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Stability of Cooperating Manipulators with Hybrid Position/Force Control and Time Delay

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The mathematical model and stability of motion of two cooperating manipulators is considered. The high order equations of this model make the mathematical analysis of the stability more difficult.
By using a symmetrical control scheme with time delay in a feedback loop, we have obtained the domains of stability and non-stability for such parameters of the system as coefficients of gains, stiffness of force sensors, time delay in the control loop, and mass of load.

1 Introduction

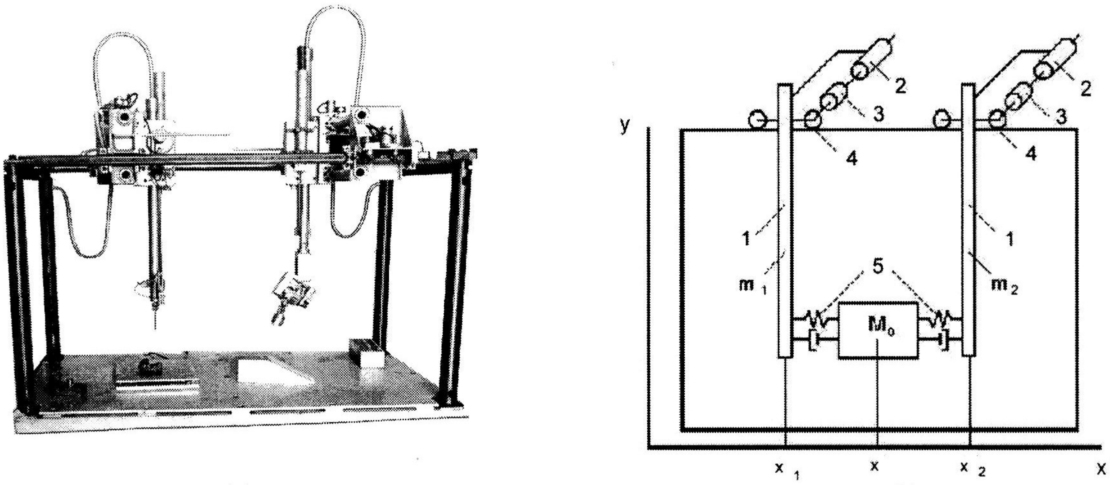
The problems of coordinated control of a few manipulators, which are coupled through an external object, acquire actual meaning in accordance with the appearance of new tasks in the area of automation of technological processes.
Some examples of such possible tasks include:
1. The transfer of a bulky or heavy object, when the power handling capacity of one manipulator is not sufficient, or when the grip can not provide reliable handling of the object.
2. Manipulation with objects. Such tasks include mounting and assembling operations, and machining operations. For example each arm holds one of the coupled parts, or one arm holds the tool and the second holds the part (Tao et al., 1990; Dauchez and Delebarre, 1991; Kosuge and Ishikawa, 1994).
The technological operation present a special interest, which realises the manipulation of non-rigid objects (such as film materials, thin metal sheet and so on), their expansion, bending, and attachment to another object with defined tension (Zheng and Chen, 1994; Bouffard et al.,1991; Von Albrichsfeld, 1996).
In all of these cases for co-ordinated movement of the coupled manipulators, it is necessary to control and correct their movements in accordance with information about reaction forces acting on the system. It is possible to measure these forces with the force sensors mounted between manipulators and the objects they hold, and to use their signals in the control loop.
By synthesis of control laws it is possible to use various control schemes based on measurements of the reaction forces. The most widespread schemes are position/force (Raibert and Craig, 1981) and impedance (Whitney, 1977) control laws.
Correction of manipulator movement by position/force control is done with the help of feedback on position and force. A hybrid position/force (Uchiyama and Dauchez, 1988; Kim and Zheng, 1989; Pujas et al., 1995; Perdereua and Drouin, 1996) and parallel (Hayati et al., 1998) systems of control are used.
With impedance control, the velocity of manipulator movement is proportional to the force error. Such methods of control are used frequently, when one of the manipulators (leading) is controlled only by the position, and second one only by force.
The most widespread case of consideration of dynamic movement is the movement of two manipulators connected through an elastic or lumped-elastic element, whose mass is negligible. In such cases, as a rule, the time delay in the control system is not taken into account, reducing the order of the characteristic equation dawn to four.
In Kim and Zheng (1989) a system of two master-slave manipulators is considered, coupled with each other through a lumped-elastic force sensor by two types of control, positioning control and positioning/force control, and without time delay in the control system. Applying the direct Lyapunov method it is shown that the system is stable in both cases.
In Kazerooni and Tsay (1988), Kopf (1989), Kokkinis (1989), Wen and Kreutz-Delgado (1992) more general formulations of the problem are shown, but the influence of the time delay on the system's stability is not considered.

