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The Prediction of Guaranteed Life Characteristics due to Fatigue Failures with Incomplete Information about Random Loading

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The guaranteed life prediction problem for structural elements is investigated. Random external loading and fatigue failures are taken into account. Firstly, the mean life definition problem is solved. Secondly, the turbine blades guaranteed life prediction with incomplete information about random loads is considered.

1 Introduction

In solving the problems of reliability theory at the design stage there is often the situation when the necessary statistical information about the vector of external load is lacking. In this case the reliability prediction has to be realized under conditions of statistical indeterminacy.

2 Formulation of the Problem

Let us consider a case, when the initial information is incomplete regarding the vector of external load, which is the vector of broadband random processes. As a lot of machine-building structural members have pronounced filtering properties, the power spectral density (PSD) $\mathbf{S}_x(\omega)$ of the external load vector $\mathbf{X}(t)$ can be considered as constant within the limits of narrow frequency ranges $|\omega-\omega_i|\le\Delta_i$ corresponding to the resonance frequencies. It is supposed, that experimentally it's possible to define the boundaries of modification of the PSD $\mathbf{S}_x(\omega)$ and variances $\boldsymbol{\sigma}_x^2$ of the external load vector $\mathbf{X}(t)$. Let us specify the set $\mathbf{M}_x$ of the piecewise constant PSDs of the vector $\mathbf{X}(t)$:

$$\mathbf{M}_x=\left[\begin{array}{l}\mathbf{S}_x(\omega)=\begin{cases}\mathbf{S}_i=const, |\omega-\omega_i|\le\Delta_i;\\ \gamma_{xi}<\mathbf{S}_i<\mathbf{\Gamma}_{xi}, (i=\overline{1,n});\\ 0, |\omega-\omega_i|>\Delta_i;\end{cases}\\ \boldsymbol{\sigma}_x{}^2=\int\limits_0^\infty \mathbf{S}_x(\omega)d\omega<\mathbf{\Gamma}_\sigma;\end{array}\right] \tag{1}$$

where γ_{xi}, $\mathbf{\Gamma}_{xi}$ are accordingly lower and upper vector change restrictions of the vector $\mathbf{S}_x(\omega)$ at the natural frequency ω_i; $\mathbf{\Gamma}_\sigma$ is the vector change boundary of the variances vector $\boldsymbol{\sigma}_x^2$. The components of the external load vector are assumed to be mutually independent.

The reliability prediction problem in the case of incomplete information about the external load vector may be turned into determination of the lower or guaranteed evaluation of one of the system reliability parameters for the most unfavorable possible values of the PSDs vector $\mathbf{S}_x(\omega)$ taken from the set $\mathbf{M}_x$. Where the mean life m_T is involved it is necessary to define the lower evaluation of the life according to the set $\mathbf{M}_x$, i.e. the guaranteed life m_{TH}:

$$m_{TH}=\min_{\mathbf{S}_x(\omega)\in\mathbf{M}_x} m_T \tag{2}$$

The stated problem is a problem of the target function optimization (2) with the parameters variation of the vector $\mathbf{S}_x(\omega)$ having the restrictions in the form (1).

3 The Mean Life Determination

As the first step of the mean life prediction the stochastic dynamics problem is considered. The connection between the external load vector $\mathbf{X}(t)$ and the vector $\mathbf{Y}(t)$ of stress-strain state parameters may be presented in the operating form

$$L\mathbf{Y}(t) = \mathbf{X}(t) \tag{3}$$

where L is a linear determined operator.

For broadband random loading and low damping the components of the vector $\mathbf{y}(t)$, representing stress-strain state parameters are the superposition of the narrow-band random processes $\mathbf{y}_k(t)$ with the fundamental frequencies ω_k

$$\mathbf{y}(t) = \sum_{k=1}^{n} \mathbf{y}_k(t) = \sum_{k=1}^{n} \mathbf{y}_{ak}(t)\cos[\omega_k t + \varphi_k(t)] \tag{4}$$

Here $\mathbf{y}_{ak}(t)$ is the envelope (amplitude) of the narrow-band random process $\mathbf{y}_k(t)$; ω_k is the system vibration natural frequency; $\varphi_k(t)$ is the vibration phase, uniformly distributed over the interval $[0,2\pi]$.

