TECHNISCHE MECHANIK, Band 19, Heft 3, (1999), 187-195
Manuskripteingang: 30. November 1998

Parametric Vibrations of Viscoelastic Cylindrical Shell under Static and Periodic Axial Loads

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Low-frequency parametric vibrations of a viscoelastic cylindrical shell subjected to axial static and additional periodic loads are studied. It is assumed that the shell is noncircular and the load is non-uniform in the circumferential direction. It is supposed that for the weak parametric excitation the shell vibrations are localized near the weakest generatrix on the shell surface. By using Fourier transformations over the circumferential co-ordinate and the multiple scale method with respect to time, the solutions of the shell equations are constructed in the form of functions that decrease quickly outside a small neighbourhood of the weakest line. The region of instability of the shell is determined with regard to the viscosity.

1 Introduction

To the present time a lot of investigations on the parametrical instability of thin cylindrical shells have been carried out. For example, vibrations of shells subjected to various combinations of static and periodic loads have been considered by Yao (1963), Wenzke (1963), Vijayaraghavan and Evan-Iwanowski (1967), and Grundman (1970). However, the majority of the obtained results concern ideal shells with constant parameters. It has been known that in this case the parametric vibrations are accompanied by the formation of waves covering the whole surface of the shell, and the problem of dynamic instability reduces to the Mathieu equation with coefficients that are functions of a static bifurcation load and a fundamental frequency of a shell.

In this paper we examine the parametric instability of non-circular thin cylindrical shells, which experience static and additional periodic axial loads. Both load components are inhomogeneous in the circumferential direction, and the frequency of excitation is close to double the fundamental frequency of the shell. The case is considered when vibrations are characterized by the localization of modes near the weakest (Tovstik, 1995) generatrix on the shell surface.

This investigation is a continuation of Mikhasev's article (1997). The main goal of this paper is to study the special case that cannot be examined by the methods that have been used by Mikhasev (1997). In addition, the influence of a viscous damping coefficient on the main instability region is studied here.

2 Setting a Problem

We consider the thin viscoelastic non-circular cylindrical shell that is sufficiently thin for the applicability of both the assumptions of the classical shell theory and the asymptotic methods. The orthogonal co-ordinate system (x, φ) is assumed as shown in Figure 1 so that the first quadratic form of the middle surface has the form $R^2 (ds^2 + d\varphi^2)$. Here $s = x R^{-1}$ is a dimensionless longitudinal co-ordinate ($0 \leq s \leq l = L/R$), R is the characteristic size of the middle surface (it will be defined below), L is the shell length, φ is a circumferential co-ordinate ($\varphi_1 \leq \varphi \leq \varphi_2$). In this case the curvature radius $R_2 = R\,\chi^{-1}(\varphi)$ is variable.

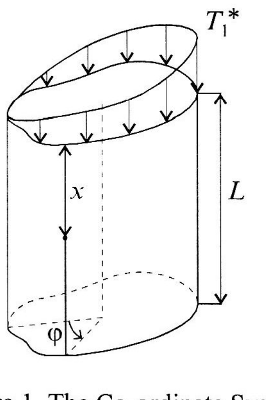

Figure 1. The Co-ordinate System

Let the shell be subjected to the combined non-uniform axial load (see Figure 1)

$$T_1^* = \varepsilon^3 E h F(\varphi, t) \qquad F(\varphi, t) = F_0(\varphi) + \varepsilon^2 \tilde{F}_1(\varphi) \cos \Omega^* t^* \tag{1}$$

where E is the Young's modulus, h the shell thickness, $\varepsilon^6 = h^2 / [12 R^2 (1 - \nu^2)]$ a small parameter, ν Poisson's ratio, Ω^* the circular frequency of the additional periodic axial load, and t^* the time.

