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Some Remarks on Dissipation Postulate in Anisotropic Finite Elasto–Plasticity

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In the paper a new version of the dissipation postulate is properly formulated for anisotropic (isothermic) finite elasto- plastic materials, in the context of Mandel's type multiplicative decomposition of the deformation gradient. Our postulate requires that the work done by internal forces (in the initial configuration) is non-negative on small cycles of strains only. The definitions are built in terms of the yield function in the elastic strain space, via the solutions of the differential system (rate-independent), which governs the evolution of the irreversible variables. Some properties that are postulated or tacitly accepted in approaches used by different authors, are proved in this approach. I emphasize some important consequences of the formulated postulate: the existence of the smooth stress potential, certain dissipation inequalities, which hold only for accessible pairs in the appropriate elastic range, some modified flow rules compatible with the dissipation postulate. Some comparison with the existing results in the domain is also presented.

1 Introduction

There exists several generalizations to large elasto-plastic deformations of Drucker's (1952) and Il'yushin's (1954) dissipation postulates, formulated within the framework of infinitesimal (isothermal) theory of plasticity, for small cycles in stress and respectively in strain space. The discussions related to the constitutive restrictions inferred in the models by the work assumptions proposed in Green and Naghdi (1965) approach to finite elasto-plasticity can be found in the papers by Naghdi (1990) and Srinivasa (1997).
In multiplicative finite elasto-plasticity:
Krawietz (1981) proposed that the work during cyclic processes of stress (in terms of symmetric Piola-Kirchhoff stress in a relaxed configuration) or elastic strain to be non-negative, under the supposition of an invertible elastic law, the state of the material element being characterized by relaxed configuration, elastic strain, and internal variables.
Lubliner (1986), (1990) has shown that the maximum dissipation principle or Il'yushin's postulate leads to certain kind of six- dimensional flow rule and not to a nine-dimensional flow rule as it was suggested by Mandel (1972).
Marigo (1989) formalized Il'yushin's postulate in the general framework of mechanical systems with domains of the reversibility and the fundamental inequalities are directly applied to a class of elasto-plastic materials, developed by Sidoroff (1984).
Lucchesi and Podio-Guidugli (1990) have extended a variation of Il'yushin's postulate to a certain class of isotropic materials (plastically incompressible, without internal variables) with elastic range, under the form of axiom that requires the small cycles of the deformation gradient to be dissipative.
Within the constitutive framework introduced in Cleja-Ţigoiu and Soós (1990), Cleja- Ţigoiu, Part I (1990), I postulate that the work done by internal forces (in the initial configuration) is non-negative on small cycles of strains. The limitation to isotropic materials is removed and the extension to certain material symmetries, pre-existing in the material, becomes possible. Just the existence and the uniqueness of the solutions of the differential system are the key point in my approach to finite plasticity.
We remark that Mandel's stress measure (see Mandel (1972) or equivalently Eshelby's stress tensor (see Epstein and Maugin (1990), Maugin (1994), Cleja-Ţigoiu and Maugin (1999)), generally non-symmetric, as well as the conjugates forces to the internal variables (see Halphen and Nguyen (1975)) naturally appear in the dissipation inequality.
On the other hand, I observe here that the possible explicit dependence on the relaxed configurations of the constitutive and evolution functions, presented in a paper by Krawietz (1981), was eliminated in our constitutive framework by the adopted assumptions.
I compare my results with other results related to the subject. All details are omitted in this paper,

since I wish to put into evidence some general results and to form a basis for further comparisons and developments.
The following notations will be used: $Lin-$ the set of all second order tensor; Lin^+- the set of elements of Lin with positive determinant; $Sym-$ the set of symmetric elements of Lin; Sym^+ all positive definite tensors of Sym; $Ort-$ the orthogonal group; Ort^+- the proper orthogonal group; $\mathbf{a}\cdot\mathbf{b}-$ the scalar product of the vectors, and $\mathbf{A}\cdot\mathbf{B} := \mathrm{tr}\,\mathbf{A}\mathbf{B}^T-$ the scalar product of $\mathbf{A},\mathbf{B}\in Lin$.

2 Constitutive Framework

We briefly recall constitutive equations for elasto-plastic materials with relaxed (plastically deformed configuration) and internal variables.
For an arbitrary given motion χ, defined in a certain neighborhood of a material point $\mathbf{X}$, we consider the deformation gradient $\mathbf{F}(t)$, $\det\mathbf{F}(t) > 0$ for $t\in[0,d]$ with $d>0$, $\mathbf{F}(0)=\mathbf{I}$, and the multiplicative decomposition of the deformation gradient into its elastic and plastic parts:

$$\mathbf{F}(t) = \mathbf{E}(t)\mathbf{P}(t) \tag{1}$$

The complete set of constitutive and evolution functions will be represented with respect to the current relaxed configuration, K_t, in the so-called elastic strain space:

- The elastic constitutive equation in terms of Piola- Kirchhoff stress tensor Π in K_t is characterized by

$$\Pi/\tilde{\rho} = \mathbf{h}(\mathbf{G},\alpha) \quad \text{with} \quad \mathbf{G}=\mathbf{E}^T\mathbf{E} \tag{2}$$

$\tilde{\rho}$ is the mass density in the relaxed configuration, and α being internal variables. The elastic constitutive function $\mathbf{h}$ has the relaxation property:

$$\mathbf{h}(\mathbf{S},\alpha) = 0 \quad \text{for} \quad \mathbf{S}\in Sym^+ \quad \Longleftrightarrow \quad \mathbf{S}=\mathbf{I} \tag{3}$$

- There is a smooth function $\widetilde{\mathcal{F}}$, called yield function, of the class C^1, depending on the elastic strain tensor $\mathbf{G}$ (or the right Cauchy- Green elastic tensor) and on internal variables $\alpha\in R^n$, which has the properties:

i) $\widetilde{\mathcal{F}} : \mathcal{D}_{\mathcal{F}} \subset Sym^+ \times \mathbf{R}^n \longrightarrow \mathbf{R}$, $\widetilde{\mathcal{F}}(\mathbf{I},\alpha) < 0$ for all α

ii) for all fixed $\alpha \in pr_2\,\mathcal{D}_{\mathcal{F}}-$ the projection on the space of internal variables, the set

$$\{\mathbf{G}\in Sym^+ \mid \widetilde{\mathcal{F}}(\mathbf{G},\alpha)\leq 0, \quad (\mathbf{G},\alpha)\in\mathcal{D}_{\mathcal{F}}\}$$

is the closure of a non-empty, connected open set, i.e. if necessary we restrict the yield function to the connected set that contains $\mathbf{I}\in pr_1\,\mathcal{D}_{\mathcal{F}}\subset Sym^+$;

iii) for all $\alpha\in pr_2(\mathcal{D}_{\mathcal{F}})$ the set $\{\mathbf{G}\in Sym^+ \mid \widetilde{\mathcal{F}}(\mathbf{G},\alpha)=0\}$ defines a C^1 differential manifold, called the current yield surface. Hence $\partial_{\mathbf{G}}\widetilde{\mathcal{F}}(\mathbf{G},\alpha) \neq 0$ on the yield surface.