The present paper considers not only the cooperating manipulators, but also the load by linear positioning/force laws of control. The model takes into account the time delay in the control loop. The symmetrical scheme of control is investigated, in which control laws of both manipulators are similar. This condition allows us to reduce the characteristic polynomial of eighth order to a multiplication of corresponding polynomials of third and fifth orders. In this case it is possible to receive areas of stability and non-stability of the dynamic system depending on its parameters, such as coefficients of gains, stiffness of force sensors, time delay in the control loop, and mass of load.

2 Model of Connected Manipulators

The researches are carried out on a gantry type configuration of manipulator with two arms (Gorinevsky et al., 1997). It consists of a rigid box frame mounted on a base, and two arms, identical in kinematics and design (Figure 1(a)). The frame height is 0.95 m, its horizontal dimensions are 1.4 × 0.76 m. The manipulator arm is moved along the horizontal axes by a carriage riding on the bridge. The first degree of freedom corresponds to the linear motion of the bridge riding on runway rails mounted on the manipulator frame along one of the horizontal axes. Another set of runways is installed on the bridge normal to this axis. The second degree of freedom corresponds to the linear motion along another horizontal axes of the carriage carrying the manipulator arm of length 0.7 m in the runways on the bridge. The vertical motion of the manipulatorarm must correspond to the third degree of freedom. Drive reduction gears of the translational degrees of freedom are connected through cylindrical pinions with racks mounted on the runways.

We will study the one-dimensional, translational motion of two manipulators which hold a load (Figure 1). Each manipulator consists of an arm (1) with mass, m_1 (m_2), a gear train (DC motor (2) and reductor (3)). The output power gear (4) is connected to the rack attached to the arm. The manipulator arm is moved along the horizontal axes OX .



(a) (b)

Figure 1. General View of the Research Manipulator (a) and Schematics for the One-dimensional Motion of the Manipulators (b)

A single-component force sensor (5) is mounted to the lower end of each manipulator arm to measure the horizontal force component acting on it.

The force sensor ends have plates, which hold an undeformable object with mass M_0 . We will neglect the moments produced by the forces, which act from side of the manipulator relative to the centre of gravity of the load. We will model a single-component force sensor by a massless lumped elastic element.

2.1 Mathematical Model

By x_1, x_2 we denote the coordinates of the point where the sensor is mounted on the appropriate manipulator and by x the coordinate of the centre of gravity of the load. We use the Lagrange second order equations to derive the equations of motion for the system:

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_\nu}\right)-\frac{\partial T}{\partial q_\nu}=Q_\nu \qquad \nu=1,....,n \tag{1}$$

where T – kinetic energy of the system, Q_ν – generalised forces, q_ν – Lagrange coordinates, n – number of degrees of freedom.

We take Cartesian coordinates x, x_1, x_2 for the Lagrange coordinates. Thus the kinetic energy of the system can be expressed as:

$$T=\tfrac{1}{2}(m_1\dot{x}_1^2+m_2\dot{x}_2^2+M_0\dot{x}^2+J_{G1}\dot{\varphi}_1^2+J_{G2}\dot{\varphi}_2^2) \tag{2}$$

where φ_i $(i=1,2)$ - is a rotational angle of the motor rotor, J_{G_i} – is the moment of inertia of the rotor of each motor.

The linear displacement x_1 (x_2) of the manipulator arm is related to the rotational angle φ of the motor as:

$$x_i=(r_i\ /\ j_i)\varphi_i$$

where r_i is a radius of the outer pinion in the gear train, and j_i is the gear train reduction ratio.

