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Similarity Solutions for Boundary Layer Flows on a Moving Surface in Non-Newtonian Power-Law Fluids

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A similarity analysis of the boundary layer flow caused by the motion of a semi-infinite flat surface in a non-Newtonian power-law fluid at rest is made in this paper. These similar solutions fall into two categories: similarity solutions corresponding to steady boundary layers over moving surfaces and similarity solutions corresponding to unsteady boundary layers past moving flat surfaces, respectively. Except in the special case $n = 1/2$ (pseudoplastic) and $n = 1$ (Newtonian) fluids, solutions of the first category problems must be obtained numerically. However, for the second category analytical solutions are possible for a large class of pseudoplastic fluids ($n < 1$), including the case of a Newtonian fluid ($n = 1$).

1 Introduction

For the past years, the problem of the classical boundary layer flow over a surface has been studied in two different types. One type is the problem of a boundary layer flow past a stationary surface, while the other type is the problem of a boundary layer flow over a solid surface continuously moving in a fluid at rest. Boundary layer behavior on a moving surface is an important type of flow that occurs in a number of engineering processes. A polymer sheet or filament extruded continuously from a die or a long thread traveling between a feed roll and a wind-up roll, are examples of continuous moving surfaces in an ambient fluid. Boundary layer flows of a viscous incompressible fluid on a surface moving with a constant velocity in a Newtonian fluid at rest were first studied by Sakiadis (1961). He showed that due to the entrainment of the ambient fluid, this situation represents a class of boundary layer flow problems, which has a solution substantially different from that of a boundary layer flow over a stationary semi-infinite flat plate known as Blasius problem (1908). Some particular aspects of this problem have also been mentioned in the papers by Pop et al. (1992, 1995).

It is well-known that a number of fluids which occur in practical applications, such as molten plastics, polymers, pulps, foods, etc. exhibit a non-Newtonian fluid behavior. Due to the growing use of these non-Newtonian substances in various manufacturing and processing industries, considerable effort has been directed towards understanding their friction and heat transfer characteristics. However, many of the inelastic non-Newtonian fluids encountered in chemical engineering processes are known to follow the so-called Ostwald-de-Waele power-law model in which the shear varies according to a power-function of the strain rate. But for this relatively simple power-law model, the mathematical complexity of the governing equations increase because of the extra nonlinearity in the viscous term. A fundamental question then arises: do the governing boundary layer equations of power-law fluids flowing over static or moving solid surfaces possess similarity solutions?

Similarity solutions of the boundary layer equations of non-Newtonian fluids past static surfaces were examined in detail by Acrivos et al. (1960), Schowalter (1960), Wells (1964), Hayasi (1965), Lee and Ames (1966), Na and Hansen (1967), Hansen and Na (1968), Timol and Kalthia (1986) and Pakdemiri (1994). However, there are only few papers, which are devoted to the corresponding problem of similarity solutions of the boundary layer equations for a moving surface in a power-law fluid at rest. Thus, we mention the papers by Fox et al. (1969), Lin and Shih (1980), Pop and Gorla (1990), Gorla and Pop (1990) and Howell et al. (1997). There are also several studies by Jadhav and Waghmode (1990), Andersson and Dandapat (1991), Andersson et al. (1996) and Rao et al. (1999) on the problem of boundary layer flow over a flat surface which is stretched with a velocity proportional to the distance from a fixed point on it in an infinite power-law fluid at rest.

The aim of this paper is to present conditions for the existence of similarity solutions of the boundary layer equations of an incompressible flow over a moving surface in a non-Newtonian power-law fluid at rest for both steady and unsteady flows. The method employed is similar to that followed by Hayasi (1965) for the boundary layer flow of a power-law fluid past fixed surfaces. It consists of defining a new independent variable and a new dependent similarity variable in terms of generalized transformation functions. When these similarity variables are substituted into the boundary layer equations, they are reduced to a single ordinary differential equation only if the coefficients of the similarity variables and their derivatives can be made constant. These coefficients when made constant are referred to as similarity solutions; they usually contain derivatives of the transformation variables and thus are differential equations themselves. Any solution to the similarity conditions, which will result in explicit forms for the transformation variables, will transform the original continuity and momentum equations into an ordinary differential equation and this is called similarity equation. After this similarity equation is obtained, several examples of the similarity solutions are determined analytically for both steady and unsteady flow of pseudoplastic fluid. The known solutions for the Newtonian fluid are also recovered.

