TECHNISCHE MECHANIK, Band 21, Heft 2, (2001), 121-129
Manuskripteingang: 01. Oktober 1999

Horizontal Oscillations of a Vibrational Platform

H. M. Kllapija, F. F. Rodyukov, R. I. Alidema

The mathematical model of a vibrational platform with horizontal vibrations is obtained by means of a complete system of splitted equations of an asynchronous motor. This model is reduced to the dimensionless form and then the stability of steady state modes is analyzed. The condition for the selection of an asynchronous motor, for which the steady state mode may be provided, is obtained.

1 Introduction

Vibrational platforms with vertical and horizontal vibrations are very commonly used in mechanical tests, in many technological processes, and also in civil constructions (Collection of articles, 1971). These platforms have been examined in a number of both analytical and applied researches (Collection of articles, 1971; Alifov and Frolov, 1985).

If the vibrational platform is intended, for example, for mechanical tests of medium-sized items, for which the demanded power is considerably less than the power of the forced motor, then the influence of oscillations of the vibrational platform on the motor is neglected. In this case the study is usually limited to the models of an oscillator with harmonic exiting force. This case is considered in details in all monographs on vibration theory.

If the power of the mechanical vibrations of the vibroplatform is comparable with the power of the motor, then in the mathematical model one should take into account the influence of vibrations on the processes on the motor. Usually the static mechanical characteristic of the motor is used (Alifov and Frolov, 1985; Kononenko, 1964). The more exact approach is in developing the complete mathematical model of a motor. Such an approach may be found in the works by Lvovich (Lvovich, 1989).

In Rodyukov (1994), Rodyukov and Lvovich (1997) it was shown that the splitted equations describe more exactly the physical processes in the asynhronous motors (AM). At the same time from a mathematical point of view such equations may be studied more easily than nonsplitted equations. The present paper is the first attempt to describe the AM that excites the vibroplatform with the complete splitted equations. By means of special transformations the equations are reduced to the simplest form. For the first time the complete mathematical model of the vibroplatform with the AM is represented in the dimensionless form, which is one of the requirements for studying the differential equations with small parameters.

The purpose of this investigation is to obtain the stability conditions for the stable state of the vibroplatform and provide the approximate relations that helps to select the AM by the parameters of the vibroplatform.

2 Construction of the Mathematical Model

The scheme of the vibroplatform is given in Figure 1. According to this scheme the mass of the vibroplatform with the attached body, m, moves horizontally under the eccenter of the radius e that is attached to the shaft of the rotor AM (not shown in the scheme) and a spring c_1 of the length l in the non-deformed state. The total stiffness of the springs of the vibroplatform is c_0. It is assumed that $e/l \ll 1$, therefore the total extension of the spring c_1 is replaced approximately by its horizontal projection. The viscous resistance r characterize the energy loss in the system. The AM is assumed to be short circuit and two-phases.

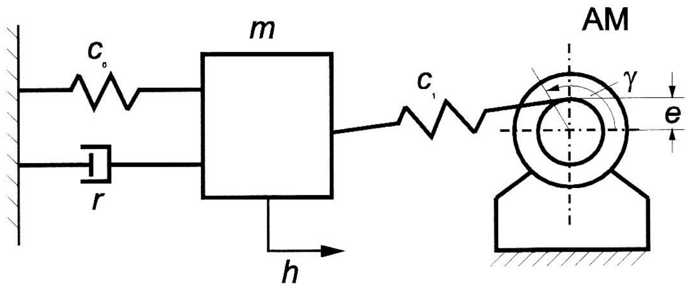

Figure 1: Scheme of the Vibroplatform

The computational scheme (see Figure 1) is described by the Lagrange equations of the second kind

$$\frac{d}{dt}\frac{\partial T}{\partial \dot{q}} - \frac{\partial T}{\partial q} + \frac{\partial \Pi}{\partial q} + \frac{\partial D}{\partial \dot{q}} = Q_q \tag{1}$$

where T is the kinetic energy of the system, Π is the potential energy of the system, D is the dissipative energy of the system, q is the generalized coordinate, Q_q is the generalized force corresponding to the generalized coordinate q, t is the time.

