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Planet Absorption by a Gas Giant

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There is mounting evidence of many extra–solar planetary systems in our galaxy, consisting typically of a Sun–like star and Jupiter–like planets on highly elliptic orbits. These systems are characterised by a dearth of smaller Earth–like planets. The present paper describes the swallowing–up of a small rock dwarf by a large gas giant, and shows that this behaviour is as predicted by the collinearity principle.

1 Young and Old Planetary Systems

According to the collinearity principle (Rimrott, 1998) planetary systems are dissipative and characterised by a tendency to lose orbital energy (e.g. by gravitational interaction and/or collisions), which manifests itself by an

- adjustment of their individual angular momenta toward unidirectional collinearity
- and an adjustment of their orbit shapes toward circular

As a consequence we conclude that systems that do not yet exhibit these characteristics are young, while systems that are close to collinearity and circularity are old.

Apparently, the extra–solar systems discovered so far (Stampf, 2001) are relatively young because they have highly elliptic orbits. The likeliness of collisions is still relatively great.

On the other hand e.g. our own solar system is apparently very old: All planetary orbits are nearly in one plane and they are almost circular. There is little likelihood of further collisions and thus little likelihood of further orbital energy reduction.

2 Extra–solar Planetary Systems

Celestial observations are providing astronomers with more and more evidence of planetary systems around distant stars in our galaxy (Stampf, 2001). Astronomers reason that these planetary systems ought to be similar to our own solar system. And indeed they are in many ways. Their planets have angular momenta about the star that are nearly unidirectionally collinear. Astronomers also reason that there should be Jupiter–like gas giants and Earth–like rock dwarfs among the planets. This, however, is apparently not the case. There appear to be huge gas giant on highly elliptical orbits, and no rock dwarfs.

Astronomers are thus faced with the dilemma of how to explain the apparent disappearance of the rock dwarfs in these systems. Are they swallowed up by the gas giants or are they catapulted out of the system by them? According to the collinearity principle, absorption is more likely because it is associated with an energy loss, while catapulting is not. The absorption process is examined in some detail in the present paper.

3 Model

We choose a model system as simple as possible, yet containing all elements necessary to describe a planet absorption by a larger fellow planet. So let us investigate a central star of gravitational attraction μ, orbited by a gas giant of mass m_1 on an orbit with eccentricity ε_1, and a rock dwarf of mass m_2, for convenience on a circular orbit of radius r_2. Both orbit the central star in one and the same orbital plane and in one and the same direction. They also have a common periapsis point PE (Figure 1). Sooner or later they will be passing the periapsis point at the same moment in time. There will be a collision. This model scenario produces the smallest collision energy loss of many other imaginable configurations. The subsequent theoretical treatment assumes that all masses, i.e. the central star, the rock dwarf, and the gas giant, are point masses, and that any gravitational attraction between the two planets is negligble.

