sin\frac{r\pi x}{l}\right] +$$

$$\mu\left[\sum_{r=1}^{n}\ddot{q}_r \sin\frac{r\pi x}{l} + \beta\sum_{r=1}^{n}\dot{q}_r \sin\frac{r\pi x}{l}\right] = p(x,z,t)$$

$$\sum_{r=1}^{n}\left\{\ddot{q}_r + \left[\frac{EI\alpha}{\mu}\left(\frac{r\pi}{l}\right)^4 + \beta\right]\dot{q}_r + \frac{EI}{\mu}\left(\frac{r\pi}{l}\right)^4 q_r\right\}\sin\frac{r\pi x}{l} = \frac{1}{\mu}\, p(x,z,t) \tag{18}$$

Multiplying equation (18) by $\sin\frac{s\pi x}{l}$, integrating from 0 to l, and using the orthogonality condition

$$\int_0^l \sin\frac{r\pi x}{l}\sin\frac{s\pi x}{l}\,dx = \begin{cases} 0 & \text{when } r \neq s \\ \frac{1}{2} & \text{when } r = s \end{cases}$$

we obtain the ordinary differential equations

$$\ddot{q}_s + \left[\frac{EI\alpha}{\mu}\left(\frac{s\pi}{l}\right)^4 + \beta\right]\dot{q}_s + \frac{EI}{\mu}\left(\frac{s\pi}{l}\right)^4 q_s = \frac{2}{l\mu}\int_0^l p(x,z,t)\sin\frac{s\pi x}{l}\,dx \qquad (s = 1,\ldots,n) \tag{19}$$

Using the characteristics of the Dirac-function, we have

$$\int_0^l p(x,z,t)\sin\frac{s\pi x}{l}dx = \int_0^l p_1(x,z,t)\sin\frac{s\pi x}{l}dx + \int_0^l p_2(x,z,t)\sin\frac{s\pi x}{l}dx + \int_0^l p_3(x,z,t)\sin\frac{s\pi x}{l}dx$$

$$= \sum_{i=1}^{N} L_i(t)\left[m_i g + G_i\sin\varphi_i - m_i\ddot{z}_i\right]\sin\frac{s\pi\eta_i}{l} - \sum_{r=1}^{n}\left[\sum_{j=1}^{J} c_j \sin\frac{r\pi b_j}{l}\sin\frac{s\pi b_j}{l}\right]q_r - \sum_{k=1}^{K} N_k(t)\sin\frac{s\pi a_k}{l} \tag{20}$$

Substituting the relation (20) into equations (19) we obtain

$$\ddot{q}_s + \left[\frac{EI\alpha}{\mu}\left(\frac{s\pi}{l}\right)^4 + \beta\right]\dot{q}_s + \frac{EI}{\mu}\left(\frac{s\pi}{l}\right)^4 q_s = \frac{2}{l\mu}\sum_{i=1}^{N} L_i(t)\left(m_i g + G_i\sin\varphi_i - m_i\ddot{z}_i\right)\sin\frac{s\pi\eta_i}{l}$$
$$-\frac{2}{l\mu}\sum_{k=1}^{K} N_k(t)\sin\frac{s\pi a_k}{l} - \frac{2}{l\mu}\sum_{r=1}^{n}\left[\sum_{j=1}^{J} c_j\sin\frac{r\pi b_j}{l}\sin\frac{s\pi b_j}{l}\right]q_r, \qquad (s = 1,\ldots,n) \tag{21}$$

Substituting the relations (16) and (17) into equation (7) we have

$$L_i(t)\left[m_i\ddot{z}_i + d_i\dot{z}_i + k_i z_i\right] = L_i(t)\left[m_i g + G_i\sin\varphi_i\right] + L_i(t)\sum_{r=1}^{n}\left(d_i\sin\frac{r\pi\eta_i}{l}\right)\dot{q}_r$$
$$+ L_i(t)\sum_{r=1}^{n}\left(k_i\sin\frac{r\pi\eta_i}{l} + d_i\frac{r\pi v_i}{l}\cos\frac{r\pi\eta_i}{l}\right)q_r, \qquad (i = 1,\ldots,N) \tag{22}$$

