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Identification of Rigid Body Parameters Using Experimental Modal Analysis Data

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A simple direct method is presented here to identify the rigid body parameters of a structure under a free-free condition using the measured vibration data and geometrical co-ordinates of the measurement points relative to an arbitrarily selected general co-ordinate system. These parameters consist of mass, co-ordinates of mass centre, mass-moment of inertia, and the corresponding required principal values and axes. The test structure should be weakly suspended or soft mounted to ground. The rigid body motion should be carefully selected from the measured transfer functions. Practical considerations like the selection of general co-ordinate system, the measurement and excitation points, the minimum set of measurements etc., to be noted during performing the vibration tests or evaluating the rigid body parameters are illustrated with the help of three practical examples. The accuracy of the identified parameters depends, to a great extent, on these considerations. Comparisons between identified and theoretical results are also given.

1 Introduction

The experimentally based procedures for identifying the rigid body parameters have attracted the attention of vibration engineers during the last decade. These procedures, which are iterative or direct, are based on the determination of the lower residual properties from measured frequency response functions (FRFs). Due to the vast development of measurement techniques and equipment in the past years the accurate identification of rigid body parameters has become possible. This could overcome the difficulties in performing the traditional pendulum tests for determining the rigid body inertia properties of a test structure, since these tests are often liable to large experimental errors and can be time consuming, dangerous, and difficult to perform if large structures are to investigate.

The identified parameters can be used to update the accuracy of a system model subjected to a simulation, to optimise the dynamic properties of a structure, e.g. engine mounts, suspension of an automobile, and to simulate the frequency response functions including the contribution of rigid body modes between any pair of measurement points.

Several researchers have made different contributions in the field of identification of rigid body parameters using rigid body modes and mode shapes from experimental data. Some papers contain procedures to identify the inertia parameters of a rigid body and the damping and stiffness matrices of the supports using vibration data of grounded structures (Pandit and Hu, 1994; Pandit, Yao and Hu, 1994).

Other papers use a direct method to obtain the mass properties from the lower residual characteristics of experimental FRFs (Bretl and Conti, 1987; Fregolent and Sestieri, 1996). The non-linear problem is then divided into a series of simpler linear problems. Artificial test data were used as input by Bretl and Conti (1987). In this paper the mass of the body is assumed to be known, whilst a conditioning analysis of the solving equations is missing. This problem was overcome by Fregolent and Sestieri (1996) by using experimental data and the method does not require the prior knowledge of the mass value.

Niebergall and Hahn (1997) apply an algorithm for the simultaneous automatic experimental identification of the ten inertia parameters of a rigid body using the complete information hidden in the non-linear model equations of the test body.

In this paper a direct approach, based on the theory developed by Okuma and Shi (1996), has been extensively investigated and important practical considerations have been discussed regarding the selection of general co-ordinate system, determination of the excitation and measurement points, minimum sets of measurements necessary for the evaluation of rigid body parameters, suspension of test structures, and correct determination of rigid body motion or lower residuals from the measured FRFs. Three different practical examples have been implemented to demonstrate the capability and validity of the identification method. The method does not require the knowledge of any of the rigid body parameters, since these parameters are directly obtained by measuring the required information. The test results show that the method can be recommended because of its simplicity even for complicated structures in practice.

2 Theory

2.1 Description of the Method

Consider a three-dimensional structure and let $O(x, y, z)$ be a general co-ordinate system with respect to which the selected excitation and measurement points are determined.

An arbitrary rigid body motion of a point on the structure can be described in six independent degrees of freedom. This point, which will be denoted later as "representative point" RP, is assumed as the origin of the general co-ordinate system. For very small oscillations, the motion of the rigid body parts of the structure can be described by the three amplitudes a_{ox}, a_{oy}, and a_{oz} of the linear accelerations of the representative point RP and the three amplitudes α_{ox}, α_{oy}, and α_{oz} of the rotational accelerations in the co-ordinate system $O(x, y, z)$.

If G_x, G_y and G_z are the co-ordinates of the mass centre in this co-ordinate system, then the equations of motion of the structure under a free-free condition is given in linear form by:

$$\begin{bmatrix} m & 0 & 0 & 0 & m\cdot G_z & -m\cdot G_y \\ 0 & m & 0 & -m\cdot G_z & 0 & m\cdot G_x \\ 0 & 0 & m & m\cdot G_y & -m\cdot G_x & 0 \\ 0 & -m\cdot G_z & m\cdot G_y & I_{oxx} & -I_{oxy} & -I_{oxz} \\ m\cdot G_z & 0 & -m\cdot G_x & -I_{oxy} & I_{oyy} & -I_{oyz} \\ -m\cdot G_y & m\cdot G_x & 0 & -I_{oxz} & -I_{oyz} & I_{ozz} \end{bmatrix} \begin{Bmatrix} a_{ox} \\ a_{oy} \\ a_{oz} \\ \alpha_{ox} \\ \alpha_{oy} \\ \alpha_{oz} \end{Bmatrix} = \begin{Bmatrix} F_{ox} \\ F_{oy} \\ F_{oz} \\ M_{ox} \\ M_{oy} \\ M_{oz} \end{Bmatrix} \quad (1)$$