We finally mention that the present paper will serve as a reference against which other exact or approximate solutions can be compared in the future.

2 Basic Equations

The physical problem being considered is shown in Figure 1. A thin solid surface is extruded from a die slot at $x = y = 0$ on a fixed coordinate system and moves in x-direction with an arbitrary surface

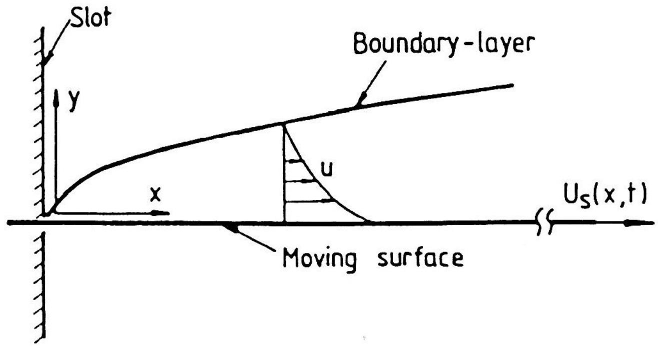

Figure 1. Physical Model and Coordinate System

velocity $U_s(x, t)$ in a non-Newtonian fluid of constant physical properties which is at rest. The basic equations governing the unsteady boundary layer flow of this problem are

$$\frac{\partial u}{\partial x}+\frac{\partial \upsilon}{\partial y} = 0 \tag{1}$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+\upsilon\frac{\partial \upsilon}{\partial y} = \frac{1}{\rho}\frac{\partial \tau_{xy}}{\partial y} \tag{2}$$

where u and υ are the velocity components along the x and y directions, respectively, t is the time, ρ is the density and τ_{xy} represents the shear stress. In the present problem, we have $\partial u/\partial y \leq 0$, which gives the shear stress as

$$\tau_{xy} = -\kappa\left(-\frac{\partial u}{\partial y}\right)^n \tag{3}$$

where κ is called the consistency coefficient and n (> 0) is the power-law index. For the particular parameter value $n = 1$, one can retrieve the Newtonian fluid model with dynamic coefficient of viscosity μ. As n deviates from unity, deviations from Newtonian behavior occur. For example, $n < 1$ and $n > 1$ correspond to pseudoplastic and dilatant fluids, respectively. However, it has been shown by Acrivos et al. (1960) that when $n > 2$ the boundary layer flow is not of much practical interest. This is motivated by the definition of a generalized Reynolds number for a power-law fluid $\mathrm{Re} = U_0^{2-n}L^n / \nu$ where U_0 and L are characteristic velocity and length, respectively, and $\nu = \kappa / \rho$ is the fluid viscosity.

Combining equations (2) and (3), we have

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + \upsilon\frac{\partial \upsilon}{\partial y} = \nu n\left(-\frac{\partial u}{\partial y}\right)^{n-1}\frac{\partial^2 u}{\partial y^2} \qquad (4)$$

which is subject to the boundary conditions

$$u = U_s(x, t) \quad \upsilon = 0 \quad \text{at} \quad y = 0 \qquad u \to 0 \quad \text{at} \quad y \to \infty \qquad (5)$$

where $U_s(x, t)$ is an arbitrary function, which is undetermined as yet.

3 Similarity Conditions

We now examine the conditions for which similarity solutions exist. For this purpose we assume

$$u = U_s(x, t) f'(\eta) \qquad (6)$$

where

$$\eta = \frac{y}{\nu^{1/(1+n)} g(x,t)} \qquad (7)$$

is a non-dimensional variable, $g(x, t)$ is an arbitrary function and the prime denotes differentiation with respect to η.