- The evolution equations for the plastic part of the deformation and for the internal variables, both supposed to be rate independent, are represented by the form

$$\dot{\mathbf{P}}\mathbf{P}^{-1} = \lambda\,\mathcal{B}(\mathbf{G},\alpha); \quad \dot{\alpha} = \lambda\,\mathbf{m}(\mathbf{G},\alpha) \tag{4}$$

with the initial condition

$$\mathbf{P}(0)=\mathbf{I}, \quad \alpha(0)=0 \tag{5}$$

when the reference configuration is plastically undeformed and stress free.

The plastic multiplier λ satisfies the requirements $\lambda\geq 0$, $\lambda\widetilde{\mathcal{F}}=0$, $\widetilde{\mathcal{F}}\leq 0$, and the consistency condition $\lambda\dot{\widetilde{\mathcal{F}}}=0$. The superposed dot denotes material time derivative.

- The pre-existing material symmetry is characterized by the symmetry group $g_k \subset Ort^+-$ the proper orthogonal group, which renders the material functions invariant (see Cleja- Ţigoiu and Soós (1989))

$$\begin{array}{ll} \mathbf{h}(\mathbf{QGQ}^T, \mathbf{Q}[\alpha]) = \mathbf{Qh}(\mathbf{G},\alpha)\mathbf{Q}^T \,, & \mathcal{F}(\mathbf{QGQ}^T, \mathbf{Q}[\alpha]) = \mathcal{F}(\mathbf{G},\alpha) \\ \mathbf{m}(\mathbf{QGQ}^T, \mathbf{Q}[\alpha]) = \mathbf{Q}[\mathbf{m}(\mathbf{G},\alpha)] \,, & \mathcal{B}(\mathbf{QGQ}^T, \mathbf{Q}[\alpha]) = \mathbf{Q}\mathcal{B}(\mathbf{G},\alpha)\mathbf{Q}^T \end{array} \tag{6}$$

$\forall \mathbf{Q} \in g_k$. Here we used the following notations for tensorial and scalar variables:

$$\mathbf{Q}[\alpha] = \mathbf{Q}\alpha\mathbf{Q}^T \quad \text{for} \quad \alpha \in Lin \,, \quad \mathbf{Q}[\alpha] = \alpha \quad \text{for} \quad \alpha \in R \tag{7}$$

3 Evolution of the Irreversible Behaviour

Let us denote by $\mathbf{Y} \in Lin \times R$ the set of variables which characterize the irreversible behaviour of the body, at the fixed material point, i.e.

$$\mathbf{Y} := (\mathbf{P}^{-1}, \alpha) \,, \quad \text{i.e.} \quad \mathbf{Y} = (\mathbf{Y}_1, \mathbf{Y}_2), \quad \mathbf{Y}_1 = \mathbf{P}^{-1}, \; \mathbf{Y}_2 = \alpha \tag{8}$$

Proposition 1. The evolution in time of $\mathbf{Y}$ is governed by the solutions of Cauchy problem:

$$\begin{array}{l} \dot{\mathbf{Y}} = -\dfrac{1}{\gamma(t,\mathbf{Y})} < \beta(t,\mathbf{Y}) > \mathcal{Y}(\mathbf{C}(t),\mathbf{Y}) H(\overline{\mathcal{F}}(\mathbf{C}(t),\mathbf{Y})) \\ \mathbf{Y}(0) = \mathbf{Y}_0 \end{array} \tag{9}$$

for a given strain history, $\hat{\mathbf{C}}$, $t \in [0,d] \to \mathbf{C}(t) \in Sym^+$, with

$$\mathbf{C} = \mathbf{F}^T\mathbf{F} \tag{10}$$

and associated to the yield function

$$\overline{\mathcal{F}}(\mathbf{C},\mathbf{Y}) := \tilde{\mathcal{F}}(\mathbf{P}^{-T}\mathbf{C}\mathbf{P}^{-1},\alpha) \equiv \tilde{\mathcal{F}}(\mathbf{G},\alpha) \quad \text{with} \quad \mathbf{Y} \equiv (\mathbf{P}^{-1},\alpha) \tag{11}$$

Here $\beta-$ the complementary plastic factor and $\gamma-$ the hardening parameter (supposed to be $\gamma > 0$) are defined for any $\mathbf{C}$ on the current yield surface $\overline{\mathcal{F}}(\mathbf{C},\mathbf{Y}) = 0$ by

$$\beta := \partial_{\mathbf{C}}\overline{\mathcal{F}}(\mathbf{C},\mathbf{Y}) \cdot \dot{\mathbf{C}} \quad , \quad \gamma := \partial_{\mathbf{Y}}\overline{\mathcal{F}}(\mathbf{C},\mathbf{Y}) \cdot \mathcal{Y}(\mathbf{C},\mathbf{Y}) \tag{12}$$

In the system (9) $<\beta>=\beta$ if $\beta > 0$ and $<\beta>=0$ if $\beta \leq 0$. H denotes Heaviside function, i.e. $H(x) = 1$ for all $x \geq 0$, $H(x) = 0$ for all $x < 0$, and the right hand side is defined by

$$\mathcal{Y}(\mathbf{C},\mathbf{Y}) := (\mathbf{P}^{-1}\mathcal{B}(\mathbf{P}^{-T}\mathbf{C}\mathbf{P}^{-1},\alpha) \,, -\mathbf{m}(\mathbf{P}^{-T}\mathbf{C}\mathbf{P}^{-1},\alpha)) \tag{13}$$

