TECHNISCHE MECHANIK, Band 21, Heft 4, (2001), 313-318
Manuskripteingang: 20. September 2001

Free Convection over a Vertical Flat Plate with a Variable Wall Temperature and Internal Heat Generation in a Porous Medium Saturated with a Non-Newtonian Fluid

T. Groşan, I. Pop

A boundary-layer analysis is performed in this paper for the free convection flow over a vertical flat plate embedded in a porous medium saturated by a power-law non-Newtonian fluid. It is assumed that the temperature is a function of the distance from the origin, and that there is an internal heat source within the porous medium. Similarity solutions are derived for the governing equations and then used to study the effects of the power-law temperature parameter and power-law fluid index on the heat transfer characteristics.

1 Introduction

The subject of thermal convection in porous media has gained increasing research interest during the past several decades. This is due to the presence of porous media in a wide range of geophysical and engineering applications of current interest. These applications include, but are not limited to, geothermal energy extraction drying processes (wood and food products), groundwater contamination, thermal energy storage, heat pipes, building insulation, separation processes in chemical industry, filtration processes heat transfer enhancement especially in high heat flux applications such as cooling of electronic equipment, to name just a few applications. Review of the extensive work that has gone into this subject is available in the recent books by Ingham and Pop (1998), Nield and Bejan (1999), Vafai (2000), and Pop and Ingham (2001).

However, a number of fluids, which could come in contact with porous media, show non–Newtonian flow behaviour, especially in ceramic processing, enhanced oil recovery and filtration, see Nakayama and Shenoy (1993).

Thermal convection in a fluid saturated porous medium with internal energy sources is very important in the theory of thermal ignition when heat sources within the fluid saturated porous medium are driven by an exothermic chemical reaction. Here the thermal gradients originated by the chemical reaction can be the driving force for the onset of free convection, which markedly enhance the rate of heat transfer in comparison to a purely conductive mechanism. The purpose of this paper is the analytical study of the free convection flow over a vertical surface with variable surface temperature, placed in a fluid-saturated porous medium with a non-Newtonian fluid and containing a uniform internal heat source. Appropriate transformation of variables are attempted in order to seek similarity solution of the governing equations. These equations are solved numerically using the shooting technique.

2 Basic Equations

We start from the conservation equations of mass, momentum (Darcy), and energy for a porous medium saturated with a non-Newtonian power-law fluid model, originally suggested by Christopher and Middelman (1965) and later modified by Dharmadhikari and Kale (1985). Under the Boussinesq and boundary layer approximation, the basic equation can be written as

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y} = 0 \qquad (1)$$

$$u^n = \frac{gK^*(n)\beta}{\nu^*}(T-T_\infty) \qquad (2)$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y} = \alpha_m\frac{\partial^2 T}{\partial y^2}+\frac{q_w}{\rho C_p} \tag{3}$$

where x and y are the Cartesian co-ordinates along and normal to the plate, respectively, u and v are the velocity components along the x and y axes, T is the fluid temperature, T_∞ is the temperature of the ambient fluid, g is the acceleration due to gravity, ρ is the density, β is the coefficient of thermal expansion, α_m is the effective thermal diffusivity, ν^* is the modified kinematic viscosity, C_p is the specific heat at constant pressure, q_w is the internal heat generation, n is the power law index and $K^*(n)$ is the modified permeability, which is given by

$$K^*(n) = \begin{cases} \dfrac{6}{25}\left(\dfrac{n\varepsilon}{3n+1}\right)^n\left(\dfrac{\varepsilon d}{3(1-\varepsilon)}\right)^{n+1} & \text{Christopher and Middleman (1965)} \\ \dfrac{2}{\varepsilon}\left(\dfrac{d\varepsilon^2}{8(1-\varepsilon)}\right)^{n+1}\dfrac{6n+1}{10n-3}\left(\dfrac{16}{75}\right)^{\frac{3(10n-3)}{10n+11}} & \text{Darmadhikari and Kale (1985)} \end{cases} \tag{4}$$

where d is the particle diameter and ε is the porosity. We notice that $n < 1$ corresponds to pseudoplastic fluids, $n = 1$ to Newtonian fluids and $n > 1$ to dilatant fluids.