Integrating equation (1) with respect to y and using equations (6) and (7), we obtain

$$\nu^{-1/(1+n)}\frac{\upsilon}{U_s} = -f\frac{g}{U_s}\frac{\partial U_s}{\partial x} + (\eta f' - f)\frac{\partial g}{\partial x} \qquad (8)$$

Equation (4) thus becomes

$$n\frac{U_s^{n-1}}{g^{n+1}}(-f'')^{n-1}f''' + \left(\frac{\partial U_s}{\partial x} + \frac{U_s}{g}\frac{\partial g}{\partial x}\right)ff'' - \frac{\partial U_s}{\partial x}f'^2 + \frac{1}{g}\frac{\partial g}{\partial t}\eta f'' - \frac{1}{U_s}\frac{\partial U_s}{\partial t}f' = 0 \qquad (9)$$

The conditions for this boundary layer equation to be similar comes out to be

$$\frac{1}{U_s^n}\frac{\partial U_s}{\partial t} = \frac{a}{g^{1+n}} \qquad (10)$$

$$g^n\frac{\partial g}{\partial t} = bU_s^{n-1} \qquad (11)$$

$$U_s^{1-n}\frac{\partial U_s}{\partial x} = \frac{d}{g^{1+n}} \qquad (12)$$

$$\frac{1}{g}\frac{\partial g}{\partial x} = e\frac{U_s^{n-2}}{g^{1+n}} \qquad (13)$$

where a, b, d and e are certain constants with respect to x and t. Then, equation (9) reduces to

$$n(-f'')^{n-1}f''' + (d+e)ff'' - df'^2 + b\eta f'' - af' = 0 \qquad (14)$$

and the boundary conditions (5) become

$$f(0) = 0 \qquad f'(0) = 1 \qquad f'(\infty) = 0 \tag{15}$$

To examine the feasibility of the solutions, several factors must be considered. The first concerns whether the solutions describe physically sensible flows. It can be assumed that the velocity along the plate and the boundary layer thickness behave in the same fashion, either both increase or both decrease or at least they will be constant. A second factor affecting feasibility is whether the resulting similarity equations can satisfy the boundary condition at infinity. It is worth noting that higher derivatives of f' approach zero outside the boundary layer as is known from mathematical considerations. A third factor, also related to satisfying boundary conditions, occurs for the solutions where the similarity equation (9) reduces to $f''' = 0$, which cannot truly satisfy the boundary conditions (15). Thus, any solution resulting in either of these forms for the function $f(\eta)$ is considered infeasible and will not be presented in the following as a possible solution.

As it was mentioned before, $U_s(x, t)$ and $g(x, t)$ are arbitrary functions. However, they take special forms when we look for similarity solutions of equation (14). To get these special forms of $U_s(x, t)$ and $g(x, t)$, we are seeking some special forms for some values of the parameters a, b, d and e. This can be done by first differentiating equation (10) with respect to x and using equations (11) and (12). Then, we differentiate equation (11) with respect to x and use equations (10), (11) and (12). After some manipulations, we get

$$\{a + (1+n)b\}d = (1+n)aU_s^{2-n} g^n \frac{\partial g}{\partial x} \tag{16}$$

$$\{(2-n)a + (1+n)b\}e + (n-1)bd = (1+n)bU_s^{2-n} g^n \frac{\partial g}{\partial x} \tag{17}$$

Further, differentiating (10) with respect to t and using equations (10), (11) and (12), we obtain

$$\left(\frac{2-n}{1+n}a + b\right)ad = 0 \tag{18}$$

Finally, eliminating $\partial g / \partial x$ from equations (16) and (17) and using equation (18), we have

$$\left(\frac{2-n}{1+n}a + b\right)a\,e = 0 \tag{19}$$

In the reminder of this paper, we will present the possible solutions of equation (14). The steady state solutions will be considered first, followed by the unsteady state solutions. Also, some examples of analytical solutions will be given.

4 The Case $a = 0$

In this case it follows from the relation (16) that $bd = 0$, which leads to the following subcases:

4.1 The case $b = 0$. It is seen from equations (10) and (11) that this case corresponds to the steady flow over a moving surface. We therefore have two subcases:

4.1.1 The case $d = 0$ and $e \neq 0$ gets

$$U_s = \text{const.} \qquad g = \{(n+1)\, e\, U_s^{n-2} x + A\}^{1/(1+n)} \tag{20}$$

where A is an integration constant. Thus, equation (14) reduces to

$$N(-f'')^{n-1} f''' + e f f'' = 0 \tag{21}$$

4.1.2 The case $d \neq 0$ leads to

$$U_s^{\frac{1}{m}-1} \frac{dU_s}{dx} = A \tag{22}$$

where $A \neq 0$ and $1/m = (1+n)(e/d) + 2 - n$. Equation (14) then becomes

$$n(-f'')^{n-1} f''' + (d+e) f f'' - d f'^2 = 0 \tag{23}$$

Here we have the following two subcases:

1. $1/m = 0$ for which

$$e = -(2-n)\frac{d}{1+n} \qquad U_s = B\exp(Ax) \qquad g = \left(\frac{A}{d} U_s^{2-n}\right)^{-1/(1+n)} \tag{24}$$

where B is another integration constant.

