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Optimal Control of the Stretching Process of Solar Arrays on a Spacecraft Using a Genetic Algorithm

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The optimal control problem of spacecraft attitude during its solar arrays stretching process is discussed in the present paper. It points out the results of nonholonomic behavior of the system. Instead of traditional Newton iteration methods, the genetic algorithm of optimal control is proposed. The authors study the optimal control algorithm of nonholonomic behavior according to the genetic algorithm. The results of the numerical simulation show that this approach is effective for the control problem of spacecraft attitude in the stretching process of solar arrays.

1 Introduction

With the development of space technology, the mechanisms of the spacecraft become more and more complex, and large stretchable multi-wing solar arrays have became indispensable parts of spacecraft. When the large solar arrays are stretching in space, the attitude of the spacecraft is changed due to the coupling of stretching motion and attitude motion (Liu, 1995). In order to ensure that the spacecraft is located in the designed position, we must study the control law in the stretching process of solar arrays. Li and Wang (1996) have studied the influence of the stretching motion of the spacecraft with appendages on the attitude of the spacecraft. Ge and Liu (1997) have discussed the optimal control problem in the solar arrays stretching process. They used the Gauss – Newton iteration method (Fernandes et al., 1991, 1994) in the optimal control to search for the optimal solution. Recently, a new genetic optimization algorithm (Holland, 1975, 1992) is rapidly developed, and is widely used in a variety of fields. The genetic algorithm has several merits, such as it employs the target function only, needs no other assistant condition, searches for multi-point values in the solution space simultaneously, has the character of a parallel process, and so on. Therefore it has a wide adaptability. The present paper uses the genetic algorithm to study the optimal control problem of the solar arrays stretching process, and provides the numerical method of optimal control. Firstly we use multibody dynamics to obtain the dynamics equations a spacecraft with solar arrays. While the nonholonomic behavior of the system is considered, we set up the control target function of the system, define the genotype of chromosome and the function of fitness, and design the corresponding genetic operation. By the results of a numerical simulation it is shown that this method can solve the control problem of the spacecraft attitude efficiently in the solar arrays stretching process.

2 Dynamics Equations

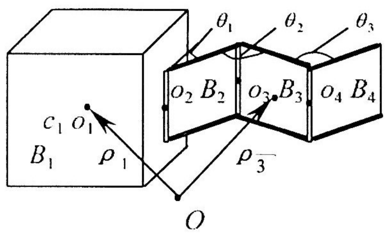

Figure 1. Spacecraft with Solar Arrays

The spacecraft with solar arrays is supposed as a chain multibody system composed by four rigid bodies $B_i(i = 1,\cdots,4)$ connected with cylindical joints (Figure 1). B_1 is the spacecraft, B_2, B_3 and B_4 are the solar arrays. In Figure 1, (O-*XYZ*) is the translation frame whose origin point O is the mass center of the system, $(o_i - x_i y_i z_i)$ are the body-fixed frames of each body $B_i(i=1,\cdots,4)$. $l_i(i=1,\cdots,4)$ are the distances from the internal joint o_i to the external joint o_{i+1}. $c_i(i=1,\cdots,4)$ are the distances from the mass center to the internal joints o_i, and $c_1=0$. The mass of the spacecraft is m_1, the moment of inertia is J_1, the mass of each solar array is $m_i(i = 2,3,4)$. The augmented body-hinge vector of the system is (Liu, 1995)

$$
\mathbf{b}_{ij} = \begin{cases} -b_i\mathbf{e}_i & (i > j) \\ (c_i - b_i)\mathbf{e}_i & (i = j) \\ (l_i - b_i)\mathbf{e}_i & (i < j) \end{cases} \tag{1}
$$

and

$$
b_i = \frac{1}{m_s}\left(m_i c_i + l_i \sum_{k=i+1}^{4} m_k \right) \qquad (i = 1,\cdots,4) \tag{2}
$$

where m_s is the total mass of the system. The vectors $\boldsymbol{\rho}_i (i = 1,\cdots,4)$, pointing from the total mass center O to the mass centers c_i of each body, can be inferred from equation (1) as

$$
\boldsymbol{\rho}_i = c_i\mathbf{e}_i + \sum_{k=1}^{i-1} l_k\mathbf{e}_k - \sum_{k=1}^{4} b_k\mathbf{e}_k \qquad (i = 1,\cdots,4) \tag{3}
$$