Substituting the relation (15) into the constraints (9) we obtain

$$w(a_k,t) = \sum_{r=1}^{n} q_r(t)\sin\frac{r\pi a_k}{l} = 0, \qquad (k = 1,\ldots,K) \tag{23}$$

The second derivative with respect to time of the equations (23) reads

$$\sum_{r=1}^{n} \ddot{q}_r(t) \sin\frac{r\pi a_k}{l} = 0, \qquad (k = 1,\ldots, K) \tag{24}$$

We can prove that the equation system (23) is equivalent to equation system (24) when the initial values of the generalized coordinates $q_r(\tau_1)$ and of the generalized velocities $\dot{q}_r(\tau_1)$ satisfy the conditions

$$\sum_{r=1}^{n} q_r(\tau_1) \sin\frac{r\pi a_k}{l} = 0, \quad \sum_{r=1}^{n} \dot{q}_r(\tau_1) \sin\frac{r\pi a_k}{l} = 0, \qquad (k = 1,\ldots, K) \tag{25}$$

Therewith the mixed equation system (5), (7), and (9) describing the transverse vibration of a continuous beam on J elastic, and K rigid supports under the action of moving bodies is transformed into the ordinary differential equations (21), (22) and (24). Thus we have $(n+N+K)$ equations with $(n+N+K)$ unknowns which are $q_s\,(s = 1,\ldots,n)$, $z_i\,(i = 1,\ldots,N)$, and $N_k\,(k = 1,\ldots,K)$.

4 Numerical Computation of the Transverse Vibrations of the Continuous Beam

For the integration of the ordinary differential equations (21), (22), and (24) we rewrite the equations (22) in the form

$$\begin{aligned} L_i(t)\ddot{z}_i = L_i(t)\Bigg[&\sum_{r=1}^{n}\left(\frac{d_i}{m_i}\sin\frac{r\pi\eta_i}{l}\right)\dot{q}_r - \frac{d_i}{m_i}\dot{z}_i + \sum_{r=1}^{n}\left(\frac{k_i}{m_i}\sin\frac{r\pi\eta_i}{l} + \frac{d_i}{m_i}\frac{r\pi v_i}{l}\cos\frac{r\pi\eta_i}{l}\right)q_r \\ &-\frac{k_i}{m_i}z_i + g + \frac{G_i}{m_i}\sin\varphi_i\Bigg] \qquad (i = 1,\ldots, N) \end{aligned} \tag{26}$$

Substituting the relation (26) into the equation (21) we obtain

$$\begin{aligned} \ddot{q}_s = &-\sum_{r=1}^{n}\left\{\delta_r^s\left[\frac{EI\alpha}{\mu}\left(\frac{\pi}{l}\right)^4 s^4 + \beta\right] + \frac{2}{l\mu}\sum_{i=1}^{N} L_i(t) d_i \sin\frac{s\pi\eta_i}{l}\sin\frac{r\pi\eta_i}{l}\right\}\dot{q}_r + \frac{2}{l\mu}\sum_{i=1}^{N} L_i(t)\left(d_i\sin\frac{s\pi\eta_i}{l}\right)\dot{z}_i - \\ &\sum_{r=1}^{n}\left\{\delta_r^s\frac{EI}{\mu}\left(\frac{\pi}{l}\right)^4 s^4 + \frac{2}{l\mu}\sum_{i=1}^{N} L_i(t)\left(d_i\frac{r\pi v_i}{l}\cos\frac{r\pi\eta_i}{l} + k_i\sin\frac{r\pi\eta_i}{l}\right)\sin\frac{s\pi\eta_i}{l}\right\}q_r + \frac{2}{l\mu}\sum_{i=1}^{N} L_i(t)\left(k_i\sin\frac{s\pi\eta_i}{l}\right)z_i \\ &-\frac{2}{l\mu}\sum_{k=1}^{K} N_k(t)\sin\frac{s\pi a_k}{l} - \frac{2}{l\mu}\sum_{r=1}^{n}\left[\sum_{j=1}^{J} c_j\sin\frac{r\pi b_j}{l}\sin\frac{s\pi b_j}{l}\right]q_r, \qquad (s = 1,\ldots, n) \end{aligned} \tag{27}$$