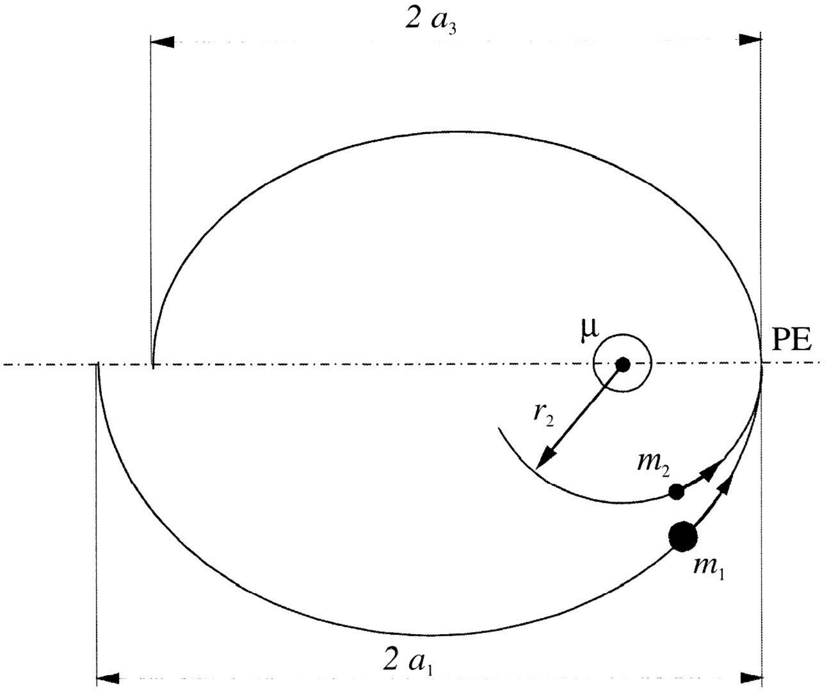

Figure 1. Planetary Orbits

The smaller rock dwarf m_2 will be absorbed by the larger gas giant m_1. The new planet after absorption will have a mass of

$$m_3 = m_1 + m_2 \tag{1}$$

It will leave the periapsis point at a new velocity v_3, which is obtained by equating the linear momenta after and before collision

$$m_3 v_3 = m_1 v_1 + m_2 v_2 \tag{2}$$

We have for the radius of the periapsis point (Rimrott, 1998)

$$r_2 = a_1(1-\varepsilon_1) = a_3(1-\varepsilon_3) \tag{3}$$

resulting in a semi–major axis for the orbit of the new planet of

$$a_3 = a_1 \frac{1-\varepsilon_1}{1-\varepsilon_3} \tag{4}$$

The eccentricity of the original gas giant (Rimrott, 1989) is

$$\varepsilon_1 = \frac{r_2 v_1^2}{\mu} - 1 \tag{5}$$

The eccentricity of the new planet after collision is

$$\varepsilon_3 = \frac{r_2 v_3^2}{\mu} - 1 \tag{6}$$

The periapsis velocity (Rimrott, 1989) of the rock dwarf is

$$v_2 = \sqrt{\frac{\mu}{r_2}} \tag{7}$$

the periapsis velocity of the gas giant is

$$v_1 = \sqrt{\frac{\mu}{a_1}\frac{1+\varepsilon_1}{1-\varepsilon_1}} = \sqrt{\frac{\mu}{r_2}}\sqrt{1+\varepsilon_1} = v_2\sqrt{1+\varepsilon_1} \tag{8}$$

The periapsis velocity of the gas giant, after the absorption of the rock dwarf, is from equations (1) and (2)

$$v_3 = \frac{1+\frac{m_2}{m_1}\frac{1}{\sqrt{1+\varepsilon_1}}}{1+\frac{m_2}{m_1}} v_1 = \frac{\sqrt{1+\varepsilon_1}+\frac{m_2}{m_1}}{1+\frac{m_2}{m_1}} v_2 \tag{9}$$

From equations (7), (8) and (9) we observe that

$$v_2 < v_3 < v_1 \tag{10}$$

From inequality (10) and equations (5) and (6) we find that

$$\varepsilon_3 < \varepsilon_1 \tag{11}$$

and conclude that the gas giant's orbit ellipticity is reduced as a result of the absorption of the rock dwarf.

The new eccentricity is, from equations (5) and (6),

$$\varepsilon_3 = \frac{v_3^2}{v_1^2}(1+\varepsilon_1)-1 \tag{12}$$

or with the help of equation (9)

$$\varepsilon_3 = \frac{\left(1+\frac{m_2}{m_1}\frac{1}{\sqrt{1+\varepsilon_1}}\right)^2}{\left(1+\frac{m_2}{m_1}\right)}(1+\varepsilon_1)-1 \tag{13}$$

or

$$\varepsilon_3 = \frac{\left(1+\frac{m_2}{m_1}\frac{1}{\sqrt{1+\varepsilon_1}}\right)^2}{\left(1+\frac{m_2}{m_1}\right)^2}\varepsilon_1 - \left[1-\frac{\left(1+\frac{m_2}{m_1}\frac{1}{\sqrt{1+\varepsilon_1}}\right)^2}{\left(1+\frac{m_2}{m_1}\right)}\right] \tag{14}$$

