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Application of the Modified Murakami's Anisotropic Creep-Damage Model to 3D Rotationally-Symmetric Problem

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This paper demonstrates a modification of the creep-damage equations, proposed by Murakami, Kawai and Rong (MKR). The goals of analysis are: verification of the MKR creep-damage equations and checking the validity of the Reissner theory in the case of a very thick structure of copper under creep-damage conditions.

1 Introduction

Damage anisotropy is experimentally evident mainly in the case of brittle damage response where the appropriate modification of the fourth rank constitutive tensors, stiffness or compliance, is used (cf. Litewka (1985), Murakami and Kamiya (1997)). Under high temperature creep conditions it is usually assumed that the damage response is of isotropic nature (cf. Kachanov (1958), Hayhurst (1972), Lemaitre and Chaboche (1985), to mention only some). Nevertheless, even if creep damage is concerned, for some materials it is necessary to account for the effect of damage anisotropy. There exist simple proposals to formulate damage anisotropy under creep conditions described by some modifications of isotropic creep damage models, by introducing additional terms being sensitive to damage anisotropy.

An interesting system of constitutive equations, containing the combined McVetty and the Mises-type isotropic creep flow rule together with the strain hardening hypothesis

$$\dot{\varepsilon}^{\mathrm{c}} = \frac{3}{2}\left[A_1 \sigma_{\mathrm{eq}}^{n_1-1} \overline{\alpha} \exp\left(-\overline{\alpha} t\right) \mathbf{s} + A_2 \widetilde{\sigma}_{\mathrm{eq}}^{n_2-1} \widetilde{\mathbf{s}}\right] \tag{1}$$

coupled with the anisotropic damage evolution law, as an extension of the classical Hayhurst-type rule,

$$\begin{aligned}\dot{\mathbf{D}} = {} & B\left[\xi\sigma_1 + \zeta\sigma_{\mathrm{eq}} + (1-\xi-\zeta)\operatorname{Tr}(\boldsymbol{\sigma})\right]^k \left\{\operatorname{Tr}\left[(\mathbf{1}-\mathbf{D})^{-1}\left(\mathbf{n}^{(1)} \otimes \mathbf{n}^{(1)}\right)\right]\right\}^l \\ & \times \left[(1-\eta)\,\mathbf{1} + \eta\mathbf{n}^{(1)} \otimes \mathbf{n}^{(1)}\right]\end{aligned} \tag{2}$$

was developed and identified for copper by Murakami, Kawai and Rong (1988).

In the above damage evolution equation, the first term is the Hayhurst stress function that describes multiaxiality and nonlinearity, the second term includes anisotropic net area reduction due to damage in direction $\mathbf{n}^{(1)}$ of the principal (tensile) stress exclusively, whereas the third term determines the direction of $\dot{\mathbf{D}}$ as a linear combination between the isotropic damage growth and the direction of maximum anisotropy $\mathbf{n}^{(1)}$. Note the additional material parameter η in eq. (2), which may be recognized as the damage anisotropy parameter (weight parameter). For the particular cases $\eta = 0$ and $\eta = 1$, eq. (2) reduces to a purely isotropic damage evolution and a purely orthotropic microcrack growth in a plane perpendicular to the maximum tensile stress, respectively, whereas for $0 < \eta < 1$ a mixed isotropic/orthotropic damage growth mechanism occurs, in which damage is controlled by the maximum principal stress. The symmetric effective stress tensor and its deviator are of the following form (Murakami et al., 1988)

$$\widetilde{\boldsymbol{\sigma}} = \frac{1}{2}\left[\boldsymbol{\sigma}\left(\mathbf{1}-\mathbf{D}\right)^{-1} + \left(\mathbf{1}-\mathbf{D}\right)^{-1}\boldsymbol{\sigma}\right] \qquad \widetilde{\mathbf{s}} = \widetilde{\boldsymbol{\sigma}} - \frac{1}{3}\left(\operatorname{Tr}\widetilde{\boldsymbol{\sigma}}\right)\mathbf{1} \tag{3}$$

whereas appropriate equivalent stresses are given by the definitions

$$\sigma_{\mathrm{eq}} = \sqrt{\frac{3}{2}\mathbf{s}\mathbf{s}} \qquad \widetilde{\sigma}_{\mathrm{eq}} = \sqrt{\frac{3}{2}\widetilde{\mathbf{s}}\widetilde{\mathbf{s}}} \tag{4}$$

A similar equation, but expressed by the fourth rank damage tensor $\widehat{\mathbf{D}}$, fourth rank unit tensor $\widehat{\mathbf{1}}$ and anisotropy tensor $\widehat{\mathbf{\Gamma}}$ is due to Chaboche (1999)

$$\dot{\widehat{\mathbf{D}}} = \left[\frac{\chi(\tilde{\boldsymbol{\sigma}})}{A}\right]^r \left[\frac{\chi(\tilde{\boldsymbol{\sigma}})}{\chi(\boldsymbol{\sigma})}\right]^k \left[(1-\gamma)\,\widehat{\mathbf{1}} + \gamma\widehat{\mathbf{\Gamma}}\right] \tag{5}$$

In what follows the first of the above defined models will be used to develop a modified model for creep damage anisotropy.

