

# Effect of Surface Rheology on Contact Displacement

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*The paper deals with the mathematical modelling of the effect of surface rheology on the greatest contact displacement. The displacement defined by contact rigidity is determined. It is established that roughness creep has an essential influence on the greatest contact displacement for some combinations of materials of cylinders and the rheology parameters of their surface.*

## 1 Introduction

Plastic deformation has a leading role in the creation of the real contact area in some cases of loading. In general, the plastic yield of an asperity is accompanied by creep. The rheology processes take place for a highly loaded contact of machine parts and lead to modification of the contact area, pressure, and rigidity. The increased accuracy of mechanisms stimulates the development of methods of the theoretical analysis of the influence of basic geometric parameters of machine parts and their surfaces on the rigidity of joints (Demkin, 1981; Levina, 1971). It is necessary to note that many theoretical and experimental works are devoted to the solution of this problem. But these empirical and semiempirical solutions are established only for some combinations of geometric and technological parameters of the body surfaces (Levina, 1971).

It is necessary to point out that the mechanics of rough surfaces has a number of peculiarities. So the problem of contact of two rough bodies can be reduced to the problem of smooth and rough bodies (Demkin, 1981). In addition the account of the contact of real surfaces has been done for the element of surface in the limits of which it is possible to consider pressure as constant in this theory (Demkin, 1981). It is important to note that we can define the probability characteristics of surface roughness on this element. This explains methodical complexities of solving contact problems for real surfaces.

In this paper we describe a method of theoretical definition of the greatest contact displacement of rough cylinders which allow to take into account the rheology parameters of their surfaces.

## 2 Rough Surface Problem

Consider an elastic isotropic plate with a rough cylindric hole of radius  $R$ . A smooth elastic disk of radius  $r$  is put into it. It will be assumed that  $\varepsilon^2$  and  $\varepsilon/R$  with  $(\varepsilon = R - r > 0)$  are small values. A force  $Y$  acts along the  $y$ -axis (Figure 1). Since the displacement defined by creep of roughness slowly varies in time, this problem can be considered as a static problem concerning a parameter of additional displacements which depend on time  $t$  (Chigarev, 1998). The temperature  $T$  inside the hole is assumed to be constant. Friction has little influence on the normal contact pressure. Therefore, we shall assume that it is absent in the contact area  $L$  (Teply, 1983).

In the area of contact we have

$$\varepsilon + u_{1,t}^{**} \cos(\zeta) + v_{1,t}^{**} \sin(\zeta) = u_{2,t}^{**} \cos(\zeta) + (v_{2,t}^{**} - \delta_t - \varepsilon) \sin(\zeta) \quad (1)$$

where  $u_{n,t}^{**}, v_{n,t}^{**}$  ( $n = \overline{1,2}$ ) are components of the displacements of the plate with the hole ( $n = 1$ ) and for the elastic disk ( $n = 2$ );  $\delta_t = v_{2,t}^{**}(3\pi/2) - v_{1,t}^{**}(3\pi/2)$  is the displacement of the disk center.

Let

$$u_{n,t}^{**} = u_{n,t}^* + u_{n,t} \quad v_{n,t}^{**} = v_{n,t}^* + v_{n,t}$$

where  $u_{n,t}^*, v_{n,t}^*, u_{n,t}^*, v_{n,t}^*$  are displacements of the basic material and additional displacements determined by roughness creep, respectively.

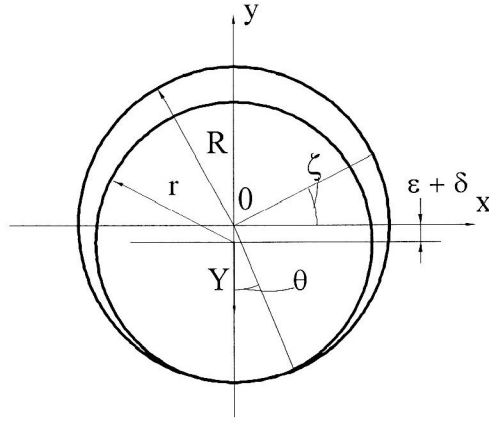


Figure 1. Scheme of Body Locations

It is important to note that the creep of roughness does not essentially depend on developing of contact area (entrance in contact new micro irregularities) (Demkin, 1981). We shall assume that the elastic radial displacement in the area of contact, being determined by the deformation of a micro-irregularity, is given by the following expression (Chigarev, 1998):