or

$$[M_o]_{6\times6}\{a_o\}_{6\times1} = \{F_o\}_{6\times1} \quad (2)$$

$[M_o]_{6\times6}$ is the mass matrix whose elements are the rigid body parameters, which have to be determined. $\{a_o\}_{6\times1}$ is the acceleration vector of the representative point RP and the force vector $\{F_o\}_{6\times1}$ results from transferring all forces applied to the structure to the origin of the representative point RP.

The basic principle for the determination of the rigid body parameters is very simple: By a defined force excitation of the structure, the acceleration vector of the representative point is measured. If now the vectors $\{F_o\}_{6\times1}$ and $\{a_o\}_{6\times1}$ are known, then it is possible to determine the unknown rigid body parameters by manipulation of equation (1).

However, the practical realisation of this idea is not so simple. The reason is that the acceleration vector $\{a_o\}_{6\times1}$ of the rigid body motion can not be measured directly, because all the measured values of accelerations result from the superposition of the rigid body modes and the elastic modes of the structure. In order to avoid a superposition of different modes, one attempts to select the frequency of the excitation force in such a way that the rigid body motion lies far below the frequency range of the elastic modes. In doing so, problems arise always concerning the accuracy of the measurement techniques.

A solution of this problem is possible with the help of experimental modal analysis. As will be shown below, with this method the rigid body accelerations can be extracted from the measured total accelerations of the elastic structure.

In performing the experimental modal analysis a co-ordinate system $O(x, y, z)$ with N measurement DOF's is generally determined (a measurement point on a three-dimensional structure consists of 3 measurement DOF's). The relation between the excitation forces and the response accelerations of the structure in a particular frequency range is expressed by:

$$\{a\}_{N\times1} = \left[H^a(\omega)\right]_{N\times N} \cdot \{F\}_{N\times1} \quad (3)$$

Here $\{a\}_{N\times1}$ is the acceleration vector of the measured DOF's in the co-ordinate system $O(x, y, z)$, $\lfloor H^a(\omega)\rfloor_{N\times N}$ is the inertance matrix (or accelerance matrix) and $\{F\}_{N\times1}$ is the vector of the excitation forces. The modal identification enables us to approximate the experimentally determined inertance matrix, in a frequency interval $\Delta\omega = \omega_{max} - \omega_{min}$, by the following expression:

$$\lfloor H^a(\omega)\rfloor_{N\times N} = [LR]_{N\times N} - \omega^2 \sum_{r=1}^{M} \left[\frac{[R_r]_{N\times N}}{j\omega - \lambda_r} + \frac{[R_r]^*_{N\times N}}{j\omega + \lambda^*_r} \right] - \omega^2 [UR]_{N\times N} \quad (4)$$

where

M	number of modes of vibration data that contributes to the structure's dynamic response within the frequency range under consideration,
$[R_r]_{N\times N}$	residual matrix for mode r,
λ_r	pole value for mode r,
$[LR_r]_{N\times N}$	lower residual matrix (residual mass) used to approximate modes at frequencies below ω_{min},
$[UR_r]_{N\times N}$	upper residual matrix (residual stiffness) used to approximate modes at frequencies above ω_{max} and
*	designates complex conjugate.

The frequency ω_{min} in the experimental modal analysis is selected in such a way that it lies below the first eigenfrequency of the elastic modes of the structure. In this way it is possible to describe the inertance matrix $\lfloor H^a(\omega)\rfloor_{N\times N}$ of the rigid body motion of the structure for $\omega \to 0$ by the following expression:

$$\lfloor H^a(\omega)\rfloor_{N\times N} \approx [LR]_{N\times N} \quad (5)$$

Therefore, the acceleration of the extracted rigid body motion can be computed using:

$$\{a\}_{N\times1} = [LR]_{N\times N} \cdot \{F\}_{N\times1} \quad (6)$$

In the following it will be assumed that the structure is excited only by a single force $\{F_q\}_{3\times1} = \{F_{qx}, F_{qy}, F_{qz}\}^T$ at point $Q(q_x, q_y, q_z)$. Then equation (6) will be modified to:

$$\{a\}_{N\times1} = \lfloor LR_q\rfloor_{N\times3} \cdot \{F_q\}_{3\times1} \quad (7)$$

where the matrix $\lfloor LR_q\rfloor_{N\times3}$ is composed of 3 columns, which correspond to the measurement DOF's of the point Q respectively.