On the other hand, damage growth accompanying creep at elevated temperature is a more anisotropic phenomenon in case when the deformation is less constrained. To emphasize this anisotropic response, a very thick rotationally symmetric 3D plate-like problem is investigated. Release of the inner constraints, such as Love-Kirchhoff's or Reissner's assumptions, when a full 3D formulation without any additional geometric constraints is used, allows to determine a limitation of the Reissner theory under creep-damage conditions.

2 Basic Equations

2.1 Assumptions

The displacement field fulfils the general rotational symmetry

$$\partial_\varphi \mathbf{u} = 0 \tag{6}$$

It is postulated that the linearized total strain tensor $\boldsymbol{\varepsilon} = \frac{1}{2}(\nabla\mathbf{u} + \mathbf{u}\nabla)$ may be decomposed into elastic, creep and thermal parts

$$\boldsymbol{\varepsilon} = \boldsymbol{\varepsilon}^{\mathrm{e}} + \boldsymbol{\varepsilon}^{\mathrm{c}} + \boldsymbol{\varepsilon}^{\mathrm{th}} \qquad \boldsymbol{\varepsilon}^{\mathrm{th}} = \mathbf{1}\alpha\theta \tag{7}$$

where $\theta = T - T_{\mathrm{ref}}$, and, additionally, the creep part is incompressible

$$\mathrm{tr}\boldsymbol{\varepsilon}^{\mathrm{c}} = 0 \tag{8}$$

2.2 Equations of Equilibrium

The system of displacement equations of the 3D continuum enriched by the creep term takes the form (Ganczarski, 2000)

$$G\nabla^2\mathbf{u} + \frac{G}{1-2\nu}\mathbf{grad}\,(\mathrm{div}\mathbf{u}) = 2G\mathrm{Div}\boldsymbol{\varepsilon}^{\mathrm{c}} \qquad \nabla^2 = \mathbf{grad}\,(\mathrm{div}) - \mathbf{rot}\,(\mathbf{rot}) \tag{9}$$

and the stress is defined as follows

$$\boldsymbol{\sigma} = 2G\left\{\boldsymbol{\varepsilon} - \boldsymbol{\varepsilon}^{\mathrm{c}} + \left[\frac{\nu\mathrm{Tr}\boldsymbol{\varepsilon} - (1+\nu)\,\alpha\theta}{1-2\nu}\right]\mathbf{1}\right\} \tag{10}$$

2.3 Modified Murakami's Anisotropic Creep-Damage Equations

The direction of the damage rate $\dot{\mathbf{D}}$ described by eq. (2) at a point depends on the linear combination of isotropic and anisotropic tensors where the weight parameter η is used and calibrated as the additional material constant. There are, however, certain regions in a structure where it is not possible to specify $\mathbf{n}^{(1)}$ uniquely. For instance, the stress at the z axis of rotational symmetry fulfills the condition of symmetry in the plane (r, φ). Therefore, all unit vectors belonging to this plane may be treated as $\mathbf{n}^{(1)}$. In such a case a convenient solution is a modification of the parameter η, which introduces a certain "degree of freedom" in the modelling of the type of anisotropy. Namely, the material parameter η is replaced by the new material function dependent on the positive eigenvalues of the stress tensor, in order to allow for damage isotropy in the region neighbouring the axis of symmetry. Hence, the following two modifications of the damage law are proposed:

- in a 2D case ($\langle\sigma_1\rangle \geq \langle\sigma_2\rangle > 0$)

$$\eta_{\mathrm{M}} = \eta\left(1 - \frac{\langle\sigma_2\rangle}{\langle\sigma_1\rangle}\right) \tag{11}$$

- in a 3D case ($\langle\sigma_1\rangle \geq \langle\sigma_2\rangle \geq \langle\sigma_3\rangle > 0$)

$$\eta_{\mathrm{M}} = \eta\left[1 - \left(1 - \frac{\langle\sigma_3\rangle}{2\langle\sigma_2\rangle}\right)\frac{\langle\sigma_2\rangle + \langle\sigma_2\rangle}{\langle\sigma_1\rangle}\right] \tag{12}$$

where η stands for the original parameter of anisotropy in eq. (2). The above proposals allow to describe a smooth transition from the strictly isotropic damage growth at the symmetry axis to the anisotropic response in a point neighbouring this axis. In this way the intrinsic inconsistency of the MKR model may be overcome.