in which

$$\delta_r^s = \begin{cases} 1 & \text{when } r = s \\ 0 & \text{when } r \neq s \end{cases}$$

$$\varphi_i = \Omega_i t + \alpha_i \; ; \; \eta_i = v_i(t - \tau_i)$$

Using the vectors

$$\begin{aligned} \mathbf{q} &= [q_1 \; q_2 \; \ldots \; q_n]^T \\ \mathbf{z} &= [z_1 \; z_2 \; \ldots \; z_N]^T \\ \mathbf{N} &= [N_1 \; N_2 \; \ldots \; N_K]^T \end{aligned} \tag{28}$$

the equations (27) can also be expressed in matrix form as

$$\ddot{\mathbf{q}} = \mathbf{U}\dot{\mathbf{q}} + \mathbf{S}_2\dot{\mathbf{z}} + \mathbf{V}\mathbf{q} + \mathbf{S}_3\mathbf{z} - \frac{2}{l\mu}\mathbf{S}_1\mathbf{N} \tag{29}$$

In this new expression the matrices $\mathbf{S}_1$, $\mathbf{S}_2$, $\mathbf{S}_3$, $\mathbf{U}$, and $\mathbf{V}$ are defined as

$$\mathbf{S}_1 = \left\{ s_{rk}^{(1)} \right\}, \quad s_{rk}^{(1)} = \sin\frac{r\pi a_k}{l}, \quad (r = 1,...,n;\ k = 1,...,K) \tag{30}$$

$$\mathbf{S}_2 = \left\{ s_{ri}^{(2)} \right\}, \quad s_{ri}^{(2)} = \frac{2}{l\mu} L_i(t) d_i \sin\frac{r\pi\eta_i}{l}, \quad (r = 1,...,n;\ i = 1,...,N) \tag{31}$$

$$\mathbf{S}_3 = \left\{ s_{ri}^{(3)} \right\}, \quad s_{ri}^{(3)} = \frac{2}{l\mu} L_i(t) k_i \sin\frac{r\pi\eta_i}{l}, \quad (r = 1,...,n;\ i = 1,...,N) \tag{32}$$

$$\mathbf{U} = \{u_{sr}\}, \quad u_{sr} = -\delta_r^s \left[\frac{EI\alpha}{\mu}\left(\frac{\pi}{l}\right)^4 s^4 + \beta \right] - \frac{2}{l\mu}\sum_{i=1}^{N} L_i(t) d_i \sin\frac{s\pi\eta_i}{l}\sin\frac{r\pi\eta_i}{l} \qquad (s, r = 1,...,n) \tag{33}$$

$$\mathbf{V} = \{v_{sr}\}, v_{sr} = -\frac{2}{l\mu}\sum_{i=1}^{N} L_i(t)\left(d_i \frac{r\pi v_i}{l}\cos\frac{r\pi\eta_i}{l} + k_i \sin\frac{r\pi\eta_i}{l} \right)\sin\frac{s\pi\eta_i}{l} - \delta_r^s \frac{EI}{\mu}\left(\frac{\pi}{l}\right)^4 s^4$$
$$-\frac{2}{l\mu}\sum_{j=1}^{J} c_j \sin\frac{r\pi b_j}{l}\sin\frac{s\pi b_j}{l}, \qquad (s, r = 1,...,n) \tag{34}$$

Equation (29) is transformed into

$$\mathbf{S}_1^{\mathrm{T}}\mathbf{S}_1\mathbf{N} = \mathbf{S}_1^{\mathrm{T}} \frac{l\mu}{2}\left[-\ddot{\mathbf{q}} + \mathbf{U}\dot{\mathbf{q}} + \mathbf{S}_2\dot{\mathbf{z}} + \mathbf{V}\mathbf{q} + \mathbf{S}_3\mathbf{z} \right] \tag{35}$$

Solving the equation (35) for $\mathbf{N}(t)$ and using the conditions (24) $\mathbf{S}_1^{\mathrm{T}}\ddot{\mathbf{q}} = 0$ we have the expression for the reaction forces of the rigid supports

$$\mathbf{N} = \mathbf{A}^{-1}\mathbf{S}_1^{\mathrm{T}} \frac{l\mu}{2}\left[\mathbf{U}\dot{\mathbf{q}} + \mathbf{S}_2\dot{\mathbf{z}} + \mathbf{V}\mathbf{q} + \mathbf{S}_3\mathbf{z} \right] \tag{36}$$

in which we put $\mathbf{A} = \mathbf{S}_1^{\mathrm{T}}\mathbf{S}_1$.