We assume that the temperature of the plate varies as a power function of the coordinate x along the plate, see Cheng and Minkowycz (1977). Thus, the boundary conditions of equations (1) – (3) are

$$\begin{array}{llllll} v = 0 & T = T_\infty + Ax^\lambda & \text{on} & y = 0 \\ u \to 0 & T \to T_\infty & \text{as} & y \to \infty \end{array} \tag{5}$$

where A and λ are positive constants.

To solve equations (1) – (3), we introduce the following similarity variables

$$\psi = \alpha_m Ra_x^{1/2} f(\eta) \qquad \theta(\eta) = \frac{T-T_\infty}{T_w-T_\infty} \qquad \eta = Ra_x^{1/2}(y/x) \tag{6}$$

where ψ is the stream function which is defined in the usual way as $u = \partial\psi/\partial y$ and $v = -\partial\psi/\partial x$, and Ra_x is the generalised local Rayleigh number which is defined as

$$Ra_x = \left(\frac{g\beta K^*(n)(T_w-T_\infty)x^n}{\nu^*\alpha_m{}^n}\right)^{\frac{1}{n}} \tag{7}$$

In order that similarity solutions of equations (1) – (3) exist, we assume, following Crepeau and Clarksean (1994), that the internal heat generation is given by

$$q_w = \frac{k_m(T_w-T_\infty)}{x^2}Ra_x e^{-\eta} \tag{8}$$

where k_m is the effective thermal conductivity of the porous medium. On using (6) and (8), equations (2) and (3) reduce to the following ordinary differential equations

$$(f')^n = \theta \tag{9}$$

$$\theta'' + \frac{1}{2}\frac{n+\lambda}{n} f\theta' - \lambda f'\theta + e^{-\eta} = 0 \tag{10}$$

and the boundary conditions (5) become

$$f(0) = 0 \qquad \theta(0) = 1 \qquad \theta \to 0 \qquad \text{as } \eta \to \infty \tag{11}$$

where primes denote differentiation with respect to η.
The main physical quantity of interest in this problem is the local Nusselt number, which is given by

$$Nu/Ra_x^{1/2} = -\theta'(n,\lambda,0) \tag{12}$$

3 Results and Discussions

Computations for the similarity equations (9) and (10) subject to the boundary conditions (11) are carried out for the power-law index n ranging from 0.5 to 2.5 with $\lambda = 0$ (isothermal plate), 1/3 and 1 using the shooting method as proposed by Chakraborthy (1998). The obtained results for the local Nusselt number given by equation (12) are shown in Tables 1 and 2. Some known results from the literature are also included in these tables. It is seen that the present results are in very good agreement with the known results. We are therefore confident that our results are correct. Further, we notice from these tables that the heat transfer from the plate increases with the index n when the internal heat generation is absent or present. However, the heat transfer is negative for all values of n for an isothermal plate ($\lambda = 0$) when the internal heat generation is present. It means that the heat transfer goes from the fluid to the plate, i.e. the plate is cooled by internal heat generation rates.

Figures 1 and 2 illustrate the non-dimensional temperature profiles for different values of the parameters n and λ for the both cases when the internal heat generation is absent and when it is present. We notice that the temperature profiles decreases with the increasing of n and the boundary layer thickness decreases with the increases of λ. Finally Figure 3 represents the variation of the local Nusselt number with n for some values of λ. We can see that the heat transfer increases with the increasing of power-law index n and also monotonically increases with λ.

	$\lambda = 0$		$\lambda = 1/3$		$\lambda = 1$	
n	Chen and Chen (1988)	present results	Cheng (1977)	present results	Cheng (1977)	present results
0.5	0.3768	0.377670		0.616256		0.928710
0.8	0.4238	0.423999		0.659114		0.978515
1.0	0.4437	0.443885	0.6776	0.677707	1.0000	0.999747
1.5	0.4752	0.475379		0.697294		1.033988
2.0	0.4938	0.493804		0.724662		1.053750
2.5	0.5059	0.505912		0.736076		1.067187