3 Boundary Problem

A constant temperature field accompanies a homogeneous displacement field, but no stress fields (see eq. (10)). Therefore a simple support is assumed to carry the mechanical load p (Fig. 1). The appropriate boundary conditions are collected in Table 1.

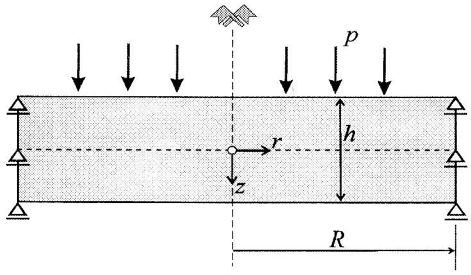

Figure 1: Simply Supported Plate Loaded by Constant Pressure p

Table 1: Boundary Conditions

boundary	condition	
top surface	$\sigma_z = -p$	$\sigma_{rz} = 0$
bottom surface	$\sigma_z = 0$	$\sigma_{rz} = 0$
symmetry axis	$u_r = 0$	$\partial_r u_z = 0$
supported edge	$\sigma_r = 0$	$u_z = 0$

4 Numerical Algorithm for Creep-Damage Problem

To solve the initial-boundary problem by FDM, we discretize the time by inserting N time intervals Δt_k, where $t_0 = 0$, $\Delta t_k = t_k - t_{k-1}$, and $t_N = t_I$ (macrocrack initiation) (Skrzypek and Ganczarski, 1999). Hence, the initial-boundary problem is reduced to a sequence of the quasistatic boundary-value problems, the solution of which determines an unknown displacement function at a given time t_k, e.g., $\mathbf{u}(\mathbf{x}, t_k) = \mathbf{u}^k(\mathbf{x})$ with the appropriate elastic solution taken as the initial condition. To account for primary and tertiary creep regimes, a dynamically controlled time step Δt_k is required, the length of which is defined by the bounded maximum damage increment

$$\Delta D^{\mathrm{lower}} \leq \max_{(\mathbf{x})}\left\{\left\|\dot{\mathbf{D}}^k(\mathbf{x}) - \dot{\mathbf{D}}^{k-1}(\mathbf{x})\right\|\Delta t_k\right\} \leq \Delta D^{\mathrm{upper}} \tag{13}$$

Discretizing also the spatial coordinates $\mathbf{x} = [r, z]_{i,j}$ by inserting a mesh $\Delta r = r_i - r_{i-1}$, $\Delta z = z_j - z_{j-1}$, we rewrite the equation of motion eq. (9) for a time step t_k in terms of FDM with respect to r_i and z_j, respectively

$$
\begin{aligned}
&2(1-\nu)\left(\frac{1}{\Delta r} - \frac{1}{2r_i}\right)\frac{u_{i-1,j}}{\Delta r} + (1-2\nu)\frac{u_{i,j-1}}{(\Delta z)^2} - \left\{2(1-\nu)\left[\frac{2}{(\Delta r)^2} - \frac{1}{2r_i^2}\right]\right. \\
&\left. + (1-2\nu)\frac{2}{(\Delta z)^2}\right\} u_{i,j} + (1-2\nu)\frac{u_{i,j+1}}{(\Delta z)^2} + 2(1-\nu)\left(\frac{1}{\Delta r} + \frac{1}{2r_i}\right)\frac{u_{i+1,j}}{\Delta r} \\
&+ \frac{w_{i-1,j-1}}{4\Delta r \Delta z} - \frac{w_{i+1,j-1}}{4\Delta r \Delta z} + \frac{w_{i-1,j+1}}{4\Delta r \Delta z} - \frac{w_{i+1,j+1}}{4\Delta r \Delta z} \\
&= (1-2\nu)\left\{\frac{-\varepsilon^{\mathrm{c}}_{r_{i-1,j}} + \varepsilon^{\mathrm{c}}_{r_{i+1,j}}}{\Delta r} + \frac{-\gamma^{\mathrm{c}}_{rz_{i,j-1}} + \gamma^{\mathrm{c}}_{rz_{i,j+1}}}{2\Delta z} + 2\frac{\varepsilon^{\mathrm{c}}_{r_{i,j}} - \varepsilon^{\mathrm{c}}_{\varphi_{i,j}}}{r_i}\right\} \\
&(1-2\nu)\left(\frac{1}{\Delta r} - \frac{1}{2r_i}\right)\frac{w_{i-1,j}}{\Delta r} + 2(1-\nu)\frac{w_{i,j-1}}{(\Delta z)^2} - \left\{(1-2\nu)\frac{2}{(\Delta r)^2}\right. \\
&\left. + 2(1-\nu)\frac{2}{(\Delta z)^2}\right\} w_{i,j} + 2(1-\nu)\frac{w_{i,j+1}}{(\Delta z)^2} + (1-2\nu)\left(\frac{1}{\Delta r} + \frac{1}{2r_i}\right)\frac{w_{i+1,j}}{\Delta r} \\
&+ \frac{u_{i-1,j-1}}{4\Delta r \Delta z} - \frac{u_{i+1,j-1}}{4\Delta r \Delta z} + \frac{u_{i-1,j+1}}{4\Delta r \Delta z} - \frac{u_{i+1,j+1}}{4\Delta r \Delta z} \\
&- \frac{u_{i,j-1}}{2r_i\Delta z} + \frac{u_{i,j+1}}{2r_i\Delta z} = (1-2\nu)\left\{\frac{-\gamma^{\mathrm{c}}_{rz_{i,j-1}} + \gamma^{\mathrm{c}}_{rz_{i,j+1}}}{2\Delta z} + \frac{-\varepsilon^{\mathrm{c}}_{z_{i,j-1}} + \varepsilon^{\mathrm{c}}_{z_{i,j+1}}}{\Delta z} + \frac{\gamma^{\mathrm{c}}_{rz_{i,j}}}{r_i}\right\}
\end{aligned}
\tag{14}
$$