Substituting the relation (36) into the equation (29) we obtain

$$\ddot{\mathbf{q}} = \mathbf{H}\left[\mathbf{U}\dot{\mathbf{q}} + \mathbf{S}_2\dot{\mathbf{z}} + \mathbf{V}\mathbf{q} + \mathbf{S}_3\mathbf{z} \right] \tag{37}$$

In equation (37) the matrix $\mathbf{H}$ is defined by

$$\mathbf{H} = \mathbf{I} - \mathbf{S}_1\mathbf{A}^{-1}\mathbf{S}_1^T \tag{38}$$

where $\mathbf{I}$ is the identity matrix. If we describe the new vector

$$\mathbf{y} = \left[q_1 \ q_2 \ \ldots \ q_n \ z_1 \ z_2 \ \ldots \ z_N \right]^T \tag{39}$$

then the differential equations (37) and (26) can be written in the following matrix form

$$\ddot{\mathbf{y}} = \mathbf{B}(t)\dot{\mathbf{y}} + \mathbf{C}(t)\mathbf{y} + \mathbf{f}(t) \tag{40}$$

The matrices $\mathbf{B}(t)$, $\mathbf{C}(t)$ and the vector $\mathbf{f}(t)$ are defined as

$$\mathbf{B}(t)=\begin{bmatrix} & \mathbf{B}_1 & & & \mathbf{B}_2 & \\ b_{n+1,1} & \cdots & b_{n+1,n} & b_{n+1,n+1} & \cdots & b_{n+1,n+N} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ b_{n+N,1} & \cdots & b_{n+N,n} & b_{n+N,n+1} & \cdots & b_{n+N,n+N} \end{bmatrix}$$

where $\mathbf{B}_1 = \mathbf{H}\,\mathbf{U},\ \mathbf{B}_2 = \mathbf{HS}_2$

$$b_{n+i,r} = L_i(t)\frac{d_i}{m_i}\sin\frac{r\pi\eta_i}{l}, \quad (r=1,\ldots,n;\ i=1,\ldots,N)$$

$$b_{n+i,n+j} = -L_i(t)\delta_i^j\frac{d_i}{m_i}, \quad (i,j=1,\ldots,N)$$

$$\mathbf{C}(t)=\begin{bmatrix} & \mathbf{C}_1 & & & \mathbf{C}_2 & \\ c_{n+1,1} & \cdots & c_{n+1,n} & c_{n+1,n+1} & \cdots & c_{n+1,n+N} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ c_{n+N,1} & \cdots & c_{n+N,n} & c_{n+N,n+1} & \cdots & c_{n+N,n+N} \end{bmatrix}$$

where $\mathbf{C}_1 = \mathbf{H}\mathbf{V},\ \mathbf{C}_2 = \mathbf{H}\,\mathbf{S}_3$

$$c_{n+i,r} = L_i(t)\left(\frac{d_i}{m_i}\frac{r\pi v_i}{l}\cos\frac{r\pi\eta_i}{l}+\frac{k_i}{m_i}\sin\frac{r\pi\eta_i}{l}\right), \quad (r=1,\ldots,n;\ i=1,\ldots,N)$$

$$c_{n+i,n+j} = -L_i(t)\delta_i^j\frac{k_i}{m_i}, \quad (i,j=1,\ldots,N)$$

$$\mathbf{f} = \begin{bmatrix} f_1 & f_2 & \cdots & f_{n+N}\end{bmatrix}^T$$

where $f_s=0$, $(s=1,\ldots,n)$

$$f_{n+i}(t) = L_i(t)\left[g+\frac{G_i}{m_i}\sin(\Omega_i t+\gamma_i)\right], \quad (i=1,\ldots,N)$$

The Runge-Kutta method is used for calculating the solutions of the ordinary differential equations (40). From this algorithm, a computer program for calculating transverse vibrations of continuous beam (VIBEAM) is created using C^{++} language at the Hanoi University of Technology.