Equation (7) gives us the rigid body accelerations of all measured DOF's by exciting the structure by the force $\{F_q\}_{3\times1}$. As an example the acceleration vectors $\{a\}_{N\times1}$, using equation (7), of a right-angled plate are illustrated in Figure 1 by performing experimental modal analysis at three different excitation points.

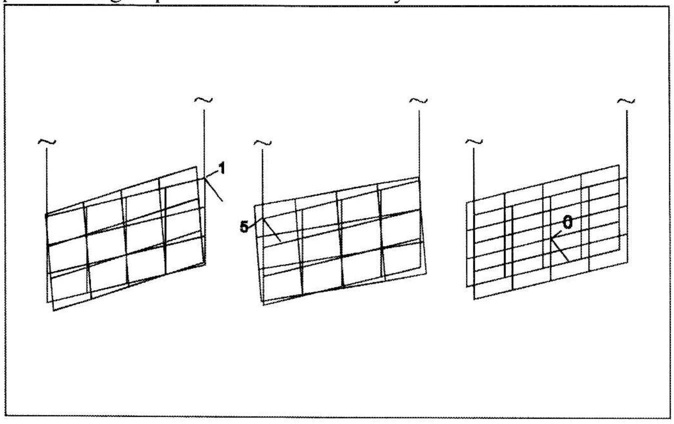

Figure 1. Lower Residuals of Plate 1 at 60 Hz

In order to calculate the required acceleration vector $\{a_o\}_{6\times1}$ of equation (1), then the calculated acceleration vector $\{a\}_{N\times1}$ from equation (7) must be transformed relative to the origin of the coordinate system $O(x,y,z)$ of the representative point. The transformation is executed as follows:

The vector $\{a\}_{N\times1}$ can be calculated from the vector $\{a_o\}_{6\times1}$ by the relation:

$$\{a\}_{N\times1} = [T_a]_{N\times6} \cdot \{a_o\}_{6\times1} \tag{8}$$

The matrix $[T_a]_{Nx6}$ consists only of the co-ordinates of the measurement points $P_i(p_{ix}, p_{iy}, p_{iz})$, $i = 1,2,...,NP, NP = N/3$, relative to the RP and has the following form (each measurement point has 3 measurement DOF's):

$$[T_a]_{Nx6} = \begin{bmatrix} 1 & 0 & 0 & 0 & p_{1z} & -p_{1y} \\ 0 & 1 & 0 & -p_{1z} & 0 & p_{1x} \\ 0 & 0 & 1 & p_{1y} & -p_{1x} & 0 \\ . & . & . & . & . & . \\ 1 & 0 & 0 & 0 & p_{NPz} & -p_{NPy} \\ 0 & 1 & 0 & -p_{NPz} & 0 & p_{NPx} \\ 0 & 0 & 1 & p_{NPy} & -p_{NPx} & 0 \end{bmatrix} \tag{9}$$

Therefor the acceleration vector $\{a_o\}_{6\times1}$ of the RP can be obtained from equation (8) by the least square method as:

$$\{a_o\}_{6\times1} = \left[[T_a]^T_{6\times N} \cdot [T_a]_{N\times6} \right]^{-1}_{6\times6} \cdot [T_a]^T_{6\times N} \cdot \{a\}_{N\times1} \tag{10}$$

Using equations (7) and (10) the acceleration vector of the rigid body motion can be obtained, transformed relative to the representative point RP, as:

$$\{a_o\}_{6\times1} = \left[[T_a]^T_{6\times N} \cdot [T_a]_{N\times6} \right]^{-1}_{6\times6} \cdot [T_a]^T_{6\times N} \cdot [LR_q]_{N\times3} \cdot \{F_q\}_{3\times1} \tag{11}$$

To get the force vector $\{F_o\}_{6\times1}$ corresponding to equation (1) the force vector $\{F_q\}_{3\times1}$ must be transferred relative to the RP with the geometrical transformation matrix $[T_F]_{6\times3}$ as:

$$\{F_0\}_{6\times1} = [T_F]_{6\times3} \cdot \{F_q\}_{3\times1} \tag{12}$$

$$[T_F]_{6\times3} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -q_z & q_y \\ q_z & 0 & -q_x \\ -q_y & q_x & 0 \end{bmatrix} \tag{13}$$

Therefore, using equations (11) and (12) the vectors $\{a_o\}_{6\times1}$ and $\{F_o\}_{6\times1}$ can be determined and they can be used further to evaluate equation (1). It is to be noted that the vector of the excitation force $\{F_q\}_{3\times1}$ appears on either sides of equation (1) whose magnitude can be cancelled out. For the numerical application of equations (11) and (12) we substitute therefore $|F_q| = 1$.