The kinetic energy of the system $T = T_m + T_e$ consists of the mechanokinetic energy

$$T_m = \tfrac{1}{2} J\dot{\gamma}^2 + \tfrac{1}{2} m\dot{h}^2$$

and the electrokinetic energy

$$T_e = \tfrac{1}{2} L_s i_A^2 + \tfrac{1}{2} L_s i_B^2 + \tfrac{1}{2} L_r i_a^2 + \tfrac{1}{2} L_r i_b^2 +$$

$$+M(i_A i_a \cos\gamma - i_A i_b \sin\gamma + i_B i_a \sin\gamma + i_B i_b \cos\gamma)$$

Here J is the moment of inertia of the rotor AM, γ is the rotor angle, m is the total mass of the vibrational platform and the mass of the body, h is the horizontal displacement of the centre of mass of the vibrational platform with the body; L_s is the inductance of the stator windings, i_A, i_B are the currents in the stator windings, L_r is the inductance of the rotor windings, i_a, i_b are the currents in the rotor windings, M is the amplitude of the mutual inductance between stator and rotor windings.

The potential energy of the system Π is given as

$$\Pi = \tfrac{1}{2} c_0 h^2 + \tfrac{1}{2} c_1 (e\cos\gamma - h)^2$$

where c_0 is the stiffness of the springs of the vibrational platform, c_1 is the stiffness of the forced spring, e is the radius of the eccentric that is clung to the shaft of the AM.

The dissipative function of the system D is written as:

$$D = \tfrac{1}{2} r\dot{h}^2 + \tfrac{1}{2} R_s i_A^2 + \tfrac{1}{2} R_s i_B^2 + \tfrac{1}{2} R_r i_a^2 + \tfrac{1}{2} R_r i_b^2$$

where r is the resistance of the viscous dampers, R_s is the resistance of the stator windings, R_r is the resistance of the rotor windings.

The following variables are the generalized coordinates q of the system: h, γ, i_A, i_B, i_a, i_b.

Before one writes the Lagrange equations in appropriate generalized coordinates, it is useful to introduce a notion of a flux linkage of the winding Ψ

$$\Psi_j = \frac{\partial T_e}{\partial i_j} \qquad j = A, B, a, b$$

Then

$$
\begin{aligned}
\Psi_A &= L_s i_A + M(i_a \cos\gamma - i_b \sin\gamma) \\
\Psi_B &= L_s i_B + M(i_a \sin\gamma + i_b \cos\gamma) \\
\Psi_a &= M(i_A \cos\gamma + i_B \sin\gamma) + L_r i_a \\
\Psi_b &= M(-i_A \sin\gamma + i_B \cos\gamma) + L_r i_b
\end{aligned}
\tag{2}
$$

Now the Lagrange equations in the generalized coordinates are

$$
\begin{aligned}
&\text{in coordinate } h \\
&m\ddot{h} + r\dot{h} + ch = c_1 e \cos\gamma \qquad\qquad c = c_0 + c_1 \\
&\text{in coordinate } \gamma \\
&J\ddot{\gamma} = \Psi_A i_B - \Psi_B i_A + c_1 e^2 \sin\gamma \cos\gamma - c_1 h e \sin\gamma \\
&\text{in coordinate } i_A \\
&\dot{\Psi}_A = -R_s i_A + u_A \\
&\text{in coordinate } i_B \\
&\dot{\Psi}_B = -R_s i_B + u_B \\
&\text{in coordinate } i_a \\
&\dot{\Psi}_a = -R_r i_a \\
&\text{in coordinate } i_b \\
&\dot{\Psi}_b = -R_r i_b
\end{aligned}
\tag{3}
$$

In the equations (3), u_A, u_B are the voltages of a two-phase network, applied to stator windings

$$
u_A = u_m \sin\omega_s t \qquad\qquad u_B = -u_m \cos\omega_s t \tag{4}
$$

where u_m is the amplitude of the voltage, ω_s is the angular frequency of the network voltage (ω_s= 2 = 314 s^{-1} for the network with the frequency f = 50 Hz).

Equations (2), (3) and (4) form the closed system, desribing the dynamic processes of the vibrational platform. The study of this system can be considerably simplified if it is reduced to a dimensionless form and for the electrical part the new coordinates are introduced.

