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The Inversion System Control of Chaotic Oscillations

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The inversion control of chaotic oscillators is investigated in this paper. The inversion system control law is proposed for single-degree-of-freedom chaotic oscillations. A mathematical model derived from spacecraft attitude dynamics is treated as an example to demonstrate its application. Finally the control law is extended to multi-degree-of-freedom oscillation systems.

1 Introduction

Much attention has been paid to controlling chaos because of its theoretical importance and possible applications (Kapitaniak, 1996) , (Chen and Dong, 1997). Developments and applications of nonlinear system theory to control chaos is one of the main aspects of research on controlling chaos (Chen and Dong, 1997). The inversion system method is a recently developed direct method that designs nonlinear control systems by linearization via feedback based on the concept of inversion dynamical systems (Fliess, 1984, Li and Feng, 1991). It has a definite physical interpretation, and is easy to adopt. For discrete time-variable systems, it is shown that the inversion system method can be modified to control chaos (Chen and Ge, 1997). Here a modification of the method is presented for nonlinear oscillation systems. The expected closed-loop equation governing dynamical behavior of the system, which is needed in establishing the inversion system control law, is altered, so that the method can be employed to control chaos in single-degree-of-freedom oscillation system. A chaotic oscillation system derived from spacecraft attitude dynamics (Chen and Liu, 1998) is treated as an example to demonstrate the application of the modified inversion system control law.

2 The Inversion System Control Law for Nonlinear Oscillations

Consider a single-degree-of-freedom nonlinear oscillator with a control parameter

$$\ddot{y} = f(y, \dot{y}, t, u) \tag{1}$$

where y, $\dot{y}$ and $\ddot{y}$ are the generalized coordinate, velocity and acceleration respectively, t is the time variable, and u is a control parameter. The system has chaotic motion if no control is applied $(u = 0)$. Assume that the system satisfies the invertibility condition, that is, $f'_u \neq 0$. Then solving from equation (1), one obtains the explicit expression of u

$$u = f^{-1}(y, \dot{y}, t, \ddot{y}) \tag{2}$$

For a given tracking goal $r(t)$, the existing inversion system method introduces an expected closed-loop equation governing the input-output dynamical behavior (Li and Feng,1991).

$$\ddot{y} + \alpha\dot{y} + \beta y = r(t) \tag{3}$$

Thus the corresponding inversion system control law is

$$u = f^{-1}(y, \dot{y}, t, r(t) - \alpha\dot{y} - \beta y) \tag{4}$$

For chaotic systems with drastic and irregular change, numerical experiments indicate that the inversion system controller designed based on the expected closed-loop equation (3) cannot achieve the desired results. Hence the expected closed-loop equation is modified as

$$\ddot{y} + \alpha\dot{y} + \beta y = \ddot{r}(t) + \alpha\dot{r}(t) + \beta r(t) \tag{5}$$

Now based on the expected closed-loop equation (5), the nonlinear state-feedback control law derived from the inversion system method is

$$u = f^{-1}\big(y,\ \dot{y},\ t,\ \ddot{r}(t) - \alpha(\dot{y} - \dot{r}(t)) - \beta(y - r(t))\big) \tag{6}$$

where coefficients α and β can be determined by normal design principles such as pole placement, linear-quadratic optimal regulator, or robust service regulator.

3 Controlling Chaotic Attitude Motion of Spacecraft

Consider a spacecraft on an elliptic orbit in the gravitational field with air drag and internal damping. There may occur chaotic attitude motion. An arbitrarily shaped spacecraft, whose principal inertia moments are A, B and C, moves in an elliptic orbit with one principal axis z normal to the orbital plane XY. Without loss of generality, suppose that $B > A$. The spacecraft has an actuator that can provide the control torque u. Denote φ as the libration angle in the orbital plane as measured from the local vertical, and t as position angle of the spacecraft in its orbit as measured from perifocus. Assume that the internal damping and the atmosphere resistance are proportional to the angular velocity and the square of the angular velocity respectively, whose coefficients are γ and c.

