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Non-uniqueness and Non-existence in the Statistical Theory of Elastic Composites. The Correlational Approximation

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I give the boundary conditions corresponding to the phenomenological model of a statistically homogeneous and isotropic elastic body, in the correlational approximation. I show that the involved strain energy density is not positive definite. By a simple example I show that a correctly formulated traction boundary value problem can have a unique solution, can have an infinity of solutions, or can have no solution, according to the relation existing between the geometrical dimension of the body and the correlation radius of the considered material.

1 Introduction

A basic problem of the theory of composite materials is the determination of the phenomenological constitutive equation connecting the mean stress and the mean strain, if the mechanical and geometrical characteristics of the phases are known. Also it is necessary to get the equilibrium equation satisfied by the mean stress. In a situation in which the mechanical and geometrical properties have a *random* character, the above problems are solved using the mathematical apparatus of statistical mechanics. However, the corresponding boundary conditions which must be satisfied by the mean stress or the mean displacement, involved in the obtained phenomenological model are not generally given. As an illustration of the above recalled facts, I give the fundamental results obtained by Beran and McCoy (1970a, 1970b) corresponding to the correlational approximation, and concerning a statistically homogeneous and isotropic linearly elastic material. To obtain the adequate phenomenological model the two authors assume that the elastic coefficients of the material are random functions of the place, the correlations of these functions are much smaller than their statistical mean values, and the space variations of the infinitesimal strain are slow, in relation to the fast fluctuations of the elastic coefficients. In the papers by Beran and McCoy (1970a, 1970b) the obtained phenomenological constitutive equation and equilibrium equation are not followed by the determination of the corresponding phenomenological boundary conditions which must be satisfied by the mean stress or by the mean displacement. However, I stress the fact that the authors discuss in great detail the formal analogies, as well as the essential differences existing between the obtained phenomenological model and the existing second order generalized elastic models, given for instance by Mindlin (1964) using only a phenomenological approach.

In this paper, using methods due to Mindlin (1964), I shall give the possible boundary conditions, adequate to the phenomenological model obtained by Beran and McCoy (1970a, 1970b) in the correlational approximation. Also, I shall establish some essential features of the obtained constitutive equation, following in an inevitable manner from the random characteristics of the the material, assumed by Beran and McCoy. As it will be seen, the involved strain energy density will not be a positive definite quadratic form of the strain and its second gradient. Hence uniqueness theorems of Kirchhoff's type can not be proved for various correctly formulated boundary value problems. Moreover, as I shall show by a simple example, a well posed traction boundary value problem, depending on the relation existing between the dimensions of the body and correlational radius of the material, can have a unique solution, can have an infinity of solutions, or the solution may not exist.

The results will be presented in a concentrated manner. All details are given in the paper by Soós (1984). I stress the fact that other interesting properties of the phenomenological model due to Beran and McCoy (1970a, 1970b) are given in Tucsnak's paper (1986).

2 The Phenomenological Model in the Correlational Approximation

Following Beran and McCoy (1970a, 1970b) I assume that the constitutive equation of the material is

$$T_{ij}(\mathbf{x}) = \lambda(\mathbf{x})\delta_{ij}E_{mm}(\mathbf{x}) + 2\mu(\mathbf{x})E_{ij}(\mathbf{x}) \tag{1}$$

where T_{ij} and E_{ij} are the Cartesian components of the stress and infinitesimal strain respectively, $\mathbf{x}$ is the position vector of a place, having Cartesian components x_1, x_2, x_3, and Lamé's coefficients $\lambda(\mathbf{x})$, $\mu(\mathbf{x})$ are *random* function of the place $\mathbf{x}$, and satisfy the usual restrictions concerning the positive definiteness of the involved strain energy density; δ_{ij} is the Kronecker delta, and all indices takes values $1, 2, 3$.