Remark 1. The appropriate form of the differential system, which defines the evolution of $(\mathbf{P}^{-1},\alpha)$, is independent of the change of frame in the actual configuration, as it is derived in the reference configuration, being dependent on the strain history $\hat{\mathbf{C}}$ only.
We assume that the admissible strain histories $\hat{\mathbf{C}} : [0,d] \to Sym^+$, are continuous and piecewise continuously differentiable mappings over $[0,d]$ (i.e. $\dot{\mathbf{C}}$ is well defined and continuous with possible exception of a finite number of points, where it is right-continuous). Due to the fact that the evolution equations are rate independent it follows that $\hat{\mathbf{C}}$ can be considered on $[0,1]$, only.
The differential system (9) must be broken into several systems, generated by continuously differentiable restrictions of the strain process. The initial conditions are chosen to be just the final values reached in the previous system. We assume that the constitutive functions satisfy the requirements of existence and uniqueness of the solutions (see Cleja-Ţigoiu, Part II (1990)). In our rate-independent system it can be proved that no critical points of the system exist, i.e. there are no $(t,\mathbf{Y})$ such that $\beta(t,\mathbf{Y})$ and $\dot{\beta}(t,\mathbf{Y})$ are simultaneously vanishing.
Remark 2. We denote by $\mathcal{G}_s$ the set of all admissible strain histories, defined on [0, 1], having the current values $\hat{\mathbf{C}}(t) \equiv \mathbf{C}(t) \in Sym^+-$ symmetric and positive definite tensors and $\mathbf{C}(0) = \mathbf{I}$.
For all sufficiently smooth histories of deformation gradient $\hat{\mathbf{F}} : [0,1] \to Lin^+$ with $\mathbf{F}(0) = \mathbf{I}$, we associate

the admissible strain histories $\hat{\mathbf{C}} \in \mathcal{G}_s$. Conversely, using the polar decomposition for invertible tensors, for a given strain history $\hat{\mathbf{C}}$ we construct the deformation gradient history $\hat{\mathbf{F}}$ such that

$$\mathbf{F}(t) = \mathbf{R}(t)\mathbf{U}(t), \quad \mathbf{U}(t) \in Sym^+, \quad \mathbf{U}^2(t) = \mathbf{C}(t) \tag{14}$$

and $\mathbf{R}(t) \in Ort^+$, with $\mathbf{R}(0) = \mathbf{I}$. We denote by $\mathcal{G}$ the set of admissible histories of deformation gradient. Let $\hat{\mathbf{C}} \in \mathcal{G}_s$ and $t \in (0, d]$. We denote by $\hat{\mathbf{C}}_t$ the restriction of $\hat{\mathbf{C}}$ to the interval $[0, t]$, i.e. $\hat{\mathbf{C}}_t(\tau) = \hat{\mathbf{C}}(t\tau)$ for all $\tau \in [0, 1]$.

4 Elastic Range and Stress Functionals

For the models with relaxed configurations we define the **elastic range** $\mathcal{U}(\hat{\mathbf{C}}_t)$ and the **reduced elastic range** $\mathcal{U}_{\mathcal{R}}(\hat{\mathbf{C}}_t)$ at time t, corresponding to $\hat{\mathbf{C}} \in \mathcal{G}_s$ as

$$\mathcal{U}(\hat{\mathbf{C}}_t) := \{\mathbf{B} \in Sym^+ \mid \overline{\mathcal{F}}(\mathbf{B}, \mathbf{Y}(t)) \leq 0\}, \quad \mathcal{U}_{\mathcal{R}}(\hat{\mathbf{C}}_t) := \{\mathbf{A} \in Sym^+ \mid \widetilde{\mathcal{F}}(\mathbf{A}, \alpha(t)) \leq 0\} \tag{15}$$

where $\mathbf{Y}(t) \equiv (\mathbf{P}(t)^{-1}, \alpha(t))$ represents the solution of (9), at time t, for the given strain history.

1. Based on (11) it follows that

$$\mathcal{U}_{\mathcal{R}}(\hat{\mathbf{C}}_t) \equiv \{\mathbf{A} \in Sym^+ \mid \mathbf{P}^T(t)\mathbf{A}\mathbf{P}(t) \in \mathcal{U}(\hat{\mathbf{C}}_t)\} \tag{16}$$

and conversely

$$\mathcal{U}(\hat{\mathbf{C}}_t) \equiv \{\mathbf{B} \in Sym^+ \mid \mathbf{P}^{-T}(t)\mathbf{B}\mathbf{P}^{-1}(t) \in \mathcal{U}_{\mathcal{R}}(\hat{\mathbf{C}}_t)\} \tag{17}$$

2. The **boundaries** $\partial\mathcal{U}(\hat{\mathbf{C}}_t)$ and $\partial\mathcal{U}_{\mathcal{R}}(\hat{\mathbf{C}}_t)$ of the elastic ranges are given by

$$\begin{aligned} \partial\mathcal{U}(\hat{\mathbf{C}}_t) &:= \{\mathbf{B} \in Sym^+ \mid \overline{\mathcal{F}}(\mathbf{B}, \mathbf{Y}(t) = 0\} \\ \partial\mathcal{U}_{\mathcal{R}}(\hat{\mathbf{C}}_t) &:= \{\mathbf{A} \in Sym^+ \mid \widetilde{\mathcal{F}}(\mathbf{A}, \alpha(t)) = 0\} \end{aligned} \tag{18}$$

Hence for instance $\mathbf{B} \in \partial\mathcal{U}(\hat{\mathbf{C}}_t) \iff \mathbf{P}^{-T}(t)\mathbf{B}\mathbf{P}^{-1}(t) \in \partial\mathcal{U}_{\mathcal{R}}(\hat{\mathbf{C}}_t)$. The boundaries of the elastic ranges define the current yield surfaces in the strain and in the elastic strain spaces, respectively.

3. The **interior** of the elastic ranges, i.e. $\overset{\circ}{\mathcal{U}}(\hat{\mathbf{C}}_t) := \{\mathbf{B} \mid \overline{\mathcal{F}}(\mathbf{B}, \mathbf{Y}(t)) < O\}$ and $\overset{\circ}{\mathcal{U}}_{\mathcal{R}}(\hat{\mathbf{C}}_t) := \{\mathbf{A} \mid \widetilde{\mathcal{F}}(\mathbf{A}, \alpha(t)) < 0\}$, are not empty since

$$\mathbf{I} \in \overset{\circ}{\mathcal{U}}_{\mathcal{R}}(\hat{\mathbf{C}}_t) \quad \text{and} \quad \mathbf{P}^T(t)\mathbf{P}(t) \in \overset{\circ}{\mathcal{U}}(\hat{\mathbf{C}}_t) \tag{19}$$

as a consequence of the stipulated property $\widetilde{\mathcal{F}}(\mathbf{I}, \alpha) < 0$ for all α, and of (11).

4. For any given $\mathbf{P}^{-1} \in Lin^+$, the map $\mathbf{C} \in Sym^+ \longrightarrow \mathbf{P}^{-T}\mathbf{C}\mathbf{P}^{-1} = \mathbf{G} \in Sym^+$ is a homeomorphism (i.e. the map and its inverse are continuous). Hence $\mathcal{U}(\hat{\mathbf{C}}_t)$ and $\mathcal{U}_{\mathcal{R}}(\hat{\mathbf{C}}_t)$ have the same topological properties, i.e. they are the closure of a non-empty, connected open set.