Table 1. Values of the Local Nusselt Number $-\theta'(n, \lambda, 0)$ when Internal Heat Generation is Absent

n	$\lambda = 0$		$\lambda = 1/3$		$\lambda = 1$	
	Postelnicu et al. (2000)	present results	Postelnicu et al. (2000)	present results	Postelnicu et al. (2000)	Present results
0.5		-0.257442		0.080008		0.469726
0.8		-0.228813		0.102851		0.508398
1.0	-0.2152	-0.215164	0.1141	0.114111	0.5240	0.525390
1.5		-0.192077		0.133529		0.553076
2.0		-0.177757		0.145721		0.569921
2.5		-0.168029		0.154037		0.581054

Table 2. Values of the Local Nusselt Number $-\theta'(n, \lambda, 0)$ when Internal Heat Generation is Present

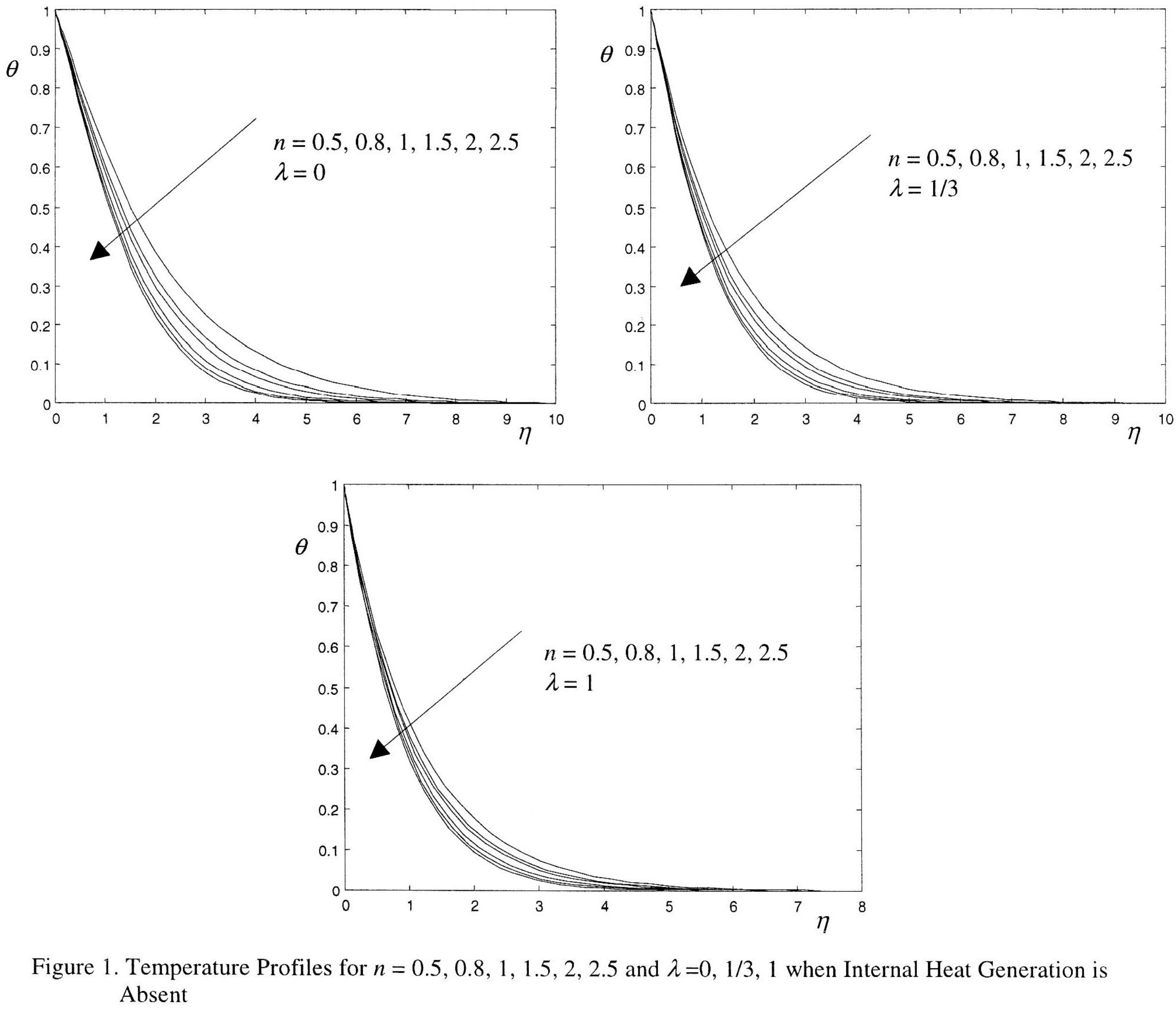

Figure 1. Temperature Profiles for n = 0.5, 0.8, 1, 1.5, 2, 2.5 and λ =0, 1/3, 1 when Internal Heat Generation is Absent

