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Perfect and Imperfect Collinearity in Multibody Gyro Systems

F.P.J. Rimrott, F. A. Salustri

The behaviour of multibody gyroscopic systems is governed by the collinearity principle. In the present paper the collinearity principle is stated and its origin, history and applications are discussed. It is then used in two examples of spacecraft with momentum wheels to demonstrate its validity and applicability.

1 The Collinearity Principle

Observation of gyroscopic systems and interpretation of experimental findings have led to the establishment of a collinearity principle (Rimrott, 1998) which may be stated as follows:

Each independent multibody system of interacting gyros tends toward unidirectional collinearity of the individual angular momenta and the global angular momentum, at the lowest possible energy level.

By independent we mean that a system is globally torque–free, i.e. not affected by any outside influence, e.g. the Earth–Moon system without the influence of the Sun or any other celestial body. A multibody system can consist of a single gyro with flexible appendages or a system consisting of more than one gyro, e.g. a spacecraft with momentum wheels. Interacting means that the individual gyros of the system are connected in some fashion, be it structural, by gravitation or by some other means. The term unidirectional collinearity means that the angular momenta of each elemental gyro are parallel and pointing in the same direction, and that at the lowest possible mechanical (kinetic and potential) energy level, e.g. the angular momentum of Saturn and the angular momenta of its moons and rings, are parallel and point in the same direction, and their orbits are circular (Rimrott and Cleghorn, 2000).

The process of collinearization is initiated and continues by a loss of mechanical energy through interaction between the individual gyros, such as through bearing friction, collision, interbody gravitational action (tides), solar wind, atmospheric resistance or some other effect. In the present paper a rigid spacecraft with two momentum wheels and a rigid spacecraft with three momentum wheels are studied for the case where one wheel is slowing down to a complete standstill due to bearing friction.

2 Origin, History and Application of the Collinearity Principle

The collinearity principle originated as a result of studies into attitude stability and attitude drift of gyrosatellites. It found its first tentative mention as a theorem in a monograph (Rimrott, 1989) as did the attitude diagram. In 1992 Rimrott and Janabi–Sharifi studied a torque–free flexible model gyro. Then in Rimrott and Sczcygielski (1993) discussed attitude diagrams in detail, which represent a significant tool for the interpretation of the attitude behaviour of single solid gyroscopes. Since planetary systems are also gyroscopic, successful attempts were made to include planetary systems, and in 1998 a seminal paper (Rimrott, 1998a) by Rimrott was published enunciating the principle and outlining some pertinent applications. Also in 1998 (Rimrott, 1998b) the principle was used by Rimrott to study the Earth–Moon system to show how it expands, how rotation and revolution slow down, and that the system loses energy in the process.

In a 1999 dissertation Saitov used the collinearity principle to study the effect of resonances upon the attitude of elastic–dissipative gyrosatellites. In 2000 the collision of coplanar satellites was investigated by Rimrott, then minimum energy orbits were obtained by Rimrott and Cleghorn. Also Rimrott and Sperling (2000) extended the collinearity principle to very small satellites (comets) on highly elliptical orbits assessing their tendency to disintegrate and lose orbital energy in the process.