Applying the stage algorithm first for the damage $[\mathbf{D}]_{i,j} \equiv 0$, the system of equations of motion eq. (9) and the boundary conditions (cf. Table 1) are numerically solved by use of the procedure `linbcg.for` of the Conjugate Gradient Method for sparse systems (Press et al., 1993), and the elastic displacements are determined $[\mathbf{u}^{\mathrm{e}}]_{i,j}$. Next, the program enters the creep-damage loop where the damage and the strain rates $[\dot{\mathbf{D}}, \dot{\boldsymbol{\varepsilon}}^{\mathrm{c}}]_{i,j}$, Eqs (1, 2) are calculated. Applying again the stage algorithm to the solution of the creep damage problem, the rates of displacement $[\dot{\mathbf{u}}]_{i,j}$ are computed. In the next time step the "new" displacement $[\mathbf{u}]_{i,j}$ is found, and the process is continued until the maximum value of damage reaches the critical level $\max[\mathbf{D}]_{i,j} = D_{\mathrm{crit}}$. A rupture criterion can be simultaneously derived from the instability condition of the stress-strain relation. Namely, the rupture occurs if either $\det\{\mathbf{H}\} = 0$ or any of its subdeterminants $\mathrm{minor}\{\mathbf{H}\} = 0$, where the matrix $\mathbf{H}$ is expressed as

$$
\mathbf{H} = \left\{
\begin{array}{cccc}
\dfrac{1-\nu}{1-2\nu} - \dfrac{\partial \varepsilon^{\mathrm{c}}_r}{\partial \varepsilon_r} & \dfrac{\nu}{1-2\nu} - \dfrac{\partial \varepsilon^{\mathrm{c}}_r}{\partial \varepsilon_\varphi} & \dfrac{\nu}{1-2\nu} - \dfrac{\partial \varepsilon^{\mathrm{c}}_r}{\partial \varepsilon_z} & 0 \\
 & \dfrac{1-\nu}{1-2\nu} - \dfrac{\partial \varepsilon^{\mathrm{c}}_\varphi}{\partial \varepsilon_\varphi} & \dfrac{1-\nu}{1-2\nu} - \dfrac{\partial \varepsilon^{\mathrm{c}}_\varphi}{\partial \varepsilon_z} & 0 \\
 & & \dfrac{1-\nu}{1-2\nu} - \dfrac{\partial \varepsilon^{\mathrm{c}}_z}{\partial \varepsilon_z} & 0 \\
\text{symmetry} & & & 1 - \dfrac{\partial \gamma^{\mathrm{c}}_{rz}}{\partial \gamma_{rz}}
\end{array}
\right\}
\tag{15}
$$

and the partial derivative $\partial\boldsymbol{\varepsilon}^{\mathrm{c}}/\partial\boldsymbol{\varepsilon}$ needs to be calculated from eq. (1). More precisely, this is the requirement that the constitutive relation must always be of the Green (hyperelastic) type, and, consequently, the quadratic form associated with the strain energy $\mathbf{H} = \partial^2 W/\partial\boldsymbol{\varepsilon}\partial\boldsymbol{\varepsilon}$ must be positive definite $(\forall\boldsymbol{\varepsilon} \quad \boldsymbol{\varepsilon}^{\mathrm{T}}\mathbf{H}\boldsymbol{\varepsilon} > 0)$ (Murakami et al., 1988, Chen and Han, 1988). For special cases the classical Kachanov-Rabotnov critical damage condition $D_\nu \leq D_{\nu\mathrm{crit}}$ $(\nu = 1, 2, 3)$ may also be used.