To reduce the system (2) - (4) to a dimensionless form, we use the dimensionless (synchronous) time $\tau = \omega_s t$ and the horizontal displacement $\bar{h}$ such as

$$
h = \bar{h}e
$$

For the stator flux linkages and currents (marked with "s") the corresponding dimensionless variables are

$$
\Psi^s = \frac{u_m}{\omega_s}\,\overline{\Psi}^s \qquad\qquad i^s = \frac{u_m}{\omega_s L_s}\,\bar{i}^s
$$

For similar rotor variables (marked with "r)

$$
\Psi^r = \frac{L_r}{M}\frac{u_m}{\omega_s}\,\overline{\Psi}^r \qquad\qquad i^r = \frac{u_m}{\omega_s M}\,\bar{i}^r
$$

It is also useful to introduce the sliding s

$$
s = \frac{\omega_s - \omega}{\omega_s}\omega = \dot{\gamma}
$$

$$
\omega = \omega_s(1-s) \qquad\qquad \frac{d\gamma}{dt} = \omega \qquad\qquad \frac{d\gamma}{d\tau} = 1 - s
$$

In the dimensionless variables the system (2) - (4) has the following form

$$
\begin{aligned}
\Psi_A &= i_A + i_a \cos\gamma - i_b \sin\gamma \\
\Psi_B &= i_B + i_a \sin\gamma + i_b \cos\gamma \\
\Psi_a &= (1-\mu)(i_A \cos\gamma + i_B \sin\gamma) + i_a \\
\Psi_b &= (1-\mu)(-i_A \sin\gamma + i_B \cos\gamma) + i_b
\end{aligned}
\tag{5}
$$

$$
\begin{aligned}
&\ddot{h} + 2n\dot{h} + \omega_0^2 h = \nu \cos\gamma \\
&\dot{\gamma} = 1 - s \\
&\dot{s} = -\delta(\Psi_A i_B - \Psi_B i_A + \kappa \sin\gamma \cos\gamma - \kappa h \sin\gamma) \\
&\dot{\Psi}_A = -\varepsilon_s i_A + \sin\tau \\
&\dot{\Psi}_B = -\varepsilon_s i_B - \cos\tau \\
&\dot{\Psi}_a = -\varepsilon_r i_a \\
&\dot{\Psi}_b = -\varepsilon_r i_b
\end{aligned}
\tag{6}
$$

Here we omit the bars over the dimensionless variables. The dot over the dimensionless variables means the derivatives with respect to the dimensionless time, τ.

In the system (5), (6) the following dimensionless parameters are used:

$$
\begin{array}{llll}
\mu = 1 - \frac{M^2}{L_s L_r} & 2n = \frac{r}{m\omega_s} & \omega_0^2 = \frac{c}{m\omega_s^2} & \nu = \frac{c_1}{m\omega_s^2} \\
\delta = \frac{u_m^2}{J\omega_s^4 L_s} & \kappa = \frac{c_1 e^2 \omega_s^2 L_s}{u_m^2} & \varepsilon_s = \frac{R_s}{\omega_s L_s} & \varepsilon_r = \frac{R_r}{\omega_s L_r}
\end{array}
\tag{7}
$$

The real AM have three phase stators and the number of the pole pairs (p) may be larger than 1. In this case flux linkages and currents may be brought into a dimensionless form

$$
\Psi^s = \frac{3}{2}\frac{u_m}{\omega_s}\overline{\Psi}^s \qquad i^s = \frac{2}{3}\frac{u_m}{\omega_s L_s}\overline{i}^s
$$

For similar rotor variables (marked with "r")

$$
\Psi^r = \frac{3}{2}\frac{L_r}{M}\frac{u_m}{\omega_s}\overline{\Psi}^r \qquad i^r = \frac{2}{3}\frac{u_m}{\omega_s M}\overline{i}^r
$$

and for the coefficients

$$
\delta = \frac{p^2 u_m^2}{J\omega_s^4 L_s} \qquad \varepsilon_s = \frac{2}{3}\frac{R_s}{\omega_s L_s} \qquad \varepsilon_r = \frac{2}{3}\frac{R_r}{\omega_s L_r}
$$

hold. Note that the coefficients have the multipliers 2/3 and p^2 (compare to equations (7)).

For these dimensionless variables the form of equations (5) - (6) remains the same. Hence, the selected system of the basic variables and denotations for the transition from 3-phase AM with $p > 1$ to the equivalent 2-phase AM makes the dimensionless equations for the 2-phase AM invariant.