Taking into consideration the above equation, the expression for the kinetic energy (2) can be rewritten in the form:

$$T=\tfrac{1}{2}(m_1\dot{x}_1^2+m_2\dot{x}_2^2+M_0\dot{x}^2+\frac{J_{G1}}{\rho_1^2}\dot{x}_1^2+\frac{J_{G2}}{\rho_2^2}\dot{x}_2^2) \tag{3}$$

where $\rho_i=r_i\ /\ j_i, (i=1,2)$.

The generalised forces Q_ν may be presented as:

$$Q_\nu=\frac{\partial U}{\partial x_\nu}+\frac{\partial R}{\partial \dot{x}_\nu}+X_\nu \qquad \nu = 1, 2, 3$$

The force function reads

$$U=-\tfrac{1}{2}k_1\left[\ x-a-(x_1+l_{01})\right]^2-\tfrac{1}{2}k_2\left[\ x_2-l_{02}-(x+a)\right]^2 \tag{4}$$

where k_i is the stiffness of each force sensor, $2a$ is the size of the load, l_{0i} is the length of the elastic element of the force sensor in unloaded state.

The energy dissipation in the system may be expressed by the Rayleigh dissipation function

$$R=-\tfrac{1}{2}\ [\ d_{21}\dot{x}_1^2+d_{22}\dot{x}_2^2+b_1(\dot{x}-\dot{x}_1)^2+b_2(\dot{x}_2-\dot{x})^2\ ] \tag{5}$$

where the positive constants $d_{2i}=\frac{C_{2i}}{\rho_i^2}$, $C_{2i}=\frac{M_{pi}-M_{ni}}{\dot{\varphi}_i}$ for a specific motor may be calculated by using the values for the starting torque M_{pi}, nominal voltage U_i, nominal torque M_{ni}, nominal angular velocity $\dot{\varphi}_i$, and b_i is a coefficient of the force sensor viscosity.

The generalised forces $X_i=d_{1i}U_i$ are proportional to the voltage U_i applied to each of the motors. They perform work along the possible displacement δx_i $(i=1,2)$.

Here $d_{1i} = \dfrac{C_{1i}}{\rho_i}, C_{1i} = \dfrac{M_{ni}}{U_{ni}}$.

By substituting equations (3), (4) and (5) into equation (1) we obtain the equation system:

$$\begin{aligned} &M_0\ddot{x}+(b_1+b_2)\dot{x}+(k_1+k_2)x-b_1\dot{x}_1-k_1x_1-b_2\dot{x}_2-k_2x_2-k_1l_1+k_2l_2=0 \\ &M_1\ddot{x}_1+(d_{21}+b_1)\dot{x}_1+k_1x_1-b_1\dot{x}-k_1x+k_1l_1-d_{11}U_1=0 \\ &M_2\ddot{x}_2+(d_{22}+b_2)\dot{x}_2+k_2x_2-b_2\dot{x}-k_2x-k_2l_2-d_{12}U_2=0 \end{aligned} \tag{6}$$

where $M_i = m_i + \dfrac{J_{Gi}}{\rho_i^2} = \dfrac{J_i}{\rho_i^2}$, $l_i = a + l_{0i}$, $i = 1,2$.

To study the dynamics of such a system we need the control laws for the voltage, U_1 and U_2 applied to the motors.

2.2 Control Laws

The general relation, including the signals of feedback of position, velocity, and force, and the delay in feedback loop may be expressed in the form:

$$U_i(t+T_i) = -k_{pi}(x_i(t)-x_{pi}) - k_{Vi}\dot{x}_i(t) - k_{Fi}(F_i(t)-F_{pi}) \qquad i = 1,2 \tag{7}$$

where $k_{pi}, k_{Vi}, k_{Fi} \geq 0$ are coefficients of feedback about position, velocity, and force for each manipulator, x_{pi} is the programmed position of the manipulator, F_{pi} are programmed values of forces acting from side of the manipulator on the load, whereas $F_{p1} = -F_{p2} = F_p \geq 0$, T_i is the time delay in each feedback loop.
From now on we consider the position of the manipulator and the load related to the coordinate system as shown in Figure 1.
To linearize the equations (6) we express $U(t+T)$ in the form of a series in T