5 Results

5.1 Material Data

Numerical results presented in this paper deal with plates made of copper of the following thermo-mechanical properties at temperature 523 K (Murakami, 1988): $T = 300°\mathrm{C}$, $T_{\mathrm{ref}} = 0°\mathrm{C}$, $E = 60.24$ MPa, $\sigma_0 = 11.0$ MPa, $\nu = 0.3$, $B = 4.46 \times 10^{-13}$ MPa^{-k}h^{-1}, $l = 5.0$, $k = 5.55$, $\xi = 1.0$, $A_1 = 2.40 \times 10^{-17}$ MPa$^{-n_1}$, $n_1 = 2.60$, $A_2 = 3.00 \times 10^{-16}$ MPa$^{-n_2}$h^{-1}, $n_2 = 7.10$, $\overline{\alpha} = 0.05$ h^{-1}, $\alpha = 2.5 \times 10^{-5}$. Characteristic parameters of the plate (thickness to diameter ratio) and the magnitude of pressure are: $h/R = 0.7$, $p = 0.1\sigma_0$, respectively. The parameter controlling the type of damage anisotropy is $\eta = 0.5$, which means that the mixed isotropic/maximum damage growth mechanism is considered, in which damage is controlled by the principal stress.

5.2 Example 1 - Original MKR Formulation

In case of the original Murakami, Kawai and Rong formulation of the damage evolution law eq. (2) the maximal principal stress type sensitivity to damage of the copper causes the first macrocrack to appear at the center of the lower surface of the plate with respect to hoop direction D_φ (Fig. 2). Two other locally principal directions at the points belonging to the symmetry axis, radial and axial, reveal a damage advance equal to half of the hoop direction $D_r = D_z = 0.5 D_\varphi$ (Fig. 3), as a consequence of $\eta = 0.5$. This point, however, belongs to the axis of symmetry, where the stress exhibits symmetry $\sigma_r = \sigma_\varphi$ so the damage should also be equal $D_r = D_\varphi$. Unfortunately, this is possible for $\eta = 0.0$ (isotropic case) exclusively.

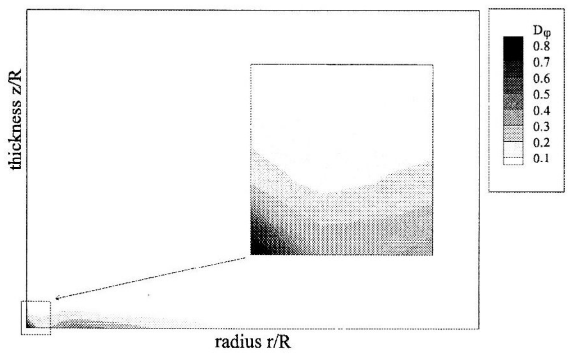

Figure 2: Hoop Damage D_φ - Original Formulation by Murakami, Kawai and Rong

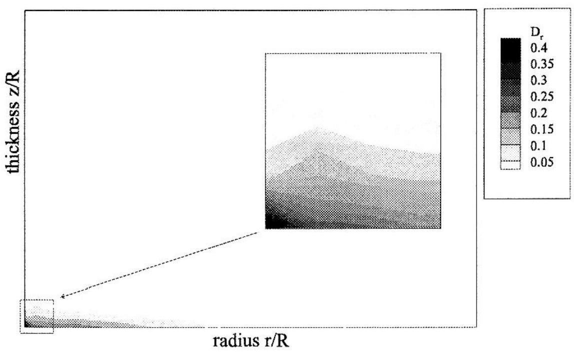

Figure 3: Radial Damage D_r - Original Formulation by Murakami, Kawai and Rong

5.3 Example 2 - Modified MKR Formulation

The above mentioned defect of the original Murakami, Kawai and Rong formulation can be successfully eliminated when the modification given by eq. (11) is taken into account. As a result, a smooth transition of damage anisotropy to damage isotropy is observed in the zone neighbouring the axis of symmetry (Figs 4, 5).