We assume a *statistically homogeneous and isotropic* material. In this case the statistical means $\langle\lambda(\mathbf{x})\rangle$, $\langle\mu(\mathbf{x})\rangle$ do not depend on $\mathbf{x}$, and will be designed by $\langle\lambda\rangle$ and $\langle\mu\rangle$, respectively. Also, the correlation functions $\langle\mu'(\mathbf{x})\mu'(\mathbf{y})\rangle$, $\langle\lambda'(\mathbf{x})\mu'(\mathbf{y})\rangle$, $\langle\mu'(\mathbf{x})\lambda'(\mathbf{y})\rangle$, $\langle\lambda'(\mathbf{x})\lambda'(\mathbf{y})\rangle$, depend on the places $\mathbf{x}$ and $\mathbf{y}$ only through the distance $r = \|\mathbf{y}-\mathbf{x}\|$ between these places. In the above relations $\lambda'(\mathbf{x}) = \lambda(\mathbf{x}) - \langle\lambda\rangle$ and $\mu'(\mathbf{x}) = \mu(\mathbf{x}) - \langle\mu\rangle$ are the fluctuations of the elastic coefficients. To designate the above correlation functions we use the notations:

$$\begin{aligned} &C_{\lambda\lambda}(r) = \langle\lambda'(\mathbf{x})\lambda'(\mathbf{x}+\mathbf{r})\rangle \qquad\qquad C_{\mu\mu}(r) = \langle\mu'(\mathbf{x})\mu'(\mathbf{x}+\mathbf{r})\rangle \\ &C_{\lambda\mu}(r) = \langle\lambda'(\mathbf{x})\mu'(\mathbf{x}+\mathbf{r})\rangle = \langle\mu'(\mathbf{x})\lambda'(\mathbf{x}+\mathbf{r})\rangle = C_{\mu\lambda}(r) \qquad \mathbf{r} = \mathbf{y} - \mathbf{x} \end{aligned} \tag{2}$$

Also, I denote by $t_{ij}(\mathbf{x}) = \langle T_{ij}(\mathbf{x})\rangle$, $\varepsilon_{ij}(\mathbf{x}) = \langle E_{ij}(\mathbf{x})\rangle$, the statistical means of the random functions $T_{ij}(\mathbf{x})$ and $E_{ij}(\mathbf{x})$, respectively. These mean values generally depend on the place $\mathbf{x}$. By $u_i(\mathbf{x}) = \langle U_i(\mathbf{x})\rangle$ I design the mean value of the random displacement $U_i(\mathbf{x})$.

Using the classical relation existing between $U_i(\mathbf{x})$ and $E_{ij}(\mathbf{x})$ it is easy to see that

$$2\varepsilon_{ij}(\mathbf{x}) = u_{i,j}(\mathbf{x}) + u_{j,i}(\mathbf{x}) \tag{3}$$

where, ",j" means partial derivative in relation to x_j.

I denote by

$$\chi_{ijk}(\mathbf{x}) = \chi_{ikj}(\mathbf{x}) = \varepsilon_{jk,i}(\mathbf{x}) \tag{4}$$

the components of the first gradient of $\varepsilon_{jk}(\mathbf{x})$.

Following Beran and McCoy (1970a, 1970b), I assume that the mean values $t_{ij}(\mathbf{x})$ and $\varepsilon_{ij}(\mathbf{x})$ have slow space variations. More exactly, the wave lengths characterizing these variations are much larger than the correlation radius of the random elastic coefficients $\lambda(\mathbf{x})$ and $\mu(\mathbf{x})$. Moreover, I suppose that these coefficients satisfy the following restrictions:

$$\langle\lambda'^2\rangle/\langle\lambda\rangle^2, \langle\mu'^2\rangle/\langle\mu\rangle^2, \langle\lambda'\mu'\rangle/\langle\lambda\mu\rangle << 1 \tag{5}$$

Taking into account the above assumptions, Beran and McCoy show that in the correlational approximation the phenomenological constitutive equations have the following form:

$$t_{ij} = \sigma_{ij} + \tau_{ij} \tag{6}$$

where

$$\sigma_{ij} = \lambda^*\varepsilon_{mm}\delta_{ij} + 2\mu^*\varepsilon_{ij} \tag{7}$$

and

$$\tau_{ij} = \tau_{ji} = -\mu_{mij,m} \tag{8}$$

with

$$\mu_{mij} = \mu_{mji} = -\alpha\chi_{mkk}\delta_{ij} - \beta\chi_{mij} - \gamma(\chi_{ijm} + \chi_{jim}) \tag{9}$$