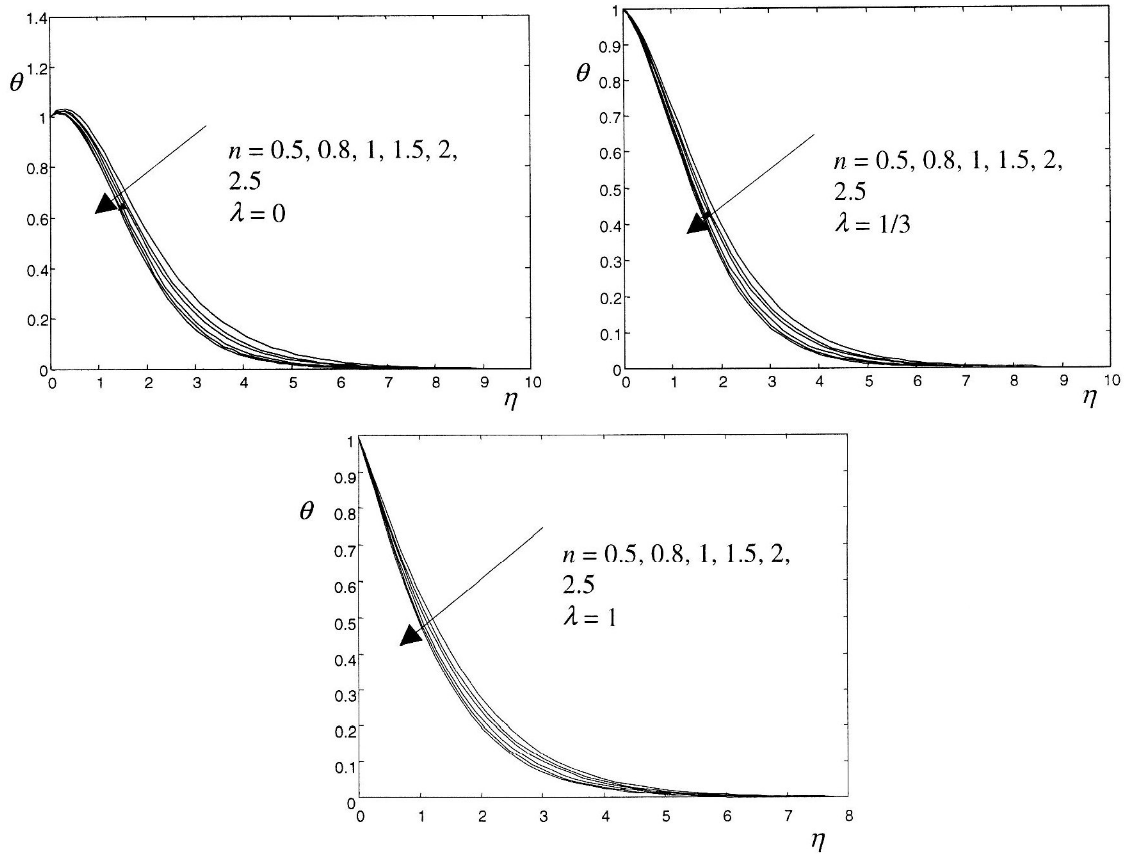

Figure 2. Temperature Profiles for n = 0.5, 0.8, 1, 1.5, 2, 2.5 and λ =0, 1/3, 1 when Internal Heat Generation is Present

