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On the Disintegration of Very Small Satellites

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Small Satellites will disintegrate, if they find a means, such as a separation force, to do so. If inside their Roche limit they are indeed subject to a separation force, that might be sufficient to bring about fragmentation. It is shown in the present paper that satellite disintegration leads to a reduction of orbital energy. The result can be looked upon as a generalization of the collinearity principle, according to which planetary systems strive towards a minimum energy state, for a given constant angular momentum.

Introduction

The radius around the primary master (Sun) within which a satellite tends to disintegrate is known as the Roche limit (Rimrott, 1989). The Roche limit is a function ot the third root of the ratio of primary master mass to secondary master mass. If the secondary master mass (comet) is very small then the ratio becomes very large, and so does the radius of the Roche limit. As a consequence, very small satellites have their whole orbit, or at least the greater part of it, inside their Roche limit, i.e. they have the tendency to disintegrate.

A straightforward explicit solution process for the present problem seems rather elusive. Consequently we base our analysis on certain a priori conditions, the reasons for which shall become apparent in the course of the investigation.

Optimum conditions for fragmentation and subsequent fragment motion on orbits that diverge only slightly from the original orbit of a dumbbell satellite separating into two equal point satellites apparently prevail if fragmentation takes place at the moment when

1. the dumbbell satellite happens to be aligned radially
2. the dumbbell satellite happens to pass the periapsis of its orbit
3. the liberation of the dumbbell satellite happens to have a slight backwards rotation of $\omega = -\dot{\theta}/2$.

Radial alignment makes slightly diverging new orbits possible, and it and the periapsis location imply *a* maximum separation force. The backward liberation secures the proper initial conditions that provide for *a* smooth insertion into the new orbits.

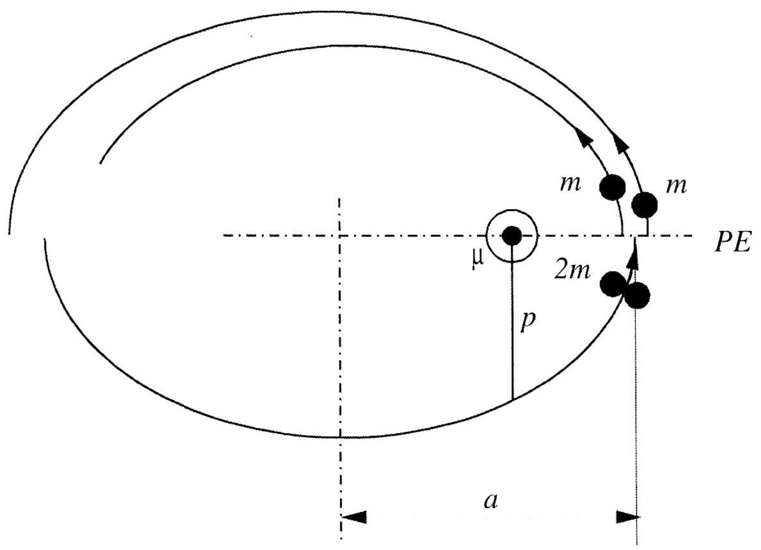

Figure1. A Dumbbell Satellite Disintegration into Two Point Satellites in the Orbit Periapsis

Let a dumbbell satellite, which consists of two point mass boulders of mass m each held together by an adhesive (ice) as shown in Figure 1, orbit a primary point master of gravitational attraction parameter μ. As the dumbbell satellite reaches the periapsis point of its orbit, it is assumed to be arranged in radial direction and to break into two.

After Disintegration

Let us assume that the adhesion between the radially arranged boulders has failed, and the two have separated. We stipulate that the new orbits and the original orbit are coplanar, or in other words that the angular momenta are collinear. We further assume that the two boulders can be treated as point masses after separation.
From experimental evidence (comets) we deduce that the satellite halves continue to move along the original orbit, obviously though with slight divergencies. We thus assume that one boulder of mass m now moves on a slightly larger orbit (Figure 1) of semi-major axis

$$a_1 = a + \Delta a_1 = a(1+x) \tag{1}$$

with the orbit change ratio

$$x = \Delta a_1 / a \tag{2}$$

and the second boulder, also of mass m, now moves on a slightly smaller orbit of semi-major axis

$$a_2 = a(1-x) \tag{3}$$

with

$$\Delta a_2 = -xa \tag{4}$$

The condition

$$|\Delta a_2| = \Delta a_1 \tag{5}$$

means that the common mass centre continues on its path during the separation process.
We further assume, that the eccentricities of the new orbits are equal