$$v_r^* = u_1^* \cos(\zeta) + v_1^* \sin(\zeta) - u_2^* \cos(\zeta) - v_2^* \sin(\zeta) = h\varepsilon_r(\theta) = \Delta_r(\cos(\theta) - \cos(\alpha_r)) \quad \theta \in [-\alpha_r, \alpha_r] \quad (2)$$

where  $h$  is the greatest height of roughness ( $h/R$  is a small value);  $\varepsilon_r(\theta)$  is a correction of radial deformation of the profile of hole defined by creep of roughness,  $\alpha_r$  is a contact half-angle.

The averaged Steclov's values of deformations and pressure in limits of base length (Figure 2) are

$$\varepsilon_{\beta,r}^*(\theta) = \begin{cases} \frac{1}{2\beta} \int_{\alpha_r-2\beta}^{\alpha_r} \varepsilon_r^*(\eta) d\eta & \theta \in [\alpha_r - \beta, \alpha_r] \\ \frac{1}{2\beta} \int_{\theta-\beta}^{\theta+\beta} \varepsilon_r^*(\eta) d\eta & \theta \in [-\alpha_r + \beta, \alpha_r - \beta] \\ \frac{1}{2\beta} \int_{-\alpha_r}^{-\alpha_r+2\beta} \varepsilon_r^*(\eta) d\eta & \theta \in [-\alpha_r, -\alpha_r + \beta] \end{cases}$$

$$p_{\beta,r}^*(\theta) = \begin{cases} \frac{-1}{2\beta} \int_{\alpha_r-2\beta}^{\alpha_r} \sigma_r^*(\eta) d\eta & \theta \in [\alpha_r - \beta, \alpha_r] \\ \frac{-1}{2\beta} \int_{\theta-\beta}^{\theta+\beta} \sigma_r^*(\eta) d\eta & \theta \in [-\alpha_r + \beta, \alpha_r - \beta] \\ \frac{-1}{2\beta} \int_{-\alpha_r}^{-\alpha_r+2\beta} \sigma_r^*(\eta) d\eta & \theta \in [-\alpha_r, -\alpha_r + \beta] \end{cases}$$

where  $2R\beta$  ( $\alpha_r \geq \beta$ ) is a base length of a measurement of characteristics of asperity,  $\sigma_r^*(\eta), \varepsilon_r^*(\eta)$  are distributions of microstress and microdeformations for asperity ( $\sigma_r^*, \varepsilon_r^*$  are integrable functions). The roughness is assumed to be in the condition of

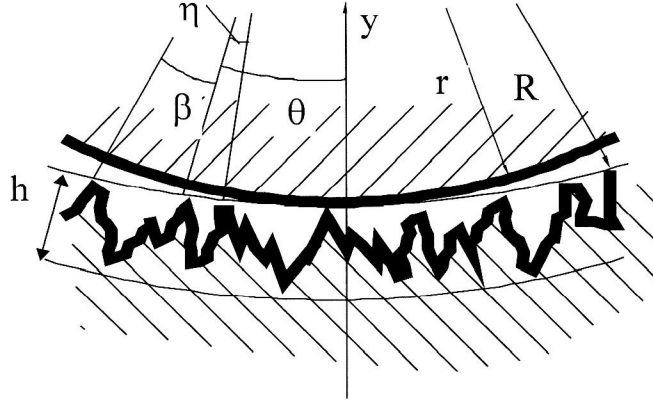


Figure 2. Scheme of Interaction.