It is supposed that the vibrations are accompanied by the formation of a large number of short waves covering the shell surface. It is assumed also that $F(\varphi, t)$ is a slowly varying function so that the stress state of the shell due to the axial stress T_1^* may be considered as the membrane one. Taking into account these assumptions, for the analysis of parametric vibrations the semi-membrane shell equations (Bolotin, 1956; Vlasov, 1958; Tovstik, 1995)

$$\begin{aligned} &\varepsilon^6 \Delta^2 w - \varepsilon^3 \chi(\varphi) \frac{\partial^2 \Phi}{\partial s^2} + \varepsilon^3 F(\varphi; t) \frac{\partial^2 w}{\partial s^2} + \frac{\partial^2 w}{\partial t^2} + \varepsilon^2 \gamma \frac{\partial w}{\partial t} = 0 \\ &\varepsilon^3 \Delta^2 \Phi + \chi(\varphi) \frac{\partial^2 w}{\partial s^2} = 0 \end{aligned} \tag{2}$$

written in the dimensionless form may be used. Here $\Delta = \partial^2 / \partial s^2 + \partial^2 / \partial \varphi^2$. The parameters ε and $\tilde{F}_1$ are connected with the corresponding magnitudes introduced in Mikhasev's paper (1997) by the following relations

$$\varepsilon = \mu^{2/3} \qquad \tilde{F}_1 = \varepsilon^{-1/2} F_1$$

The dimensionless magnitudes are introduced as follows:

$$\begin{aligned} &w = w^* / R \qquad \Phi = \Phi^* / (\varepsilon^3 E h R^2) \\ &t = t^* / t_c \qquad \Omega = \Omega^* t_c \qquad \gamma = \varepsilon^{-1/2} t_c \delta / \rho \qquad t_c = R \sqrt{\rho / E} \end{aligned} \tag{3}$$

where w^* is the normal deflection, Φ^* is the stress function, t_c is the characteristic time, δ is the viscous damping coefficient, ρ is the mass density. The functions $\chi(\varphi)$, $F_0(\varphi)$, $\tilde{F}_1(\varphi)$ are supposed to be infinitely differentiable.

Let the shell edges be joint supported so that at the edges $s = 0$, $s = l$ Navier's conditions

$$w = \frac{\partial^2 w}{\partial s^2} = \Phi = \frac{\partial^2 \Phi}{\partial s^2} = 0 \tag{4}$$

are posed. To satisfy boundary conditions (4) the solution of equations (2) is assumed to be of the form

$$\begin{aligned} &w = w_n(\varphi, t) \sin\left(\frac{p_n s}{\varepsilon^{3/2}}\right) \qquad \Phi = f_n(\varphi, t) \sin\left(\frac{p_n s}{\varepsilon^{3/2}}\right) \\ &p_n = \frac{\varepsilon^{3/2} n \pi}{l} \qquad n = 1, 2, \ldots \end{aligned} \tag{5}$$

The cases $p_n < 1$ and $p_n > 1$ have been investigated by Mikhasev (1997). Now we consider the case $p_n \approx 1$. Substituting (5) into (2) yields the sequence of equations

$$\begin{aligned} &\varepsilon^6 \frac{\partial^4 w_n}{\partial \varphi^4} - 2 \varepsilon^3 p_n^2 \frac{\partial^2 w_n}{\partial \varphi^2} + p_n^4 w_n + p_n^2 \chi(\varphi) f_n - p_n^2 F(\varphi, t) w_n + \frac{\partial^2 w_n}{\partial t^2} + \varepsilon^2 \gamma \frac{\partial w_n}{\partial t} = 0 \\ &\varepsilon^6 \frac{\partial^4 f_n}{\partial \varphi^4} - 2 \varepsilon^3 p_n^2 \frac{\partial^2 f_n}{\partial \varphi^2} + p_n^4 f_n - p_n^2 \chi(\varphi) w_n = 0 \end{aligned} \tag{6}$$

with respect to w_n, f_n for $n = 1, 2, \dots$. The subscript n in p_n, w_n, f_n will be omitted below.

It is assumed here again (Mikhasev, 1997) that vibrations are concentrated near some weakest generatrix $\varphi = \varphi_0$ that will be found below. We shall scale near this generatrix and introduce a slow time (Nayfeh, 1973):

$$\zeta = \varepsilon^{-1} (\varphi - \varphi_0) \qquad t_0 = t \qquad t_1 = \varepsilon^2 t$$

The problem is to construct a solution of equations (6) under the conditions

$$w,\ f \to 0 \quad \text{as} \quad |\zeta| \to \infty$$

Then the condition of periodicity in φ or the influence of the edges $\varphi = \varphi_1$, $\varphi = \varphi_2$ can be neglected (at least in some initial time interval in the case of the parametrical resonance).