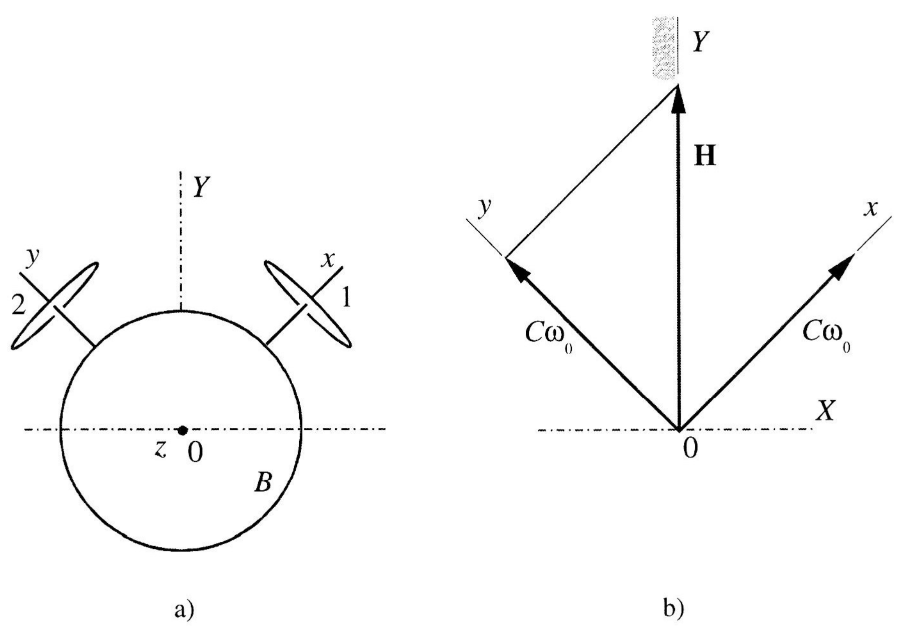

Figure 1. Initial Attitude of Gyrosatellite with Momentum Wheels 1 and 2

3 Perfect Collinearity

In Figure 1a torque–free gyrosatellite is shown, whose attitude behaviour is to be analyzed. It consists of a large axisymmetric bus body B and two small momentum wheels. The bus body, including the momentum wheels, has an inertia moment in body–fixed $Oxyz$–coordinates of

$$[I_B] = \begin{bmatrix} A_B & 0 & 0 \\ 0 & A_B & 0 \\ 0 & 0 & C_B \end{bmatrix} \tag{1}$$

Initially the bus body is stable, i.e. it is not rotating. Furthermore, it has no energy source (battery) on board. Each of the two momentum wheels has an inertia moment C and an angular speed of ω_0. Thus the global angular momentum is

$$\mathbf{H} = [\mathbf{e}_x\ \mathbf{e}_y\ \mathbf{e}_z] \begin{bmatrix} C\omega_0 \\ C\omega_0 \\ 0 \end{bmatrix} = [\mathbf{e}_X\ \mathbf{e}_Y\ \mathbf{e}_Z] \begin{bmatrix} 0 \\ \sqrt{2}C\omega_0 \\ 0 \end{bmatrix} \tag{2}$$

For the present problem we specifiy $C_B < A_B$. We also keep in mind that C is typically very small. Because there is no external torque, the angular momentum retains its constant magnitude

$$H = \sqrt{2}\,C\omega_0 \tag{3}$$

and its constant direction, which has been chosen to coincide with the space–fixed Y–axis. The kinetic energy of the system is initially

$$T_i = 2\left(\frac{1}{2}C\omega_0^2\right) \tag{4}$$

Of the two momentum wheels, let wheel 2 be subject to bearing friction, while wheel 1 is friction free and continues to spin at a constant ω_0. The latter is possible as the bus body itself takes up rotation and changes its attitude. The process is described in detail in Rimrott (1998b). Eventually the final attitude is reached as depicted in Figure 2. The bus body's x–axis now points in the direction of the space–fixed Y–axis, with

$$\mathbf{H} = \left[\mathbf{e}_X\ \mathbf{e}_Y\ \mathbf{e}_Z\right]\begin{bmatrix} 0 \\ C\omega_0 + H_B - C\omega_B \\ 0 \end{bmatrix} \tag{5}$$

Because the bus body's inertia moment (1) includes the momentum wheels the angular momentum portion $C\omega_B$ must be subtracted from H_B.