2.2 Identification of the Rigid Body Mass Matrix

In order to apply the above theory and identify the rigid body parameters experimentally as none of these parameters are generally known in advance, then equation (1) can be transformed into the following simultaneous equations with respect to the elements of the rigid body mass matrix as shown below (Okuma and Shi, 1996):

$$[A_o]_{6\times 10}\{M_o\}_{10\times 1} = \{F_o\}_{6\times 1} \tag{14}$$

where $\{M_o\}_{10\times 1}$ is the vector of the rigid body parameters formed from the rigid body mass matrix $[M_o]_{6\times 6}$, i.e.

$$\{M_o\}_{10\times 1} = \{m, mG_x, mG_y, mG_z, I_{oxx}, I_{oyy}, I_{ozz}, I_{oxy}, I_{oxz}, I_{oyz}\}^T \tag{15}$$

and the matrix

$$[A_o]_{6x10} = \begin{bmatrix} a_{ox} & 0 & -\alpha_{oz} & \alpha_{oy} & 0 & 0 & 0 & 0 & 0 & 0 \\ a_{oy} & \alpha_{oz} & 0 & -\alpha_{ox} & 0 & 0 & 0 & 0 & 0 & 0 \\ a_{oz} & -\alpha_{oy} & \alpha_{ox} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & a_{oz} & -a_{oy} & \alpha_{ox} & 0 & 0 & -\alpha_{oy} & -\alpha_{oz} & 0 \\ 0 & -a_{oz} & 0 & a_{ox} & 0 & \alpha_{oy} & 0 & -\alpha_{ox} & 0 & -\alpha_{oz} \\ 0 & a_{oy} & -a_{ox} & 0 & 0 & 0 & \alpha_{oz} & 0 & -\alpha_{ox} & -\alpha_{oy} \end{bmatrix} \tag{16}$$

is the coefficient matrix resulted from the elements of the acceleration vector $\{a_o\}_{6\times 1}$ by the transformation of equation (1).

For a single excitation the number of equations and unknown parameters are 6 and 10 respectively (see equation (14)). Then a second set of 6 equations, obtained by exciting the structure at a different point, seems to be sufficient to make the system of 12 over-determined. However, Okuma and Shi (1996) exhibited that in this case the set of equations is still under-determined and that at least three independent sets of equations with respect to different excitation points are required.

If the structure is excited at m different points then equation (14) has the following form:

$$\begin{bmatrix} [A_o]_1 \\ [A_o]_2 \\ \cdot \\ \cdot \\ [A_o]_m \end{bmatrix}_{6m\times 10} \cdot \{M_o\}_{10x1} = \begin{Bmatrix} \{F_o\}_1 \\ \{F_o\}_2 \\ \cdot \\ \cdot \\ \{F_o\}_m \end{Bmatrix}_{6m\times 1} \tag{17}$$

Then the vector of the rigid body parameters $\{M_o\}_{10\times 1}$ can be obtained from equation (17) by the least square method.

The accuracy of the identified parameters depends strongly on the following points:

- Estimation of the rigid body motion or lower residuals from measured FRFs
- Effects of elastic modes on the rigid body motion
- Method of suspension of the structure
- Selection of excitation points

The identification method has been investigated thoroughly as will be seen in the following examples.

3 Practical Examples

The inertance matrix, which is a symmetric matrix, can be obtained by measuring a row or a column. In this paper, a column has been measured by varying the excitation DOF's and fixing the measurement DOF's. For conducting the experimental modal analysis, acceleration transducers of type 4393 Brüel & Kjær, impact hammer type 8202 Brüel & Kjær, charge amplifier type 2635 Brüel & Kjær and the modal analysis system LMS CADA-X have been used throughout this work.

Excitation points in the following tables denote points used for the evaluation of equation (11). Note that the excitation points should be distributed over either sides of the selected representative point RP for the test structure under investigation. Values shown in parentheses in the attached tables denote the corrected identified rigid body parameters when the mass of the structure is known, while the numbers 1 and 2 in the legends of Figures 5.2 and 5.3 represent the identified and the corrected identified parameters.

3.1 Beam Model

The simplest way to understand the theory demonstrated in section 2 is to apply it to a one-dimensional structure, a beam model, as shown in Figure 2. In order to apply equation (14), which is reduced to a one-dimensional case, two sets of measurements are required. These are obtained by fixing two accelerometers at two different points, i.e. points 1 and 9 as shown in Figure 2. The FRFs are measured in a suitable frequency range of interest including both rigid body modes and elastic modes. Consequently modal parameters are extracted from the measured data and theoretical FRF curves are regenerated. Rigid body motion or lower residuals are then extracted from the measured modal data to identify the inertia parameters of the beam model, as will be discussed in detail in the next example.