5. For the material characterized by the positive hardening parameter, $\gamma > 0$, defined in $(12)_2$, due to the continuity of the yield function and of the solution $\mathbf{Y}$ of the differential system (9), the elastic ranges evolve smoothly, i.e. for all $\hat{\mathbf{C}} \in \mathcal{G}_s$, and for each $\mathbf{B} \in \overset{\circ}{\mathcal{U}}(\hat{\mathbf{C}}_t)$ there is an $\epsilon > 0$ such that

$$\mathbf{B} \in \bigcap_{\tau \in [t, t+\epsilon)} \overset{\circ}{\mathcal{U}}(\hat{\mathbf{C}}_\tau) \iff \widetilde{\mathcal{F}}(\mathbf{P}^{-T}(\tau)\mathbf{B}\mathbf{P}^{-1}(\tau), \alpha(\tau)) < 0 \quad \forall \tau \in [t, t+\epsilon) \tag{20}$$

In our approach to finite elasto-plasticity it can be proved:
Insensitivity of the plastic deformation and of the internal variables to elastic strain cycles. Let $\hat{\mathbf{C}}$ be in $\mathcal{G}_s$ a continuously differentiable strain history and let $\mathbf{A}$ be in $\mathcal{U}(\hat{\mathbf{C}}_t)$ for a certain $t \in (0, 1)$.

Let $\hat{\mathbf{H}}$ be in $\mathcal{G}_s$ a strain process having the properties: there exist $t_0, \bar{t}$ with $t < t_0 < \bar{t} < 1$ such as

$$\hat{\mathbf{H}}(u) = \begin{cases} \hat{\mathbf{C}}(u) & u \in [0,t] \\ \in \mathcal{U}(\hat{\mathbf{C}}_t) \quad , \hat{\mathbf{H}}(t) = \hat{\mathbf{H}}(\bar{t}),\ \hat{\mathbf{H}}(t_0) = \mathbf{A}, & u \in [t,\bar{t}] \\ \hat{\mathbf{C}}(t - \bar{t} + u) \quad (\text{with } \delta > 0) & \bar{t} < u \leq \bar{t} + \delta \end{cases} \tag{21}$$

with $\delta > 0$ and $\bar{t} + \delta \leq 1$. We denote by $\mathbf{Y_H}$ the solution of the differential system (9) constructed via the given strain history $\hat{\mathbf{H}}$. Thus

$$\mathbf{Y_H}(u) = \begin{cases} \mathbf{Y}(u) & u \in [0,t] \\ \mathbf{Y}(t) & u \in (t,\bar{t}] \\ \mathbf{Y}(t - \bar{t} + u) & \bar{t} < u \leq \bar{t} + \delta \end{cases} \tag{22}$$

Insensitivity of plastic deformation and **internal variables to elastic strain continuations.** For all $\hat{\mathbf{C}}$ in $\mathcal{G}_s$, $\mathcal{U}(\hat{\mathbf{C}}_t)$ and $\mathcal{U}_{\mathcal{R}}(\hat{\mathbf{C}}_t)$ are invariant under elastic continuation of $\hat{\mathbf{C}}_t$, i.e. for all strain histories $\hat{\mathbf{H}} \in \mathcal{G}_s$, such that there exists $\bar{t} \in (0,1)$ and $\hat{\mathbf{H}}_{\bar{t}} = \hat{\mathbf{C}}_t$ (which means $\hat{\mathbf{H}}(\bar{t}\tau) = \hat{\mathbf{C}}(t\tau)$, $\forall \tau \in [0,1]$) and $\hat{\mathbf{H}}(\tau) \in \mathcal{U}(\hat{\mathbf{C}}_t)$ for all $\tau \in [\bar{t}, 1]$, then $\mathcal{U}(\hat{\mathbf{H}}) = \mathcal{U}(\hat{\mathbf{C}}_t)$ and $\mathcal{U}_{\mathcal{R}}(\hat{\mathbf{H}}) = \mathcal{U}_{\mathcal{R}}(\hat{\mathbf{C}}_t)$.
We put into evidence three type of stress constitutive functionals which define the current values of the appropriate stress measures, Cauchy stress tensor $\mathbf{T}$, Piola-Kirchhoff stress tensors Π and Π_0, in the plastically deformed (relaxed) configuration and in the initial configuration respectively . All of them are generated by the elastic constitutive function (2), via the solution of the differential system (9), taking into account the relation existing between these stress measure:

$$\frac{\Pi_0(t)}{\rho_0} = \mathbf{F}^{-1} \frac{\mathbf{T}}{\rho} \mathbf{F}^{-T} = \mathbf{P}^{-1} \frac{\Pi}{\tilde{\rho}} \mathbf{P}^{-T} \tag{23}$$

$\rho_0, \tilde{\rho}, \rho$ are mass densities in initial, relaxed, and actual configurations.
We exemplify our results by the Theorem:

1. There exist the stress constitutive functionals $\tilde{\mathbf{K}}_0$, which give the current value of the Piola- Kirchhoff stress tensor Π_0 corresponding to the strain history $\hat{\mathbf{C}} \in \mathcal{G}_s$, being defined by

$$\tilde{\mathbf{K}}_0(\hat{\mathbf{C}}_t) = \rho_0\, \mathbf{P}^{-1}(t)\, \mathbf{h}(\mathbf{P}^{-T}(t)\mathbf{C}(t)\mathbf{P}^{-1}(t), \alpha(t))\, \mathbf{P}^{-T}(t) \tag{24}$$

where $\mathbf{Y}$ denotes the solution of the differential system (9), constructed via the strain history $\hat{\mathbf{C}}$.
Here $\mathbf{P}^{-T}(t)\mathbf{C}(t)\mathbf{P}^{-1}(t) \equiv \mathbf{G}(t)$ is the elastic strain (see $(2)_2$ together with (1)), measured in the relaxed configuration (or plastically deformed configuration).

2. The stress functional $\tilde{\mathbf{K}}_0$ is frame invariant, since $\hat{\mathbf{F}} \in \mathcal{G}$ and $\hat{\mathbf{Q}}\hat{\mathbf{F}} \in \mathcal{G}$, with $\hat{\mathbf{Q}}-$ history of proper orthogonal tensors, lead to the same strain history $\hat{\mathbf{C}}$.