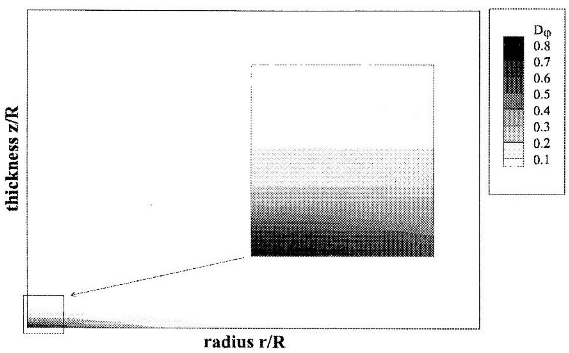

Figure 4: Hoop Damage D_φ - Modified Formulation

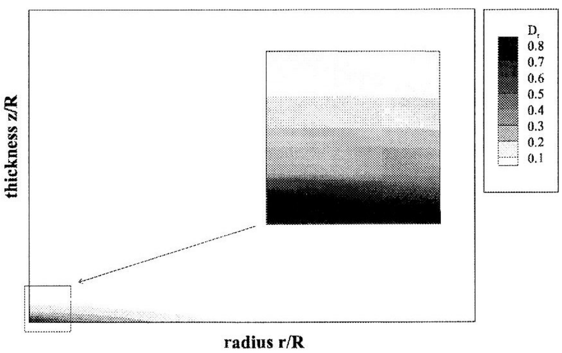

Figure 5: Radial Damage D_r - Modified Formulation

5.4 Comparison of the 3D vs. Reissner's Approaches with the Modified MKR Model Used

The shape of the supported edge does not obey any of the generally applied hypotheses (Fig. 6). Detailed analysis reveals an essential deformation of the mid-surface, which no longer coincides with the neutral surface. Although the mechanical moduli are assumed to not be affected by damage in the model under

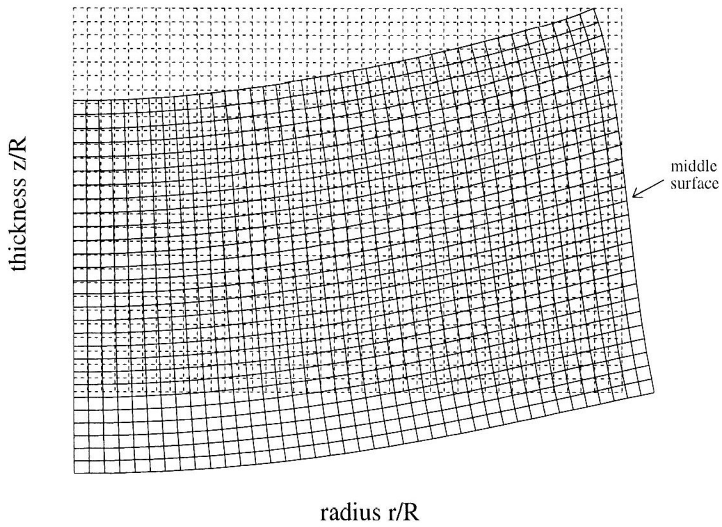

Figure 6: Deformation of the Plate under a Purely Mechanical Load p (Magnification $\times$ 1250)

consideration, the hoop stress is subjected to a change. Contrary to Reissner's approach (Ganczarski and Skrzypek, 2000), in the present 3D rotationally symmetric formulation the hoop stress is no longer linear (Fig. 7). The other stress components, namely the axial σ_z and the shear stress σ_{rz}, exhibit qualitative and quantitative changes. If the axial stress σ_z can be well approximated by the Reissner cubic parabola $-\frac{3}{4}p\left[\frac{2}{3} - \frac{2z}{h} + \frac{1}{3}\left(\frac{2z}{h}\right)^3\right]$ during primary and secondary creep in the central part of the plate, the tertiary creep and the boundary effect require a full 3D analysis (Fig. 8).

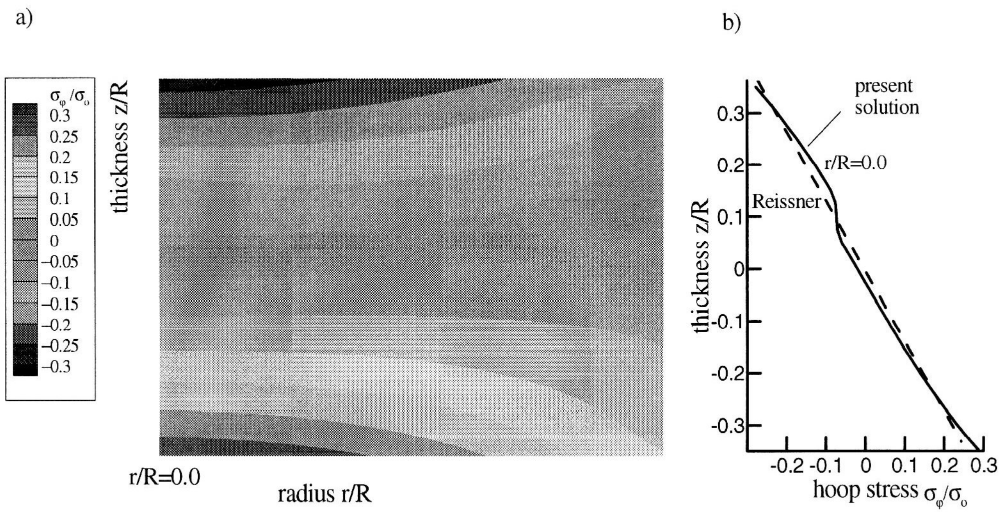

Figure 7: Hoop Stress σ_φ/σ_0 at the Instant of First Macrocrack Initiation a), Comparison to the Reissner Theory b)