3 Approach

The uniformly valid asymptotic solution has the form

$$w(\varphi, t, \varepsilon) \cong \sum_{k=0}^{\infty} \varepsilon^k \, w_k(\varsigma, t_0, t_1) \qquad f(\varphi, t, \varepsilon) \cong \sum_{k=0}^{\infty} \varepsilon^k f_k(\varsigma, t_0, t_1) \tag{7}$$

The functions $\chi(\varphi)$, $F_0(\varphi)$, $\widetilde{F}_1(\varphi)$ are expanded into series in the neighbourhood of the generatrix $\varphi = \varphi_0$. For example,

$$\chi(\varphi) = \chi(\varphi_0) + \varepsilon\, \chi'(\varphi_0)\, \zeta + 0{,}5\, \varepsilon^2 \chi''(\varphi_0)\, \zeta^2 + \dots \tag{8}$$

Let $\chi(\varphi_0) = 1$, then the characteristic size is the curvature radius $R = R_2(\varphi_0)$ at the weakest generatrix. As the case $p \approx 1$ is considered here, it may be assumed

$$p = 1 + \varepsilon^2 \widetilde{p} \tag{9}$$

where $\widetilde{p} \sim 1$ as $\varepsilon \to 0$.

Substituting equations (7)—(9) into equation (6), with eliminating the functions f_k, produces the sequence of equations

$$\sum_{j=0}^{k} D_j\, w_{k-j} = 0 \qquad k = 0, 1, 2, \dots \tag{10}$$

Here

$$D_0 = \frac{\partial^2}{\partial t_0^2} + [2 - F_0(\varphi_0)]$$

$$D_1 = [2\, \chi'(\varphi_0) - F_0'(\varphi_0)]\, \zeta$$

$$D_2 = 4 \frac{\partial^4}{\partial \zeta^4} + 4\, \chi'(\varphi_0)\, \zeta \frac{\partial^2}{\partial \zeta^2} + 4\, \chi'(\varphi_0) \frac{\partial}{\partial \zeta} + \frac{1}{2} [2\, \chi''(\varphi_0) + 2\, \chi'^2(\varphi_0) - F_0''(\varphi_0)]\, \zeta^2 +$$

$$+ 2\widetilde{p}[2 - F_0(\varphi_0)] + 2 \frac{\partial^2}{\partial t_0\, \partial t_1} + \gamma \frac{\partial}{\partial t_0} - \widetilde{F}_1(\varphi_0) \cos \Omega t$$

3.1 Zeroth and First Order Approximations

In the zeroth order approximation ($k = 0$), one has the homogeneous equation $D_0\, w_0 = 0$ which has the solution

$$w_0\,(\zeta, t_0, t_1) = w_{0,s}\,(\zeta, t_1) \sin \omega_0\, t_0 + w_{0,c}\,(\zeta, t_1) \cos \omega_0\, t_0 \tag{11}$$

$$\omega_0^2 = 2 - F_0\,(\varphi_0)$$

Here ω_0 / t_c is the zeroth order approximation of the fundamental frequency (Thompson and Willson, 1979) of the thin cylinder under the axial load $T_1{}^* = \varepsilon^3\, E\, h\, F_0\,(\varphi_0)$. It can be seen that $F_0 < F_b = 2$, where the value $F_b = 2$ corresponds to the classical axial buckling force $T_1{}^{cr} = 2\, \varepsilon^3\, E\, h$ (Timoshenko, 1936).