From eqution (14) we readily conclude again that

$$\varepsilon_3 < \varepsilon_1 \tag{11}$$

4 Orbital Energy

The original orbital energy (Rimrott, 1998) of the gas giant was

$$E_1 = -\frac{\mu m_1}{2a_1} = -\frac{\mu m_1}{2r_2}(1-\varepsilon_1) \tag{15}$$

and of the rock dwarf it was

$$E_2 = -\frac{\mu m_2}{2r_2} = -\frac{\mu m_1}{2r_2}\frac{m_2}{m_1} \tag{16}$$

Together the initial orbital energy of the two planets was consequently

$$E_i = E_1 + E_2 = -\frac{\mu m_1}{2r_2}\left(1-\varepsilon_1+\frac{m_2}{m_1}\right) \tag{17}$$

The final orbital energy, after the absorption of the rock dwarf by the gas giant, is

$$E_f = -\frac{\mu(m_1+m_2)}{2a_3} = -\frac{\mu m_1}{2r_2}\left(1+\frac{m_2}{m_1}\right)(1-\varepsilon_3) \tag{18}$$

The change in orbital energy is

$$\Delta E = E_f - E_i = -\frac{\mu m_1}{2r_2}\left[\varepsilon_1-\varepsilon_3\left(1+\frac{m_2}{m_1}\right)\right] \tag{19}$$

Because the rock dwarfs absorption process involves a collision, there is an energy loss. This in turn means that ΔE is negative, which means that the expression in square brackets must be positive. Indeed with the help of equation (13) and after some lengthy manipulation (Sperling, 2001) it can be shown that

$$\varepsilon_1-\varepsilon_3\left(1+\frac{m_2}{m_1}\right) = \frac{m_2}{m_1+m_2}\left(\sqrt{1+\varepsilon_1}-1\right)^2 \tag{20}$$

Consequently,

$$\Delta E = -\frac{\mu m_1}{2r_2}\frac{m_2}{m_1+m_2}\left(\sqrt{1+\varepsilon_1}-1\right)^2 \tag{21}$$

is found to be negative. Thus it represents an energy loss.

5 Numerical Example

We choose a gas giant with an orbit eccentricity $\varepsilon_1 = 0.9$. For the mass ratio of rock dwarf to gas giant we select, for lack of better data, the mass ratio Earth/Jupiter, i.e. $m_2/m_1 = 1/318 = 0.0031446$.

The periapsis velocities are then, from equations (7) and (8),

$$\begin{aligned} v_2 &= \sqrt{\frac{\mu}{r_2}} \\ v_1 &= \sqrt{\frac{\mu}{a_1}}\sqrt{\frac{1+\varepsilon_1}{1-\varepsilon_1}} = \sqrt{\frac{\mu}{r_2}}\sqrt{1+\varepsilon_1} = 1.378\,404\,9\, v_2 \end{aligned} \tag{22}$$

and from equation (9)

$$v_3 = \sqrt{\frac{\mu}{a_3}}\sqrt{\frac{1+\varepsilon_3}{1-\varepsilon_3}} = \sqrt{\frac{\mu}{r_2}}\sqrt{1+\varepsilon_3} = 1.377\,218\,7\,v_2 \tag{23}$$

We thus find

$$v_2 < v_3 < v_1 \tag{24}$$

as expected. The new eccentricity is, from equation (12)

$$\varepsilon_3 = \left(\frac{v_3}{v_1}\right)^2 (1+\varepsilon_1)-1 = 0.896\,731\,2 \tag{25}$$

For later use we form

$$\left(1+\frac{m_2}{m_1}\right)\varepsilon_3 = 1.003\,144\,6\,(0.896\,731\,2) = 0.899\,551\,1 \tag{26}$$

and find that

$$\varepsilon_3 < \left(1+\frac{m_2}{m_1}\right)\varepsilon_3 < \varepsilon_1 \tag{27}$$

The orbital energy change (19) or (21) is

$$\Delta E = -\frac{\mu m_1}{2r_2}[0.9-0,899\,551\,1] = -0.0004489\frac{\mu m_1}{2r_2} \tag{28}$$

The orbital energy change is negative, thus it represents an energy loss, as expected.

6 Conclusion

In the present paper is has been shown how absorption of a rock dwarf by a gas giant leads to a reduction of global orbital energy. Therefore, according to the collinearity principle, an absorption is a more likely phenomenon in a young planetary system, than a catapulting of a rock dwarf out of the system, which would take place without energy loss.

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