TECHNISCHE MECHANIK, Band 19, Heft 4, (1999), 249-258
Manuskripteingang: 26. Mai 1999

Solution of Creep–Damage Problems for Beams and Rectangular Plates Using the Ritz and the Finite Element Method

H. Altenbach, G. Kolarow, K. Naumenko

Starting from a constitutive model with a single damage parameter, creep-damage problems for beams and rectangular plates in bending are solved using the Ritz and the finite element method. The creep-damage material model is incorporated into the ANSYS finite element code by writing a user material subroutine. A comparison of finite element results with results based on special solution techniques is presented. Various numerical tests show the sensitivity of numerical long-term predictions to the mesh sizes and element types using standard finite elements available in the ANSYS code for creep analysis.

1 Introduction

Engineering structures operating under creep conditions are designed with respect to increased requirements of safety. One of the factors which must be considered in structural analysis is time dependent material behaviour coupled with damage evolution (Roche et al., 1992). Incorporating creep–damage models into the finite element code, predictions of time dependent stresses, strains, and damage fields can be performed with the help of numerical solution of nonlinear initial–boundary value problems (e.g. Hayhurst, 1994). The first problem arising in a creep damage analysis is the formulation of a phenomenological material model that is able to describe the sensitivity of creep strain and damage rate to the stress level, stress state, temperature level, etc. The second problem can be related to the quality of the finite element predictions, particularly in the analysis of structures of complex shapes.

Numerous finite element simulations considering creep damage effects have been made. Examples are discussed by Othman et al. (1994) and Becker et al. (1994) for notched bars, by Saanouni et al. (1989) and Murakami and Liu (1995) for thick–walled tubes and notched plates, and by Fleig (1996) for notched plates under in–plane loading. Since these examples confirm the ability of finite element simulations to predict stress redistributions and failure times with sufficient accuracy for engineering applications, a little effort has been made for the analysis of transversely loaded thin–walled structures. The structural analysis of beams and plates can be performed using the equations of beam or plate theory. By some simplifications, e.g. for geometry or loading, it is possible to formulate the creep problem, which can be solved by direct variational methods using the shape functions defined for the whole domain. Such solutions can be used for the verification of user defined material subroutines and finite element predictions, whereas the following questions require a special consideration. The first: how sensitive is the long–term prediction of plates with predominating bending stresses to the mesh sizes and the type of elements available in finite element codes for shell analysis. Even for steady state creep the deflection functions for beams and circular plates are polynomials of order significantly higher than these for the elastic solution, whereas the order of the polynomial depends on the order of the power in the classical Norton creep law (e.g. Boyle and Spence, 1983). The second question is related to the ability of the shell theory used in finite element codes (usually shear deformable theories) to represent the time dependent stress redistributions caused by damage evolution.

In this paper we solve creep–damage problems for beams and rectangular plates in bending using the Ritz and the finite element method. First we construct special solutions incorporating damage effects based on the Ritz method. These solutions are used for the verification of the ANSYS user defined creep material subroutine which we modify by including damage evolution. Based on various numerical tests we show the mesh sensitivity of long–term solutions in creep. Finally we compare the results based on different plane stress, shell and solid elements available in the ANSYS–code for the plasticity and creep analysis. For the simplicity we consider steady state loads and temperatures only. Further we do not discuss the problems of damage localization which can cause of spurious mesh dependency and which requires special regularization techniques (e.g. Saanouni et al., 1989; Murakami and Liu, 1995).