For $k = 1$, equation (10) leads to the non-homogeneous differential equation

$$D_0\, w_1 = -\, D_1\, w_0 \tag{12}$$

The right part of this equation, with (11) in mind, generates secular terms with respect to t_0. To eliminate these terms it should be assumed

$$2\chi'(\varphi_0) - F_0'(\varphi_0) = 0 \tag{13}$$

This equation allows to determine the weakest line $\varphi = \varphi_0$. Let us consider the special case $\chi'(\varphi_0) = F_0'(\varphi_0) = 0$ here. Then equation (12) is transformed into the following one

$$D_0\, w_1 = 0 \tag{14}$$

Its solution can be found in form (11)

$$w_1\,(\zeta, t_0, t_1) = w_{1,s}\,(\zeta, t_1) \sin \omega_0\, t_0 + w_{1,c}\,(\zeta, t_1) \cos \omega_0\, t_0 \tag{15}$$

3.2 Second Order Approximation

In the second order ($k = 2$) approximation, we obtain the non-homogeneous differential equation

$$D_0\, w_2 = -\, D_2\, w_0 - D_1\, w_1 \tag{16}$$

It can be seen that condition (13) yields identically $D_1\, w_1 \equiv 0$. Therefore equation (16) can be rewritten as

$$D_0\, w_2 = -\, D_2\, w_0 \tag{17}$$

It is known that the parametric instability occurs in the case when the ratio Ω^* / ω^* is equal or close to one of the following values (Yao, 1963)

$$\frac{\Omega^*}{\omega^*} = \frac{2}{1};\ \frac{2}{2};\ \frac{2}{3};\ \frac{2}{4};\ \frac{2}{5};\ \ldots$$

where ω^* is the fundamental frequency of the shell.

Let us consider here the basic case when the excitation frequency is close to the double fundamental frequency:

$$\Omega = 2\omega_0 + \varepsilon^2 \sigma \qquad \sigma \sim 1 \text{ as } \varepsilon \to 0 \tag{18}$$

Then equation (17) takes the form

$$D_0 w_2 = -(N_s \sin \omega_0 t_0 + N_c \cos \omega_0 t_0) + \\ + \frac{1}{2} \widetilde{F}_1(\varphi_0) [w_{0,s} \sin(3\omega_0 t_0 + \sigma t_1) + w_{0,c} \cos(3\omega_0 t_0 + \sigma t_1)] \tag{19}$$

where

$$N_s = 4\frac{\partial^4 w_{0,s}}{\partial \varsigma^4} + \frac{1}{2}[2\chi''(\varphi_0) - F_0''(\varphi_0)]\varsigma^2 w_{0,s} + 2\widetilde{p}[2 - F_0(\varphi_0)]w_{0,s} - \\ -\left(2\frac{\partial}{\partial t_1} + \gamma\right)\omega_0 w_{0;c} + \frac{1}{2}\widetilde{F}_1(\varphi_0)\left(w_{0,s} \cos \sigma t_1 + w_{0,c} \sin \sigma t_1\right)$$

$$N_c = 4\frac{\partial^4 w_{0,c}}{\partial \varsigma^4} + \frac{1}{2}[2\chi''(\varphi_0) - F_0''(\varphi_0)]\varsigma^2 w_{0,c} + 2\widetilde{p}[2 - F_0(\varphi_0)]w_{0,c} + \\ +\left(2\frac{\partial}{\partial t_1} + \gamma\right)\omega_0 w_{0,s} + \frac{1}{2}\widetilde{F}_1(\varphi_0)\left(w_{0,s} \sin \sigma t_1 - w_{0,c} \cos \sigma t_1\right) \tag{20}$$

The first term of the right part in equation (19) generates secular terms. The absence condition of these terms yields a system of equations

$$N_s = N_c = 0 \tag{21}$$

The last ones may be rewritten as the differential equation with respect to the vector $\mathbf{X} = (w_{0,s}, w_{0,c})^T$

$$4\frac{\partial^4 \mathbf{X}}{\partial \zeta^4} + \frac{1}{2}[2\chi''(\varphi_0) - F_0''(\varphi_0)]\zeta^2 \mathbf{X} + 2\widetilde{p}\,\omega_0^2 \mathbf{X} + \left(2\frac{\partial}{\partial t_1} + \gamma\right)\omega_0 \mathbf{E}' \mathbf{X} + \mathbf{G}\mathbf{X} = 0 \tag{22}$$

$$\mathbf{E}' = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \qquad \mathbf{G} = \frac{1}{2}\widetilde{F}_1(\varphi_0)\begin{pmatrix} \cos \sigma t_1 & \sin \sigma t_1 \\ \sin \sigma t_1 & -\cos \sigma t_1 \end{pmatrix}$$