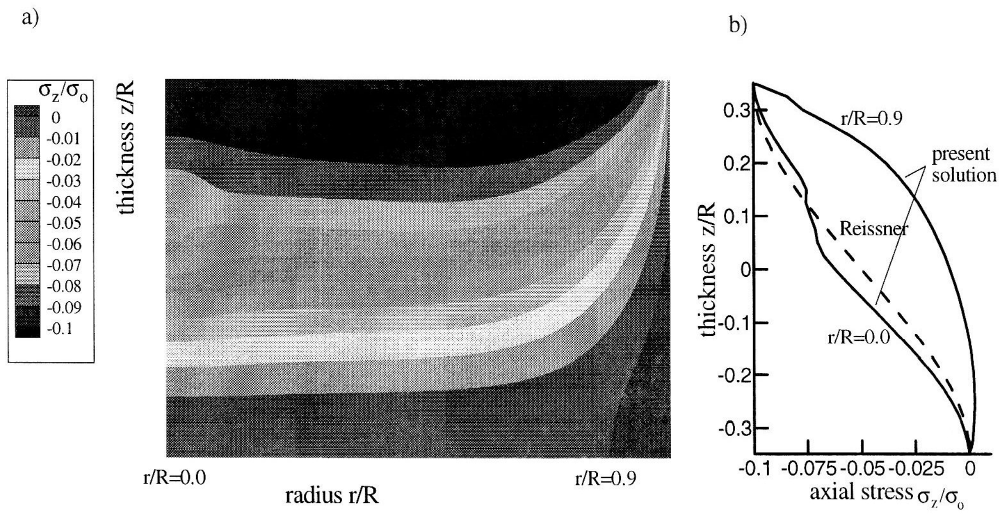

Figure 8: Axial Stress σ_z/σ_0 at the Instant of First Macrocrack Initiation a), Comparison to the Reissner Theory b)

Similarly, the shear stress σ_{rz} cannot be properly described by the Reissner parabola $-\frac{3p}{4h}\left[1-\left(\frac{2z}{h}\right)^2\right]$ even in the elastic case because it looses its symmetry with respect to mid-surface and, what is most essential, it follows the creep-damage process (Okumura and Oguma, 1998)(Fig. 9).

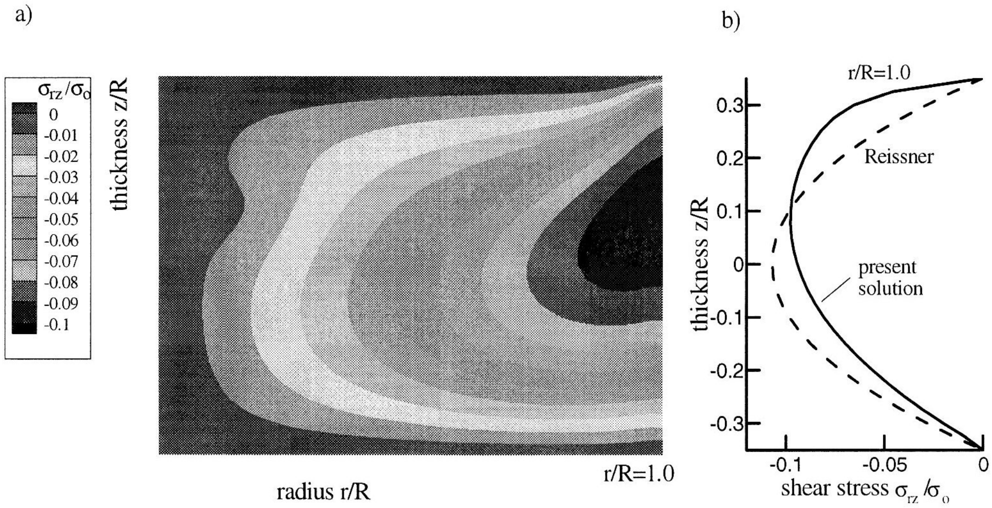

Figure 9: Shear Stress σ_{rz}/σ_0 at the Instant of First Macrocrack Initiation a), Comparison to the Reissner Theory b)

6 Conclusions

- The Damage evolution described by the original MKR model eq. (2) depends locally on the combination of the isotropy and anisotropy tensors $\mathbf{1}$, $\mathbf{n}^{(1)} \otimes \mathbf{n}^{(1)}$, respectively. This turns out to be intrinsically inconsistent in the case of plane stress isotropy $\sigma_r(0) = \sigma_\varphi(0)$, which always exists at the symmetry axis. The stress isotropy results here from the "local" isotropy of the damage growth. In order to fulfill the required damage isotropy at points belonging to the axis of symmetry, the modification given by eq. (11) can be introduced. It allows for a smooth transition between the strictly isotropic damage growth at all points at the axis and an anisotropic one in the neighbourhood.