$$\varepsilon_1 = \varepsilon_2 \tag{6}$$

Eccentricity

Semi-major axis and periapsis radius r_0 are related by (Rimrott, 1989)

$$r_0 = a(1-\varepsilon) \tag{7}$$

For the new outer orbit

$$r_{01} = a_1(1-\varepsilon_1) \tag{8}$$

From Figure 2 we note that

$$r_{01} = r_0 + \frac{d}{2} \tag{9}$$

Thus, we have from equations (1), (7), (8), (9)

$$a(1+x)(1-\varepsilon_1) = a(1-\varepsilon) + \frac{d}{2} \tag{10}$$

By the same token

$$a(1-x)(1-\varepsilon_2) = a(1-\varepsilon) - \frac{d}{2} \tag{11}$$

Because of equation (6) the sum of equations (10) and (11) immediately yields

$$\varepsilon_1 = \varepsilon \qquad\qquad \varepsilon_2 = \varepsilon \tag{12}$$

Thus we find that the eccentricities of the two new orbits and of the original orbit are equal.
From equations (1), (7), (8) and (9) we have also

$$r_{01} = r_0(1+x) \tag{13}$$

$$xr_0 = \frac{d}{2} \tag{14}$$

Orbit Insertion

For the point mass on the larger of the new orbits to have a semi-major axis as given by equation (1) the insertion velocity must be (Rimrott, 1989)

$$v_1 = \sqrt{\frac{\mu}{a_1}\frac{1+\varepsilon}{1-\varepsilon}} = \sqrt{\frac{\mu}{a}\frac{1+\varepsilon}{1-\varepsilon}}\,(1-\frac{x}{2}+\frac{3}{8}x^2) \tag{15}$$

For the point mass on the smaller of the two new orbits the insertion velocity must be

$$v_2 = \sqrt{\frac{\mu}{a_2}\frac{1+\varepsilon}{1-\varepsilon}} = \sqrt{\frac{\mu}{a}\frac{1+\varepsilon}{1-\varepsilon}}\,(1+\frac{x}{2}+\frac{3}{8}x^2) \tag{16}$$

This in turn means that the still intact dumbbell stellite must rotate at an angular speed of

$$\omega = \frac{v_1 - v_2}{d} = -\sqrt{\frac{\mu}{a}\frac{1+\varepsilon}{1-\varepsilon}}\;\frac{x}{d} \tag{17}$$

i.e. in opposite direction of the orbital angular speed, which in the orbit periapsis is

$$\dot{\theta} = \frac{\sqrt{\mu p}}{r_0^2} = \sqrt{\frac{\mu}{a}\;\frac{1+\varepsilon}{1-\varepsilon}}\;\frac{1}{r_0} \tag{18}$$

From

$$\frac{\omega}{\dot{\theta}} = -\frac{xr_0}{d} \tag{19}$$

and with equation (14) we obtain

$$\omega = -\frac{1}{2}\dot{\theta} \tag{20}$$

Thus for a smooth insertion into the new orbits the dumbbell's (backward) angular speed must be ½ of the (forward) orbital angular speed.

Angular Momentum

After disintegration we have for the angular momentum of the point mass boulder on the outer orbit (Rimrott, 1989)

$$H_1 = m\sqrt{\mu p_1} = m\sqrt{\mu a(1+x)(1-\varepsilon^2)} = m\sqrt{\mu a(1-\varepsilon^2)}(1+\frac{x}{2}-\frac{x^2}{8}) \tag{21}$$

and for that of the second boulder

$$H_2 = m\sqrt{\mu p_2} = m\sqrt{\mu a(1-x)(1-\varepsilon^2)} = m\sqrt{\mu a(1-\varepsilon^2)}(1-\frac{x}{2}-\frac{x^2}{8}) \tag{22}$$

The sum of the two is

$$H_f = H_1 + H_2 = m\sqrt{\mu a(1-\varepsilon^2)}(2-\frac{x^2}{4}) \tag{23}$$

Before disintegration the combined linear momentum, from equations (15) and (16), is

$$(mv_1 + mv_2) = m\sqrt{\frac{\mu}{a}\frac{1+\varepsilon}{1-\varepsilon}}(2+\frac{3}{4}x^2) \tag{24}$$

Its moment about the point master is

$$(mv_1 + mv_2)r_0 = mr_0\sqrt{\frac{\mu(1+\varepsilon)}{a(1-\varepsilon)}}(2+\frac{3}{4}x^2) = m\sqrt{\mu a(1-\varepsilon^2)}(2+\frac{3}{4}x^2) \tag{25}$$

To this there must be added the angular momentum

$$I\omega = -\frac{md^2}{2}(\frac{1}{2}\dot{\theta}) = -\frac{md^2}{4r_0^2}\sqrt{\mu a(1-\varepsilon^2)} = -m\sqrt{\mu a(1-\varepsilon^2)}\ x^2 \tag{26}$$

resulting in

$$H_i = (mv_1 + mv_2)r_0 + I\omega = m\sqrt{\mu a(1-\varepsilon^2)}(2-\frac{x^2}{4}) \tag{27}$$

Compared to equation (23) we see that it is confirmed

$$H_f = H_i \tag{28}$$

or, in words, the angular momentum of the system has been conserved during the disintegration process.