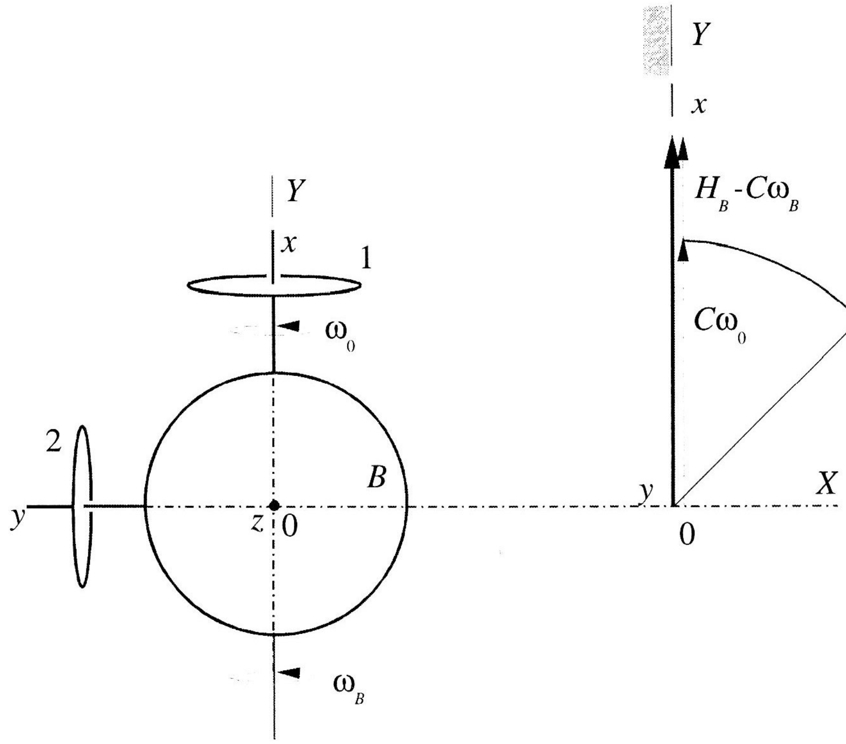

Figure 2. Final Attitude

A comparison of equations (2) and (5) now reveals that the bus body's final angular momentum, with $H_B = A_B\omega_B$, is

$$H_B = \frac{A_B}{A_B - C}\left(\sqrt{2} - 1\right)C\omega_0 \tag{6}$$

For the bus body's final angular speed we obtain

$$\omega_B = \frac{C}{A_B - C}\left(\sqrt{2} - 1\right)\omega_0 \tag{7}$$

The final kinetic energy is

$$T_f = \frac{1}{2}C\omega_0^2 + \frac{1}{2}\left(A_B - C\right)\omega_B^2 \tag{8}$$

resulting, with the help of equation (7), in

$$T_f = \frac{1}{2}C\omega_0^2\left[1 + \frac{C}{A_B - C}\left(3 - 2\sqrt{2}\right)\right] \tag{9}$$

The kinetic energy change that took place during the attitude drift is

$$\Delta T = T_f - T_i = -\frac{1}{2}C\omega_0^2\left[1 - \frac{C}{A_B - C}\left(3 - 2\sqrt{2}\right)\right] \tag{10}$$

The kinetic energy change is negative as long as $A_B > \left(4 - 2\sqrt{2}\right)C = 1.171C$, a condition always satisfied (equation (2)). Thus equation (10) represents an energy loss, as is required by the collinearity principle. It is perhaps of interest to note that the energy loss is not just simply $-C\omega_0^2/2$ as one might expect if one considers that the bearing friction leads to a complete standstill of the spinning of momentum wheel 2.