In the system (5)-(6) we introduce the following coordinates:

$$
\begin{aligned}
&i_{sx} = i_A \cos\tau + i_B \sin\tau \qquad i_{sy} = -i_A \sin\tau + i_B \cos\tau \\
&i_{rx} = i_a \cos(\tau - \gamma) + i_b \sin(\tau - \gamma) \\
&i_{ry} = -i_a \sin(\tau - \gamma) + i_b \cos(\tau - \gamma) \\
\\
&\Psi_{sx} = \Psi_A \cos\tau + \Psi_B \sin\tau \qquad \Psi_{sy} = -\Psi_A \sin\tau + \Psi_B \cos\tau \\
&\Psi_{rx} = \Psi_a \cos(\tau - \gamma) + \Psi_b \sin(\tau - \gamma) \\
&\Psi_{ry} = -\Psi_a \sin(\tau - \gamma) + \Psi_b \cos(\tau - \gamma) \\
\\
&u_x = u_A \cos\tau + u_B \sin\tau = 0 \qquad u_y = -u_A \sin\tau + u_B \cos\tau = -1
\end{aligned}
\tag{8}
$$

The transformation (8) means that the variables of the stator and the rotor are projected on the same orthogonal axes of coordinates x, y that rotates synchronously with a magnetic field of the stator of AM.

In the new variables the system (5)-(6) is transformed to

$$
\begin{aligned}
&\Psi_{sx} = i_{sx} + i_{rx} \qquad \Psi_{sy} = i_{sy} + i_{ry} \\
&\Psi_{rx} = (1-\mu) i_{sx} + i_{rx} \qquad \Psi_{ry} = (1-\mu) i_{sy} + i_{ry}
\end{aligned}
\tag{9}
$$

$$
\begin{aligned}
&\ddot{h} + 2n\dot{h} + \omega_0^2 h = \nu \cos\gamma \\
&\dot{\gamma} = 1 - s \\
&\dot{s} = -\delta(\Psi_{sx} i_{ry} - \Psi_{sy} i_{rx} + \kappa \sin\gamma\cos\gamma - \kappa h \sin\gamma) \\
&\dot{\Psi}_{sx} = \Psi_{sy} - \varepsilon_s i_{sx} \\
&\dot{\Psi}_{sy} = -\Psi_{sx} - \varepsilon_s i_{sy} - 1 \\
&\dot{\Psi}_{rx} = s\Psi_{ry} - \varepsilon_r i_{rx} \\
&\dot{\Psi}_{ry} = -s\Psi_{rx} - \varepsilon_r i_{ry}
\end{aligned}
\tag{10}
$$

Using the expressions (9) we shall pass in the equations (10) to the flux linkages and currents only of the stator windings (the hybrid variables of stator). In the hybrid variables the system (9), (10) is equivalent to the following closed system (we omit "s" in the lower indeces)

$$
\begin{aligned}
&\ddot{h} + 2n\dot{h} + \omega_0^2 h = \nu \cos\gamma \\
&\dot{\gamma} = 1 - s \\
&\dot{s} = -\delta(\Psi_x i_y - \Psi_y i_x + \kappa \sin\gamma\cos\gamma - \kappa h \sin\gamma) \\
&\dot{\Psi}_x = \Psi_y - \varepsilon_s i_x \qquad \dot{\Psi}_y = -\Psi_x - \varepsilon_s i_y - 1 \\
&\mu \dot{i}_x = \dot{\Psi}_x - s(\Psi_y - \mu i_y) + \varepsilon_r(\Psi_x - i_x) \\
&\mu \dot{i}_y = \dot{\Psi}_y + s(\Psi_x - \mu i_x) + \varepsilon_r(\Psi_y - i_y)
\end{aligned}
\tag{11}
$$

Now we substitute the new variables x, y instead of the currents i_x, i_y in system (11) according to the relations

$$
i_x = \tfrac{1}{\mu}\Psi_x - \alpha_r b x \qquad i_y = \tfrac{1}{\mu}\Psi_y - \alpha_r b y \qquad b = \tfrac{1-\mu}{\mu} \qquad \alpha_r = \varepsilon_r/\mu
$$