The phenomenological material parameters λ^* and μ^* can be expressed through the statistical and mechanical characteristics of the material and are given by the following equations:

$$\begin{aligned} \lambda^* &= \langle\lambda\rangle - \frac{\langle\lambda'^2\rangle + \frac{4}{3}\langle\lambda'\mu'\rangle}{\langle\lambda + 2\mu\rangle} + \frac{4\langle\lambda + \mu\rangle\langle\mu'^2\rangle}{15\langle\mu\rangle\langle\lambda + 2\mu\rangle} \\ \mu^* &= \langle\mu\rangle - \frac{2\langle 3\lambda + 8\mu\rangle\langle\mu'^2\rangle}{15\langle\mu\rangle\langle\lambda + 2\mu\rangle} \end{aligned} \tag{10}$$

From the assumed properties of the elastic coefficients $\lambda(\mathbf{x})$, $\mu(\mathbf{x})$, and from the restrictions (5), it follows that

$$\mu^* > 0 \qquad\qquad 3\lambda^* + 2\mu^* > 0 \tag{11}$$

The phenomenological material parameters α, β, γ specific to the correlational approximation also can be expressed by the statistical and mechanical characteristics of the material, being given by the following relations:

$$\begin{aligned} \alpha &= \frac{2}{15\langle\lambda + 2\mu\rangle}k_{\lambda\mu} - \frac{4\langle\lambda + \mu\rangle}{105\langle\mu\rangle\langle\lambda + 2\mu\rangle}k_{\mu\mu} \\ \beta &= -\frac{2}{5\langle\lambda + 2\mu\rangle}k_{\lambda\mu} - \frac{28\langle\mu\rangle}{105\langle\mu\rangle\langle\lambda + 2\mu\rangle}k_{\mu\mu} \\ \gamma &= \frac{2}{5\langle\lambda + 2\mu\rangle}k_{\lambda\mu} + \frac{2\langle\lambda + 8\mu\rangle}{35\langle\mu\rangle\langle\lambda + 2\mu\rangle}k_{\mu\mu} \end{aligned} \tag{12}$$

where $k_{\lambda\mu}$ and $k_{\mu\mu}$ can be expressed by the correlation functions (2), using the equations:

$$k_{\lambda\mu} = \int_0^\infty rC_{\lambda\mu}(r)\,dr \qquad\qquad k_{\mu\mu} = \int_0^\infty rC_{\mu\mu}(r)\,dr \tag{13}$$

Using the equations (6) and (8) we get

$$t_{ij} = \sigma_{ij} - \mu_{mij,m} \tag{14}$$

Now we can conclude that the constitutive equation of the phenomenological model corresponding to the correlational approximation and obtained by Beran and McCoy, formally coincides with a special variant of the constitutive equation of a second order elastic material, obtained by Mindlin (1964) using a pure phenomenological approach.

The phenomenological model obtained by Beran and McCoy is characterized by 5 material parameters λ^*, μ^*, α, β, γ which are *completely* determined by the statistical and mechanical characteristics of the statistically homogeneous and isotropic elastic material, taken into account.

Ending this section, I observe, that as Beran and McCoy show, the mean stress $t_{ij}(\mathbf{x})$ satisfies Cauchy's equilibrium equation

$$t_{ij,j} + f_i = 0 \tag{15}$$

where $f_i(\mathbf{x})$ are the components of the body force, assumed to be usual deterministic functions.

3 Possible Boundary Value Problems

In order to get the adequate boundary conditions corresponding to Beran and McCoy's phenomenological model, I introduce the strain energy density

$$2w = \sigma_{ij}\varepsilon_{ij} + \mu_{ijk}\chi_{ijk} \tag{16}$$

involved in the model. Starting from the equilibrium equation (15) and using the techniques taken into account by Mindlin (1964) it can be shown that the following *work theorem* is true:

$$2\int_V w\, dv = \int_V f_j u_j\, dv + \int_S T_j u_j\, da + \int_S H_j u_{j,n}\, da \tag{17}$$

where V is the domain occupied by the body, and S is the boundary of V.

Also on S we have:

$$T_j = t_{jk}n_k + (b_{ik} - b_{ll}n_i n_k)\mu_{ijk} - n_i\mu_{ijk,k} + n_i n_k n_l \mu_{ijk,l} \tag{18}$$

$$H_j = n_i \mu_{ijk} n_k \tag{19}$$

In the above relations n_j are the Cartesian components of the external unit normal to S, b_{ij} are the space components of the curvature tensor corresponding to S, and $u_{j,n}$ is the normal derivative of u_j, on the boundary S.