Further the average time before fracture due to fatigue damage accumulation is determined. For this, on the basis of the random processes schematization method the broadband process $\mathbf{y}(t)$ is adjusted to the narrow-band process $y_e(t)$ having the equivalent damaging effect

$$y_e(t) = y_{ae}(t)\cos[\omega_e t + \varphi_e(t)] \tag{5}$$

Let us use the one-dimensional schematization of random processes according to the methods of peaks or complete cycles. The analytical expressions for one-dimensional probability density of the equivalent amplitudes $y_{ae}(t)$ of the Gaussian random process are obtained earlier (Gusev, 1984; Kogaev, 1977). The equivalent process frequency ω_e in the first method is equated to the number of peaks - in the second method to the number of zeros of the initial broadband process.

For the linear hypothesis of damage accumulation and degree approximation of the fatigue curve the fatigue damages measure $Z(t)$, accumulated at a certain point, is described by means of the kinetic equation of the following kind (see Gusev, 1984):

$$\frac{dZ(t)}{dt} = \frac{\omega_e y_{ae}^m(t)}{2\pi N_0 \sigma_{-1}^m} \tag{6}$$

Here N_0, m, σ_{-1} are the parameters of the fatigue curve. To define the damage measure at the time t let us integrate the equation (6) within the limits $[0,t]$ provided that $Z(0) = 0$. Averaging the obtained equation and equating the damage measure to one, we shall get the formula for the definition of the mean life m_T

$$m_T^{-1} = \frac{\omega_e}{2\pi N_0 \sigma_{-1}^m} \int_{\sigma_{-1}}^{\infty} y_{ae}^m f(y_{ae}) dy_{ae} \tag{7}$$

4 The Guaranteed Life Prediction

Let us consider the solution of the given problem as applied to the blades of axial turbines. The dynamic loads, acting on the blades of turbines and compressors, are considered as a space - time random field, allowing to take into account all main factors of the nonuniformity and randomness of an aerodynamic flow. Mutual independence of circular and radial nonuniformity is supposed that allows to present the load field as the product of the determined load distribution function along the blade $R(r)$ and the field $\Phi(t,\varphi)$, describing the circular flow nonuniformity as well as the random pulses of impact pressure before the stage under study.

$$Q(r,\varphi,t) = R(r)\Phi(t,\varphi+\Omega t) \qquad (8)$$

where Ω is the angular velocity of the turbine wheel.

The pressure field $\Phi(t,\varphi)$ can be expanded into a complex Fourier series in terms of the variable φ

$$\Phi(t,\varphi+\Omega t) = \sum_{k=-\infty}^{\infty} C_k(t)\exp(ik(\varphi+\Omega t)) \qquad (9)$$

Here $C_k(t)$ are the random broadband processes with the PSDs $S_{c_k}(\omega)$. As a result of the solution of the problem on random vibrations the loads, represented as equation (9) allow to obtain the stress spectrum, qualitatively representing the amplitude-frequency response, i.e. having pronounced peaks at natural frequencies. In the general case the functions $C_k(t)$ are complex and the correlation between the real and imaginary parts of the process $C_k(t)$ is supposed to be unvariable. Thus, the correlation function and the PSD are real. Let us introduce the random process $\xi_k(t) = C_k(t)\exp(ik\Omega t)$. The PSD $S_{\xi_k}(\omega)$ of this process will be identical to $S_{c_k}(\omega)$ but with the ω-shift by the amount $k\Omega$. These with the load field may be presented as

$$Q(r,\varphi,t) = R(r)\sum_{k=-\infty}^{\infty}\xi_k(t)\exp(ik\varphi) \qquad (10)$$