Establishing the dynamical equation and changing the independent variable lead to (Chen and Liu,1998)

$$\ddot{\varphi} - \frac{2e\sin t(1+\dot{\varphi})}{1+e\cos t} + \frac{K\sin 2\varphi}{1+e\cos t} + c\dot{\varphi}^2 + \frac{\gamma\dot{\varphi}}{(1+e\cos t)^2} = \frac{u}{C(1+e\cos t)} \tag{7}$$

where
$$K = \frac{3(B-A)}{2C} \tag{8}$$

Both analytical and numerical study has shown that chaotic motion may appear in equation (7) if $u = 0$. The above inversion system control law is applied to control it.

Comparing equation (1) with equation (7), one gets

$$f(\varphi,\ \dot{\varphi},\ t,\ u) = \frac{2e\sin t(1+\dot{\varphi})}{1+e\cos t} - \frac{K\sin 2\varphi}{1+e\cos t} - c\dot{\varphi}^2 - \frac{\gamma\dot{\varphi}}{(1+e\cos t)^2} + \frac{u}{C(1+e\cos t)} \tag{9}$$

which satisfies the invertibility condition

$$f_u' = \frac{1}{C(1+e\cos t)} \neq 0 \tag{10}$$

The inversion of system (9) is

$$\begin{aligned} f^{-1}(\varphi,\dot{\varphi},t,\ddot{\varphi}) = {} & C(1+e\cos t)\ddot{\varphi} - 2eC\sin t(1+\dot{\varphi}) + KC\sin 2\varphi + \\ & cC(1+e\cos t)\dot{\varphi}^2 + \frac{\gamma C\dot{\varphi}}{1+e\cos t} \end{aligned} \tag{11}$$

Let $K = 0.75$, $\gamma = 0.05$, $c = 0.04$, $e = 0.14$, $C = 1.0$ in equation (7). In this case, system (7) without control leads to chaotic behavior (Chen and Liu, 1998). The control goals successively are taken as a fixed point and a period 2 motion

$$r_1(t) = 0 \tag{12}$$

$$r_2(t) = \sin 0.5t \tag{13}$$

Coefficients α and β in the controller are determined by the pole placement. Let $\alpha = 2.8$, $\beta = 4.0$. Activate control when $t_0 = 80.0$. The control results are respectively shown in Figures 1 and 2. The corresponding control signals $u = u(t)$ are respectively shown in Figures 3 and 4.

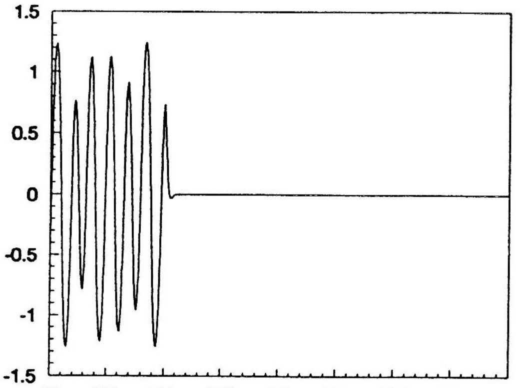

Figure 1. Control of System (8) to Goal (12)

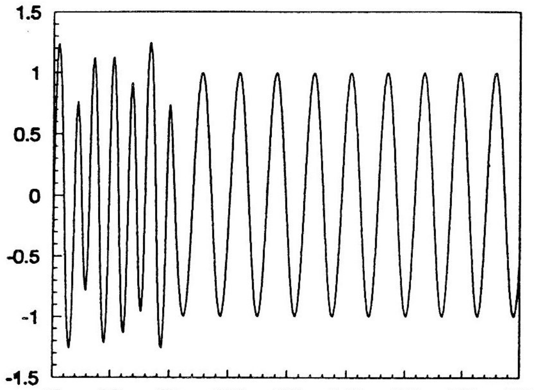

Figure 2. Control of System (8) to Goal (13)