2. $1/m \neq 0$ and then we have

$$U_s = (m)^{-m}(Ax+B)^m \qquad g = \left(\frac{d}{A}\right)^{\frac{1}{1+n}} U_s^{\frac{e}{d}} \tag{25}$$

4.2 The case $b \neq 0$. This corresponds to the unsteady flow over a moving surface.

4.2.1 The case $d = 0$, where

$$U_s = \text{const.} \qquad g = \{(n+1)\, b\, U_s^{n-1} t + A\}^{1/(1+n)} \tag{26}$$

From equation (14) we then get the equation

$$n f''' - (ef + b\eta)(-f'')^{2-n} = 0 \tag{27}$$

5 The Case $a \neq 0$

$$\left(\frac{2-n}{1+n}a + b\right)d = \left(\frac{2-n}{1+n}a + b\right)e = 0$$

5.1 The case $(2-n)a + (1+n)b = 0$ leads to

$$b = -\frac{(n-2)a}{1+n} \qquad U_s = -\frac{dx+B}{at+A} \qquad g = (dx+B)^{\frac{n-1}{n+1}}(-at-A)^{\frac{2-n}{1+n}} \tag{28}$$

Now, equation (14) becomes

$$n(-f'')^{n-1} f''' + (d+e) f f'' - d f'^2 - \frac{2-n}{1+n} a\eta f'' - a f' = 0 \tag{29}$$

5.2 The case $(2-n)a + (1+n)b \neq 0$, $d = e = 0$

In this case equation (14) reduces to

$$N(-f'')^{n-1} f''' + b\eta f'' - a f' = 0 \tag{30}$$

and there are the following subcases:

$$1.\, q \;=\; (1+n)\frac{b}{a}+1-n \;=\; 0$$

$$U_s \;=\; B\exp\left(\frac{at}{A}\right) \qquad g \;=\; A^{\frac{1}{1+n}} B^{\frac{b}{a}} \exp\left(\frac{bt}{A}\right) \tag{31}$$

2. $q \neq 0$

$$U_s \;=\; A\{q(at+B)\}^{\frac{1}{q}} \qquad g \;=\; A^{\frac{n-1}{1+n}} \{q(at+B)\}^{\frac{b}{aq}} \tag{32}$$

6 Examples of Similar Solutions

In this section, we shall consider three examples of the similarity equations established in the previous sections, which admit closed form (analytical) solutions.

6.1. Steady flow case 4.1.2 where $a = b = 0$. Additionally, we assume that $d + e = 0$ and $d > 0$. In this case there exist analytical solutions for $n = ½$ (pseudoplastic) and $n = 1$ (Newtonian) fluids only. Thus, for $n = ½$ equation (23) can be written as

$$(F')^{1/2} F'' + 2F^2F' = 0 \tag{33}$$

where $F = -df/d\eta = -u/U_s$ and primes denote differentiation with respect to

$$\xi = (d)^{\frac{2}{3}}\eta \tag{34}$$

The boundary conditions of equation (33) are

$$F(0) = -1 \qquad F(\infty) = 0 \tag{35}$$

The solution of equation (33) is

$$\frac{u}{U_s} = -F = \frac{1}{1+\xi} \tag{36}$$

However, for $n = 1$ (Newtonian fluid) equation (23) can be reduced to

$$F' - 6F^2 = 0 \tag{37}$$

subject to conditions (35) and primes now signify the differentiation with respect to

$$\xi = \left(\frac{d}{6}\right)^{\frac{1}{2}}\eta \tag{38}$$

The solution of equation (37) is given by

$$\frac{u}{U_s} = -F = \frac{1}{(1+\xi)^2} \tag{39}$$

The velocity profiles u/U_s given by equations (36) and (39) are shown in Figure 2. It is seen that the velocity profiles are bigger for a pseudoplastic fluid ($n = 1/2$) than for a Newtonian fluid ($n = 1$).