$$U(t+T) = U(t) + T\dot{U}(t) + \frac{1}{2}T^2\ddot{U}(t) + \ldots \tag{8}$$

We consider only the linear part of equation (8), so

$$U(t+T) = U(t) + T\dot{U}(t) \tag{9}$$

The considered manipulators are mechanically connected to each other through the lumped elastic elements of the corresponding sensors and through the load, and that is why the programmed values of positions x_{p1} and x_{p2} corresponding to the equation system (6) must be related by the equation:

$$x_{p2} - x_{p1} = l_1 + l_2 - (\Delta x_1 + \Delta x_2)$$

Here $l_1 + l_2$ is the distance between manipulators in the steady state and in case the elastic elements of force sensors are not deformed, Δx_i is the deformation of each sensor ($i = 1,2$). The values of Δx_i are restricted by the maximal linear deformation.
By substituting equation (9) into equation (7) and taking into consideration, that

$$\begin{aligned} &F_1(t) = -k_1(x - x_1 - l_1) - b_1(\dot{x} - \dot{x}_1) \\ &F_2(t) = -k_2(x - x_2 + l_2) - b_2(\dot{x} - \dot{x}_2) \end{aligned}$$

we obtain the equations for control laws:

$$U_1 + T_1 \dot{U}_1 = -(k_{F1} \cdot k_1 + k_{p1})x_1 - (k_{V1} + k_{F1} b_1)\dot{x}_1 + k_{F1} k_1 x + k_{F1} b_1 \dot{x} + k_{F1}(F_p - k_1 l_1) + k_{p1} x_{p1}$$

$$U_2 + T_2 \dot{U}_2 = -(k_{F2} \cdot k_2 + k_{p2})x_2 - (k_{V2} + k_{F2} b_2)\dot{x}_2 + k_{F2} k_2 x + k_{F2} b_2 \dot{x} + k_{F2}(F_p - k_2 l_2) + k_{p2} x_{p2} \qquad (10)$$

The relations (10) present the general case of linear control laws for two connected manipulators with different characteristics and coefficients of position, velocity and force.

3 Stability of the System of Motion

To study the stability we rewrite the equations of motion (6) and the control law (10) in deviations from the stationary regime holding for the deviations from the old symbols x, x_1, x_2, U_1, U_2. We obtain the following equation system:

$$\begin{aligned} &M_0 \ddot{x} + (b_1 + b_2)\dot{x} + (k_1 + k_2)x - b_1 \dot{x}_1 - k_1 x_1 - b_2 \dot{x}_2 - k_2 x_2 = 0 \\ &M_1 \ddot{x}_1 + (d_{21} + b_1)\dot{x}_1 + k_1 x_1 - b_1 \dot{x} - k_1 x - d_{11} U_1 = 0 \\ &M_2 \ddot{x}_2 + (d_{22} + b_2)\ddot{x}_2 + k_2 x_2 - b_2 \dot{x} - k_2 x - d_{12} U_2 = 0 \end{aligned} \qquad (11)$$

and the control law:

$$\begin{aligned} &U_1 + T_1 \dot{U}_1 = -(k_{F1} k_1 + k_{p1})x_1 - (k_{V1} + k_{F1} b_1)\dot{x}_1 + k_{F1} k_1 x + k_{F1} b_1 \dot{x} \\ &U_2 + T_2 \dot{U}_1 = -(k_{F2} k_2 + k_{p2})x_2 - (k_{V2} + k_{F2} b_2)\dot{x}_2 + k_{F2} k_2 x + k_{F2} b_2 \dot{x} \end{aligned} \qquad (12)$$

The stability of the system (11) and (12) is given by the position of the roots of the corresponding characteristic polynomial in the complex plane. This polynomial is of 8-th order, which is a consequence of the fact that:
1) There are three second order differential equations describing the motion of the load and of both manipulators (with three degrees of freedom).
2) There are another two, first-order polynomials which make the mathematical analysis of the stability even more difficult. In spite of this, the analysis is possible in some special cases.