2 Constitutive Model

The constitutive model for creep behaviour can be formulated as a set of first order differential equations for the creep rate as a function of the stress tensor, the temperature, and possible internal state variables with appropriate evolution equations (Altenbach et al., 1997b). The first step in the conventional creep modelling is the formulation of an empirical function describing the sensitivity of the minimum creep rate to the stress level and temperature. Using the von Mises type potential the secondary creep constitutive equation can be written as follows (e.g. Altenbach, 1999)

$$\dot{\varepsilon}^{cr} = \frac{3}{2}\frac{f_\sigma(\sigma^{vM})}{\sigma^{vM}}\boldsymbol{s} \qquad \sigma^{vM} = \sqrt{\frac{3}{2}\boldsymbol{s}\cdot\cdot\,\boldsymbol{s}} \qquad \boldsymbol{s} = \boldsymbol{\sigma} - \frac{1}{3}\mathrm{tr}\boldsymbol{\sigma}\boldsymbol{I} \tag{1}$$

where $\dot{\varepsilon}^{cr}$ is the creep strain rate tensor, $\boldsymbol{\sigma}, \boldsymbol{s}$ are the stress tensor and deviator, respectively. σ^{vM} denotes the von Mises stress, $\boldsymbol{I}$ is the unit tensor. In most cases, the stress dependence f_σ is the power law function $f_\sigma = A\sigma^n$ with the material constants A and n determined from uniaxial creep tests at constant temperature conditions. After verification of secondary creep behaviour, the minimum creep rate functions can be modified by suitable hardening and damage variables for the description of primary and tertiary creep. Following the classical concept proposed by Kachanov (1986) and Rabotnov (1969), the creep rate equation (1) can be extended by a scalar damage parameter ω, and the damage rate is postulated to be a function of the stress and the current damage states. The constitutive and evolution equations for secondary and tertiary creep behaviour can be written using the power law functions and assuming the temperature to be constant during the creep process (e.g. Leckie and Hayhurst, 1977; Hyde et al., 1996). Modifying these equations by introducing of a time–hardening function which allows the additional description of the primary creep we finally get

$$\begin{aligned} &\dot{\varepsilon}^{cr} = \frac{3}{2}\frac{A(\sigma^{vM})^{n-1}}{(1-\omega)^n}\boldsymbol{s}t^m \qquad \dot{\omega} = \frac{B\left(\langle\sigma_\omega^{eq}\rangle\right)^\chi}{(1-\omega)^\phi}t^m \qquad \sigma_\omega^{eq} = \alpha\sigma_I + (1-\alpha)\sigma^{vM} \\ &\langle\sigma_\omega^{eq}\rangle = \sigma_\omega^{eq} \quad \text{for} \quad \sigma_\omega^{eq} > 0 \qquad \langle\sigma_\omega^{eq}\rangle = 0 \quad \text{for} \quad \sigma_\omega^{eq} \le 0 \qquad 0 \le \omega \le \omega_* \end{aligned} \tag{2}$$

Here σ_I is the maximum principal stress, B, χ, ϕ, ω_*, α, are material constants. The coefficient α controls different damage mechanisms. At the same time the model reflects different sensitivity of the damage rate to tension and compression loads.

Assuming the elastic properties to be influenced by creep-damage, the elasticity constitutive equation coupled with damage can be presented as follows

$$\sigma_{ij} = C_{ijkl}(\omega)(\varepsilon_{kl} - \varepsilon_{kl}^{cr}) \tag{3}$$

For the isotropic elastic material the elasticity tensor can be written as (Lemaitre and Chaboche, 1990)

$$C_{ijkl} = \frac{E(\omega)}{2(1-\nu^2)}\left[(1-\nu)(\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk}) + 2\nu\delta_{ij}\delta_{kl}\right] \tag{4}$$

For numerical convenience, we will use the following specification for the "damaged" Young's modulus

$$E(\omega) = E(1 - \zeta\omega) \qquad 0 \le \zeta \le \frac{1}{\omega_*} \tag{5}$$

where E is the Young modulus in the virgin state. The constant ζ controls the influence of the creep induced deterioration. For $\zeta = 0$ we obtain the classical Hooke law, for $\zeta = 1/\omega_*$ – the fully coupled approach that means a vanishing stiffness ($E = 0$) for the critical damage state.