Separation Force

As a result of conditions 1 and 2 there will be a separation force between the two boulders (Figure 2)

$$F = C_1 - K_1 \tag{29}$$

with the centrifugal force

$$C_1 = \frac{K_1 + K_2}{2} + m\frac{d}{2}\omega^2 \tag{30}$$

and Kepler forces of magnitude (Rimrott 1989)

$$K_1 = \frac{\mu m}{(r_0 + \frac{d}{2})^2} = \frac{\mu m}{r_0^2}(1 - \frac{d}{r_0}) \tag{31}$$

$$K_2 = \frac{\mu m}{(r_0 - \frac{d}{2})^2} = \frac{\mu m}{r_0^2}(1 + \frac{d}{r_0}) \tag{32}$$

and with the help of equations (17) and (14) we have eventually

$$F = \frac{9+\varepsilon}{8}\frac{\mu m}{r_0^2}\frac{d}{r_0} = \frac{9+\varepsilon}{4}\frac{\mu m}{r_0^2}x \tag{33}$$

It is interesting to note that the higher the eccentricity (comet orbits!) the greater the separation force.

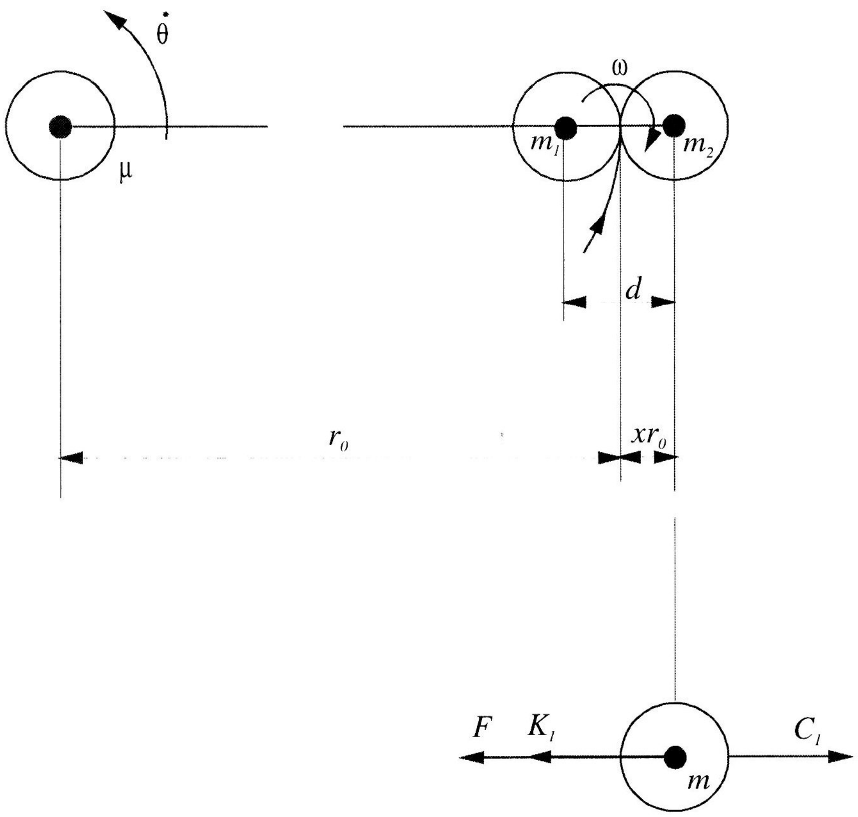

Figure 2. Satellite at Disintegration

Orbital Energy

The orbital energy of the point mass on the outer of the two new orbits after disintegration is (Rimrott, 1989)

$$E_1 = -\frac{\mu m}{2a_1} = -\frac{\mu m}{2a(1+x)} = -\frac{\mu m}{2a}(1 - x + x^2) \tag{34}$$

and of the point mass on the inner orbit it is

$$E_2 = -\frac{\mu m}{2a_2} = -\frac{\mu m}{2a(1-x)} = -\frac{\mu m}{2a}(1 + x + x^2) \tag{35}$$