By applying the Fourier transformation

$$\mathbf{X}(\zeta; t_1) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} \mathbf{X}^F(\eta; t_1)\, e^{i\eta\zeta}\, d\eta \qquad \mathbf{X}^F(\eta; t_1) = (\omega_s^F(\eta, t_1); \omega_c^F(\eta; t_1))^T$$

to equation (22), the lower order differential equation

$$4\eta^4 \mathbf{X}^F - \frac{1}{2}[2\chi''(\varphi_0) - F_0''(\varphi_0)]\frac{\partial^2 \mathbf{X}^F}{\partial \eta^2} + 2\widetilde{p}\,\omega_0^2 \mathbf{X}^F + \left(2\frac{\partial}{\partial t_1} + \gamma\right)\omega_0 \mathbf{E}' \mathbf{X}^F + \mathbf{G}\mathbf{X}^F = 0 \tag{23}$$

is obtained. The substitution of

$$x = \tau\, \eta \qquad \tau = \sqrt{2}\, [2\, \chi''(\varphi_0) - F_0''(\varphi_0)]^{-1/6}$$

into equation (23) gives the equation

$$4\, \tau^{-4} \left(x^4\, \mathbf{X}^F - \frac{\partial^2 \mathbf{X}^F}{\partial x^2} + \frac{1}{2} \tilde{p}\, \omega_0^2\, \tau^4\, \mathbf{X}^F \right) + \left(2 \frac{\partial}{\partial t_1} + \gamma \right) \omega_0\, \mathbf{E}'\, \mathbf{X}^F + \mathbf{G}\, \mathbf{X}^F = 0 \tag{24}$$

It is required to find a non-trivial solution of equation (24) so that $|\mathbf{X}^F| \to 0$ as $|x| \to \infty$.

We consider the problem

$$y''_{xx} + (\lambda - x^4)\, y = 0 \qquad y \to 0 \quad \text{as} \quad x \to \pm\infty \tag{25}$$

Let λ_m and y_m be eigenvalues and eigenfunctions ($m = 0, 1, 2, \ldots$) respectively for problem (25). For example, the first two eigenvalues are $\lambda_0 \approx 1{,}125$, $\lambda_1 \approx 3{,}750$.

The solution of equation (24) is

$$\mathbf{X}^F = y_m\,(x)\; \mathbf{Y}_m(t_1) \qquad \mathbf{Y}_m = (S_m(t_1),\, C_m(t_1))^{\mathrm{T}} \tag{26}$$

The substitution of equation (26) into equation (24) leads to the homogeneous system of differential equations with respect to $\mathbf{Y}_m$:

$$\dot{\mathbf{Y}}_m - \mathbf{A}_m\,(t_1)\; \mathbf{Y}_m = 0 \tag{27}$$

where

$$\mathbf{A}_m\,(t_1) = \begin{pmatrix} -\frac{\gamma}{2} - a_1 \sin \sigma\, t_1 & -\, a_2 \\ m + a_1 \cos \sigma\, t_1 & a_{2,m} + a_1 \cos \sigma\, t_1 \\ -\frac{\gamma}{2} + a_1 \sin \sigma\, t_1 & \end{pmatrix} \tag{28}$$

$$a_1 = \frac{\tilde{F}_1\,(\varphi_0)}{4\, \omega_0} \qquad a_{2,m} = \frac{\lambda_m}{2\, \omega_0}\, [2\, \chi''(\varphi_0) - F_0''(\varphi_0)]^{2/3} + \tilde{p}\, \omega_0$$

4 Free Vibrations

If $\tilde{F}_1 \equiv 0$, then system (27) has a solution in the explicit form. In this case the approximate formula

$$w^* = R \sin\left(\frac{\pi n s}{l}\right) \left[\exp\left\{ -\frac{\varepsilon^2 \gamma}{2} t \right\} \cdot \left(c_1 \sin \omega t + c_2 \cos \omega t\right) \cdot Z_m(\varphi, \varepsilon) + O(\varepsilon) \right] \tag{29}$$

for the mode of free vibrations takes place, where

$$Z_m(\varphi, \varepsilon) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} y_m(\sqrt{2}\,[2\,\chi''(\varphi_0) - F_0''(\varphi_0)]^{-1/6}\,\eta)\,\exp\{i\eta\varepsilon^{-1}(\varphi - \varphi_0)\}d\eta$$

$$\omega = \sqrt{2 - F_0(\varphi_0)} + \varepsilon^2 \left\{ \frac{\lambda_m [2\chi''(\varphi_0) - F_0''(\varphi_0)]^{2/3}}{2\sqrt{2 - F_0(\varphi_0)}} + \tilde{p}\sqrt{2 - F_0(\varphi_0)} \right\} + O(\varepsilon^4) \tag{30}$$