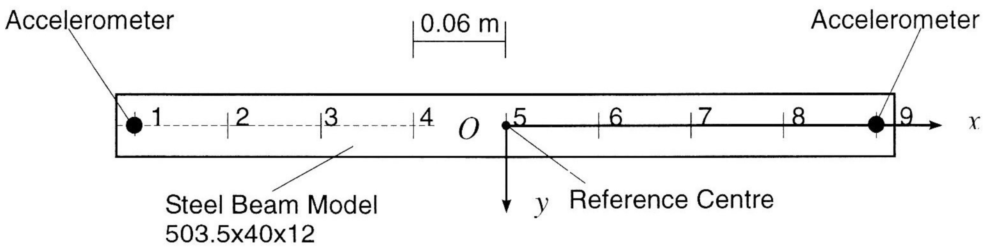

Figure 2. Beam Model Suspended Laterally

Table 1.1 shows the average values of identified rigid body parameters when the general co-ordinate origin O is taken at the mid-point of the beam model, whilst Table 1.2 indicates the same parameters when the origin O is shifted backwards by 0.06m (see Figure 2). Values shown in parentheses in the attached tables denote the corrected identified rigid body parameters when the mass of the structure is known. Results obtained in both cases show that the above method can be used safely and with a good accuracy to identify the rigid body parameters.

Parameters	**Identified data (with known mass)**	**Theoretical data**
Mass m, kg	**1.9266 (1.880)**	**1.880**
Centre distance G_x, m	**0.0002 (0.0002)**	**0.000**
Principal moment of inertia I_{yy}, kg.m²	**0.0406 (0.0397)**	**0.0397**

Table 1.1: Rigid Body Parameters of the Beam Model, Co-ordinate System's Origin $O(x,y)$ at the Centre of the Beam

Parameters	**Identified data (with known mass)**	**Theoretical data**
Mass m, kg	**1.9266 (1.880)**	**1.880**
Centre distance G_x, m	**0.0598 (0.0598)**	**0.060**
Principal moment of inertia I_{yy}, kg.m²	**0.0406 (0.0397)**	**0.0397**

Table 1.2: Rigid Body Parameters of the Beam Model, Co-ordinate System's Origin $O(x,y)$ at 0.060 m away from the Centre of the Beam, see Figure 2.

3.2 Plate 1

The theory is extended now to a two-dimensional structure, for instance a plate of uniform thickness. The plate is suspended laterally with two springs. In this case three sets of measurements are necessary to identify six rigid body parameters. The points 1, 5, and 20 are considered as excitation points (see Figure 3).

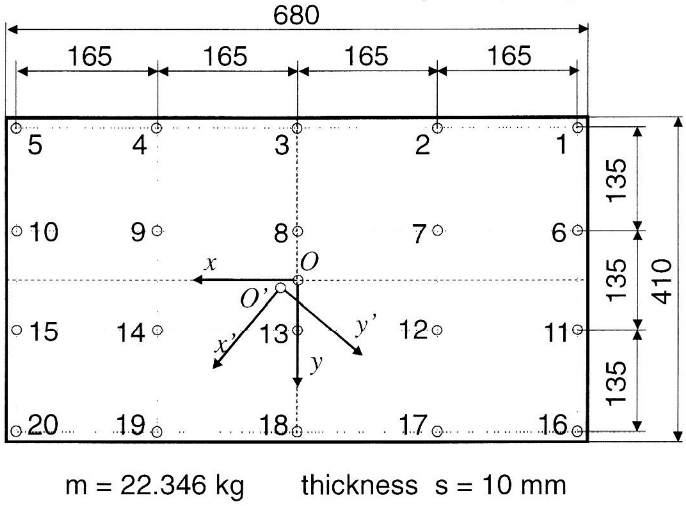

Figure 3. Plate 1 with Constant Thickness

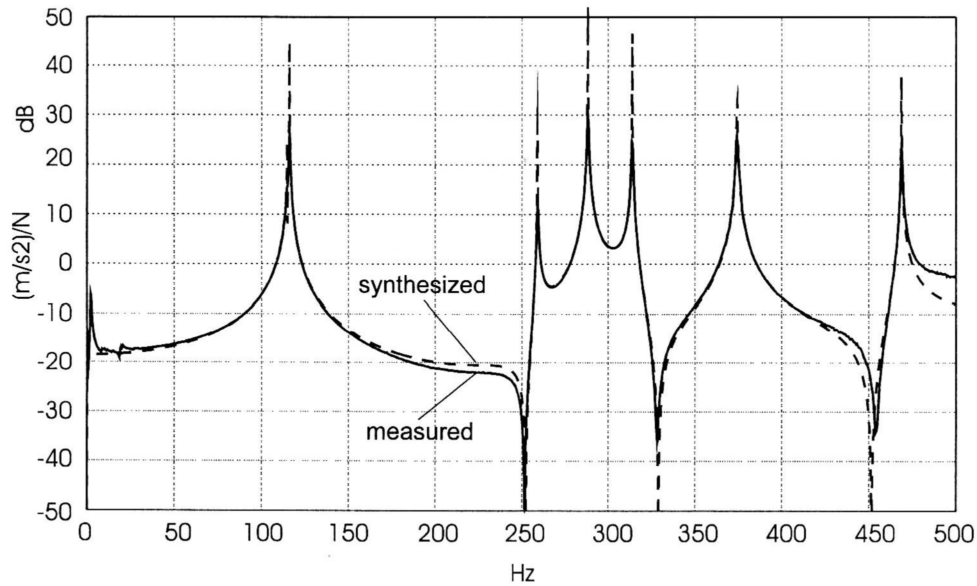

Figure 4. Frequency Response Function (Inertance) of Plate 1
Excitation at Point 1, Response at Point 11