It is supposed that the determined distribution of the external load along the blade $R(r)$ is given; the PSDs $S_{\xi_k}(\omega_i)$ of the broadband processes $\xi_k(t)$, ($k=\overline{1,n}$) at the natural frequencies ω_i, ($i=\overline{1,m}$) is constant within the limits of the resonance peaks of the frequency response; the boundary changes of the PSDs of processes $\xi_k(t)$ at natural frequencies and the upper boundary of the external load variance are known. Further, according to equation (1) the set $\mathbf{M}_x$ with power spectral densities of broadband processes $\xi_k(t)$ as components of a vector $\mathbf{S}_i$ is formed and the presented problem is reduced to the determination of the guaranteed (lower) evaluation of the blade mean life m_{TH} for the most unfavorable variant of external load for the described mathematical model.

This optimization problem can be solved in terms of stresses. The submission of loads as equation (10) and the assumption that the processes $\xi_k(t)$ are stationary allow to use spectral methods of analysis in order to solve the problem of statistical dynamics. According to the method of spectral presentations (Bolotin, 1984) the PSD of the stresses $S_y(r,\omega)$ in the given blade section is determined by the following formula

$$S_y(r,\omega) = \sum_{k=-\infty}^{\infty} S_{\xi_k}(\omega)H_k^2(r,\omega) \qquad (11)$$

The frequency response of the system $H_k(r,\omega)$ is obtained from the solution of the problem on the blade forced determined vibrations. In order to solve the determined problem we can use the design procedure for the blade forced vibrations (Petrov, 1981) according to which the turbine blades are considered as naturally twisted rods of variable non-symmetric section and with elastic restraint in the disc. This design is based on the initial parameters method in matrix form combined with the discrete models of blades. The discrete model is a system of concentrated masses connected by inertialess elastic sections. Applying the general scheme of the initial parameters method, we obtain the vibrations column vector in each blade cross-section. This vector of parameters consists of ten components: axial and tangential displacements; three turn angles, two transversal forces, two bending moments and a torque moment. These parameters allow to calculate stresses and displacements in any blade cross-section.

The stress presentation as equation (4) allows to define the stress variances at the i-th natural frequency from k-th load component at a given blade cross-section

$$\sigma_{y_{ki}}^2(r) = S_{\xi_k}\int_{\omega_i-\Delta_i}^{\omega_i+\Delta_i} H_k^2(r,\omega)d\omega = S_{\xi_k}(\omega)\alpha_{ki}(r) \qquad (12)$$

where

$$\alpha_{ki}(r) = \int_{\omega_i-\Delta_i}^{\omega_i+\Delta_i} H_k^2(r,\omega)d\omega \tag{13}$$

The restrictions $\gamma_{y_{ki}}$ and $\Gamma_{y_{ki}}$ of varied parameters $\sigma^2_{y_{ki}}(r)$ are connected with the boundaries S_{ξ_k} as

$$\begin{aligned} \gamma_{y_{ki}} &= \gamma_{\xi_{ki}}\alpha_{ki} \\ \Gamma_{y_{ki}} &= \Gamma_{\xi_{ki}}\alpha_{ki} \end{aligned} \tag{14}$$

Consequently the total stress variance may be presented as

$$\sigma_y^2(r) = \sum_k\sum_i S_{\xi_k}(\omega_i)\alpha_{ki}(r) \tag{15}$$

The set $\mathbf{M}_y$ is given similarly equation (1). In this case the components of the vector $\mathbf{S}_i$ are the stresses variances $\sigma^2_{y_{ki}}$. The restrictions $\gamma_{y_{ki}}$, $\Gamma_{y_{ki}}$ may be obtained from equations (13), (14) if the appropriate parameters for S_{ξ_k} are known. Otherwise, they can be obtained on the basis of experimental measurements of the working blades vibration characteristics.