plastic yield, therefore, we assume that its material is incompressible. In the case of nonlinear creep of asperity we get (Johnson, 1987)

$$\varepsilon_{\beta,t}^*(\theta) = \left( \frac{p_{\beta,t}^*(\theta)}{b \cdot HB} \right)^{\frac{1}{\lambda}} + \frac{1}{(t_{HB}^m \cdot b \cdot HB)^{\frac{1}{\lambda}} t_{HB}} \int_0^t (t-\tau)^{\frac{m}{\lambda}} \frac{\partial}{\partial \tau} \left[ p_{\beta,t}^*(\theta)^{\frac{1}{\lambda}} \right] d\tau$$

where  $HB$  is Brinell hardness;  $t_{HB}$  is a time for test on Brinell hardness;  $b, \lambda$  are coefficients of a bearing area curve;  $m$  is a constant defined by an experiment (Demkin, 1981);  $t$  is the time ( $t \geq t_{HB}$ ). Hence we get (Chigarev, 1998)

$$\varepsilon_{\beta,t}^*(\theta) = \left( \frac{p_{\beta,t}^*(\theta)}{b \cdot HB} \right)^{\frac{1}{\lambda}} \left( \frac{t}{t_{HB}} \right)^{\frac{m}{\lambda}} \left( 1 - O\left(\frac{m}{\lambda}\right) \right) \quad (3)$$

On the other hand, from equation (3) we obtain

$$\tilde{\varepsilon}_{\beta,t}^* \approx \left( \frac{\tilde{p}_{\beta,t}^*}{b \cdot HB} \right)^{\frac{1}{\lambda}} \left( \frac{t}{t_{HB}} \right)^{\frac{m}{\lambda}} \left( 1 + O\left( \frac{(\lambda-1)}{2\alpha_t \lambda^2} \int_0^{\alpha_t} \left( \frac{p_{\beta,t}^*(\eta)}{\tilde{p}_{\beta,t}^*} - 1 \right)^2 d\eta - \frac{m}{\lambda} \right) \right) \quad (4)$$

where

$$\tilde{\varepsilon}_{\beta,t}^* = \frac{1}{\alpha_t} \int_0^{\alpha_t} \varepsilon_{\beta,t}^*(\eta) d\eta \quad \tilde{p}_{\beta,t}^* = \frac{-1}{\alpha_t} \int_0^{\alpha_t} \sigma_{\beta,t}^*(\eta) d\eta$$

As the bodies are in the equilibrium at time  $t$  then

$$\varepsilon_{\beta,t}^*(\theta) = \varepsilon_{\beta,t}(\theta) \quad p_{\beta,t}^*(\theta) = p_{\beta,t}(\theta)$$

where  $\varepsilon_{\beta,t}(\theta), p_{\beta,t}(\theta)$  are average Steclov's values of correction of deformation  $\varepsilon_t(\theta)$  and pressure  $p_t(\theta)$  ( $p_t(\theta) = -\sigma_t(\theta)$ ) in the limits of the base length for smooth bodies. Therefore equation (4) is also valid for these functions.

Functions  $\varepsilon_t(\theta), \sigma_t(\theta)$  are continuous on  $[-\alpha_t, \alpha_t]$  and indefinite - differentiable on  $]-\alpha_t, \alpha_t[$ . Then taking into account equation (4) we can obtain the following approximation

$$\tilde{\varepsilon}_t \approx \left( \frac{\tilde{p}_t}{b \cdot HB} \right)^{\frac{1}{\lambda}} \left( \frac{t}{t_{HB}} \right)^{\frac{m}{\lambda}} (1 + O(U))$$

where

$$U = \frac{(\lambda-1)}{2\lambda^2\alpha_t} \int_0^{\alpha_t} \left( \frac{p_{\beta,t}(\eta)}{\tilde{p}_{\beta,t}} - 1 \right)^2 d\eta - \frac{m}{\lambda} + \frac{\beta^2}{2\lambda\alpha_t} \left| \frac{1}{\tilde{p}_t} \frac{dp_t(\theta)}{d\theta} \right|_{\theta=\alpha_t-\beta} + \frac{\beta^2}{2\alpha_t} \left| \frac{d\varepsilon_t(\theta)}{d\theta} \frac{1}{\tilde{\varepsilon}_t} \right|_{\theta=\alpha_t-\beta}$$

$$\tilde{p}_t = \frac{-1}{\alpha_t} \int_0^{\alpha_t} \sigma_t(\eta) d\eta \quad \tilde{\varepsilon}_t = \frac{1}{\alpha_t} \int_0^{\alpha_t} \varepsilon_t(\eta) d\eta$$

$U$  does not exceed 12% of researched values (Chigarev, 1998).