In the new variables the system (11) becomes

$$
\begin{aligned}
&\ddot{h} + 2n\dot{h} + \omega_0^2 h = \nu \cos\gamma \\
&\dot{\gamma} = 1 - s \\
&\dot{s} = -\delta\big[\alpha_r b(\Psi_y x - \Psi_x y) + \kappa \sin\gamma\cos\gamma - \kappa h \sin\gamma\big] \\
&\dot{\Psi}_x = \Psi_y - \alpha_s \Psi_x + \alpha_s \alpha_r (1-\mu) x \\
&\dot{\Psi}_y = -\Psi_x - \alpha_s \Psi_y + \alpha_s \alpha_r (1-\mu) y - 1 \\
&\dot{x} = -\alpha_r x + s y + \Psi_x \\
&\dot{y} = -\alpha_r y - s x + \Psi_y \qquad \alpha_s = \varepsilon_s/\mu
\end{aligned}
\tag{12}
$$

We split the last four equations of system (12) neglecting the small terms $\alpha_s\alpha_r(1-\mu)x$ and $\alpha_s\alpha_r(1-\mu)y$. In Rodyukov (1984), Rodyukov and Lvovich (1997) it was shown that the equations of AM splitted in such a way describe the physical processes in AM better than the initial equations. That is why the system of such splitted equations of AM have been considered as exact in the above mentioned works.

For small $\alpha_s \ll 1$ the flux linkages are $\Psi_x \approx -1$, $\Psi_y \approx 0$. Since our purpose is to study the steady state modes of the vibration platform, we substitute these value in the corresponding equations. Thus, we obtain the final mathematical model to study the steady state modes of the vibration platform

$$
\begin{aligned}
&\ddot{h} + 2n\dot{h} + \omega_0^2 h = \nu \cos\gamma \\
&\dot{\gamma} = 1 - s \\
&\dot{s} = -\delta(\alpha_r b y + \kappa \sin\gamma\cos\gamma - \kappa h \sin\gamma) \\
&\dot{x} = -\alpha_r x + s y - 1 \\
&\dot{y} = -\alpha_r y - s x
\end{aligned}
\tag{13}
$$

3 Mathematical Investigation

Theorem. *If the condition $b(1-\alpha_r) > \frac{\kappa\nu}{20n}$ holds then the system (13) has the steady state solution at any operational mode of the vibrational platform.*

Proof. System (13) has not the exact solution but it contains "fast phase" γ and the slow variable s, and therefore permits the simple way of study of static stability (local stability).

The value of δ for AM is small, $\delta \sim 10^{-2} - 10^{-3}$. The value of α_r is the critical sliding of AM. For the considered type of AM, $\alpha_r \approx 0.2$ and the factor of electromagnetic scattering of energy $\mu \approx 0.1$ (it corresponds to $b \approx 9$). Therefore, $\alpha_r b \sim 1$. Assuming $\kappa < 1$ we conclude that $\dot{s} \sim \delta \ll 1$. Indeed s is the slow variable. In this case we apply the method of freezing of a slow variable as compared to fast variables in system (13). It means that in the last two equations (13) we assume that s is a constant. Then from these equations for the steady state mode ($\dot{x} = 0$, $\dot{y} = 0$) we get

$$y = \tfrac{s}{\alpha_r^2+s^2}$$

Hence in this case the second equation in (13) may be integrated

$$\gamma = (1-s)\tau + \gamma_0$$

Then from the first equation in (13) we find for the steady state mode the partial solution

$$h = A\sin(1-s)\tau + B\cos(1-s)\tau$$

(we assume $\gamma_0 = 0$, which does not reduce the generality). After calculations we obtain

$$A = \tfrac{2n(1-s)}{4n^2(1-s)^2+[\omega_0^2-(1-s)^2]^2}\,\nu$$

$$B = \tfrac{\omega_0^2-(1-s)^2}{4n^2(1-s)^2+[\omega_0^2-(1-s)^2]^2}\,\nu$$

Finally the equation of motion of the rotor has the form

$$\dot{s} = -\delta\left[\tfrac{\alpha_r bs}{\alpha_r^2+s^2} + \kappa\sin(1-s)\tau\ \cos(1-s)\tau - \kappa A\sin^2(1-s)\tau - \kappa B\sin(1-s)\tau\ \cos(1-s)\tau\right]$$