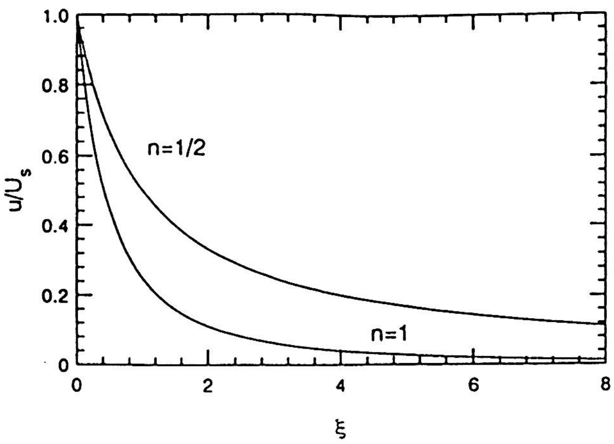

Figure 2. Velocity Profiles given by Equations (36) and (39)

6.2. Unsteady flow case

We will now consider the cases 4.2.1 and 5.2, respectively. In the first case, assuming that $e = 0$ and $b > 0$, equation (27) can be reduced to

$$F'' + 2\xi(F')^{2-n} = 0 \tag{40}$$

where $F = -df/d\eta = -u/U_s$ as before and primes now denote differentiation with respect to the variable

$$\xi = \left(\frac{b}{2n}\right)^{\frac{1}{n+1}}\eta \tag{41}$$

The boundary conditions are given by equation (35). It is worth mentioning that this problem corresponds, as Wang et al. (1997) noticed, to Stoke's first problem of the impulsively translated plate in a power-law fluid ($n \neq 1$) or in a Newtonian ($n = 1$) fluid. For $n = 1$ it is easily shown, see also Pop and Na (1996), that the solution of equation (40) can be expressed in terms of the complementary error function as

$$F = -\,\mathrm{erfc}(\xi) \tag{42}$$

Integrating equation (40) once gives

$$F' = \left\{A + (1-n)\xi^2\right\}^{\frac{1}{1-n}} \tag{43}$$

which holds only for $n < 1$ (pseudoplastic fluid), A being an integration constant. On the other hand, if we make the transformation

$$F(\xi) = \phi(\xi) - 1 \tag{44}$$

then equation (40) remains unchanged

$$\phi'' + 2\xi(\phi')^{2-n} = 0 \tag{45}$$

while the boundary conditions (35) become

$$\phi(0) = 0 \qquad \phi(\infty) = 1 \tag{46}$$

Equation (45) subject to condition (46) is identical to equation (5.1) from Hayasi (1965). This equation, which admits solutions only for $n < 1$, has been integrated analytically by Hayasi (1965) for $n = 1/3$, $1/2$, $2/3$ and $5/6$. Using Hayasi's solutions, we have for the present problem

$n = 1/3$

$$\frac{u}{U_s} = -F = 1 - \frac{\xi}{(\xi^2+\alpha)^{1/2}} \qquad \alpha = \frac{3A}{2} = \left(\frac{3}{2}\right)^{\frac{3}{2}}$$

$n = 1/2$

$$\frac{u}{U_s} = -F = 1 - \frac{2}{\pi}\left\{\frac{\alpha\xi}{\xi^2+\alpha^2} + \tan^{-1}\left(\frac{\xi}{\alpha}\right)\right\} \qquad \alpha = (2A)^{\frac{1}{2}} = \pi^{\frac{1}{3}}$$

$n = 2/3$

$$\frac{u}{U_s} = -F = 1 - \frac{2}{\pi}\left\{\frac{\alpha\xi(3\xi^2+5\alpha^2)}{3(\xi^2+\alpha^2)^2} + \tan^{-1}\left(\frac{\xi}{\alpha}\right)\right\} \qquad \alpha = (3A)^{\frac{1}{2}} = \left(\frac{3}{2}\right)^{\frac{4}{5}}\pi \tag{47}$$