Therefore we get

$$\Delta_t \approx \frac{\alpha_t}{\sin(\alpha_t) - \alpha_t \cos(\alpha_t)} h \left( \frac{P}{b HB 2R \sin(\alpha_t)} \right)^{\frac{1}{\lambda}} \left( \frac{t}{t_{HB}} \right)^{\frac{m}{\lambda}}$$

Hence

$$\delta_t = v_{2,t}(3\pi/2) - v_{1,t}(3\pi/2) + \Delta_t(1 - \cos(\alpha_t)) \quad (5)$$

### 3 Basic Formulae for Elastic Contact of Cylinders

It's known (Teplý, 1983) that the stress state of an isotropic plate can be defined as

$$\begin{aligned} \sigma_{\zeta,t}^n + \sigma_{\rho,t}^n &= 2[\Phi_{n,t}(w) + \overline{\Phi_{n,t}(w)}] \\ \sigma_{\zeta,t}^n - \sigma_{\rho,t}^n + 2i\tau_{\rho\zeta,t}^n &= 2e^{2i\zeta} [\overline{w}\Phi'_{n,t}(w) + \Psi_{n,t}(w)] \end{aligned} \quad (6)$$

where  $w = z = \rho e^{i\zeta}$  ( $\rho \in [R, +\infty]$ ),  $w = s = \rho e^{i\zeta}$  ( $\rho \in [0, r]$ ) ;  $i = \sqrt{-1}$  ;  $\Phi_{n,t}(w), \Psi_{n,t}(w)$  ( $n = 1, 2$ ) are analytical functions in the exterior of the hole ( $n = 1$ ) and the interior of the disk ( $n = 2$ );  $\sigma_{\zeta,t}^n, \sigma_{\rho,t}^n$  are the normal components of stress;  $\tau_{\rho\zeta,t}^n$  is a tangential stress. Since friction is absent in the contact area hence we obtain (Teplý, 1983)

$$\Phi_{1,t}(z) = \frac{\kappa_1}{2\pi(1+\kappa_1)} \frac{iY}{z} - \frac{1}{2\pi i} \int_L \frac{\sigma_{R,t}(\tau) d\tau}{\tau - z} = -\frac{iY}{2\pi(1+\kappa_1)} \frac{1}{z} + \sum_{k=2}^{\infty} \frac{1}{\pi} \int_0^{\alpha_0} \sigma_{R,t}(\theta) \cos(k\theta) d\theta \frac{(-iR)^k}{z^k} \quad (7)$$

$$\begin{aligned} \Phi_{2,t}(s) &= \frac{-iY}{2\pi(1+\kappa_2)} \frac{1}{s} - \frac{iY}{2\pi(1+\kappa_2)} \frac{s}{r^2} + \frac{1}{2\pi i} \int_L \frac{\sigma_{r,t}(\xi) d\xi}{\xi - s} - \frac{1}{4\pi i} \int_L \frac{\sigma_{r,t}(\xi) d\xi}{\xi} = \\ &= -\frac{iY}{2\pi(1+\kappa_2)} \frac{1}{s} + \frac{(\kappa_2-1)iY}{2\pi(1+\kappa_2)} \frac{s}{r^2} + \frac{1}{2\pi} \int_0^{\alpha_0} \sigma_{r,t}(\theta) d\theta + \sum_{k=2}^{\infty} \frac{1}{\pi} \int_0^{\alpha_0} \sigma_{r,t}(\theta) \cos(k\theta) d\theta \frac{s^k}{(-ir)^k} \end{aligned} \quad (8)$$

$$\Psi_{n,t}(w) = \frac{R_n^2}{w^2} \Phi_{n,t}(w) + \frac{R_n^2}{w^2} \overline{\Phi_{n,t}\left(\frac{R_n^2}{w}\right)} - \frac{R_n^2}{w} \Phi'_{n,t}(w) \quad (9)$$