5 Numerical Investigation

On the basis of the described approach the GTE-45-3 axial compressor 2nd step working blades mean life account was performed. The optimization problem was solved in terms of stresses, i.e. the stress variances of endangered blade section were used as variable parameters. A search for the guaranteed life and accordingly for the worst relation between stresses variances was carried out at the first four natural frequencies (ω_1 = 165Hz, ω_2 = 520 Hz, ω_3 = 1310 Hz, ω_4 =1515 Hz). The variable parameters restrictions were changed within the following limits γ_{y_i} = 0.5 to 10 (MPa)^2·s, Γ_{y_i} = 10 to 60 (MPa)^2·s. The optimization results for different varied parameters restrictions are given in Figures 1 to 4. Where S_y^* - is the most critical PSD of the stresses at prescribed restrictions, m_{TH}/T_1 - is the guaranteed life value to fundamental frequency period value relation. The «black» columns means the real calculation results of PSD peaks. The section-line columns determine the restrictions of varied parameters. The broadwidth of columns is defined by filtering properties at the every natural frequency. Numerical analyses of the guaranteed life allow to draw the following conclusion: The most critical PSD is directly related to restrictions values and to the natural frequency spectrum.

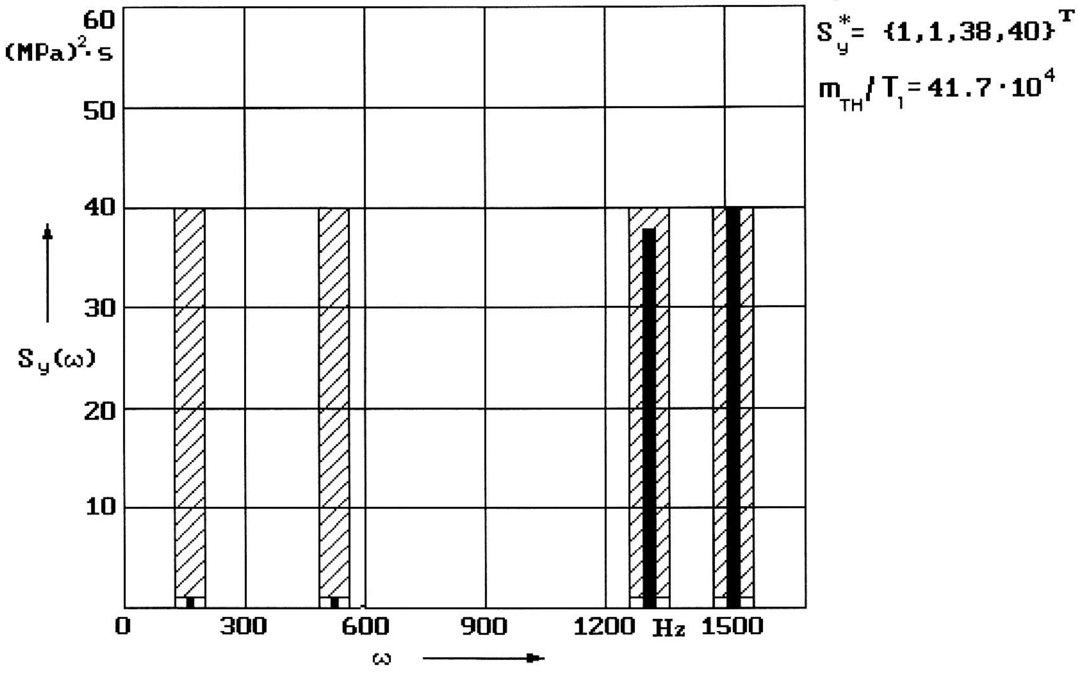

Figure 1. The most critical PSD at prescribed restrictions:
$\gamma_y = \{1,1,1,1\}^T$, $\mathbf{\Gamma}_y = \{40,40,40,40\}^T$