As no external moment of force effects the system, the momentum of the system with respect to (O-*XYZ*) is zero, and the moment of momentum with respect to point O is conservative. It is assumed that the initial moment of momentum of the system $\mathbf{H}_0 = 0$, according to the conservation principle of moment of momentum, it can be stated that

$$
\sum_{i=1}^{4} (\mathbf{J}_i \cdot \boldsymbol{\omega}_i + \boldsymbol{\rho}_i \times m_i\dot{\boldsymbol{\rho}}_i) = 0 \tag{4}
$$

Substituting equation (3) into equation (4), and considering that $\boldsymbol{\omega}_i = \sum_{j=1}^{i} \boldsymbol{\rho}_j^T \dot{\theta}_j$, we obtain

$$
I_Z\dot{\phi}_Z + \left[-(I_{2Z} + I_{3Z} + I_{4Z})\dot{\theta}_1 + (I_{3Z} + I_{4Z})\dot{\theta}_3 \right] = 0 \tag{5}
$$

where ϕ_1 is the attitude angle of the spacecraft, $\theta_j (j = 1,2,3)$ are the relative angle between solar arrays, $I_Z = \sum_{j=1}^{4} I_{jZ}$ is the total equivalent moment of inertia of the system, I_{jZ} is the equivalent moment of inertia of each body B_i with respect to point O. Equation (5) has the form of a nonholonomic constraint equation. It shows that the solar arrays can disturb the attitude of the spacecraft during the stretching process from the folded state to the stretched state.

3 Optimal Control Problem

We define the attitude $\mathbf{x} = (\varphi_1, \theta_1, \theta_2, \theta_3)^T$ of the system as a state variable, and consider the relative angle velocity $\dot{\theta}_i (i = 1,2,3)$ between each solar array as input variable, noted as $\mathbf{u}$

$$
\mathbf{u} = (\dot{\theta}_1, \dot{\theta}_2, \dot{\theta}_3)^T
$$

The state variable of the system is written as follows

$$
\dot{\mathbf{x}} = \mathbf{B}(\mathbf{x})\mathbf{u} \tag{6}
$$

where

$$
\mathbf{B}(\mathbf{x}) = \begin{bmatrix} B_1 & B_2 & B_3 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}
$$

$$B_1 = (I_{2Z} + I_{3Z} + I_{4Z})/I_Z \qquad B_2 = -(I_{3Z} + I_{4Z})/I_Z \qquad B_3 = I_{4Z}/I_Z$$

In the solar arrays stretching process, we search for the optimal control input $\mathbf{u}(t)$ according to an optimal index. We employ the principle of minimum energy control, and choose the dissipative energy of each rotation joint of the solar arrays as the optimal control index. Then the target function is

$$J(\mathbf{u}) = \int_0^T (\mathbf{u}, \mathbf{u}) dt \tag{7}$$

where $\mathbf{u}(t)$ is the measurable vector function of the Hilbert space L_2. We consider the cases of a finite dimensional space in the actual calculation, specified by the linear combination of Fourier basic vector $\{a_i\}_{i=1}^N$ (Courant and Hilbert, 1953) as

$$\mathbf{u} = \sum_{i=1}^{N} a_i \alpha_i = \boldsymbol{\phi\alpha} \tag{8}$$

where $\alpha_i (i = 1, \cdots, N)$ is the projection of the function $\mathbf{u}$ onto the basis vector $\{a_i\}_{i=1}^N$, ϕ is a $n \times N$ dimensional matrix composed of orthogonal Fourier basis vectors. Regarding α as a new control variable and considering the condition of terminal constraints of the system, the target function of equation (7) becomes (Fernandes et al. 1994)

$$J(\boldsymbol{\alpha}, \lambda) = \sum_{i=1}^{N} \alpha_i^2 + \lambda \left\| \mathbf{x}(T) - \mathbf{x}_f \right\|^2 \tag{9}$$

where λ is a penalty factor, $\mathbf{x}(T)$ is the state of the system when $t = T$ under a given control input $\mathbf{u}$. Obviously $\mathbf{x}(T)$ is a function of $\boldsymbol{\alpha}$, and noted as $\mathbf{x}(T) = f(\boldsymbol{\alpha})$, when N and λ are given, equation (9) can be written as

$$J(\boldsymbol{\alpha}) = \langle \boldsymbol{\alpha}, \boldsymbol{\alpha} \rangle + \lambda \left\| f(\boldsymbol{\alpha}) - \mathbf{x}_f \right\|^2 \tag{10}$$

Therefore, the problem of making equation (7) take a minimum by searching for the control input $\mathbf{u}$ is changed into that of making the target function (10) take a minimum by searching for $\boldsymbol{\alpha}$.