From equation (17) it follows that T_j are the Cartesian components of the traction $\mathbf{T}$ acting on the boundary S, and H_j are the Cartesian components of the hyper-stress $\mathbf{H}$, acting on the same boundary.

Also, the work relation (17) shows the possible boundary value problems which can be formulated in a rational manner in the frame-work of Beran's and McCoy's phenomenological model, and for which we can hope the uniqueness of the solution of the considered problems. For instance, on S we can prescribe the traction $\mathbf{T}$ and the hyper-stress $\mathbf{H}$ (Neumann's problem), or the displacement $\mathbf{u}$ and its normal derivative $\mathbf{u}_{,n}$ (Dirichlet's problem). However, in the first case $\mathbf{T}$ and $\mathbf{H}$ can not be given in an arbitrary manner, since the following global equilibrium conditions must be satisfied:

$$\int_V \mathbf{f}\, dv + \int_S \mathbf{T}\, da = 0, \quad \int_V \mathbf{x} \times \mathbf{f}\, dv + \int_S \mathbf{n} \times \mathbf{T}\, da + \int_S \mathbf{n} \times \mathbf{H}\, da = 0 \tag{20}$$

The above restrictions follows from the invariance of the strain energy density w relative to a rigid motion of the body.

I observe that the above global equilibrium conditions do not impose any restriction on the normal component of the hyper-stress $\mathbf{H}$.

Also, from equations (7), (9) and (16) we get the following expression of the strain energy density w, as function of ε_{ij} and χ_{ijk}:

$$2w = \lambda^*\varepsilon_{ii}\varepsilon_{jj} + 2\mu^*\varepsilon_{ij}\varepsilon_{ij} - \alpha\chi_{ijj}\chi_{ikk} - \beta\chi_{kij}\chi_{kij} - 2\gamma\chi_{kij}\chi_{ijk} \tag{21}$$

As it is well known, the work relation (17) is used to prove Kirchhoff's type uniqueness theorems for well formulated boundary value problems. To prove theorems of this type it is assumed the positive-definiteness of the involved strain energy density w , given in our case by equation (21). As it is easy to see, using Sylvester's criterion, that w is positive definite if and only if the phenomenological material parameters λ^*, μ^*, α, β and γ satisfy the following restrictions:

$$\mu^* > 0, \quad 3\lambda^* + 2\mu^* > 0 \tag{22}$$

$$\alpha > 0, \quad \beta - \gamma < 0, \quad \beta + 2\gamma < 0, \quad \alpha + \beta + 2\gamma < 0, \quad 4\alpha + 3\beta - 2\gamma < 0 \tag{23}$$

For later use I observe that from (23) it follows that β and γ satisfy the inequality

$$\beta + \gamma < 0 \tag{24}$$

if w is positive definite.

In theories using a pure phenomenological approach, for instance in Mindlin's theory (Mindlin, 1964), the restrictions of the type (22) and (23) are *imposed*, just to guarantee the uniqueness of the solution of a well formulated boundary value problem. In the framework of Beran's and McCoy's phenomenological model restrictions of this type *can not be imposed*, since the involved phenomenological parameters can be calculated using the given statistical and mechanical characteristics of the body. Hence, all what we can do is to see if the restrictions (22) and (23) are satisfied or are not satisfied. I shall realize this step in the next section, assuming a composite having two phases, represented by two homogeneous and isotropic elastic materials. To do this I shall assume that the ergodicity property takes place, hence statistical mean values can be replaced by mean values determined using representative volume elements. As is argued by Beran and McCoy, this replacement can be done in any situation in which the correlation radius of the material is much smaller than the geometrical dimensions of the body taken into account.