where  $Y$  is a principal vector of forces,  $R_1 = R$  and  $R_2 = r$  ;  $\sigma_t(\theta) = \sigma_{R,t}(\theta) = \sigma_{r,t}(\theta), \theta \in [-\alpha_0, \alpha_0]$ ,  $\tau = Re^{i\zeta}, \xi = re^{i\zeta}$  and (Kravchuk, 1998)

$$\sigma_r(\theta) = -Y \frac{\sqrt{2}}{R} \left[ \gamma_2 \frac{2}{\pi} + \frac{\gamma_1}{\alpha_r - \cos(\alpha_r) \sin(\alpha_r)} \right] \sqrt{\cos(\theta) - \cos(\alpha_r) \cos(\theta/2)} +$$

$$+ 2 \left[ Y \left( \frac{\gamma_3}{\pi} - \frac{\gamma_1 \cos(\alpha_r)}{R(\alpha_r - \cos(\alpha_r) \sin(\alpha_r))} \right) + \gamma_4 b_r + \gamma_5 (\varepsilon - \Delta_r \cos(\alpha_r)) \right] \ln \left[ \frac{\sqrt{1 + \cos(\theta)} - \sqrt{\cos(\theta) - \cos(\alpha_r)}}{\sqrt{1 + \cos(\alpha_r)}} \right]$$

$$\gamma_1 = \frac{(G_{1,2} - \nu_2 G_{2,2}) E_1 R r - (G_{1,1} - \nu_1 G_{2,1}) E_2 R^2}{2(R^2 E_2 G_{1,1} + r^2 E_1 G_{1,2})} \quad \gamma_2 = \frac{(1 + \nu_2) E_1 R r + \kappa_1 (1 + \nu_1) E_2 R^2}{4(R^2 E_2 G_{1,1} + r^2 E_1 G_{1,2})}$$

$$\gamma_3 = \frac{G_{1,2} \varepsilon R E_1}{2r(1 + \kappa_2)(R^2 E_2 G_{1,1} + r^2 E_1 G_{1,2})} \quad \gamma_4 = \frac{G_{1,1} E_2}{(R^2 E_2 G_{1,1} + r^2 E_1 G_{1,2})}$$

$$\gamma_5 = \frac{E_1 E_2 R}{2(R^2 E_2 G_{1,1} + r^2 E_1 G_{1,2})}$$

$$b_r = -\frac{R^2}{\pi} \int_0^{\alpha_r} \sigma_r(\theta) d\theta$$

$$Y = -2R \int_0^{\alpha_r} \sigma_r(\theta) \cos(\theta) d\theta$$

where  $\nu_n$  ( $n=1,2$ ) is Poisson's ratio of the basic material for plate ( $n=1$ ) and disk ( $n=2$ );  $E_n$ , ( $n=1,2$ ) is Young's modulus;  $G_{1,n} = (1 - \nu_n^2)$ ,  $G_{2,n} = (1 + \nu_n)$ , ( $n=1,2$ ) – for the state of plane deformation;  $G_{1,n} = G_{2,n} = 1$ ,  $n=(1,2)$  – for the state of plane stress;  $\kappa_n = 3 - 4\nu_n$ , ( $n=1,2$ ) – for the state of plane deformation;  $\kappa_n = (3 - \nu_n)/(1 + \nu_n)$ ,  $n=(1,2)$  – for state of plane stress. Beside that we have

$$\frac{\partial v_{\rho,t}^n}{\partial \rho} = \frac{1}{E} (G_{1,n} \sigma_{\rho,t}^n - \nu_n G_{2,n} \sigma_{\zeta,t}^n) \quad (10)$$

where  $v_{\rho,t}^n$ , ( $n=1,2$ ) are radial displacement for the basic material of plate and disk, respectively;  $\rho$  is a distance up to a zero coordinate.