3. The stress constitutive functional $\tilde{\mathbf{K}}_0$ is path independent on the elastic strain continuation of the strain history $\hat{\mathbf{C}} \in \mathcal{G}_s$, i.e. for all $\hat{\mathbf{A}}$ and $\hat{\mathbf{B}} \in \mathcal{G}_s$ elastic strain continuations of $\hat{\mathbf{C}}$ at time $t \in [0,1)$, having the same value at the time $\tau > t$ (with $\tau \leq 1$) $\hat{\mathbf{A}}(\tau) = \hat{\mathbf{B}}(\tau) \equiv \mathbf{A}_0$, i.e. $\hat{\mathbf{A}}(u), \hat{\mathbf{B}}(u) \in \mathcal{U}_{\mathcal{R}}(\mathbf{C}_t)$ for all $u \in [t, \tau]$, then

$$\tilde{\mathbf{K}}_0(\hat{\mathbf{A}}_\tau) = \tilde{\mathbf{K}}_0(\hat{\mathbf{B}}_\tau)\,, \quad \text{moreover} \quad \tilde{\mathbf{K}}_0(\hat{\mathbf{A}}_\tau) = \rho_0\, \mathbf{P}^{-1}(t)\, \mathbf{h}(\mathbf{A}_0, \alpha(t))\, \mathbf{P}^{-T}(t) \tag{25}$$

5 Work Property on Small Strain Cycles

At a given material point $\mathbf{X}$, the **work done by internal forces** in the deformation process described by $\hat{\mathbf{F}} \in \mathcal{G}$ between the instants t_1 and t_2, can be expressed in terms of the Cauchy stress tensor $\mathbf{T}$ as well as in terms of the Piola- Kirchhoff stress tensor Π_0 in the reference configuration

$$\int_{t_1}^{t_2} \frac{\mathbf{T}(\tau)}{\rho(\tau)} \cdot \{\dot{\mathbf{F}}(\tau)\mathbf{F}^{-1}(\tau)\}^s d\tau = \frac{1}{2} \int_{t_1}^{t_2} \frac{\Pi_0(\tau)}{\rho_0} \cdot \dot{\mathbf{C}}(\tau) d\tau \tag{26}$$

or in terms of Piola-Kirchhoff stress tensor Π via formula (23). The values at time τ for the stress tensor fields are calculated via the appropriate constitutive stress functionals, for instance introduced in (24), corresponding to the history $\hat{\mathbf{C}} \in \mathcal{G}_s$.
We introduce now the work property on small cycles of strains only.
Axiom of the dissipative nature of strain cycles. For all $\hat{\mathbf{C}} \in \mathcal{G}_s$ and $t_1, t_2 \in [0,1]$ such that $0 \leq t_1 < t_2 \leq 1$ and

$$\hat{\mathbf{C}}(t_1) = \hat{\mathbf{C}}(t_2) \in \bigcap_{\tau \in \in [t_1, t_2]} \mathcal{U}(\hat{\mathbf{C}}_\tau) \tag{27}$$

the work done by internal forces in the strain processes between t_1 and t_2 is non- negative

$$\frac{1}{2}\int_{t_1}^{t_2} \frac{\Pi_0(\tau)}{\rho_0} \cdot \dot{\mathbf{C}}(\tau) d\tau \geq 0 \tag{28}$$

As a consequence of the property of the elastic range, written in (20) for $t = t_1$, there is a $t_2 < t + \epsilon$ such that the intersection in (27) is non-empty. Hence the postulate of non- negative work done by internal forces is allowed for sufficiently small cycles of strains only.

6 Stress Potential and Dissipation Inequalities

Existence of the stress potential. 1. For all $\hat{\mathbf{C}} \in \mathcal{G}_s$ there exists a smooth scalar valued function $\widetilde{\varphi}(\cdot, \hat{\mathbf{C}}_t)$ (for all $t \in [0,1]$) defined over $\mathcal{U}_{\mathcal{R}}(\hat{\mathbf{C}}_t)$ such that

$$\begin{aligned} &\widetilde{\varphi}(\mathbf{G}, \hat{\mathbf{C}}_t) = \varphi(\mathbf{G}, \alpha(t)) \quad \text{and} \\ &\frac{\Pi(t)}{\tilde{\rho}(t)} = \mathbf{h}(\mathbf{G}, \alpha(t)) = 2\,\partial_{\mathbf{G}}\varphi(\mathbf{G}, \alpha(t)). \quad \text{Here} \quad \mathbf{G} = \mathbf{P}^{-T}(t)\mathbf{C}(t)\mathbf{P}^{-1}(t) \end{aligned} \tag{29}$$

and $(\mathbf{P}^{-1}(t), \alpha(t))$ is the current value of the solution of (9), corresponding to the history $\hat{\mathbf{C}}$.

2. There exists a smooth scalar valued function $\sigma(\cdot, \mathbf{Y}(t))$ defined over $\mathcal{U}(\hat{\mathbf{C}}_t)$ such that

$$\sigma(\mathbf{C}(t), \mathbf{Y}(\mathbf{t})) = \varphi(\mathbf{P}^{-\mathbf{T}}\mathbf{C}(\mathbf{t})\mathbf{P}^{-\mathbf{1}}, \alpha(\mathbf{t})) \tag{30}$$

and for all $\hat{\mathbf{F}} \in \mathcal{G}$ associated to $\hat{\mathbf{C}}_t$ by the procedure mentioned in Remark 2.,

$$\frac{\mathbf{T}(t)}{\rho(t)} = 2\,\mathbf{F}(t)\,\partial_{\mathbf{C}}\sigma(\mathbf{C}(t), \mathbf{Y}(t))\mathbf{F}^T(t) \iff \frac{\Pi_0(t)}{\rho_0} = 2\,\partial_{\mathbf{C}}\sigma(\mathbf{C}(t), \mathbf{Y}(t)) \tag{31}$$

3. The stress potentials φ and σ are invariant under a change of the observer in the actual configuration as well as under the symmetry group g_k.
Examples for stress potentials similar to the specific free energy function generally accepted in the finite elasto-plasticity, for isothermic process are given in the special form

$$\varphi(\mathbf{G}, \alpha) = \varphi_e(\mathbf{G}) + \varphi_b(\alpha) \tag{32}$$