Here $\omega^* = \omega / t_c$ is the fundamental frequency depending on the parameters λ_m and $\tilde{p}$. Equations (29), (30) supplement the analogous ones for the cases $p < 1$, $p > 1$ (Mikhasev, 1997). It should be reminded that the error of these formulas increases while $F_0 \to F_b$ (in this research $F_b = 2$).

5 Parametric Instability

The shell normal deflection is defined by the approximate formula

$$w^* = R\sin\left(\frac{\pi n s}{l}\right)\left[\left(S_m(\varepsilon^2 t)\sin\omega_0 t_0 + C_m(\varepsilon^2 t)\cos\omega_0 t_0\right)\cdot Z_m(\varphi, \varepsilon) + O(\varepsilon)\right] \tag{31}$$

Depending on the correlation of parameters a_1, $a_{2,m}$, ε, γ, formula (31) defines the unstable or stable vibrations of the shell in a small neighbourhood of the weakest line $\varphi = \varphi_0$. In the case of absence of a viscosity ($\gamma = 0$) the region of instability for equation (27) has been established by Mikhasev and Kuntsevich (1997). In Figure 2 the boundaries of this region are shown by dashed lines. We have found the region of instability of equation (27) in case of non-zero viscous damping coefficient. In Figure 2 this region is shaded. For points (σ, a_1) lying in the shaded area, the amplitudes $S_m(\varepsilon^2 t)$ and $C_m(\varepsilon^2 t)$ of parametric oscillations are functions growing infinitely with time, and outside of this area the amplitudes are bounded.