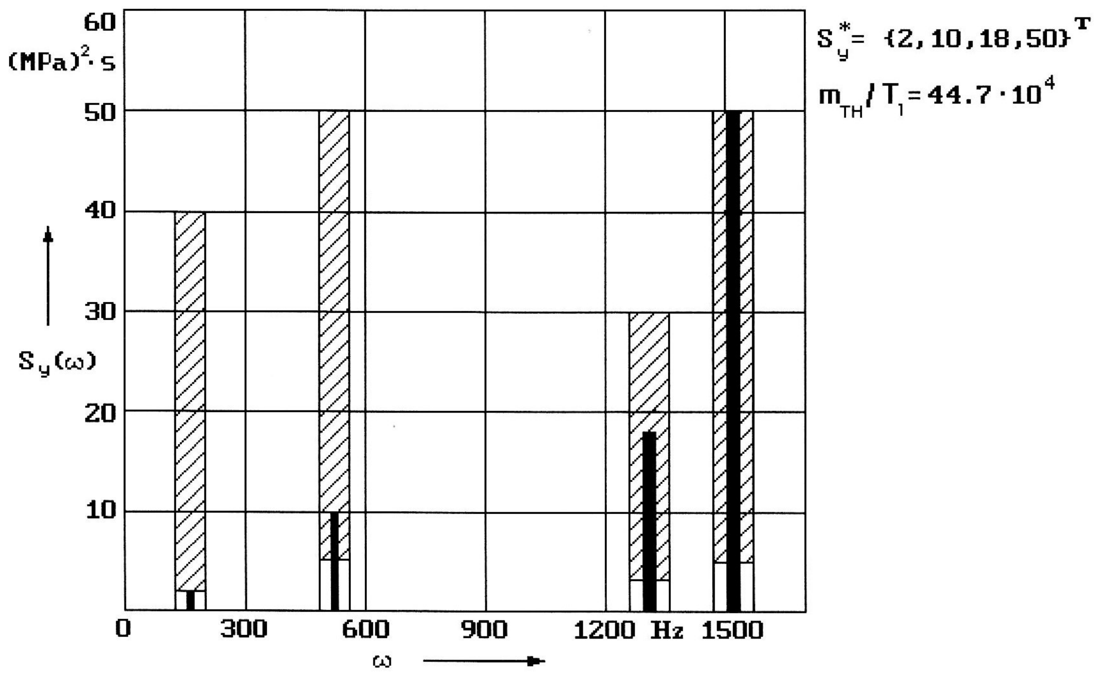

Figure 2. The most critical PSD at prescribed restrictions:
$\gamma_y = \{2,5,3,5\}^T$, $\mathbf{\Gamma}_y = \{40,50,30,50\}^T$