Here φ_e is the specific potential for the Piola- Kirchhoff symmetric stress tensor in the relaxed configuration and φ_b is the stored potential. The form (32) is justified only when there is no influence of internal variables on the elastic property of the material, i.e. when the elastic function $\mathbf{h}$ from (2) (see also (29)) is independent of α. Some particular case of (32) arises when φ_e and φ_b are quadratic forms with respect to their tensorial variables.
Under the hypothesis (32), for **structural isotropic elasto-plastic materials** defined by $g_k = Ort^+$, we derive

$$2\varphi(\mathbf{G}, \alpha, \kappa) = \frac{\lambda_e}{2}(\mathbf{I} \cdot \Delta)^2 + \mu_e \Delta \cdot \Delta + a_1(\kappa)(\alpha \cdot \mathbf{I})^2 + a_2(\kappa)\alpha \cdot \alpha + \varphi_0(\kappa) \tag{33}$$

where $\Delta = \frac{1}{2}(\mathbf{G} - \mathbf{I})$, λ_e and μ_e are the (constant) Lame elastic coefficients. Here the set of internal

variables is denoted by (α, κ) in order to distinguish between scalar, κ, and tensorial, α, variables.
When we refer to transversely isotropic materials, two symmetry groups g_1 and g_4, which preserve a symmetry direction $\mathbf{n}_1$ (see Shih- Liu (1983), are accepted in our models. Based on the recent results formulated in Cleja- Tigoiu (1999), we can prove that

1. For both symmetry groups g_1 and g_4 the same representation for φ_e

$$2\,\varphi_e(\mathbf{G}) := a(\Delta\mathbf{n}_1 \cdot \mathbf{n}_1)^2 + 2b\mathrm{tr}(\Delta)(\Delta\mathbf{n}_1 \cdot \mathbf{n}_1) + f\Delta \cdot \Delta + d(\mathrm{tr}\,\Delta)^2 + 2\,e(\Delta\mathbf{n}_1 \cdot \mathbf{n}_1)^2 \tag{34}$$

holds. Here a, b, d, e, f are (constant) elastic coefficients.

2. The stored potential, depending on α, possibly a non-symmetric tensor, can be expressed by

$$\begin{aligned}\varphi_b(\alpha) &\equiv b_1(\alpha^s \cdot (\mathbf{n}_1 \otimes \mathbf{n}_1))^2 + b_2(\alpha^s)^2 \cdot (\mathbf{n}_1 \otimes \mathbf{n}_1) + b_3(\alpha^s)^2 \cdot \mathbf{N}_1^2 + \\ &+ b_4(\alpha^s \cdot \mathbf{I})(\alpha^s \cdot (\mathbf{n}_1 \otimes \mathbf{n}_1)) + b_5(\alpha^s \cdot I)^2 + b_6\alpha^s\alpha^a \cdot \mathbf{N}_1 + b_7\alpha^a\mathbf{N}_1^2 \cdot \alpha^s + \\ &+ b_8(\alpha^a \cdot \mathbf{N}_1)(\alpha^s \cdot (\mathbf{n}_1 \otimes \mathbf{n}_1)) + b_9(\alpha^a)^2 \cdot (\mathbf{n}_1 \otimes \mathbf{n}_1) + b_{10}(\alpha^a \cdot \mathbf{N}_1)^2\end{aligned} \tag{35}$$

where $\mathbf{N}_1$ is the skew-symmetric tensor coaxial with $\mathbf{n}_1$, for g_1 transversely isotropic materials with ten material functions, and with eight scalar material functions, depending on the scalar internal variables, for g_4- transversely isotropic materials by

$$\begin{aligned}\varphi_b(\alpha) &:= a_1(\alpha^s \cdot (\mathbf{n}_1 \otimes \mathbf{n}_1))^2 + a_2(\alpha^s)^2 \cdot (\mathbf{n}_1 \otimes \mathbf{n}_1) + a_3\mathrm{tr}(\alpha^s)^2 + a_4(\mathrm{tr}\alpha^s)(\alpha^s \cdot (\mathbf{n}_1 \otimes \mathbf{n}_1)) + \\ &+ a_5(\mathrm{tr}\alpha^s)^2 + a_6\mathrm{tr}(\alpha^a)^2 + a_7(\alpha^a)^2 \cdot (\mathbf{n}_1 \otimes \mathbf{n}_1) + a_8\mathrm{tr}(\alpha^s(\mathbf{n}_1 \otimes \mathbf{n}_1)\alpha^a).\end{aligned} \tag{36}$$

Dissipation inequalities. Let $\hat{\mathbf{C}}$ be in $\mathcal{G}_s$. For all $t \in (0,1)$ such that $\mathbf{C}(t) \in \partial\mathcal{U}(\hat{\mathbf{C}}_t)$, the work property (28) implies for all $\mathbf{A} \in \mathcal{U}(\hat{\mathbf{C}}_t)$, the following equivalent dissipation inequalities

$$\begin{aligned}&[\partial_{\mathbf{Y}}\sigma(\mathbf{A}(t), \mathbf{Y}(t)) - \partial_{\mathbf{Y}}\sigma(\mathbf{C}(t), \mathbf{Y}(t))] \cdot \dot{\mathbf{Y}}(t) \geq 0 \quad \text{and} \\ &2\,[\mathbf{G}(t)\partial_{\mathbf{G}}\varphi(\mathbf{G}(t), \alpha(t)) - \mathbf{G}^*(t)\partial_{\mathbf{G}}\varphi(\mathbf{G}^*(t), \alpha(t))] \cdot \dot{\mathbf{P}}(t)\mathbf{P}^{-1}(t) + \\ &+ [\partial_\alpha\varphi(\mathbf{G}^*(t), \alpha(t)) - \partial_\alpha\varphi(\mathbf{G}(t), \alpha(t))] \cdot \dot{\alpha}(t) \geq 0\end{aligned} \tag{37}$$

where $\mathbf{G}(t)$ and $\mathbf{G}^*(t)$ are defined as in $(29)_3$ and $\mathbf{G}^*(t) = \mathbf{P}^{-T}(t)\mathbf{A}\mathbf{P}^{-1}(t)$. Let us introduce Mandel's stress measure Σ in the relaxed configuration, generally a non-symmetric tensor field, which is defined by the function $\hat{\Sigma}(\cdot, \alpha) : Sym^+ \longrightarrow Lin$, when the elastic constitutive equation (2) (or (29)) is taken into account :

$$\Sigma := \mathbf{G}\Pi/\tilde{\rho} \equiv \mathbf{G}\mathbf{h}(\mathbf{G}, \alpha) \equiv 2\,\mathbf{G}\partial_{\mathbf{G}}\varphi(\mathbf{G}, \alpha) \equiv \hat{\Sigma}(\mathbf{G}, \alpha) \tag{38}$$