3 Creep–Damage of Beams - Closed Estimations

First let us introduce the classical closed solution for steady state creep of a Bernoulli beam (e.g. Boyle and Spence, 1983). Neglecting the elastic strain and the damage evolution in the material model (2) the constitutive equation for the beam can be written as

$$\dot{\varepsilon}(x,z) \approx \dot{\varepsilon}^{cr}(x,z) = -\dot{w}(x)''z = A|\sigma(x,z)|^{n-1}\sigma(x,z) \tag{6}$$

where w is the deflection, x - the beam axis ($0 \le x \le l$) and z - the normal axis ($-h/2 \le z \le h/2$). From equation (6) the stress can be represented as a function of the beam curvature. The integration over the beam cross section A_c yields

$$M(x) = \int\limits_{A_c} \sigma z dA_c = I_n \left[-\frac{\dot{w}(x)''}{A} \right]^{\frac{1}{n}} \qquad \text{with} \qquad I_n = \int\limits_{A_c} |z|^{\frac{n+1}{n}} dA_c \tag{7}$$

In the case of a statically determinate beam the deflection function in steady state creep can be obtained from the equation (7). As an example we consider the beam simply supported on both edges and loaded by the uniformly distributed force q. In this case $M(x) = qx(l-x)/2$ and the equation (7) can be rewritten as

$$\dot{w}(x)'' = -\frac{A}{I_n^n}\frac{q^n}{2^n}x^n(l-x)^n \tag{8}$$

For integer values of the power n the solution is a polynomial of the order $2n+2$

$$\dot{w}(x) = \frac{A}{I_n^n}\frac{q^n}{2^n}x(l-x)\sum_{k=0}^{n}\sum_{i=0}^{n+k}\alpha_k l^{2n-i}x^i \qquad \alpha_k = (-1)^k \binom{n}{k}\frac{1}{(n+k+1)(n+k+2)}$$

Comparing with the elastic solution

$$w(x) = \frac{q}{24EI}x(x-l)(x^2-lx-l^2)$$

with I as the moment of inertia one can conclude that if the creep problem is numerically solved using variational methods the shape functions for the deflection or deflection rate should contain terms of the order $2n+2$. Even in analysis of steady state creep, and using the power law stress dependence, an accurate solution cannot be obtained with approximations justified from the elastic solution.

4 Creep–Damage of Beams - the Ritz Method

Starting from Hooke's law (3) applied to the beam bending problem (Pilkey and Wunderlich, 1994)

$$\sigma = E(1-\zeta\omega)(\varepsilon - \varepsilon^{cr}) = -E(1-\zeta\omega)(-u_0' + w''z + \varepsilon^{cr}) \tag{9}$$

with u_0 as the axial displacement of the beam centreline, the principle of virtual displacements yields

$$\begin{aligned}\int\limits_x q\delta w dx = \int\limits_V \sigma\delta\varepsilon dV &= E\int\limits_x (I - \omega^{zzz})w''\delta w'' dx + E\int\limits_x (A_c - \omega^z)u_0'\delta u_0' dx \\ &+ E\int\limits_x \omega^{zz}\delta(u_0'w'')dx + E\int\limits_x M^{cr}\delta w'' dx - E\int\limits_x N^{cr}\delta u_0' dx\end{aligned}$$

with

$$M^{cr} = \int\limits_{A_c}(1-\zeta\omega)\varepsilon^{cr}z dA_c \qquad N^{cr} = \int\limits_{A_c}(1-\zeta\omega)\varepsilon^{cr}dA_c$$

$$\omega^z = \int\limits_{A_c}\zeta\omega dA_c \qquad \omega^{zz} = \int\limits_{A_c}\zeta\omega z dA_c \qquad \omega^{zzz} = \int\limits_{A_c}\zeta\omega z^2 dA_c$$

Assuming the creep strain and damage parameter to be known functions of coordinates x and z for the fixed time variable t we can formulate the following functional

$$\begin{aligned}\Pi_t(w,u_0) &= \frac{1}{2}E\int\limits_x (I-\omega^{zzz})w''^2dx + \frac{1}{2}E\int\limits_x (A_c-\omega^z)u_0'^2dx + E\int\limits_x \omega^{zz}u_0'w''dx \\ &+ E\int\limits_x M^{cr}w''dx - E\int\limits_x N^{cr}u_0'dx - \int\limits_x qwdx\end{aligned}$$