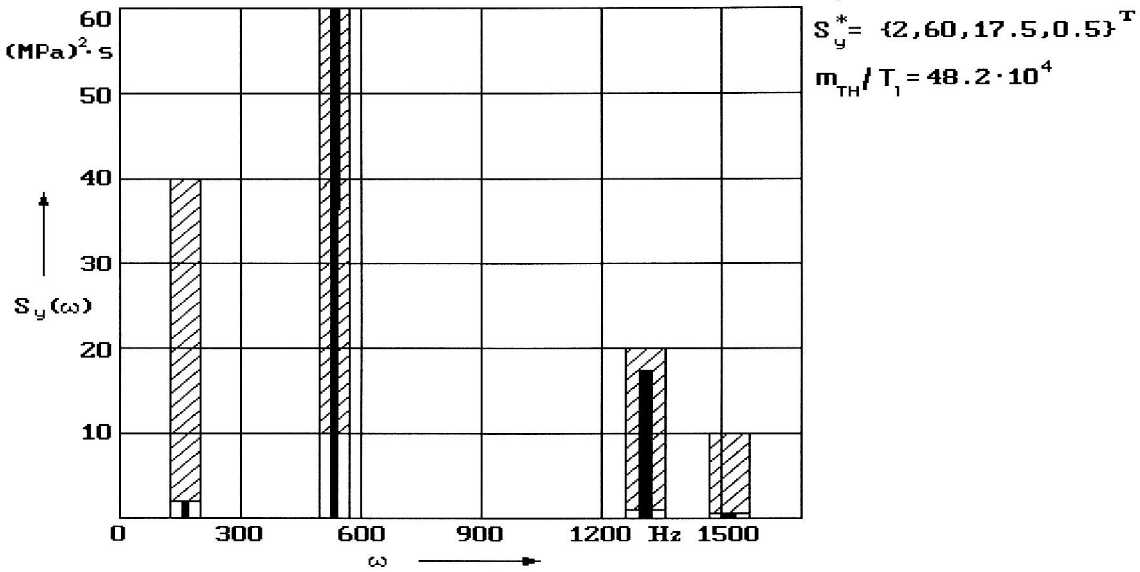

Figure 3. The most critical PSD at prescribed restrictions:

$\gamma_y = \{2,5,1,0.5\}^T$, $\Gamma_y = \{40,60,20,10\}^T$

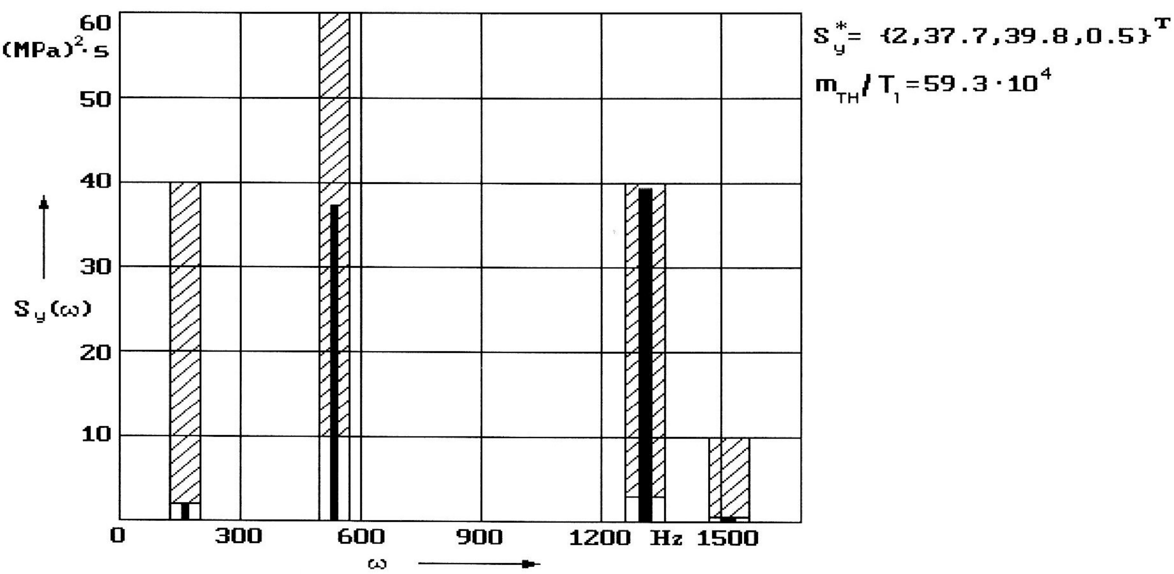

Figure 4. The most critical PSD at prescribed restrictions:

$\gamma_y = \{2,10,3,0.5\}^T$, $\Gamma_y = \{40,60,40,10\}^T$

Literature

1. Gusev, A.S.; Svetlitsky, V.A.: Structures Calculation under Random Loading. Moscow, Mashinostrojenije, (1984), 240p.

2. Kogaev V.P.: Strength Calculation under Stresses Changing inTime. Moscow, Mashinostrojenije, (1977), 232p.

3. Bolotin V.V.: Machines and Structures Resourse Prediction. Moscow, Mashinostrojenije, (1984), 321p.

4. Petrov E.P. Design of Blade Forced Vibrations in Nonuniform Flow. Dynamics and Strength of Machines, 34, (1981), p.61-68.

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