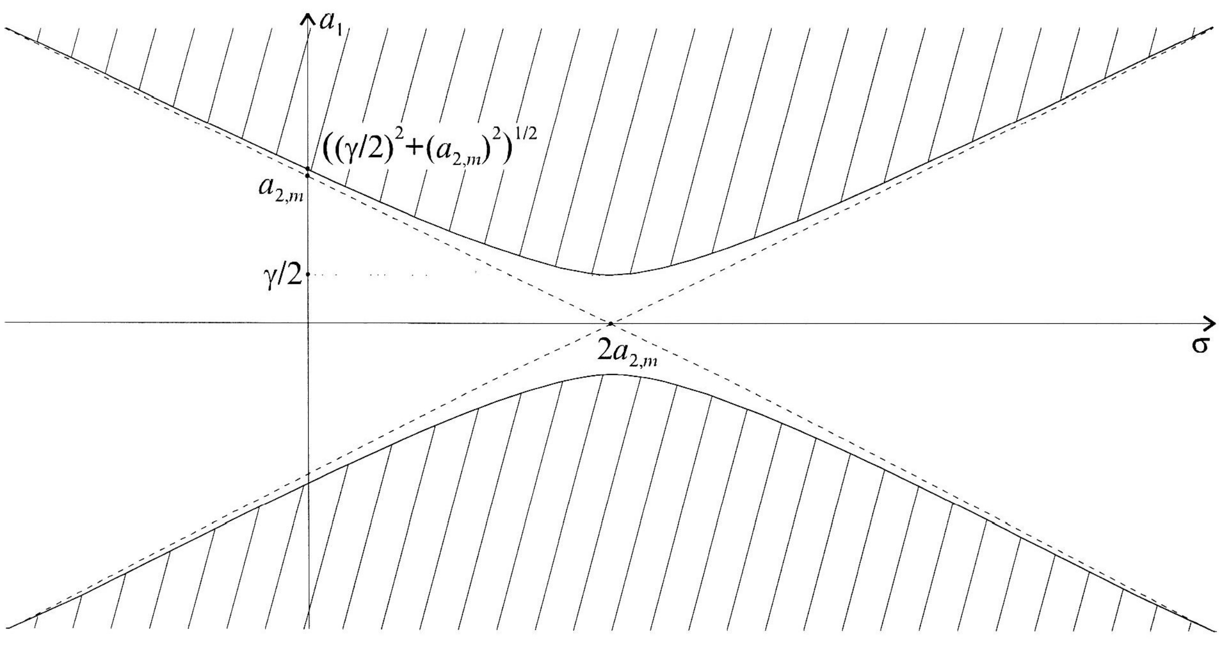

Figure 2. The Main Regions of Instability

It should be kept in mind that solution (31) is an asymptotic one, and the regions of stability and instability in Figure 2 are valid while a_1, $a_{2,m}$, $\sigma \sim 1$ as $\varepsilon \to 0$.

6 Conclusions and Example

Taking into account the results obtained earlier (Mikhasev, 1997), solutions (29), (31) allow to study both the free and parametric vibrations of the thin viscoelastic non-circular cylinder under inhomogeneous in the circumferential direction static and addition periodic axial loading in the case when the shell has the weakest generatrix and the frequency of excitation is about the double fundamental frequency. The main regions of instability in Figure 2 permit easily to determine the intervals $\Omega^- \leq \Omega \leq \Omega^+$ for the dimensionless excitation frequency $\Omega = \Omega^* \, t_c$ corresponding to the dynamic instability

$$\Omega^{\pm} = 2\,\omega_0 + \sqrt[3]{\frac{h^2}{12\,R^2\,(1-\nu^2)}}\;\sigma^{\pm} \quad \text{at} \quad p \approx 1$$

$$\Omega^{\pm} = 2\,\omega_0 + \sqrt[4]{\frac{h^2}{12\,R^2\,(1-\nu^2)}}\;\sigma^{\pm} \quad \text{at} \quad p < 1 \quad \text{or} \quad p > 1$$

where $\sigma^{\pm} = 2\,(a_{2,m} \pm a_1)$. Moreover, in the first case ($p \approx 1$), the parameters ω_0, a_1, $a_{2,m}$ are found by equations (11), (28), while in the second case ($p < 1$ or $p > 1$) the correspondence formulas derived in Mikhasev's paper (1997) should be used.

For example, let us consider the circular elastic cylinder ($\chi = 1, \gamma = 0$) under the combined non-uniform load (1), where $F_0\,(\varphi) = 0{,}5\,(1 + \cos\varphi)$, F_1 is constant. Here the weakest generatrix is the line $\varphi_0 = 0$, where the axial force T_1^* is maximum. The dependence of the parameters p, $\Omega^{\pm}$ and the dimensionless fundamental frequencies ω on numbers n and m for $F_1 = 1$ and $F_1 = 2$ are shown in Table 1. It can be seen that increasing the amplitude F_1 of the periodic component of the axial force results in extending the boundaries of instability.

n	p	$F_1 = 1$						$F_1 = 2$					
		$m = 0$			$m = 1$			$m = 0$			$m = 1$		
		Ω^-	Ω^+	ω	Ω^-	Ω^+	ω	Ω^-	Ω^+	ω	Ω^-	Ω^+	ω
1	0.332	0.693	0.712	0.351	0.769	0.787	0.389	0.684	0.721	0.351	0.760	0.769	0.389
2	0.665	1.327	1.363	0.627	1.358	1.394	0.688	1.308	1.381	0.627	1.339	1.413	0.688
3	0.997	1.981	2.036	1.004	2.016	2.071	1.022	1.954	2.064	1.004	1.988	2.099	1.022
4	1.329	3.067	3.131	1.550	3.126	3.189	1.579	3.036	3.162	1.550	3.094	3.221	1.579
5	1.662	4.835	4.898	2.433	4.882	4.945	2.457	4.804	4.929	2.433	4.851	4.976	2.457

Table 1. The Dependence of the Parameters p, $\Omega^{\pm}$, ω on Numbers n, m for the Shell with $h = 0.01$, $R = 1$, $l = 0.52$, $\nu = 0.3$

Acknowledgements

This work was supported by grant BFFI #T97—142.

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