5 The Transverse Vibration of a Cable Stayed Bridge in Vietnam

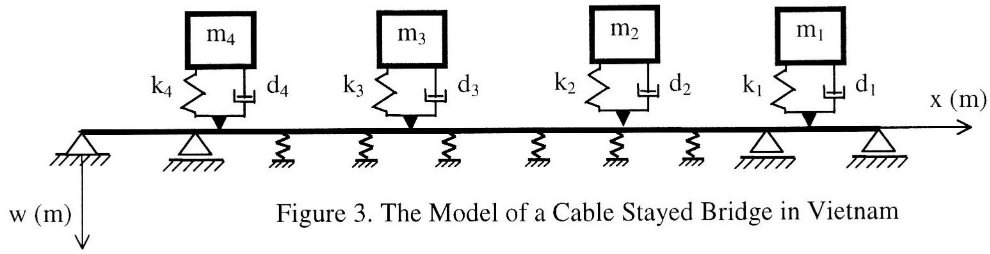

Figure 3. The Model of a Cable Stayed Bridge in Vietnam

The model of a cable stayed bridge in Vietnam influenced by the action of moving bodies is drawn in Figure 3. The data for calculating the model is given in Table 1.

l = 173.9 m	b_6 = 133 m	m_3 = 6515 kg
ρA = 3629.89 kg/m	c_6 = 10294933 N/m	k_3 = 716781.38 N/m
EI = 1928552021 Nm^2	N = 4	d_3 = 2871.74 Ns/m
Mku = 0.0162 m^3	m_1 = 6515 kg	v_3 = 19.444 m/s
α = 0.027 s	k_1 = 716781.38 N/m	G_3 = 0 N
β = 0.01 1/s	d_1 = 2871.74 Ns/m	Ω_3 = 0 rad/s
K = 2	v_1 = 19.444 m/s	γ_3 = 0 rad
a_1 = 22.5 m	G_1 = 0 N	τ_3 = 4 s
a_2 = 151.4 m	Ω_1 = 0 rad/s	m_4 = 6515 kg
J = 6	γ_1 = 0 rad	k_4 = 716781.38 N/m
b_1 = 40.9 m	τ_1 = 0 s	d_4 = 2871.74 Ns/m
c_1 = 10294933 N/m	m_2 = 6515 kg	v_4 = 19.444 m/s
b_2 = 59.2 m	k_2 = 716781.38 N/m	G_4 = 0 N
c_2 = 4344805 m	d_2 = 2871.74 Ns/m	Ω_4 = 0 rad/s
b_3 = 77.5 m	v_2 = 19.444 m/s	γ_4 = 0 rad
c_3 = 2409446 N/m	G_2 = 0 N	τ_4 = 6 s
b_4 = 96.4 m	Ω_2 = 0 rad/s	g = 9.81 m/s^2
c_4 = 2409446 N/m	γ_2 = 0 rad	n = 15
b_5 = 114.7 m	τ_2 = 2 s	t_c = 15 s
c_5 = 4344805 N/m		

Table 1. Data for Calculating the Model of a Cable Stayed Bridge in Vietnam

Computational results are presented in Figures 4, 5, 6, and 7. In Figure 4 the transverse deflection of the continuous beam at the time t = 7.5 s is demonstrated. In Figure 5, the curve of the dynamic stress of the cross section at 86.95 m is plotted. Figures 6 and 7 show the transverse vibrations of the cross sections at 10.8 m and 86.95 m.

6 Conclusion

A system of vibration equations of a continuous beam on elastic and rigid supports under the action of moving bodies has been determined by the application of the substructure method. Also an algorithm and the computer program (VIBEAM) for the numerical calculation of vibrations of beams have been created at the Hanoi University of Technology. The VIBEAM program can be used for analysing in the design of bridges that bear moving bodies.