Ending this section and for later use, I observe that taking into account the equilibrium equation (15) and the constitutive equations (7), (9) we can conclude that in the framework of the phenomenological model given by Beran and McCoy in the correlational approximation, the mean displacement $\mathbf{u}$ satisfies the following Lamé's type fourth order differential equation:

$$(\lambda^* + 2\mu^*)(1 + a\Delta)\operatorname{grad}\operatorname{div}\mathbf{u} + \mu^*(1 + b\Delta)\operatorname{rot}\operatorname{rot}\mathbf{u} + \mathbf{f} = \mathbf{0} \tag{25}$$

where the parameters a and b, having the physical dimension of the square of length, have the following expressions:

$$a = \frac{\alpha + \beta + \gamma}{\lambda^* + 2\mu^*} \qquad b = \frac{\beta + \gamma}{2\mu^*} \tag{26}$$

Using equation (12) a and b can be expressed also by the relations:

$$(\lambda^* + 2\mu^*)a = \frac{8}{15\langle\lambda + 2\mu\rangle}k_{\lambda\mu} + \frac{8\langle\lambda + 2\mu\rangle}{105\langle\mu\rangle\langle\lambda + 2\mu\rangle}k_{\mu\mu} \tag{27}$$

$$\mu^* b = \frac{\langle 3\lambda + 10\mu\rangle}{105\langle\mu\rangle\langle\lambda + 2\mu\rangle}k_{\mu\mu} \tag{28}$$

4 Composite with Two Phases

I assume a composite formed by two homogeneous and isotropic elastic phases, having Lamé's constants λ_1, μ_1 and λ_2, μ_2, respectively, and concentrations c_1, c_2, respectively. Using results given by Shermergor (1977) and taking into account the ergodicity property, for the mean values and correlation functions we get:

$$\langle\lambda\rangle = c_1\lambda_1 + c_2\lambda_2 \qquad \langle\mu\rangle = c_1\mu_1 + c_2\mu_2 \tag{29}$$

$$c_{\lambda\lambda}(r) = c_1c_2(\lambda_1 - \lambda_2)^2\varphi(r) \qquad c_{\mu\mu}(r) = c_1c_2(\mu_1 - \mu_2)^2\varphi(r) \tag{30}$$

$$c_{\lambda\mu}(r) = c_{\mu\lambda}(r) = c_1c_2(\lambda_1 - \lambda_2)(\mu_1 - \mu_2)\varphi(r) \tag{31}$$

where $\varphi(r)$, characterizing completely the correlation degree of the phases, is a non-negative function, satisfying the condition $\varphi(0) = 1$. Since the phase correlation decreases rapidly with the distance r we can take

$$\varphi(r) = \exp(-r/d) \tag{32}$$

d representing the correlation radius of the diphasic composite. According to the assumptions made, d is much smaller than the geometrical dimensions of the body.

Using equations (13) and (32) we get

$$k_{\lambda\mu} = c_1 c_2(\lambda_1 - \lambda_2)(\mu_1 - \mu_2)d^2 \qquad k_{\mu\mu} = c_1 c_2(\mu_1 - \mu_2)^2 d^2 \tag{33}$$

From the second equation we can see that $k_{\mu\mu}$ *can not be negative*; i.e.

$$k_{\mu\mu} > 0 \tag{34}$$

Consequently, from equations (11) and (28) it results that the parameter b *can not have negative values*; i.e.

$$b > 0 \tag{35}$$

In this way, analyzing the equation $(26)_2$ we can conclude that the parameters β andγ satisfy in Beran's and McCoy's phenomenological model the restriction

$$\beta + \gamma > 0 \tag{36}$$

Comparing (24) and (34) we can see that *the strain energy density* w, involved in the analyzed phenomenological model, corresponding to the correlational approximation, is *not positive definite.* Hence, the uniqueness of the solution of various boundary value problems, corresponding to the Beran's and McCoy's type phenomenological model is not guaranteed. Moreover, as we shall see in the next section, situations may occur in which the solution of various boundary value problems does not exist, which can be formulated in a rational manner in the framework of the phenomenological model.

5 Example: A Traction Problem for a Strip

I assume that the material occupies the domain

$$-L \leqslant x_1 \leqslant L \qquad -\infty < x_2, x_3 < \infty \tag{37}$$

and the body force is zero.

Also I suppose the following boundary conditions:

$$T_1 = T \quad T_2 = T_3 = 0 \quad H_1 = H \quad H_2 = H_3 = 0 \quad \text{for} \quad x_3 = L \tag{38}$$

$$T_2 = -T \quad T_2 = T_3 = 0 \quad H_1 = \bar{H} \quad H_2 = H_3 = 0 \quad \text{for} \quad x_3 = -L \tag{39}$$

Here L, T, H and $\bar{H}$ are given constants.