3.1 The Stability of a System with Position Control and without Delay in Feedback Loop

In the control laws (12), we set the coefficients of the force $k_{F1} = k_{F2} = 0$ and the delay time $T_1 = T_2 = 0$. Then by substituting the equations for the control voltages U_1, U_2 in the system (11) we rewrite it in a matrix form:

$$M\ddot{\bar{y}} + K_V \dot{\bar{y}} + K_p \bar{y} = 0 \qquad (13)$$

where the symmetric matrices and vectors have the form:

$$M = \begin{Vmatrix} M_0 & 0 & 0 \\ 0 & M_1 & 0 \\ 0 & 0 & M_2 \end{Vmatrix} \qquad K_V = \begin{Vmatrix} b_1 + b_2 & -b_1 & -b_2 \\ -b_1 & d_{21} + b_1 + d_{11} k_{V1} & 0 \\ -b_2 & 0 & d_{22} + b_2 + d_{12} k_{V2} \end{Vmatrix}$$

$$K_p = \begin{Vmatrix} k_1 + k_2 & -k_1 & -k_2 \\ -k_1 & k_1 + d_{11} k_{p1} & 0 \\ -k_2 & 0 & k_2 + d_{12} k_{p2} \end{Vmatrix} \qquad \ddot{\bar{y}} = \begin{Vmatrix} \ddot{x} \\ \ddot{x}_1 \\ \ddot{x}_2 \end{Vmatrix} \quad \dot{\bar{y}} = \begin{Vmatrix} \dot{x} \\ \dot{x}_1 \\ \dot{x}_2 \end{Vmatrix} \quad \bar{y} = \begin{Vmatrix} x \\ x_1 \\ x_2 \end{Vmatrix}$$

We use the direct method of Lyapunov and assume the Lyapunov function in the form:

$$V = \frac{1}{2}(\dot{\bar{y}}^* \cdot M \cdot \dot{\bar{y}} + \bar{y}^* \cdot K_p \cdot \bar{y})$$

where the sign " * " means the operation of transposition.
The positive definiteness of the matrix M is obvious, and the positive definiteness of the matrix K_p follows from the fact that its principal diagonal minors are positive (Gantmacher, 1960). By the same reasoning the matrix K_V is also non-negative definite. Therefore, V is a positive definite quadratic form.
The final form of derivation of Lyapunov function V, taking into consideration the system (13), has the form:

$$\frac{dV}{dt} = \dot{\bar{y}}^*(M\ddot{\bar{y}} + K_p\bar{y}) = -\dot{\bar{y}}^* K_V \dot{\bar{y}} \leq 0$$

So, we have shown, that with position control and no time delay, the motion of the system is stable for any values of the gain coefficients in the control systems of both manipulators, for any load mass, and for any characteristics of the force sensors.
This result is a generalisation of the result obtained in Kim and Zheng (1989).

3.2 The Stability of the System with Symmetrical Control Law and in Presence of Time Delay in Feedback Loop

Now we consider a system of two equal manipulators, which have symmetrical control laws and equal feedback gains of positions, velocities and forces corresponding:

$$\begin{array}{llll} M_1 = M_2 = M & k_1 = k_2 = k & b_1 = b_2 = b & d_{11} = d_{12} = d_1 \\ d_{21} = d_{22} = d_2 & k_{V1} = k_{V2} = k_V & k_{F1} = k_{F2} = k_F & T_1 = T_2 = T \end{array}$$

Then the equations (11) take the form:

$$\begin{aligned} &M_0\ddot{x} + 2b\dot{x} + 2kx - b(\dot{x}_1 + \dot{x}_2) - k(x_1 + x_2) = 0 \\ &M\ddot{x}_1 - b\dot{x} + (d_2 + b)\dot{x}_1 + kx_1 - kx - d_1 U_1 = 0 \\ &M\ddot{x}_2 - b\dot{x} + (d_2 + b)\dot{x}_2 + kx_2 - kx - d_1 U_2 = 0 \end{aligned} \tag{14}$$

and the control law (12) takes the form:

$$\begin{aligned} &U_1 + T\dot{U}_1 = -(k_F k + k_p)x_1 - (k_V + k_F b)\dot{x}_1 + k_F kx + k_F b\dot{x} \\ &U_2 + T\dot{U}_2 = -(k_F k + k_p)x_2 - (k_V + k_F b)\dot{x}_2 + k_F kx + k_F b\dot{x} \end{aligned} \tag{15}$$

In this case, even though the degree of the polynomial remains 8, the system splits up into two polynomials of 3rd and 5th degree, respectively.