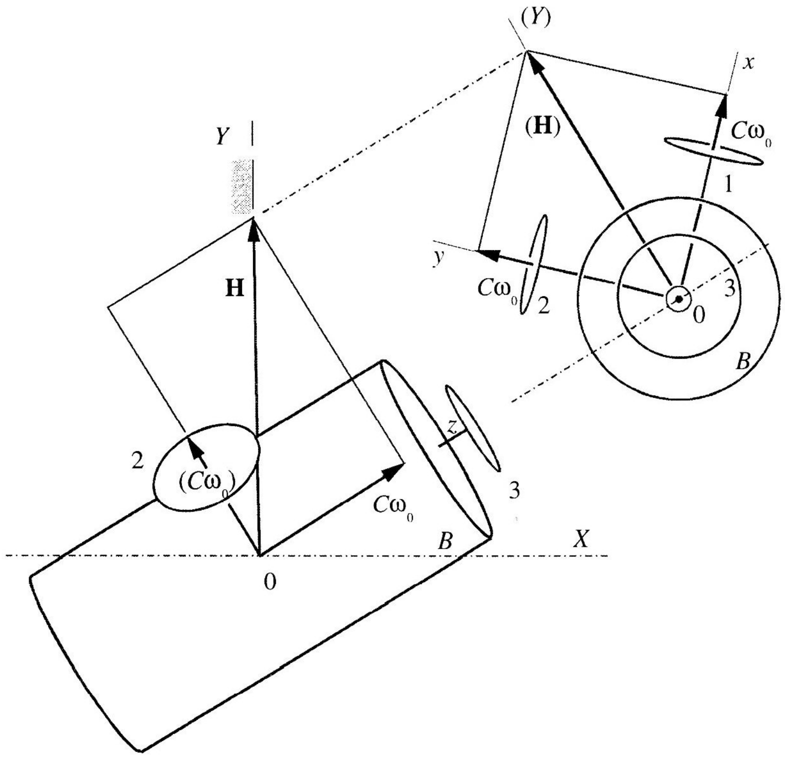

Figure 3. Initial Attitude of Gyrosatellite with Momentum Wheels 1, 2 and 3

4 Imperfect Collinearity

While for most systems perfect collinearity of all angular momenta represents the final state, there are some systems for which perfect collinearity cannot be achieved, yet such systems still adjust themselves as close to collinearity as the physical restraints allow. As a model we shall investigate a four gyro system, consisting of a large axisymmetric bus body B with three equal small momentum wheels, of which wheel 3 is subject to bearing friction and comes to a complete stop. Initially all three momentum wheels have angular momenta of equal magnitude, while the bus body is stable, i.e. not rotating. Thus

$$\mathbf{H} = \left[\mathbf{e}_x\ \mathbf{e}_y\ \mathbf{e}_z\right]\begin{bmatrix} C\omega_0 \\ C\omega_0 \\ C\omega_0 \end{bmatrix} = \left[\mathbf{e}_X\ \mathbf{e}_Y\ \mathbf{e}_Z\right]\begin{bmatrix} 0 \\ \sqrt{3}C\omega_0 \\ 0 \end{bmatrix} \tag{11}$$

where $0xyz$ is the body–fixed coordinate system, C is the inertia moment of each momentum wheel and ω_0 is the initial angular speed of each (Figure 3).

The magnitude of the global angular momentum of the system is consequently

$$H = \sqrt{3}C\omega_0 \tag{12}$$

and the direction is along the space–fixed Y–axis, which has been chosen to be along the global angular momentum, which is constant since the system is independent and free of any external torque. Any change in attitude of the individual gyros is thus brought about by internal energy losses. The bus body, including momentum wheels, has an inertia moment as given by equation (1). The initial attitude of the system is depicted in Figure 3. The initial kinetic energy is

$$T_i = 3\left(\frac{1}{2}C\omega_0^2\right) \tag{13}$$

The final attitude is reached when momentum wheel 3 has stopped spinning, and is depicted in Figure 4.

The angular momentum (12) has remained constant. The body–fixed $0xyz$–coordinate system is now changing its attitude, the bus body is rotating and has acquired an angular momentum H_B.

$$\mathbf{H} = \begin{bmatrix}\mathbf{e}_x & \mathbf{e}_y & \mathbf{e}_z\end{bmatrix}\begin{bmatrix} C\omega_0 + (A_B - C)\omega_B/\sqrt{2} \\ C\omega_0 + (A_B - C)\omega_B/\sqrt{2} \\ 0 \end{bmatrix} = \begin{bmatrix}\mathbf{e}_X & \mathbf{e}_Y & \mathbf{e}_Z\end{bmatrix}\begin{bmatrix} 0 \\ \sqrt{2}C\omega_0 + H_B - C\omega_B \\ 0 \end{bmatrix} \tag{14}$$

and, from equations (11) and (14),

$$\sqrt{3}C\omega_0 = \sqrt{2}C\omega_0 + H_B - C\omega_B \tag{15}$$

we have

$$H_B - C\omega_B = \left(\sqrt{3} - \sqrt{2}\right)C\omega_0 \tag{16}$$

and since $H_B = A_B\omega_B$, we obtain for the bus body's final angular speed

$$\omega_B = \frac{C}{A_B - C}\left(\sqrt{3} - \sqrt{2}\right)\omega_0 \tag{17}$$