Applying to the "fast phase" γ the standard operation of averaging for one turn of the magnetic field of a stator

$$\tfrac{1}{2\pi}\int_0^{2\pi}\sin(1-s)\tau\ \cos(1-s)\tau\ d\tau = 0$$

$$\tfrac{1}{2\pi}\int_0^{2\pi}\sin^2(1-s)\tau\ d\tau = \tfrac{1}{2}\int_0^{2\pi}[1-\cos 2(1-s)\tau]\,d\tau = \tfrac{1}{2}$$

we obtain the averaged equation of motion of the rotor

$$\dot{s} = -\delta\left(\frac{\alpha_r bs}{\alpha_r^2+s^2} - \frac{\kappa A}{2}\right) \tag{14}$$

To find the steady state solutions of the equation (14), we let $\dot{s} = 0$. Geometrically these solutions are the intersection of two curves:

$$f_1(s) = \frac{\alpha_r bs}{\alpha_r^2+s^2} \tag{15}$$

and

$$f_2(s,\omega_0) = \frac{2n(1-s)\kappa\pi\nu}{4n^2(1-s)^2+[\omega_0^2-(1-s)^2]^2} \tag{16}$$

Curve (15) is the static mechanical characteristic of AM, well known in the theory of asynchronous machines (see Figure 2).

To study the behaviour of curve (16) we find its extrema calculating the derivative with respect to s and setting it equals to zero. As a result we get the biquadratic equation in $1-s$:

$$3(1-s)^4 - 2(\omega_0^2-2n^2)(1-s)^2 - \omega_0^4 = 0$$

where

$(1-s)^2 = \frac{\omega_0^2-2n^2+\sqrt{(\omega_0^2-2n^2)^2+3\omega_0^4}}{3}$

(the solution with "$-$" at radix has no physical sense). Unique extreme in s, which has a physical sense is

$$s^* = 1 - \sqrt{\frac{\omega_0^2 - 2n^2 + \sqrt{(\omega_0^2 - 2n^2)^2 + 3\omega_0^4}}{3}} \tag{17}$$

Since $f_2(1) = 0$ and $f_2(-\infty) = +0$ the function $f_2(s, \omega_0)$ attains the unique maximum at s^* (see Figure 2).

We may approximately consider the extremums of the curve $f_2(s, \omega_0)$ as laying on a hyperbola (provided $n << 1$)

$\frac{\kappa\nu}{4n(1-s)}$

These extremums are the maximum amplitudes of the torque applied to the vibrational platform from AM for various values of ω, $0 < \omega_0 \leq 1$.

From Figure 2 it follows that there may be one, two, or three steady state points.

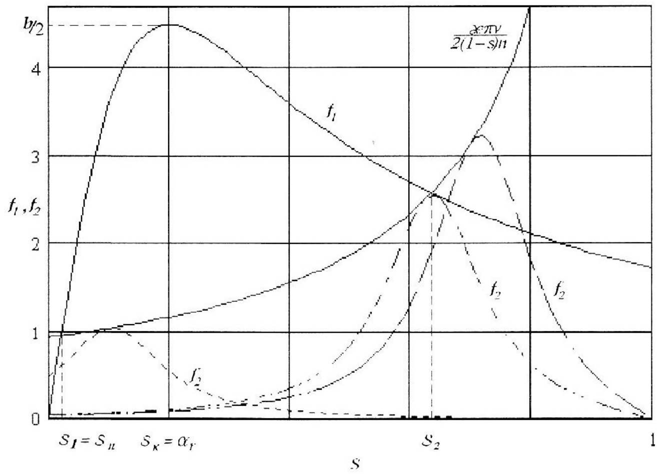

Figure 2: Graphs of the functions $f_1(s)$, $f_2(s, \omega_0)$

Theorem 1. *Let the equality*

$$\frac{\kappa\nu}{2n} = \frac{2\alpha_r b s_n (1 - s_n)}{\alpha_r^2 + s_n^2} \tag{18}$$

hold. Then at optimum set-up of the vibrational platform equation (14) has a unique absolutely stable steady state solution.

Proof. We call a set-up of a vibrational platform as optimum, if $s^* = s_n$ (the AM is inspected for a long work at $s \leq s_n$) and if thus $f_2(s_n, 1 - s_n) = f_1(s_n)$. From this condition we find the relation between the parameters of AM and those of the vibrational platform (18).