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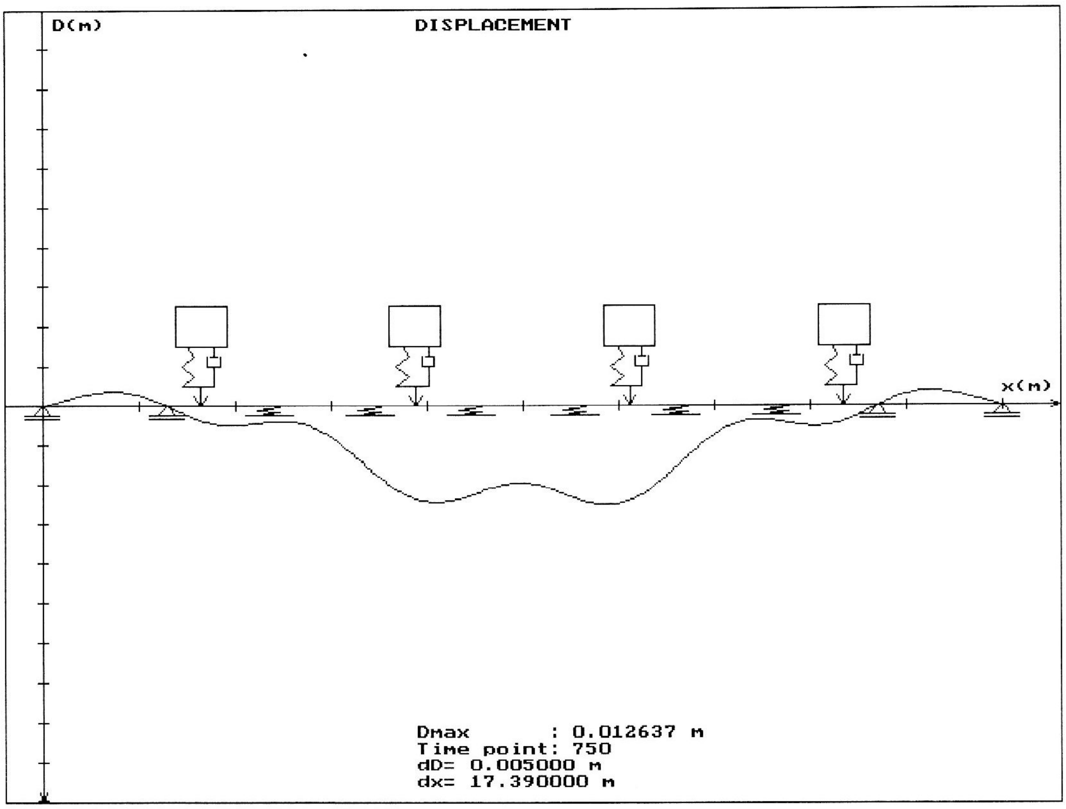

Figure 4. The Deflection of the Continuous Beam at the Time t = 7.5 (s) .