4 Genetic Algorithm of Optimal Control

We employ a genetic algorithm to solve the optimization problem. Firstly the variables are coded to form the chromosomes. The different chromosomes form a population. To suit our problem, we evaluate the value of fitness according to the state of survival of the fittest. A new generation population with better adaptability is constructed in terms of three genetic operations viz. selection, crossover, and mutation. The evolution procedure is continued until the optimal solution of problem is obtained. In general, binary strings are used to encode the optimal parameter space in the genetic algorithm. When the number of parameters becomes large and the range of parameter values becomes great, the speed of convergence of the algorithm will become slow. As the floating-point representations (Goldberg, 1991; Michalewicz et al., 1992) have the advantages of good precision and convenience to search for in a large space, floating-point coding is utilized. According to the genetic algorithm, considering the optimal control problem in the solar arrays stretching process, we design the genetic algorithm steps as follows:

1. Chromosomes representation: Utilizing the parallel searching mechanism of the genetic algorithm, we code $\boldsymbol{\alpha}$ which is the projection of function $\mathbf{u}$ on Fourier basis in equation (8). The chromosome is an N-dimensional vector composed of all $\alpha_i (i = 1, 2 \cdots, N)$.

$$\boldsymbol{\alpha} = [\alpha_1, \alpha_2, \cdots, \alpha_N] \tag{11}$$

   where α_i are real.

2. Initializing the population of parents: We generate n components randomly, the components of each parent is selected from a Gauss random variable with zero mean value and unity standard deviation.

3. The selection of fitness function: We define the fitness function as

$$g(\mathbf{a}) = \frac{1}{J(\boldsymbol{\alpha})} \tag{12}$$

where $J(\boldsymbol{\alpha})$ is the target function in equation (10), $\boldsymbol{\alpha}$ is the chromosome.

4. Selection: The fitness value $g_i(\alpha_i)(i = 1,2,\cdots,N)$ of each chromosome is calculated according to equation (12). The generation probability of the ith chromosome is given by

$$P_i = \frac{g_i(\alpha_i)}{\sum_{i=1}^{N} g_i(\alpha_i)} \tag{13}$$

The roulette wheel selection is used in the process of individual selection.

5. Crossover: A simple one-point crossover is employed. Two chromosomes are selected randomly according to the crossover probability P_s, a splice point is determined uniformly at random. The genetic codes following the splice point are interchanged, and two new chromosomes are generated as shown in Figure 2.

$$\alpha_1^k \quad \alpha_2^k \cdots \alpha_l^k \big| \alpha_{l+1}^k \cdots \alpha_N^k \;\Rightarrow\; \alpha_1^k \quad \alpha_2^k \cdots \alpha_l^k \big| \alpha_{l+1}^s \cdots \alpha_N^s$$

$$\alpha_1^s \quad \alpha_2^s \cdots \alpha_l^s \big| \alpha_{l+1}^s \cdots \alpha_N^s \;\Rightarrow\; \alpha_1^s \quad \alpha_2^s \cdots \alpha_l^s \big| \alpha_{l+1}^k \cdots \alpha_N^k$$

Figure 2. Crossover Chart

6. Mutation: Some components $\alpha_{ij}(j = 1,2,\cdots,m)$ of the chromosome α_i are chosen to mutate according to the mutation probability P_m, that is, adding a Gauss random variable on the components of individuals

$$\alpha_{ij} = \alpha_{ij} + \delta_j \tag{14}$$

where δ_j is a Gauss random variable.

7. The steps (4) - (6) are repeated until the optimal solution satisfying the condition is obtained.

5 Simulation Example

The mass and geometry of the spacecraft are $m_1 = 200$ kg, $J_1 = 322$ kgm^2, those of the strechable solar arrays are $m_2 = 5$ kg, $\frac{1}{2} \times l = 0.5 \times 1\mathrm{m}^2$; $m_3 = m_4 = 10$ kg, $l \times l = 1 \times 1$ m^2. The time used in the stretching process is $t = 5\mathrm{s}$. The control parameters of the genetic algorithm are selected as: The dimension of the chromosomes $N = 15$, the probability of crossover $P_c = 0.8$, the probability of mutation $P_m = 0.06$, the number of evolution generation $R = 2000$. In the simulation calculation, we have selected 15 terms of the Fourier basis. $\{a_i(t)\}_{i=1}^{5}$ is specificed as