For the sake of the stability analysis let us introduce non-dimensional variables by the formulas:

$$\bar{t} = \rho\sqrt{\frac{k}{J}} \cdot t \qquad \bar{x} = \frac{k\rho}{C_1 U_0} x \qquad \bar{x}_i = \frac{k\rho}{C_1 U_0} x_i \qquad \bar{U}_i = \frac{U_i}{U_0} \qquad (i = 1,2) \tag{16}$$

Here the bar points to non-dimensional variables.
By expressing dimensional variables with non-dimensional variables and substituting them into the system of equations (14) and (15) we obtain the following system in non-dimensional variables (bar will be left out):

$$
\begin{aligned}
&\mu\ddot{x}+2\beta\dot{x}+2x-\beta\dot{x}_1-x_1-\beta\dot{x}_2-x_2=0\\
&\ddot{x}_1+(\alpha+\beta)\dot{x}_1+x_1-\beta\dot{x}-x-U_1=0\\
&\ddot{x}_2+(\alpha+\beta)\dot{x}_2+x_2-\beta\dot{x}-x-U_2=0\\
&\tau\dot{U}_1+U_1+(f+s)x_1+(v+\beta f)\dot{x}_1-fx-\beta f\dot{x}=0\\
&\tau\dot{U}_2+U_2+(f+s)x_2+(v+\beta f)\dot{x}_2-fx-\beta f\dot{x}=0
\end{aligned}
\tag{17}
$$

Here $\alpha=\dfrac{d_2}{\sqrt{Mk}}=\dfrac{C_2}{\rho\sqrt{Jk}}$, $\beta=\dfrac{b\rho}{\sqrt{Jk}}$, $\mu=\dfrac{M_0\rho^2}{J}$, $f=k_Fd_1=\dfrac{k_FC_1}{\rho}$, $v=\dfrac{k_Vd_1\rho}{\sqrt{Jk}}=\dfrac{k_VC_1}{\sqrt{Jk}}$, $\tau=T\rho\sqrt{\dfrac{k}{J}}$,

$s=\dfrac{k_pd_1}{k}=\dfrac{k_pC_1}{\rho k}$ are non-dimensional expressions determining the behaviour of the system.

This mathematical model is based on the assumption of the same characteristics for both manipulators and identical positions, velocities and force feedback coefficients in both control laws. This fact makes the analysis of the 8th order characteristic polynomial simpler, because it is factored into two polynomials of 3rd and 5th order, respectively.

The characteristic polynomial is

$$
F_8(\lambda)=F_3(\lambda)\cdot F_5(\lambda) \tag{18}
$$

where

$$
\begin{aligned}
&F_3(\lambda)=\tau\lambda^3+[\tau(\alpha+\beta)+1]\lambda^2+(\alpha+\beta+\tau+v+\beta f)\lambda+f+s+1,\\
&F_5(\lambda)=\tau\mu\lambda^5+[2\beta\tau+\tau(\alpha+\beta)\mu+\mu]\lambda^4+[2\tau+2\alpha\beta\tau+\tau\mu+2\beta+(\alpha+\beta)\mu+(v+\beta f)\mu]\lambda^3\\
&+[2\alpha\tau+2+2\alpha\beta+\mu+2\beta v+\mu(f+s)]\lambda^2+2(\alpha+v+\beta s)\lambda+2s
\end{aligned}
$$