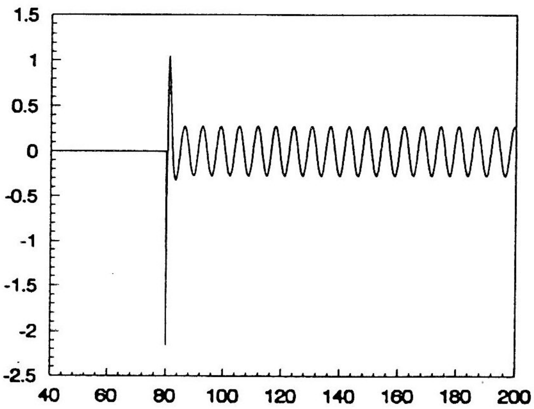

Figure 3. The Control Signal for Controlling System (8) to Goal (12)

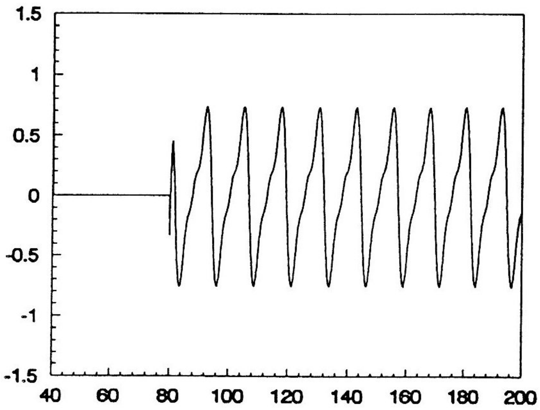

Figure 4. The Control Signal for Controlling System (8) to Goal (13)

4 Extension to Multi-Degree-of-Freedom Oscillation Systems

Let us study a controllable inertial uncoupled nonlinear oscillation system with n degrees of freedom,

$$\ddot{\mathbf{q}} = \mathbf{f}(\mathbf{q}, \dot{\mathbf{q}}, t, \mathbf{u}) \tag{14}$$

where n dimension vectors $\mathbf{q}$, $\dot{\mathbf{q}}$ and $\ddot{\mathbf{q}}$ are the generalized coordinates, velocities and accelerations respectively, t is the time variable, and the n-dimension vector $\mathbf{u}$ is a control parameter. The invertibility condition of the system is that the Jacobian of $\mathbf{f}$ with respect to $\mathbf{u}$ is nonsingular. Hence one can explicitly solve from equation (14)

$$\mathbf{u} = \mathbf{f}^{-1}(\mathbf{q}, \dot{\mathbf{q}}, t, \ddot{\mathbf{q}}) \tag{15}$$

Given a periodic control goal $\mathbf{r}(t)$. Suppose that the input-output dynamical behavior satisfies the expected closed-loop equation

$$\ddot{\mathbf{q}} + \boldsymbol{\alpha}\dot{\mathbf{q}} + \boldsymbol{\beta}\mathbf{q} = \ddot{\mathbf{r}} + \boldsymbol{\alpha}\dot{\mathbf{r}} + \boldsymbol{\beta}\mathbf{r} \tag{16}$$

where the $n \times n$ matrices $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ are diagonal

$$\boldsymbol{\alpha} = \mathrm{diag}[\alpha_1, \alpha_2, \ldots, \alpha_n] \qquad \boldsymbol{\beta} = \mathrm{diag}[\beta_1, \beta_2, \ldots, \beta_n] \tag{17}$$

Therefore the corresponding inversion system control law is

$$\mathbf{u} = \mathbf{f}^{-1}(\mathbf{q}, \dot{\mathbf{q}}, t, \ddot{\mathbf{r}} - \boldsymbol{\alpha}(\dot{\mathbf{q}} - \dot{\mathbf{r}}) - \boldsymbol{\beta}(\mathbf{q} - \mathbf{r})) \tag{18}$$

where coefficient α_i and $\beta_i (i = 1, 2, \ldots, n)$ can be determined by normal design principles such as pole placement, linear-quadratic optimal regulator, or robust service regulator.

5 Conclusions

In this paper, the inversion system method is modified to control chaos in nonlinear oscillations. The method is applied to control the chaotic planar attitude motion of a kind of rigid spacecraft on an elliptic orbit in the gravitational field with air drag and internal damping. The method is extended to multi-degree-of-freedom oscillation systems.

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