At first the general co-ordinate system $O(x, y, z)$ is fixed at the geometrical centre of the plate as shown in Figure 3. Following the same steps as in example 3.1, to obtain the lower residuals from measured modal data, locate a single point at the lower frequency range where the curve exhibits a constant behaviour (see Figure 4). The rigid body modes or lower residuals of the plate have been determined using modal analysis and are illustrated in Figure 1. Theoretically, when the excitation frequency ω tends to zero, the FRFs curve has a constant value equal to the contributions of the rigid body modes in the total FRFs function. Practically it is difficult to obtain such a condition, since the suspension of the test structure using springs, accuracy of measuring equipment, etc., affect the FRFs function in the lower frequency range. Therefore it is recommended that a series of lower residuals to be estimated at different frequencies over a part of the lower frequency range. These lower residuals are then used to identify the required inertia parameters using equations (11), (12) and (14) respectively. An inspection of FRFs curve in Figure 4 shows that a possibility exists in estimating the lower residuals in the lower frequency range from 10 Hz to 110 Hz. These estimations produce different values of rigid body parameters over the selected frequencies. These variations are illustrated with different curves as shown in Figures 5.1, 5.2 and 5.3, respectively. The question still remains, which parameters should be selected and at what frequency.

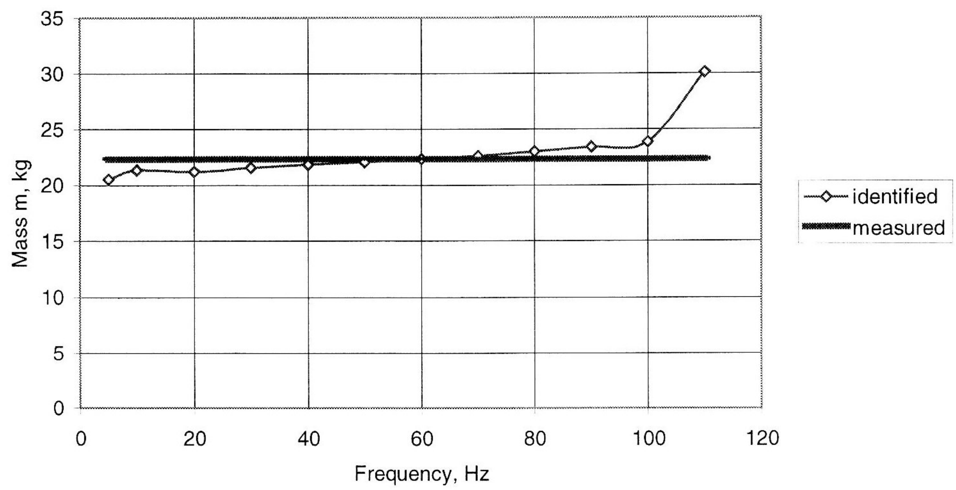

Figure 5.1. Variation of Mass m in the Lower Frequency Range

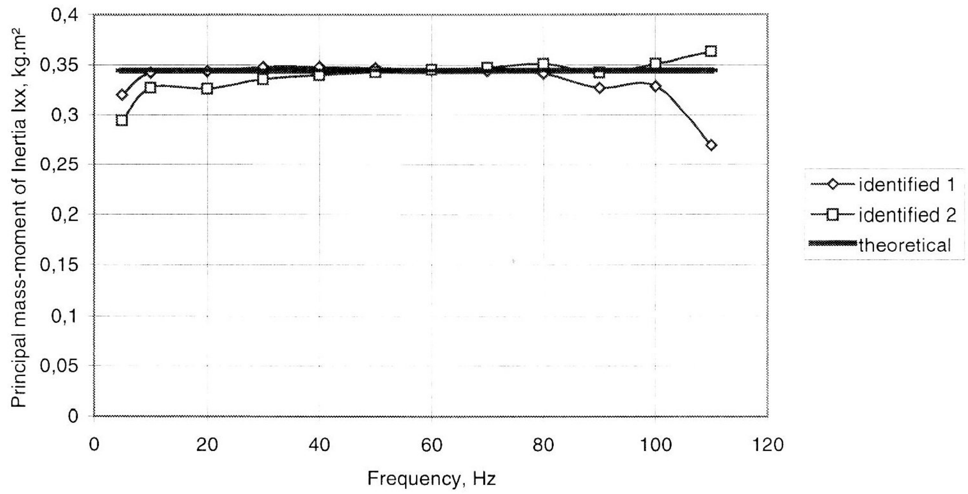

Figure 5.2. Variation of Mass-Moment I_{xx} in the Lower Frequency Range, identified 2: With Known Mass