Let us show that if condition (18) holds, then only one steady state point exists. We calculate

$f_1(s_n) = \frac{\alpha_r b s_n}{\alpha_r^2 + s_n^2}$ $\qquad$ $f_1(1) = \frac{\alpha_r b}{1 + \alpha_r^2}$

Since the value of s_n is much less than the value of the critical sliding $s_k = \alpha_r$, $\alpha_r = 0.1 \div 0.2$ then

$$f_1(s_n) \approx \frac{bs_n}{\alpha_r} < b\alpha_r \approx f_1(1)$$

and the curve $f_2(s, 1 - s_n)$ does not intersect the curve $f_1(s)$. After linearization of the equation in the neighbourhood of the steady state solutions, s_0 becomes

$$\dot{\tilde{s}} + \delta \left[\frac{\alpha_r b(\alpha_r^2 - s_0^2)}{(\alpha_r^2 + s_0^2)^2} + \frac{\kappa\nu}{2(1-s_0)^2} \right] \tilde{s} = 0$$

By the first theorem of Lyapunov for the optimum set-up ($s_0 = s_n$) its solution is absolutely stable.

In the general case one can formulate the following theorem.

Theorem 2. *If the inequality*

$$\alpha_r \frac{2nb - \sqrt{(2nb)^2 - \kappa\nu(\kappa\nu + 4\alpha_r bn)}}{\kappa\nu + 4\alpha_r bn} \le s_n \tag{19}$$

holds, then there exists an absolutely stable steady state solution of equation (14) in the working domain of the sliding $0 \le s \le s_n$.

Proof. The function $f_2(s, \omega_0)$ attains its maximum value near the resonance point $\omega_0 = 1 - s$ (provided $n << 1$) . Let us show that for the case under consideration there exists only one steady state point for which

$$f_2^{\max}(\omega_0) = \frac{\kappa\nu}{4n\omega_0}$$

The equation of the curve is the geometric place of the maximums $f_2^{\max}(\omega_0)$, which is the function $f_2^{\max}(s) = \frac{\kappa\pi\nu}{2n(1-s)}$. To find the points of intersection of the curves $f_2(s)$ and $f_1(s)$, we solve the equation

$$\frac{\alpha_r bs}{\alpha_r^2 + s^2} = \frac{\kappa\nu}{4n(1-s)}$$

to obtain

$$s_{1,2} = \alpha_r \frac{2nb \pm \sqrt{(2nb)^2 - \kappa\nu(\kappa\nu + 4\alpha_r bn)}}{\kappa\nu + 4\alpha_r bn}$$

For the existence of the operational mode of AM in the working domain of sliding $0 \le s \le s_n$ it is required that condition (19) holds. Then for any value of ω_0 the curve $f_2(s, \omega_0)$ intersects the curve $f_1(s)$ at the abscissa point which is less than s_n (see Figure 2). To show that steady state solution at this point is absolutely stable we apply the graphic stability criterion (Lvovich, 1989)

$$\frac{\partial}{\partial s}\left[f_1(s) - f_2(s, \omega_0)\right] > 0$$

From Figure 2 it follows that at the considered point $\frac{\partial}{\partial s} f_1(s) > 0$, $\frac{\partial}{\partial s} f_2(s, \omega_0) > 0$ and $\frac{\partial}{\partial s} f_1(s) > \frac{\partial}{\partial s} f_2(s, \omega_0)$. Therefore $\frac{\partial}{\partial s}\left[f_1(s) - f_2(s, \omega_0)\right] > 0$ i.e. in this point the steady state operational mode is absolutely stable.

Remark 1. Theorem 1 is a particular case of Theorem 2 with the additional condition of the optimum set-up of the vibrational platform.

Remark 2. If equation (14) has two or three steady state solutions then the corresponding points lie on a part of a curve $f_1(s)$ which is the non-working zone of AM and these points have no practical interest.

Remark 3. If in nominal operational mode of the vibrational platform is in the resonance, the following approximate equality is valid

$$f_2 \approx \frac{\kappa\nu}{4n} = f_1 \approx \frac{b}{s_k} s_n$$

The AM works stable for the steady state operational mode at any natural frequency of the vibrational platform if s_1 is larger or equal to s_n, i.e.

$$s_n < \frac{\kappa\nu}{4nb} s_k$$

The values s_k, s_n can be found in the technical certificate of any AM.

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Address: Professor Rashit I. Alidema, Hilmi M. Kllapija, PRISHTINA University, YUGOSLAVIA; Feodor F. Rodyukov, Faculty of Mathematics and Mechanics, St. Petersburg State University, 2 Bibliotechnaya sq., Stary Peterhoff, RUS-198504, St. Petersburg, RUSSIA.