The final kinetic energy is obtained from

$$T_f = 2\left(\frac{1}{2}C\omega_0^2\right) + \frac{1}{2}(A_B - C)\omega_B^2 \tag{18}$$

which, with the help of equation (17), amounts to

$$T_f = \frac{1}{2}C\omega_0^2\left[2 + \frac{C}{A_B - C}\left(5 - 2\sqrt{6}\right)\right] \tag{19}$$

The kinetic energy change is then, using equations (13) and (19),

$$\Delta T = T_f - T_i = -\frac{1}{2}C\omega_0^2\left[1 - \frac{C}{A_B - C}\left(5 - 2\sqrt{6}\right)\right] \tag{20}$$

The kinetic energy change is negative for all realistic cases for which $A_B > \left[6 - 2\sqrt{6}\right]C = 1.101C$ (equation (2)). An inspection of Figure 4 shows that the system seeks a final state as close to collinearity as physically possible.

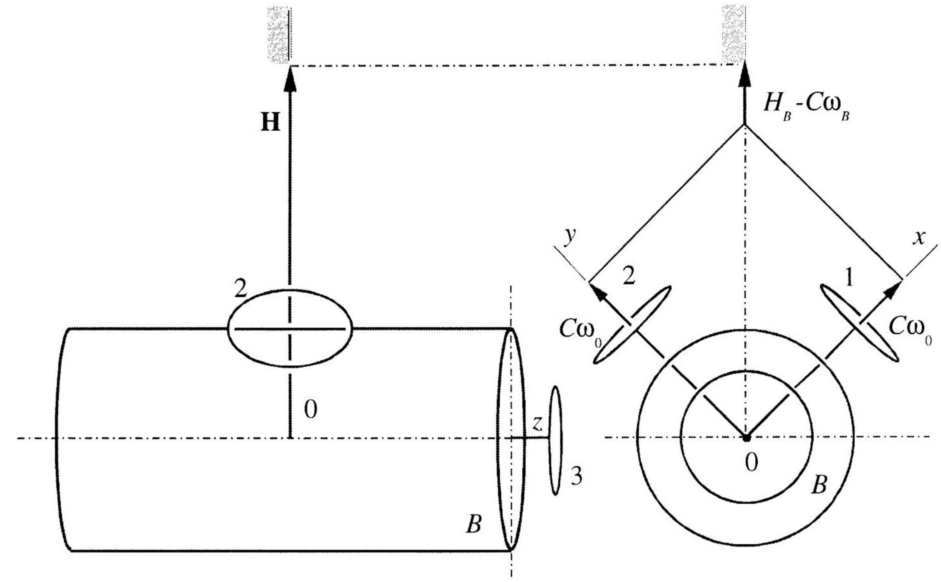

Figure 4. Final Attitude

5 Conclusion

After a short statement of the collinearity principle and its history, its application is demonstrated by two examples, chosen as simple as possible, to highlight that perfect collinearity, which is the usual final state of an independent gyroscopic system, cannot always be attained. There are indeed some systems for which perfect collinearity is impossible because of physical restraints. For such a system it is shown that it seeks attitudes as close to collinearity for all angular momenta as can physically be achieved.

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Addresses: Professor F.P.J. Rimrott, Department of Mechanical and Industrial Engineering, University of Toronto, Toronto, Ontario, Canada M5S 3G8. E–mail: frimrott@halhinet.on.ca
Professor F. A. Salustri, Department of Mechanical, Aerospace and Industrial Engineering, Ryerson University, Toronto, Ontario, Canada M5B 2K3. E–mail: salustri@ryerson.ca