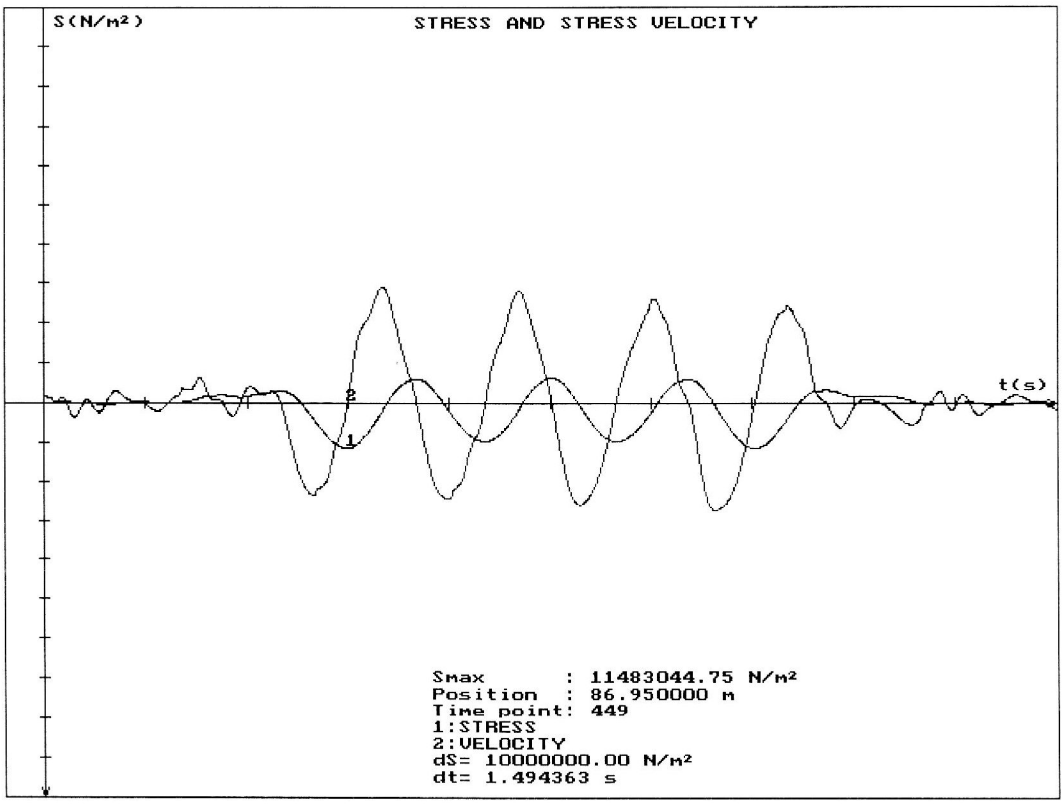

Figure 5. The Dynamic Stress of the Cross Section 86.95 (m)

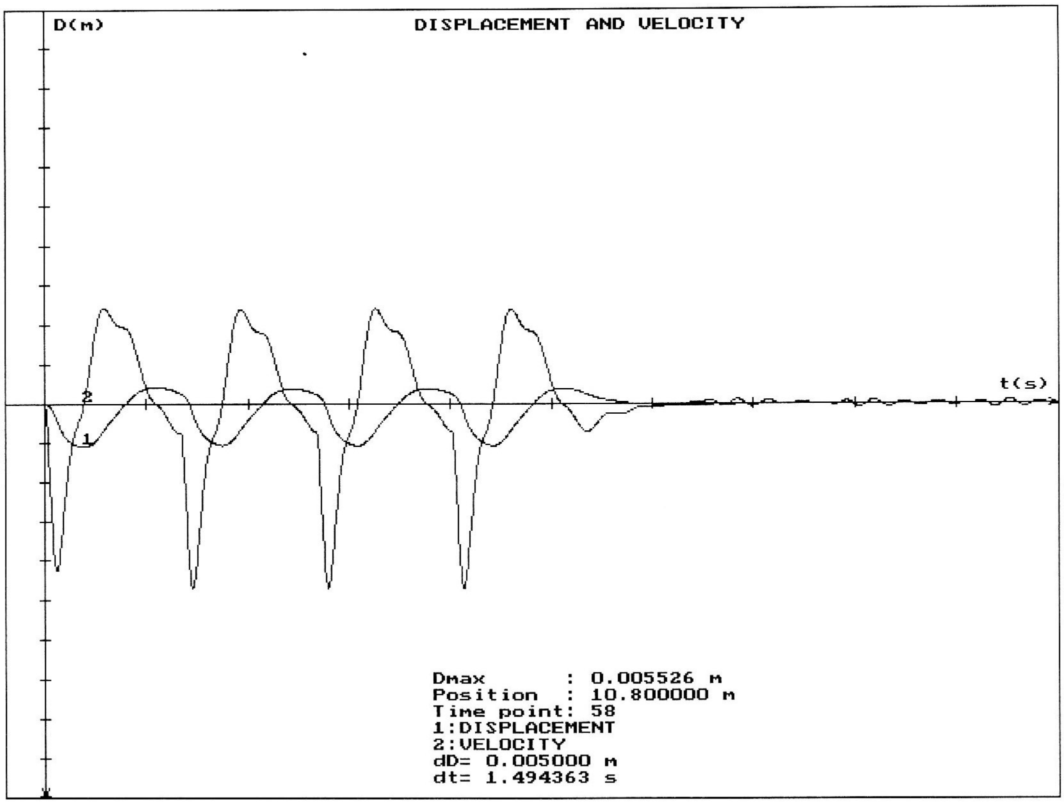

Figure 6. TheTransverse Vibrations of the Cross Section 10.8 (m)