Further, for simplicity, we will consider the force sensors as only elastic but not viscous-elastic (availability of viscosity only increases the zone of stability). According to the Hurwitz criterion, the asymptotic stability condition of the system may be written in the form:

$$
0\le S\le S^* \tag{19}
$$

where the limit value S^* is defined from the corresponding squared equation and has the form:

$$
S^*=\frac{1}{2}\left[\frac{1}{\tau}(\alpha+v)(f+2)+\alpha(\alpha+v)-(\frac{2}{\mu}+1)(f-\alpha\tau)-\sqrt{D}\right] \tag{20}
$$

where

$$
D=(f-\alpha\tau)^2\left[\frac{1}{\tau^2}(\alpha+v)^2+(\frac{2}{\mu}+1)^2\right]+2(\alpha+v)(\frac{2}{\mu}-1)\left[\alpha(2f-\alpha\tau)-\frac{1}{\tau}f^2\right]
$$

Figure 2 shows the relationship between limit gains k_p^* and time delay T in the control loop (curve 1) obtained in correspondence with equations (19) and (20).

The numerical parameters of the manipulator used for the calculation of this curve are as follows (Gorinevsky et al., 1997):

C_1 =2,64$\cdot10^{-3}$ N·m/V $\quad C_2=6\cdot10^{-5}$ N·m·s $\quad \rho=2{,}94\cdot10^{-4}$ m $\quad J=6{,}96\cdot10^{6}$ kg·m² sensor stiffness is $k = 10^5$ N/m; load $M_0 = 10$ kg; velocity gain $k_V = 5 \cdot 10^4$ V·s/m; force gain $k_F = 1{,}0$ V/N.

The range of the time delay T is selected from 0,01 s to 0,1 s which corresponds to real values of delay for various robot systems.

With $T \ll 1$, the equation of the neutral curve (20) separating the field of stability from the field of instability becomes simpler, and the corresponding condition for asymptotic stability (19) may be written in the form:

$$0 \le S < \frac{\alpha + \nu}{\tau} - f$$

or in dimensional parameters

$$0 \le k_p < \frac{1}{T}\left(\frac{1}{\rho} \cdot \frac{C_2}{C_1} + k_V\right) - k \cdot k_F \tag{21}$$

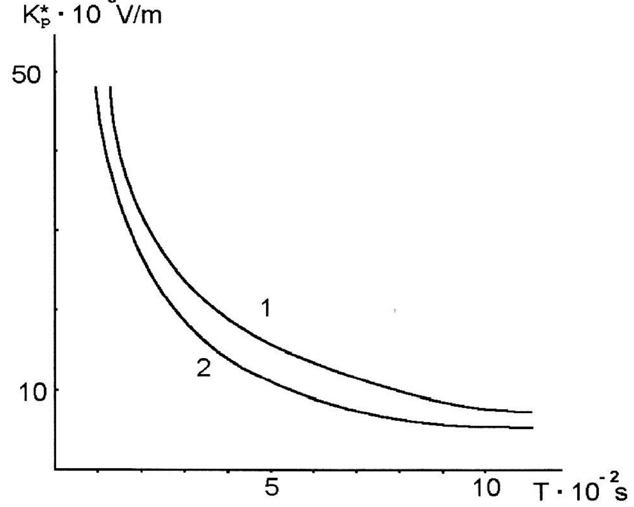


Figure 2. The Relationship between Limit Gains k_p^* and Time Delay T

Since the values k_V are much bigger than the values of $\frac{1}{\rho} \cdot \frac{C_2}{C_1}$, the inequality (21) may be written as

$$0 \le k_p < \frac{k_V}{T} - k \cdot k_F \tag{22}$$

Figure 2 displays also the corresponding curve (2) specified by the above variables $(k_V,\ k,\ k_F)$. Since the curve (2) is located lower than the curve (1), it is clear that by fulfilling the condition (22), the condition (19) is also satisfied.

As follows from condition (22) the domain of stability is diminished with the increase of gain k_F and the stiffness of force sensor k. That is to say, the system with a "soft" sensor is more stable than one with a "rigid" one. The stability domain increases with increase of velocity feedback gain k_V. The system is always stable in absence of a time delay ($T \to 0$).