#### 4 Contact Displacement in the Problem of Elastic Isotropic Disk and Plate with Hole

From equations (1), (5)-(10) (Johnson, 1987; Timoshenko, 1951) we get

$$\delta_r = \frac{Y}{\pi} \left[ \frac{\nu_1 (\kappa_1 - 1) G_{2,1} + (\kappa_1 + 3) G_{1,1}}{2(\kappa_1 + 1) E_1} \ln \left[ \frac{\rho_1}{R} \right] + \frac{\nu_2 (\kappa_2 - 1) G_{2,2} + (\kappa_2 + 3) G_{1,2}}{2(\kappa_2 + 1) E_2} \ln \left[ \frac{r}{\rho_2} \right] + \right.$$

$$\left. + \frac{(\kappa_1 - 1)(1 + \nu_1)}{4(\kappa_1 + 1) E_1} + \frac{(\kappa_2 - 1)(G_{1,2} - 3\nu_2 G_{2,2})}{4(\kappa_2 + 1) E_2} \right] + (1 + \nu_1) \frac{C_{1,t}}{E_1 R} + (G_{1,2} - \nu_2 G_{2,2}) \frac{C_{2,t} r}{E_2} + \Delta_r (1 - \cos(\alpha_r)) \quad (11)$$

where

$$C_{1,t} = -R^2 A_{0,t} - R^2 \left[ \sum_{k=2}^{\infty} \frac{2(G_{1,1} - \nu_1 G_{2,1})(k+1) + (1 + \nu_1) 2k}{(1 + \nu_1)(k^2 - 1)} A_{k,t} \right]$$

$$C_{2,t} = -A_{0,t} - \left[ \sum_{k=2}^{\infty} \frac{2(G_{2,1} - \nu_2 G_{2,2})(k-1) + (1 + \nu_2) 2k}{(G_{2,1} - \nu_2 G_{2,2})(k^2 - 1)} A_{k,t} \right]$$

$$A_{k,t} = \frac{1}{\pi} \int_0^{\alpha_r} \sigma_r(\theta) \cos(k\theta) d\theta$$

$$k = 0, 2, 3, \dots$$

$\rho_1$  is a constant which defines the distance where the displacements in the plate are equal to zero;  $\rho_2$  is a constant which defines the distance where the displacements in the disk are equal to zero (in a coordinate system connected with the disk centre) (Timoshenko, 1951).

It is important to emphasize, that first and second items of equation (11) are defined by a solution of the problem for a point force in a continuous plate. It explains the presence of a logarithmic singularity. It defines displacements which are determined by the deformation of the whole plate and disk. The third and fourth items of equation (11) are defined by absence of tangential stress on the contour of hole and disk. The other items are defined by contact rigidity (local displacements) of the plate with hole and the disk.

Hence we can define the greatest contact displacement as

$$\delta_{cont,t} = (1 + \nu_1) \frac{C_{1,t}}{E_1 R} + (G_{1,2} - \nu_2 G_{2,2}) \frac{C_{2,t} r}{E_2} + \Delta_t (1 - \cos(\alpha_t)) \quad (12)$$

It has been established that the obtained dependence of  $\delta_{cont,o} / \varepsilon$  on the contact half-angle for a smooth hole and a rigid disk defined by equation (12), is analogous to the dependence defined by Shtaerman's scheme of loading (Teply, 1983) when  $\alpha_t = \alpha_o \in [0, \pi/6]$  (Figure 3). In this case the value of stress of a separation

$$\sigma_\zeta(\rho, \zeta) \Big|_{(\rho, \zeta)=(R, 0)} = \frac{-2}{\pi} \int_0^{\alpha_o} \sigma(\theta) d\theta$$

has a small influence on the contact rigidity. Its influence on contact displacements is essentially increased when  $\alpha_o > \pi/6$  (Figure 3). It has been established that the value of the contact displacement is defined by plastic yield of asperity in a time  $t_{HB}$  and further is not changed for steel, hardened disk and plate. The results of theoretical solution of the problem show that the roughness creep has essential influence on the greatest contact displacement for some combinations of materials of cylinders and rheology parameters of their surfaces (Figure 4).