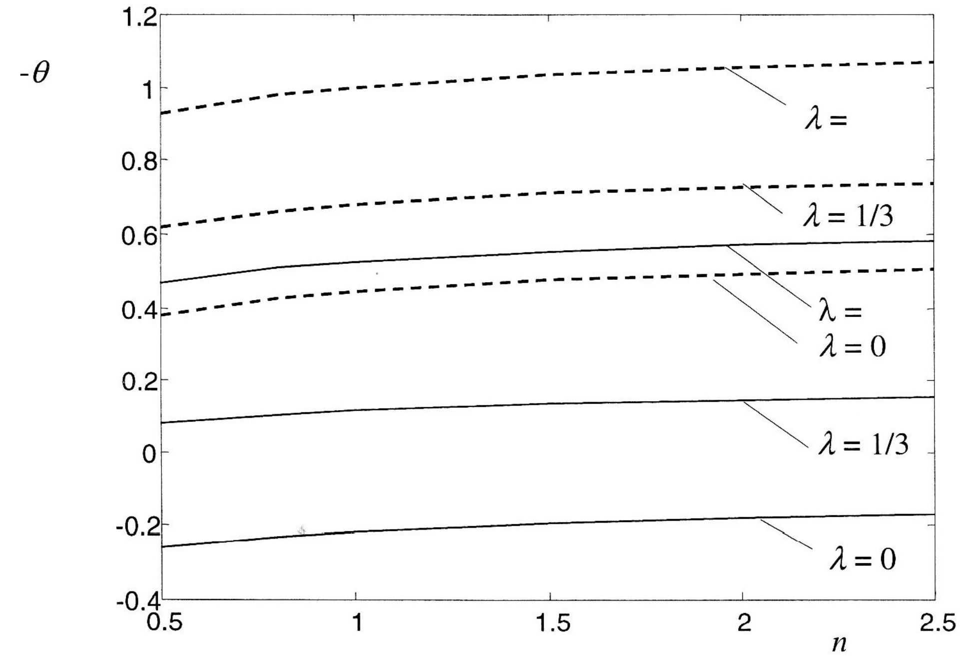

Figure 3. Variation of the Local Nusselt Number $-\theta'(n,\lambda,0)$ with n. (— with Internal Heat Generation; --- without Internal Heat Generation)

Acknowledgements

The authors would like to thank the referee for his valuable comments, which led to the improvement of the paper.

Literature

1. Chakraborty, S: Some problems of flow and heat transfer on magnetohydrodynamics. Ph.D. Thesis, Tezpur University, Assam, India, (1998).

2. Chen, H.T.; Chen, C.K.: Free convection flows of non-Newtonian fluids along a vertical plate embedded in a porous medium. ASME J. Heat Transfer, 110, (1988), 257-259.

3. Cheng, P.: The influence of lateral mass flux on free convection boundary layers in a saturated porous medium. Int. J. Heat Mass Transfer, 20, (1977), 201-206.

4. Cheng, P.; Minkowycz, W.J.: Free convection about a vertical flat plate embedded in a porous medium with application to heat transfer from a dike. J. Geophys. Res., 82, (1977), 2040-2044.

5. Christopher, R.V.; Middleman, S.: Power-law flow trough a packed tube. Ind. Engng. Chem. Fundamentals, 4,(1965),424-426.

6. Crepeau, R.V.; Clarksean, R.: Similarity solutions of natural convection with internal heat generation. J.Heat Transfer, 119,(1997), 183-185.

7. Dharmadhikari, R.V.; Kale, D.D.: Flow of non-Newtonian fluids through porous media. Chem. Engng. Sci., 40, (1985), 527-529.

8. Ingham, D.B., Pop, I. (eds.): Transport Phenomena in Porous Media, Pergamon, Oxford, (1998).

9. Nakayama, A.; Shenoy, A.V.: Combined forced and free convection heat transfer in power-law fluid-saturated porous media. Appl. Sci. Res., 50, (1993), 83-95.

10. Nield, D.A.; Bejan, A.: Convection in Porous Media (2nd edition). Springer, New York, (1999).

11. Pop, I.; Ingham, D.B.: Convective Heat Transfer: Mathematical and Computational Modelling of Viscous Fluids and Porous Media, Pergamon, Oxford, (2001).

12. Postelnicu, A.; Grosan, T.; Pop, I.: Free convection boundary-layer over a vertical permeable flat plate in a porous medium with internal heat generation. Int. Comm. Heat Mass Transfer, 27, (2000), 729-738.

13. Vafai, K.(ed.): Handbook of Porous Media, Mercel Dekker, New York, (2000).

Address: T. Groşan and Professor I. Pop, Faculty of Mathematics, University of Cluj, R-3400 Cluj, CP 253, Romania. *e-mail*: popi@math.ubbcluj.ro