Together the two represent the final total energy

$$E_f = E_1 + E_2 = -\frac{\mu m}{a}(1+x^2) \qquad (36)$$

Before disintegration we have

$$E_i = -\frac{\mu(2m)}{2a} + \frac{1}{2} I\omega^2 \qquad (37)$$

with the help of equations (17) and (14) and with $I = md^2/2$ we have

$$E_i = -\frac{\mu m}{a}(1 - \frac{1}{4}\frac{1+\varepsilon}{1-\varepsilon}x^2) \qquad (38)$$

The energy change during disintegration is consequently

$$\Delta E = E_f - E_i = -\frac{\mu m}{a}\left(1 + \frac{1}{4}\frac{1+\varepsilon}{1-\varepsilon}\right)x^2 \qquad (39)$$

The energy change (39) is negative, i.e. it represents an energy loss. This energy loss can be interpreted for our model and our assumptions as the work done by the separation force during the separation process. Very small satellites generally have the tendency to disintegrate, a circumstance well known in astronomy where the so-called meteor showers are interpreted as the remnants of former comets. From our analysis we conclude that disintegration is typically accompanied by a loss of energy. Since according to the collinearity principle gyroscopic systems tend to arrange themselves such that their mechanical energy its at a minimum, we can interpret the energy loss during disintegration as a generalization of this principle.

Orbital Period

It is interesting to realize, how little the orbital period changes for the debris of disintegration. The Kepler period is known to be (Rimrott, 1989)

$$\tau = 2\pi\sqrt{\frac{a^2}{\mu}} \qquad (40)$$

For a slightly larger orbit with $a_1 = a(1+x)$ it is then

$$\tau_1 = \tau + \Delta\tau_1 = 2\pi\sqrt{\frac{a^3(1+x)^3}{\mu}} \qquad (41)$$

Expanded into a series and broken off after the x term, we have

$$\tau_1 = 2\pi\sqrt{\frac{a^3}{\mu}}\,(1 + \frac{3}{2}x) \qquad (42)$$

and thus

$$\Delta\tau_1 = \frac{3}{2}\tau x \qquad (43)$$

By the same token, we have for the slightly smaller orbit

$$\Delta\tau_2 = -\frac{3}{2}\tau x \qquad (44)$$

In order to obtain an idea of realistic magnitudes of $\Delta\tau_1$ we shall have a look at data for Halley's comet (Hartmann, 1978), for which $\tau = 76$ years, and $a = 18$ AU. Let us assume debris at $\Delta a_1 = 100$ km from the comet nucleus orbit then

$$x = \frac{100}{18(150)10^6} = 0,037(10^{-6})$$

$$\Delta\tau_1 = \frac{3}{2} 76(365)\,24(60)\,60(0,037)10^{-6} = \quad 133\,\mathrm{s} = \quad 2,217\,\mathrm{min}$$

This comet debris on the slightly larger orbit thus arrives only 2,217 min later than the comet nucleus, after 76 years! In the course of aeons, however, the debris begins to stretch out along the orbit until the entire orbit is covered and as a result there occur annual meteor showers (e.g. Orionid) which are observed every time the Earth passes near the associated comet's orbit (e.g. Halley's).

Conclusion

It is shown that very small satellites tend to disintegrate, and that disintegration means a lowering of orbital energy. The disintegration process is looked upon as a generalized interpretation of the collinearity principle.

Literature

1. Hartmann, William K.: Astronomy, The Cosmic Journey, Wadsworth Publishing, Belmont, (1978) 536 p.
2. Rimrott, F.P.J.: Introductory Orbit Dynamics, Vieweg, Wiesbaden (1989), 193 p.
3. Rimrott, F.P.J.: Das Kollinearitätsprinzip, Technische Mechanik, 18, 1, (1998), 57-68.

List of Symbols

AU = Astronomical Unit = 150 (10^6) km
C = centrifugal force, N
E = energy, J
F = separation force, N
H = angular momentum, m^2kg/s
I = inertia moment of dumbbell, m^2kg
K = Kepler force, N

a = orbit semi-major axis, m
d = distance, m
m = mass, kg
p = orbit semi-parameter, m
r = radius, m
v = speed, m/s
x = a small ratio

ε = orbit eccentricity
θ = polar angle, rad
μ = gravitational attraction parameter, m^3/s^2
τ = orbital period, s
ω = angular speed of dumbbell, rad/s

Subscripts

$_0$ = of periapsis
$_1$ = pertaining to outer mass
$_2$ = pertaining to inner mass
$_f$ = final
$_i$ = initial

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