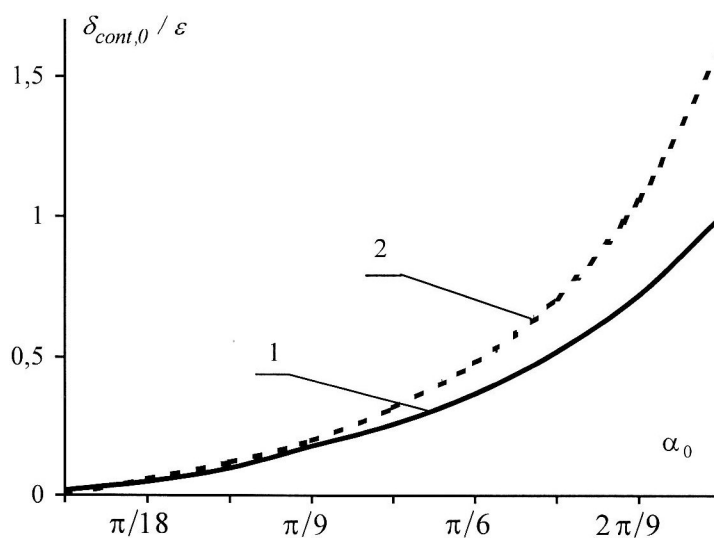


Figure 3. The connection between  $\delta_{cont,o} / \varepsilon$  and  $\alpha_o$  for smooth hole and rigid disk: 1 is defined by equation (12); 2 is defined by Shtaerman (Teply, 1983).

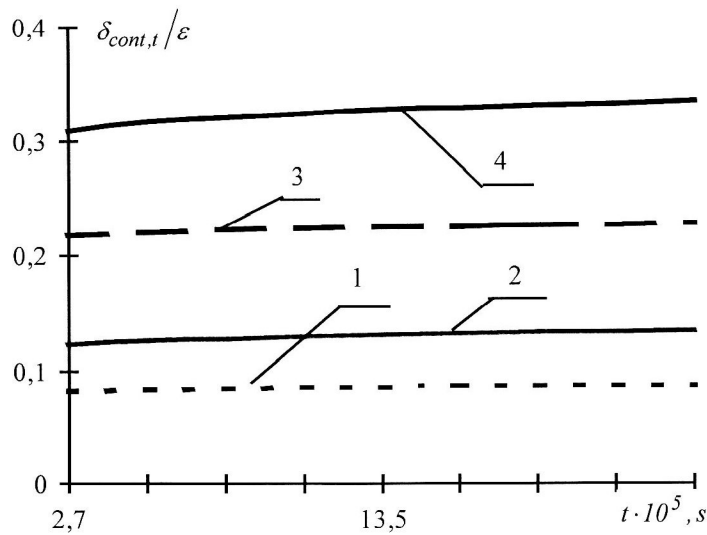


Figure 4. The theoretical connection between  $\delta_{cont,t}/\varepsilon$  and time  $t$  for smooth steel disk ( $E_1 = 210 \cdot 10^9$  N/m<sup>2</sup>,  $\nu_1 = 0.3$ ) and rough hole:

- 1 - the plate is lead ( $E_2 = 17 \cdot 10^9$  N/m<sup>2</sup>,  $\nu = 0.3$ ,  $HB = 40 \cdot 10^6$  N/m<sup>2</sup>,  $m = 0.065$ ,  $h = 12.5 \cdot 10^{-6}$  m,  $\lambda = 2$ ,  $b = 0.642$ ,  $t_{HB} = 10$  s,  $T = 293^0$  K (Demkin, 1981)) and  $P = 100$  N ;
- 2 - the plate is tin ( $E_2 = 50 \cdot 10^9$  N/m<sup>2</sup>,  $\nu_2 = 0.3$ ,  $HB = 45 \cdot 10^6$  N/m<sup>2</sup>,  $m = 0.096$ ,  $h = 12.5 \cdot 10^{-6}$  m,  $\lambda = 2$ ,  $b = 0.642$ ,  $t_{HB} = 10$  s,  $T = 293^0$  K (Demkin, 1981)) and  $P = 260$  N ;
- 3 - the plate is lead and  $P = 900$  N ;
- 4 - the plate is tin and  $P = 2400$  N ;

## 5 Conclusions

The method of theoretical definition of the greatest contact displacement of rough cylinders which allow to take into account the rheology parameters of their surfaces is worked out.

Roughness creep has an essential influence on the greatest contact displacement for some combinations of materials of cylinders and rheology parameters of their surfaces.

## Literature

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