Let us consider in detail a case of a position strategy of a manipulator control ($k_F \to 0$). In this case the relationship between the commanded positions of two manipulators has the form:

$$x_{p2} = x_{p1} + 2\left(l - \frac{F}{k}\right)$$

that is, the commanded position of the second manipulator, x_{p2}, is determined as the sum of the following terms: the commanded position x_{p1} of first manipulator, the constant $2l$ and deformations of elastic elements of the force sensors with stiffness k under the action of the force F. The inequality (19) may be rewritten in the form:

$$0 \le S < S^* \qquad (23)$$

where

$$S^* = \frac{1}{2}\left[(\alpha+\nu)(\alpha+\frac{2}{\tau})+\alpha(\frac{2}{\mu}+1)-\alpha\sqrt{D}\right]$$

$$D = (\frac{2}{\mu}+1)^2\tau^2 - 2(\alpha+\nu)(\frac{2}{\mu}-1)\tau+(\alpha+\nu)^2$$

The values S^* and D are positive for all admissible values entering expressions for these parameters.
We consider in more detail the relationships obtained in this case.
Figure 3 shows the neutral curve $k_p^* = k_p(T)$ separating the stable and the unstable domain. The values of other parameters are fixed and they are the same as in Figure 2. The curve has a flat minimum given by parameters $T \approx 0{,}6$ s, $k^*_{p\min} = 3{,}94 \cdot 10^5$ V/m. Thus, the system is stable with any time delay if the position gain k_p is smaller than $k^*_{p\min}$. Certainly, the values of time delay are limited to such values for which the relationship (9) is fulfilled. The curve has also a horizontal asymptote:

$$\lim_{T\to\infty} k_p^* = \frac{1}{C_1\tau}(C_2+\rho C_1 k_V)\cdot\frac{2C_2}{M_0\rho^2+2J}$$

With increase of the value of the load, M_0, the stable domain narrows monotonically (Figure 3). Moreover, the equations of corresponding limiting curves have the form:

$$\lim_{M_0\to\infty} k_p^* = \frac{1}{C_1\rho}(C_2+\rho C_1 k_V)\cdot\frac{1}{T}$$

$$\lim_{M_0\to 0} k_p^* = \frac{1}{C_1\rho}(C_2+\rho C_1 k_V)\cdot(\frac{C_2}{J}+\frac{1}{T})$$

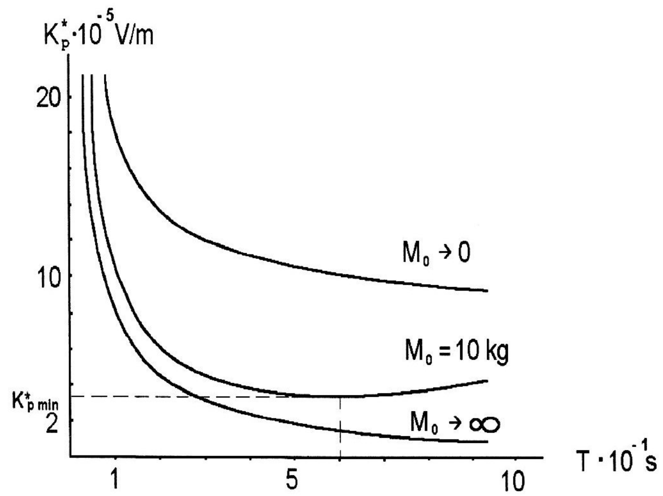


Figure 3. The Neutral Curve $k_p^* = k_p(T)$ Separating the Stable and the Unstable Domain

4 Conclusions

The analysis of a mathematical model of two, one-degree-of-freedom manipulators holding a load allows us to make the following conclusions:

1. Using position control and in the absence of time delay in the feedback loop, the system is stable with any gains, mass of load, arms, and force sensor stiffness.
2. The linear symmetrical control system with the position and force feedback loop is also stable in absence of time delay.
3. With time delay in feedback loop and linear symmetrical control laws, the range of position and force feedback gains in which the motion of manipulators are asymptotically stable has an upper bound. The stability domain diminishes with the growth of the sensor stiffness (k) and mass (M_0) of load. Thus, the system stability may be violated by an increase of position (k_p), force (k_F) feedback gains, stiffness k of force sensor, mass of load M_0 , and by an increase of time delay T . An increase of velocity feedback gain k_V stabilises the control system.

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