I observe that the considered traction boundary conditions satisfy the global equilibrium conditions (20). From symmetry considerations it follows that the solution has the following structure:

$$u_1 = u(x) \quad x = x_1 \quad u_2 = u_3 = 0 \quad \text{for} \quad -L \leqslant x \leqslant L \quad -\infty < x_2, x_3 < \infty \tag{40}$$

From Lamé's type equilibrium equation (25) it result that the unknown function $u = u(x)$ must satisfy the differential equation

$$\left(1 + a\frac{\mathrm{d}^2}{\mathrm{d}x^2}\right)\frac{\mathrm{d}^2 u}{\mathrm{d}x^2} = 0 \tag{41}$$

Let us assume now that $\lambda_1 < \lambda_2$, $\mu_1 < \mu_2$ or $\lambda_1 > \lambda_2$, $\mu_1 > \mu_2$, a rational hypothesis for our elastic phases. In this case, from the first relation (33) it results that

$$k_{\lambda\mu} > 0 \tag{42}$$

Consequently, from (22), (27) and (34) we get

$$a > 0 \tag{43}$$

Hence, we can put

$$a = l^2 \tag{44}$$

where l is a positive *real* number, having as physical dimension the dimension of a *length*. Obviously, l characterizes an *internal length* of the considered composite; this length is obviously the *correlation radius* of the material taken into account.

Now it is easy to see that the general solution of the equation (41) is

$$u = u(x) = M \cos(x/l) + N \sin(x/l) + Px + Q \tag{45}$$

where M, N, P and Q are arbitrary real constants, which must be determined using the assumed boundary conditions. For the involved stress and hyper-stress components we get the following expressions:

$$t_{11} = (\lambda^* + 2\mu^*)(u' + l^2 u''') \qquad \mu_{111} = -(\lambda^* + 2\mu^*)l^2 u'' \tag{46}$$

In this way the boundary conditions $(39)_{6,10}$ take the following form:

$$M \cos(L/l) + N \sin(L/l) = (\lambda^* + 2\mu^*)^{-1} H \qquad M \cos(L/l) - N \sin(L/l) = (\lambda^* + 2\mu^*)^{-1} \bar{H} \tag{47}$$

Now it is clear that the existence, uniqueness, non-existence or non-uniqueness of the solution depend on the relation existing between L, the geometrical characteristic of the dimensions of the body, and l, the internal geometrical characteristic of the material.

Case 1: If

$$L/l \neq k\pi/2 \qquad k = 1, 2, 3, \dots \tag{48}$$

the solution exists, is unique and has the form

$$(\lambda^* + 2\mu^*)u(x) = \frac{H + \bar{H}}{2} \frac{\cos(x/l)}{\cos(L/l)} + \frac{H - \bar{H}}{2} \frac{\sin(x/l)}{\sin(L/l)} + Tx \tag{49}$$

Case 2: If

$$L/l = \pi/2 + k\pi \qquad k = 0, 1, 2, 3, \dots \tag{50}$$

the solution exists if and only if

$$\bar{H} = -H \tag{51}$$

If the above restriction is fulfilled, the solution is not unique and we have

$$(\lambda^* + 2\mu^*)u(x) = M \cos(x/l) + (-1)^k H \sin(x/l) + Tx \tag{52}$$

where M is an *arbitrary* real constant.

Case 3: If

$$L/l = k\pi \qquad k = 1, 2, 3, \dots \tag{53}$$

our problem has a solution if and only if

$$\bar{H} = H \tag{54}$$

In such a situation we get

$$(\lambda^* + 2\mu^*)u(x) = (-1)^k H \cos(x/l) + N \sin(x/l) + +Tx \tag{55}$$

N being an *arbitrary* real constant.

Ending, I observe that the mechanical significance of the restrictions (48) (50), (51) and (53), (54) is not clear, since the assumptions leading to the analyzed phenomenological model do not exclude the possibility of the realization of the restrictions (50) or (54), for sufficient high values of k, even if according to the adopted hypothesis $l << L$.

Consequently, as a result of the analysis made the following question arises: In which circumstances can Beran's and McCoy's phenomenological model be used, corresponding to the correlational approximation, in order to describe the real behaviour of an elastic composite having random mechanical and geometrical properties? As I can see, the answer to this question is not known at this moment.

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