Proposition 2. Let $\hat{\mathbf{C}}$ be in $\mathcal{G}_s$. For all $t \in (0,1)$ such that the associated elastic strain $\mathbf{G}(t) \in \partial\mathcal{U}_{\mathcal{R}}(\hat{\mathbf{C}}_t)$, the work property (28) under the consequence $(37)_2$ implies, for all $\mathbf{G}^* \in \mathcal{U}_{\mathcal{R}}(\hat{\mathbf{C}}_t)$, the following dissipation inequality

$$(\Sigma(t) - \Sigma^*) \cdot \dot{\mathbf{P}}(t)\mathbf{P}^{-1}(t) + (\mathbf{a}(t) - \mathbf{a}^*) \cdot \dot{\alpha}(t) \geq 0 \tag{39}$$

when the conjugated forces to the internal variables (see Halphen and Nguyen (1975)) are considered

$$\mathbf{a}(t) := -\partial_\alpha\varphi(\mathbf{G}(t), \alpha(t)), \quad \mathbf{a}^* = -\partial_\alpha\varphi(\mathbf{G}^*, \alpha(t)) \tag{40}$$

Here $\Sigma(t), \Sigma^*$ are given in (38), being calculated for the elastic strains $\mathbf{G}(t)$ and $\mathbf{G}^*$.
When the potential σ is considered, the dissipation inequality $(37)_2$ can be written in the following form

For all $t \in (0,1)$ such that $\overline{\mathcal{F}}(\mathbf{C}(t), \mathbf{Y}(t)) = 0$ the following dissipation inequality

$$
\begin{aligned}
&2[\partial_{\mathbf{P}^{-1}}\sigma(\mathbf{C}^*, \mathbf{Y}(t)) - \partial_{\mathbf{P}^{-1}}\sigma(\mathbf{C}(t), \mathbf{Y}(t))] \cdot \frac{d}{dt}(\mathbf{P}^{-1}) + \\
&+[\partial_\alpha\sigma(\mathbf{C}^*, \mathbf{Y}(t)) - \partial_\alpha\sigma(\mathbf{C}(t), \mathbf{Y}(t))] \cdot \dot{\alpha}(t) \geq 0,
\end{aligned}
\tag{41}
$$

holds for all $\mathbf{C}^*$ such that $\overline{\mathcal{F}}(\mathbf{C}^*, \mathbf{Y}(t)) \leq 0$, where $\mathbf{Y}(t) = (\mathbf{P}^{-1}(t), \alpha(t))$.
Let us consider the pseudo-potential of dissipation $\hat{\mathcal{D}}_p$, given in Cleja- Tigoiu and Maugin (2000), formulae (9) and (14), but mentioning explicitly its dependence on $\mathbf{G}$ and α, i.e.

$$
\hat{\mathcal{D}}_p(\mathbf{L}_p, \dot{\alpha}; \mathbf{G}, \alpha) := 2\mathbf{G}\partial_{\mathbf{G}}\varphi(\mathbf{G}, \alpha) \cdot \dot{\mathbf{P}}\mathbf{P}^{-1} - \partial_\alpha\varphi(\mathbf{G}, \alpha) \cdot \dot{\alpha} \tag{42}
$$

where $\mathbf{L}_p = \dot{\mathbf{P}}\mathbf{P}^{-1}$. Consequently (37) becomes **the principle of maximum dissipation**

$$
\hat{\mathcal{D}}_p(\mathbf{L}_p, \dot{\alpha}; \mathbf{G}, \alpha) - \hat{\mathcal{D}}_p(\mathbf{L}_p, \dot{\alpha}; \mathbf{G}^*, \alpha) \leq 0 \tag{43}
$$

whenever $\widetilde{\mathcal{F}}(\mathbf{G}, \alpha) = 0$, for all $\mathbf{G}^*$ such that $\widetilde{\mathcal{F}}(\mathbf{G}^*, \alpha) \leq 0$.

7 Flow Rules

Further we emphasize some implications derived from the dissipation inequalities.
A modified flow rule can be derived from the dissipation inequality (39)

$$
(\partial_{\mathbf{G}}\hat{\Sigma}(\mathbf{G}, \alpha))^T[\dot{\mathbf{P}}\mathbf{P}^{-1}] = \mu\partial_{\mathbf{G}}\widetilde{\mathcal{F}}(\mathbf{G}, \alpha) + \partial^2_{\alpha\,\mathbf{G}}\varphi(\mathbf{G}, \alpha)[\dot{\alpha}] \tag{44}
$$

where $\mu \geq 0$, for all $\mathbf{G}$ on yield surface $\widetilde{\mathcal{F}}(\mathbf{C}, \mathbf{Y}) = 0$. The equivalent inequality (41) leads to

$$
-\partial_{\mathbf{Y}}[\partial_{\mathbf{C}}\sigma(\mathbf{C}, \mathbf{Y})][\dot{\mathbf{Y}}] = \bar{\mu}\partial_{\mathbf{C}}\overline{\mathcal{F}}(\mathbf{C}, \mathbf{Y}) \quad \bar{\mu} \geq 0, \tag{45}
$$

for all $\mathbf{C}$ on the yield surface $\overline{\mathcal{F}}(\mathbf{C}, \mathbf{Y}) = 0$, for fixed $\mathbf{Y}$.
For structurally isotropic materials, i.e. $g_k = Ort^+$, due to the invariance property 3. the stress potential φ allows the representation

$$
\varphi(\mathbf{G}, \alpha) = \varphi(\mathbf{B}^e, \mathbf{a}), \quad \mathbf{B}^e = \mathbf{R}^e\mathbf{G}(\mathbf{R}^e)^T, \quad \mathbf{a} = \mathbf{R}^e[\alpha] \tag{46}
$$

$\mathbf{B}^e$ and $\mathbf{G}$ are the right and left Cauchy-Green elastic tensors, and $\mathbf{R}^e \in Ort^+$, $\mathbf{V}^e \in Sym^+$ (with $\mathbf{B}^e = (\mathbf{V}^e)^2$) such that

$$
\mathbf{E} = \mathbf{V}^e\mathbf{R}^e, \quad \mathbf{F} = \mathbf{V}^e\hat{\mathbf{P}} \quad \text{with} \quad \hat{\mathbf{P}} = \mathbf{R}^e\mathbf{P}. \tag{47}
$$

When (32) holds we derive $\partial^2_{\alpha\mathbf{G}}\,\varphi(\mathbf{G}, \alpha) = 0 \iff \partial^2_{\mathbf{a}\mathbf{B}^e}\,\varphi(\mathbf{B}^e, \mathbf{a}) = 0$, and consequently the elastic constitutive equation (31) in terms of $\mathbf{T}-$ Cauchy stress tensor is written in the form

$$
\frac{\mathbf{T}}{\rho} = 2\,\mathbf{V}^e\,\partial_{\mathbf{B}^e}\varphi(\mathbf{B}^e, \mathbf{a})\mathbf{V}^e := \mathbf{f}(\mathbf{B}^e) \tag{48}
$$