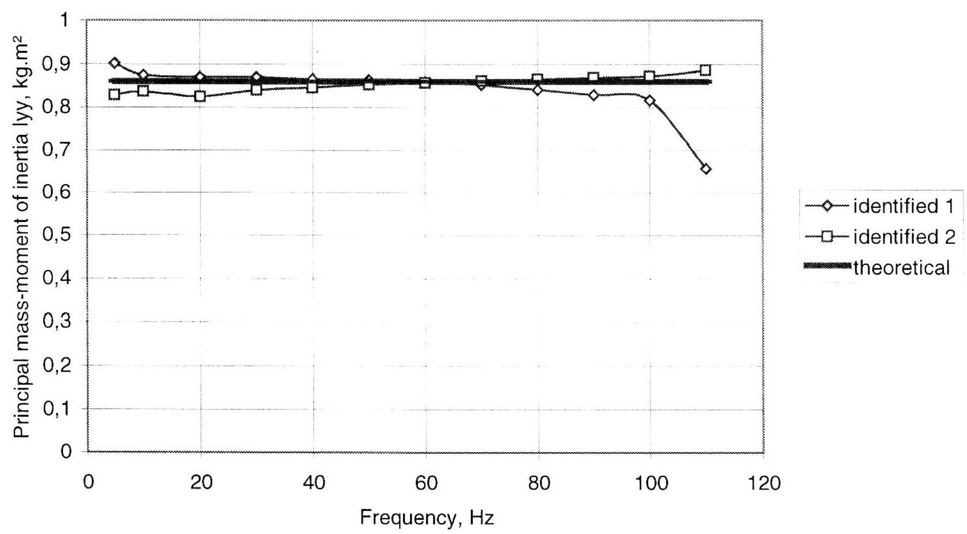

Figure 5.3. Variation of Mass-Moment of Inertia I_{yy} in the Lower Frequency Range, identified 2: With Known Mass

Proper inspection of Figures 5.1, 5.2 and 5.3 provides a means for selecting the required parameters. Below 40 Hz, the values are extremely affected by the suspensions, whilst the range above 80 Hz is influenced by the elastic modes of the structure. Considering the frequency range 40 Hz to 80 Hz, the parameters show approximately a constant trend and therefore average values of inertia parameters are calculated. The average values of the identified parameters are listed in Table 2.1. If the general co-ordinate system is now shifted to the new positions (0.0, 0.068m) and then to (0.165m, 0.068m), then the corresponding rigid body parameters are given in Tables 2.2 and 2.3 respectively. Results of the identified parameters in all the three cases are approximately the same and they are also in a good agreement with the theoretical data.

Parameters	Identified data (with known mass)	Theoretical data
Mass m, kg	22.3640 (22.345)	22.345
Centre distance G_x, m	-0.0029 (-0.0029)	0.0000
Centre distance G_y, m	-0.0013 (-0.0013)	0.0000
Principal moment of inertia I_{xx}, kg.m^2	0.3454 (0.3422)	0.3445
Principal moment of inertia I_{yy}, kg.m^2	0.8576 (0.8525)	0.8600

Table 2.1: Rigid Body Parameters of Plate 1, $O(x,y)$ at Center of Plate, see Figure 3.

Parameters	Identified data (with known dass)	Theoretical data
Mass m, kg	22.3266 (22.345)	22.345
Centre distance G_x, m	-0.0031 (-0.0031)	0.000
Centre distance G_y, m	-0.0610 (-0.0610)	-0.068
Principal moment of inertia I_{xx}, kg.m^2	0.3685 (0.3688)	0.3445
Principal moment of inertia I_{yy}, kg.m^2	0.8590 (0.8595)	0.8600

Table 2.2: Rigid Body Parameters of Plate 1, Co-ordinate System Shifted to Position (0.0, 0.068m)

Parameters	Identified data (with known mass)	Theoretical data
Mass m, kg	22.3268 (22.345)	22.345
Centre distance G_x, m	-0.1681 (-0.1681)	-0.165
Centre distance G_y, m	-0.0675 (-0.0675)	-0.068
Principal moment of inertia I_{xx}, kg.m^2	0.2822 (0.3000)	0.3445
Principal moment of inertia I_{yy}, kg.m^2	0.9222 (0.9200)	0.8600

Table 2.3: Rigid Body Parameters of Plate 1, Co-ordinate System Shifted to Position (0.165m, 0.068m)

The next problem to be investigated is to use an oriented co-ordinate system $O'(x',y')$ as shown in Figure 3. Here the co-ordinate system is rotated 50° anti-clockwise. The identified rigid body parameters obtained by the above method (see Table 2.4) are approximately the same as in Table 2.1. We note that the identified parameters are obtained with good accuracy only when the plate is suspended at two points along the x'-axis (see Figure 3). This means that the suspension of a structure should be always parallel to or at points along one of the selected co-ordinate axes.