$$\begin{bmatrix} 0{,}5 \\ 0 \\ 0 \end{bmatrix} \quad \begin{bmatrix} \sin t \\ 0 \\ 0 \end{bmatrix} \quad \begin{bmatrix} \cos t \\ 0 \\ 0 \end{bmatrix} \quad \begin{bmatrix} \sin 2t \\ 0 \\ 0 \end{bmatrix} \quad \begin{bmatrix} \cos 2t \\ 0 \\ 0 \end{bmatrix}$$

and $\{a_i(t)\}_{i=6}^{10}$, $\{a_i(t)\}_{i=11}^{15}$ are obtained by permuting the rows of the above components. The example considers that the solar arrays are fully stretched from the folded state; it is ensured that the attitude of the spacecraft in the initial and the final state remains unchanged. The initial and final configurations of the system are respectively

$$\mathbf{x}_0 = [0,0,0,0]^T \qquad \mathbf{x}_f = [0,\pi/2,\pi,\pi]^T$$

The genetic algorithm calculation is terminated when the 2000th generation is reached. The results of the simulation are shown in Figures 3-4, where the Figure 3(a) shows the optimal trajectory of the spacecraft attitude, (b)-(d) show the optimal trajectory of the relative angle of the solar arrays, the two end points of the curve are the initial and final configuration of the system, Figure 4 (a)-(c) show the trajectory of optimal control input of the solar arrays. In these figures, the solid curve is the calculation result obtained by the genetic algorithm, the dashed curve is the calculation result obtained by the Gauss-Newton iteration method (Ge and Liu, 1997).

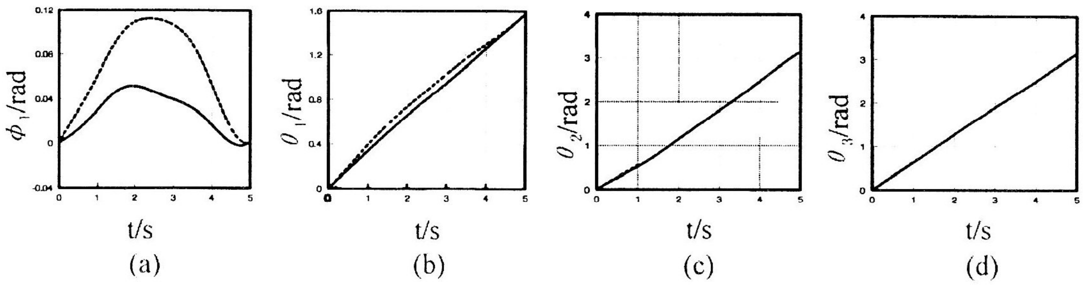

Figure 3. The Optimal Trajectory of the Spacecraft and Solar Arrays

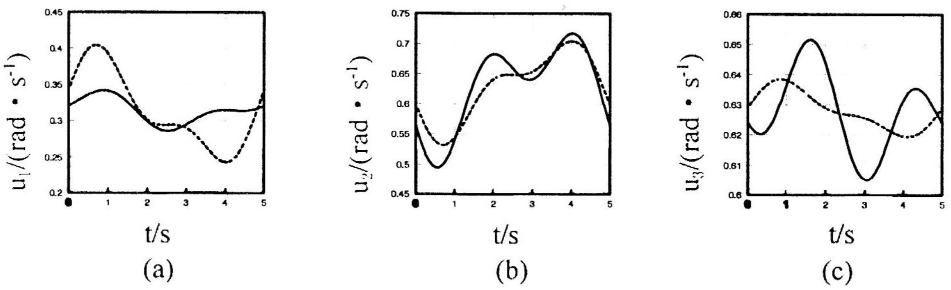

Figure 4. The Optimal Control Input for Solar Arrays

6 Conclusion

1. It is a new and useful attempt to introduce the genetic algorithm into optimal control. The results of a simulation calculation show that the genetic algorithm is effective in solving the optimal control problem of a spacecraft system with stretchable solar arrays.

2. In the genetic algorithm, the selection of floating-point representations solves the contradiction between the precision demand and the amount of calculation. Then it is benefitial to an optimal analysis of the parameters.

3. For traditional methods of optimal control search for the optimal solution, it is necessary that the target function has good continuity and differentiability. The genetic algorithm used in this paper requires one fitness function only, and no differential or other assistant information, and it has wide adaptability.

4. Genetic algorithm is used to study the optimal control problem of the attitude of a spacecraft with solar arrays in this paper, at the same time it also supplies a new way to solve other optimal control problems.

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