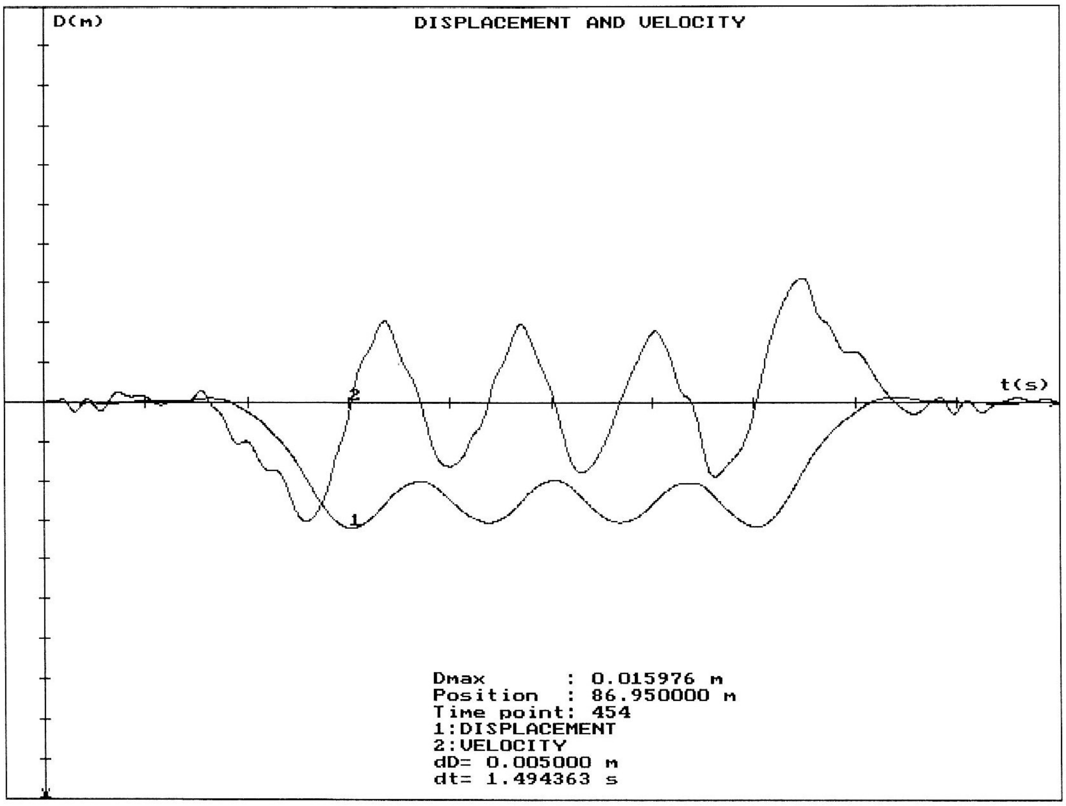

Figure 7. The Transverse Vibrations of the Cross Section 86.95 (m)

Literature

1. Filippov, A.P.; Kokhmaniuk, X.X.; Borobuv, IU.X.: Action of the dynamic loads on the elements of construction (in Russian). Naukova dumka, Kiev, (1974).
2. Hagedorn, P.: Technische Schwingungslehre, Band 2. Springer-Verlag, Berlin, (1989).
3. Nguyen Van Khang; Do Xuan Tho; Hoang Ha: Biegeschwingungen des einfachen Brückenträgers unter mehreren bewegten Körpern. Technische Mechanik, Band 19, Heft 3, 203-210, (1999).
4. Nguyen Van Khang; Hoang Ha; Vu Van Khiem; Do Xuan Tho: On the transverse vibration of beam-bridges under the action of a moving bodies. IUTAM Symposium on Recent Developments in Non-linear Oscillations of Mechanical Systems, 187-195, Kluwer Academic Publishers, (2000).
5. Petersen, C.: Dynamik der Baukonstruktion. Vieweg, Braunschweig/Wiesbaden, (1996).
6. Popp, K.; Schiehlen, W.: Fahrzeugdynamik. B.G. Teubner, Stuttgart, (1993).

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