Parameters	Identified data (with known mass)	Theoretical data
Mass m, kg	22.100 (22.345)	22.345
Centre distance G'_x, m	-0.034 (-0.034)	-0.0336
Centre distance G'_y, m	0.006 (0.006)	0.0058
Principal moment of inertia I_{xx}, kg.m^2	0.3482 (0.3693)	0.3445
Principal moment of inertia I_{yy}, kg.m^2	0.8468 (0.8980)	0.8600
Orientation angle, θ°	-51.66°	-50.0°

Table 2.4: Rigid Body Parameters of Plate 1, Co-ordinate System $O'(x',y')$ Shifted to Position (0.026m, 0.022m) and Rotated 50° anti-clockwise, see Figure 3.

3.3 Plate 2

A further extension of applications is now made to an aluminium plate of variable thickness as shown in Figure 6. Similar steps for the estimation of lower residuals and identification of rigid body parameters are also applied in this case. Here the points 16, 20 and 25 are considered as excitation points and three sets of measurements are needed to identify six inertia parameters. Average values of the identified parameters are shown in Table 3 together with the measured data obtained by oscillating the plate and finite-element model data (Zehn and Martin, 1997). The results in all the three cases are satisfactorily good.

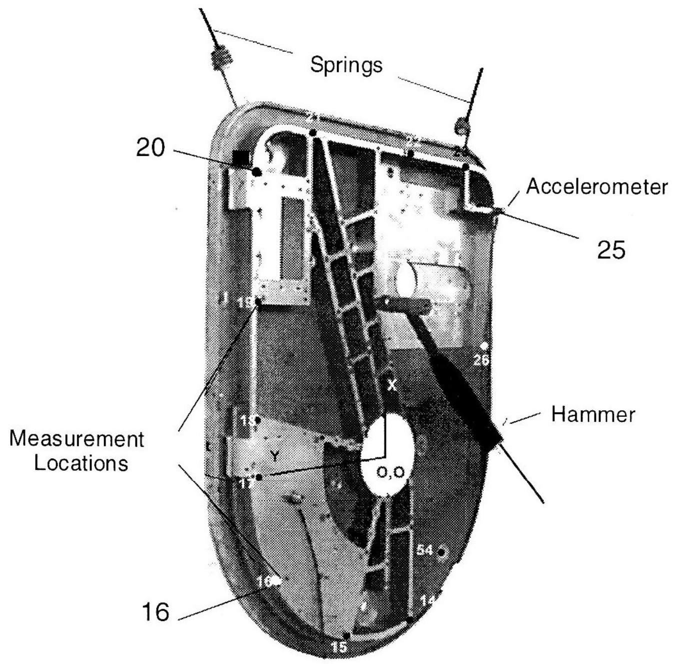

Figure 6. Plate 2 with Variable Thickness

Parameters	Identified data (with known mass)	Measurement	FEM-Analysis
Mass m, kg	**5.3230 (5.237)**	**5.237**	**5.3805**
Centre distance G_x, m	**0.0832 (0.0832)**	**0.085**	**0.0885**
Centre distance G_y, m	**-0.0120 (-0.0120)**	**-0.010**	**-0.0094**
Principal moment of inertia I_{xx}, kg.m²	**0.0637 (0.0626)**	**0.0740**	**0.0648**
Principal moment of inertia I_{yy}, kg.m²	**0.1112 (0.1002)**	**0.1070**	**0.1096**
Orientation angle, θ°	**1.07°**	-----	-----

Table 3: Rigid Body Parameters of Plate 2, Co-ordinate system's origin $O(x,y)$– at an arbitrarily selected point, see Figure 6.

4 Conclusions and Recommendations

The method of identification of rigid body parameters of a structure under a free-free condition has been proved in reference to the above practical examples as an effective practical tool for the identification purposes. The method is very simple and can be recommended because of its simplicity even for complicated structures in practice. The rigid body parameters obtained in the previous examples are in a good agreement with the theoretical and experimental data. Their accuracy depends to a great extent on the selection of excitation points, suspension of a structure, a precise estimations of the lower residuals from measured FRFs. If these factors are selected carefully then good results are expected.
It is recommended that the rigid body parameters are determined and identified over a part of the lower frequency range and not on a particular frequency, since average values of identified inertia parameters over a particular frequency range are more informative and acceptable than those at single frequencies.
Based on the results of practical examples, it can be recommended to apply this method in the practice to identify the ten rigid body parameters of a three-dimensional structure or even more complicated structures.

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