We stipulate that $\mathbf{f}$ is one to one on Sym^+, a diffeomorphism of the class C^1.
Definition 2. Let $\hat{\mathbf{F}}$ be in $\mathcal{G}$. For all $t \in (0,1)$, the elastic range in **the stress space** (in the actual configuration), associated to the reduced elastic range, $\mathcal{U}_{\mathcal{R}}(\hat{\mathbf{C}}_t)$ introduced in (15), is defined by

$$
\mathcal{K} := \{\mathbf{S} \in Sym \mid \mathcal{F}(\mathbf{S}, \mathbf{a}(t)) \leq 0\} \quad \text{where} \quad \mathcal{F}(\mathbf{S}, \mathbf{a}(t)) := \widetilde{\mathcal{F}}(\mathbf{B}, \mathbf{a}(t)), \quad \mathbf{B} = \mathbf{f}^{-1}(\mathbf{S}). \tag{49}
$$

Drucker's postulate. Let $\hat{\mathbf{F}}$ be in $\mathcal{G}$. For all $t \in (0,1)$ such that $\mathcal{F}(\dfrac{\mathbf{T}(t)}{\rho(t)}, \mathbf{a}(t)) = 0$, the dissipation inequality (39) becomes

$$
\begin{aligned}
&(\frac{\mathbf{T}}{\rho}-\mathbf{S})\cdot\{\frac{d}{dt}(\hat{\mathbf{P}})\hat{\mathbf{P}}^{-1}\}^{s}\geq 0 \quad \forall \quad \mathbf{S} \quad \text{such that} \quad \mathcal{F}(\mathbf{S},\mathbf{a}(t))\leq 0 \\
&\frac{d}{dt}(\hat{\mathbf{P}})\hat{\mathbf{P}}^{-1}=\omega^{e}+\mathbf{R}^{e}\dot{\mathbf{P}}\mathbf{P}^{-1}(\mathbf{R}^{e})^{T} \quad \text{where} \quad \omega^{e}=\dot{\mathbf{R}}^{e}(\mathbf{R}^{e})^{T}
\end{aligned}
\tag{50}
$$

under the hypotheses formulated in (32) and with the smooth elastic constitutive function $\mathbf{f}$ invertible.
Consequences. Drucker's postulate holds if and only if the following conditions hold $a)$ At the point $\mathbf{S}=\frac{\mathbf{T}}{\rho}$ of the stress yield surface, $\{\frac{d}{dt}(\hat{\mathbf{P}})\hat{\mathbf{P}}^{-1}\}^{s}$ is either null or collinear with the unit exterior normal of the stress yield surface at the given point, i.e.

$$
\{\frac{d}{dt}(\hat{\mathbf{P}})\hat{\mathbf{P}}^{-1}\}^{s}=\mu\partial_{\mathbf{S}}\,\mathcal{F}(\frac{\mathbf{T}}{\rho},\mathbf{a})
\tag{51}
$$

$b)$ The elastic range in stress space introduced in Definition 2. is a closed convex set in Sym, for every given internal variable $\mathbf{a}$.
Finally we consider an anisotropic elasto-plastic material without influence of the hardening on the elastic constitutive function, i.e. when $(11)_1$ holds, and with the yield function in the strain-space defined by

$$
\tilde{\mathcal{F}}(\mathbf{G},\alpha)=\hat{\mathcal{F}}(\Sigma,\alpha), \quad \Sigma=\hat{\Sigma}(\mathbf{G})
\tag{52}
$$

The material satisfies the dissipation inequality (39) if for every accessible state, $\mathbf{G}^{*}$, the following inequality

$$
(\hat{\Sigma}(\mathbf{G})-\hat{\Sigma}(\mathbf{G}^{*}))\cdot\dot{\mathbf{P}}\mathbf{P}^{-1}\geq 0, \quad \forall\mathbf{G}^{*}\ \text{such that}\ \hat{\mathcal{F}}(\hat{\Sigma}(\mathbf{G}^{*}),\alpha)\leq 0
\tag{53}
$$

holds.

2. Moreover, at any regular point of the yield function in strain space, the appropriate associative flow rule (referred as **Lubliner's flow rule**) takes the form

$$
\mathbf{L}^{p}\equiv\dot{\mathbf{P}}\mathbf{P}^{-1}=\mu\,\partial_{\Sigma}\hat{\mathcal{F}}(\Sigma,\alpha)+\mathbf{L}^{p*} \quad \text{with} \quad \mathbf{L}^{p*}: \quad (\partial_{\mathbf{G}}\hat{\Sigma}(\mathbf{G}))^{T}(\mathbf{L}^{p*})=0
\tag{54}
$$

where $\partial_{\mathbf{G}}\hat{\Sigma}(\mathbf{G}) : Sym \longrightarrow Lin$ is a linear mapping, obtained by the chain rule from (38), and μ is the loading parameter or plastic multiplier.

3. $\hat{\mathcal{F}}$ and $\mathbf{L}^{p*}$ are invariant relative to the considered symmetry group g_k.

8 Conclusions

The existence of the stress potential imposes a strong reduction to the number of elastic constants, as it can be seen from our examples. Indeed, transversely isotropic elasto-plastic materials with a linear elastic constitutive equation depending on eight constants for g_1 and on six constants for g_4 (see Cleja-Ţigoiu (2000)) do not satisfy the dissipation postulate.
The dissipation inequality (41) is the equivalent form of the restriction on the strain energy function derived in Casey and Tseng (1984) to our approach to finite elasto-plasticity. Inequalities similar to (53) appear in Lubliner (1986), (1990), in Marigo (1989) (where Σ is dependent on the elastic part $\mathbf{E}$ and not on the elastic strain tensor), in Krawietz (1981)(where $\Sigma \in Sym$, which is not true generally as it can be seen from our example relative to g_4- transversely isotropic material). In Srinivasa (1997) an inequality equivalent to (39) is derived, with an other interpretation for the appropriate variable Σ element in $Lin \times R$.
Our result in (54) is similar to Lubliner's result, but here we emphasized that the nine dimensional Mandel's flow rule defines the rate of the plastic deformation only up to a term which belongs to the kernel of a fourth order tensor with a precise physical meaning.
The dissipation inequality has been reformulated as a maximum dissipation postulate. This interpretation appears possible, when the free energy density (that is a potential for the stress, as a consequence of Clausius- Duhem thermodynamic restriction) is identified with the stress potential φ.

As it was proved in section 7, Il'yushin's postulate is less restrictive than Drucker's postulate, which is generally identified in finite elasto-plasticity with certain dissipation inequalities proved in an appropriate stress space. The problem which arises is to find these classes of elasto-plastic materials, allowing for stress potential, for which the dissipation postulate is a possible alternative characterization of the properties of convexity and normality alltogether.

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