- Comparison between the 3D and the Reissner formulation for a mid-thick plate determines the limitation of Reissner's approach in the tertiary creep regime. In such a case, tertiary creep of the thick-wall structures (thickness $\propto$ diameter) cannot be properly described in terms of the Reissner theory and requires a full 3D continuum model.

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Literature

1. Chaboche, J.-L.: Themodynamically founded CDM models for creep and other conditions, in Creep and damage in materials and structures, Eds. Altenbach H. and Skrzypek J., SpringerWien, (1999), 209 – 283.

2. Chen, W. F.; Han, D. J.: Plasticity for structural engineers, Springer-Verlag, Berlin Heidelberg, (1988).
3. Ganczarski, A.: Thick axisymmetric plate subjected to thermo-mechanical damage, Proc. IUTAM Symp. on Creep in Structures, Kluwer, (2000), 207 – 216.
4. Ganczarski, A.; Skrzypek, J.: Damage effect on thermo-mechanical fields in a mid-thick plate, J. Theor. Appl. Mech., 2, 38, (2000), 271 – 284.
5. Hayhurst, D. R.: Creep rupture under multiaxial state of stress, J. Mech. Phys. Solids, 20, 6, (1972), 381 – 390.
6. Kachanov, L. M.: Time of the rupture process under creep conditions, Izv. AN SSR, Otd. Tekh. Nauk, 8, (1958), 26 – 31.
7. Lemaitre, J.; Chaboche, J.-L.: Méchanique des Matériaux Solides, Dunod Publ., Paris, (1985).
8. Litewka, A.: Effective material constants for orthotropically damaged elastic solid, Arch. Mech., 37, 6, (1985), 631 – 642.
9. Murakami, S.; Kawai, M.; Rong, H.: Finite element analysis of creep crack growth by a local approach, Int. J. Mech. Sci., 30, 7, (1988), 491 – 502.
10. Murakami, S.: Mechanical modelling of material damage, J. Appl. Mech., 55, 6, (1988), 280 – 286.
11. Murakami, S., Kamiya; K.: Constitutive and damage evolution equations of elastic-brittle materials based on irreversible thermodynamics, Int. J. Solids Struct., 39, 4, (1997), 473 – 486.
12. Okumura, I. A.; Oguma, Y.: Series solutions for transversely loaded and completely clamped thick rectangular plate based on the three-dimensional theory of elasticity, Arch. Appl. Mech., 68, (1998), 103 – 121.
13. Press, W. H.; Teukolsky, S. A.; Vetterling, W. T.; Flannery, B. P.: Numerical Recipes in Fortran, Cambridge Press, (1993).
14. Skrzypek, J.; Ganczarski, A.: Modeling of Material Damage and Failure of Structures, Theory and Applications, Springer-Verlag, Berlin Heidelberg, (1999).

Symbols

Mechanical Quantities

$A_1, A_2, \overline{\alpha}, n_1, n_2$	-	constants of the creep law
$B, k, l, \xi, \zeta, \eta$	-	constants of the damage evolution law
$\mathbf{D}$	-	damage tensor
E, G, ν	-	Young's and Kirchhoff's moduli, Poisson's ratio, respectively
h	-	thickness
p	-	pressure
R	-	radius
$\mathbf{n}$	-	unit vector
$\mathbf{u}$	-	displacement
t	-	time
$\mathbf{x}\,[r, z]$	-	cylindrical coordinates
$\varepsilon, \varepsilon^{\mathrm{e}}, \varepsilon^{\mathrm{c}}, \varepsilon^{\mathrm{th}}$	-	strain: total, elastic, inelastic (creep) and thermal
$\boldsymbol{\sigma}, \mathbf{s}$	-	stress tensor and its deviator
$\widetilde{\boldsymbol{\sigma}}, \widetilde{\mathbf{s}}$	-	effective stress and its deviator
$\sigma_{\mathrm{eq}}, \widetilde{\sigma}_{\mathrm{eq}}$	-	equivalent stress and equivalent effective stress
σ_0	-	yield stress
$\mathbf{1}$	-	unit tensor

Thermal Quantities

T	-	temperature
T_{ref}	-	reference temperature
α	-	coefficient of thermal expansion
θ	-	temperature change

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