$n = 5/6$

$$\frac{u}{U_s} = -F = 1 - \frac{2}{\pi}\left\{\frac{128\alpha^9\xi}{315(\xi^2+\alpha^2)^5} + \frac{167\alpha^7\xi}{35(\xi^2+\alpha^2)^4} + \frac{8\alpha^5\xi}{15(\xi^2+\alpha^2)^3} + \frac{2\alpha^3\xi}{3(\xi^2+\alpha^2)^2} + \frac{\alpha\xi}{\xi^2+\alpha^2} + \tan^{-1}\left(\frac{\xi}{\alpha}\right)\right\}$$

$$\alpha = (6A)^{\frac{1}{2}} = \left(\frac{45927\pi}{8}\right)^{\frac{1}{11}}$$

Finally, we consider the unsteady case 5.2 where $d = e = 0$ and assume $b = 0$ and $a > 0$. If we again take $F = -df/d\xi = -u/U_s$, then equation (30) can be written as

$$\frac{1}{2}(1+n)(F')^n F'' - FF' = 0 \tag{48}$$

subject to the boundary conditions (35). Primes denote differentiation with respect to

$$\xi = \left\{(1+n)\frac{a}{2n}\right\}^{\frac{1}{1+n}}\eta \tag{49}$$

Integration of equation (48) gives

$$\frac{u}{U_s} = -F = \left\{1 + \frac{1-n}{1+n}\xi\right\}^{-\frac{1+n}{1-n}} \tag{50}$$

which is valid for $n < 1$ (pseudoplastic fluids) only. However, for a Newtonian fluid ($n = 1$) the solution of Eq. (48) has been given by Crane (1970) as

$$\frac{u}{U_s} = -F = \exp(-\xi) \tag{51}$$

which is the limiting form of equation (50) when $n \to 1$. Velocity distribution u/U_s given by equations (50) and (51) are presented in Figure 3 for n = 1/9, 1/5, 1/3,1/2, and 1. We notice from this figure that the velocity profiles again decrease as the power-law index n increases to the value n = 1. (Newtonian fluid).

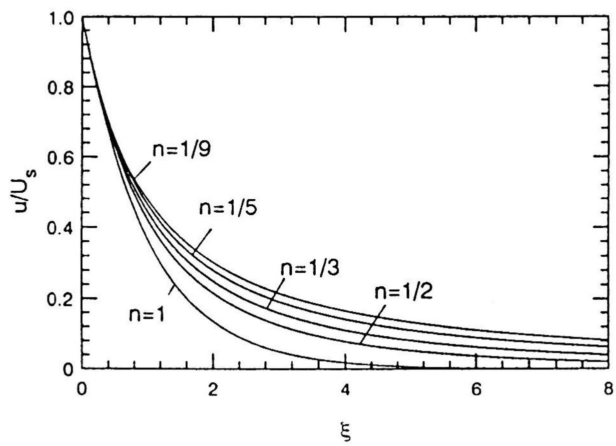

Figure 3. Velocity Profiles given by Equations (50) and (51)

Finally, we consider the steady flow case when the plate moves continuously with a constant velocity U_s in a power-law fluid at rest. This configuration corresponds to the case 4.1.1, where $e > 0$. Making the transformation

$$F(\xi) = \{(1+n)e\}^{1/(1+n)} f(\eta) \qquad \xi = \{(1+n)e\}^{1/(1+n)}\eta \tag{52}$$

equation (21) takes the form

$$n(1+n)F''' - F(-F'')^{2-n} = 0 \tag{53}$$

subject to

$$F(0) = 0 \qquad F'(0) = 1 \qquad F'(\infty) = 0 \tag{54}$$

Here primes denote again the differentiation with respect to ξ. It is worth mentioning that for $n = 1$, equation (53) along with the boundary conditions (54) reduces to that of Sakiadis (1961). Equation (53) has been solved numerically for $n = 1/3$, $1/2$, 1 and 1.5. These solutions are presented in Figure 4.

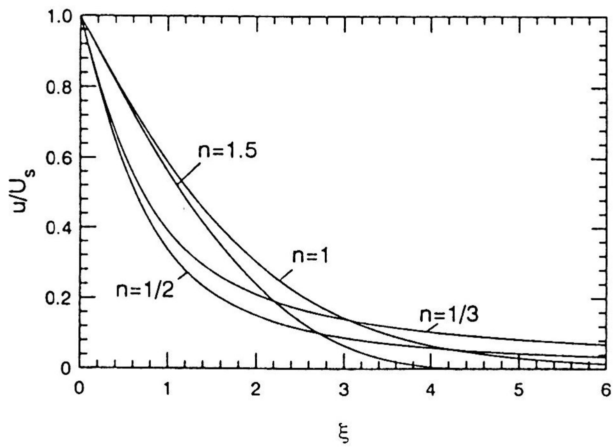

Figure 4. Velocity Profiles given by Equations (53

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