This leads to the problem to find such functions w and u_0 that yield an extremal value of the functional. The solutions for fixed time t can be represented as

$$w(x) = a_0^w\varphi_0^w(x) + \sum_{i=1}^{N} a_i^w\varphi_i^w(x) \qquad u_0(x) = \sum_{i=0}^{M} a_i^u\varphi_i^u(x) \tag{10}$$

For the simply supported beam discussed above the shape functions can be formulated as follows. $\varphi_0^w(x) = x(x-l)(x^2-lx-l^2)$ is the first approximation following from the elastic solution. For $\varphi_i^w(x)$ we use the functions satisfying the boundary conditions $w=0$ and $M=0$ for $x=0$ and $x=l$

$$\varphi_i^w(x) = x^{i+2}(l-x)^{i+2} \tag{11}$$

Assuming $u_0 = 0$ for the edge $x=0$ the functions $\varphi_i^u(x) = x^{i+1}$ can be used. The unknown coefficients a_i can be found using the Ritz method from the set of linear algebraic equations

$$\frac{\partial \Pi_t}{\partial a_k} = 0 \qquad \begin{bmatrix} \boldsymbol{R}^{ww} & \boldsymbol{R}^{wu} \\ \boldsymbol{R}^{uw} & \boldsymbol{R}^{uu}\end{bmatrix}\begin{bmatrix}\boldsymbol{a}^w \\ \boldsymbol{a}^u\end{bmatrix} = \begin{bmatrix}\boldsymbol{f}^w \\ \boldsymbol{f}^u\end{bmatrix} \tag{12}$$

with

$$\begin{aligned}
R_{kj}^{ww} &= E\int\limits_0^l (I-\omega^{zzz})\varphi_k^{w\prime\prime}\varphi_j^{w\prime\prime}dx \qquad k=1,\ldots,N \qquad j=1,\ldots,M \\
R_{kj}^{wu} &= E\int\limits_0^l \omega^{zz}\varphi_k^{w\prime\prime}\varphi_j^{u\prime}dx \qquad k=1,\ldots,N \qquad j=1,\ldots,M \\
R_{kj}^{uu} &= E\int\limits_0^l (A_c-\omega^z)\varphi_k^{u\prime}\varphi_j^{u\prime}dx \qquad k=1,\ldots,M \qquad j=1,\ldots,M \\
R_{kj}^{uw} &= E\int\limits_0^l \omega^{zz}\varphi_k^{u\prime}\varphi_j^{w\prime\prime}dx \qquad k=1,\ldots,M \qquad j=1,\ldots,N \\
f_k^w &= q\int\limits_0^l \varphi_k^w dx - E\int\limits_0^l M^{cr}\varphi_k^{w\prime\prime}dx \qquad k=1,\ldots,N \\
f_k^u &= E\int\limits_0^l N^{cr}\varphi_k^{u\prime}dx \qquad k=1,\ldots,M
\end{aligned}$$

The stress $\sigma(x,z,t)$ can be calculated from equation (9). For the known values of the stress and the damage parameter the constitutive model (2) yields the rates of creep strain and damage for the time t. From these the new values for time $t+\Delta t$ are calculated using the implicit time integration procedure

$$\begin{aligned}
&\varepsilon^{cr}(x,z,t+\Delta t) = \varepsilon^{cr}(x,z,t) + \Delta t[(1-\theta)\dot\varepsilon^{cr}(x,z,t) + \theta\dot\varepsilon^{cr}(x,z,t+\Delta t)] \\
&\omega(x,z,t+\Delta t) = \omega(x,z,t) + \Delta t[(1-\theta)\dot\omega(x,z,t) + \theta\dot\omega(x,z,t+\Delta t)] \\
&\varepsilon^{cr}(x,z,0) = 0 \qquad \omega(x,z,0) = 0 \qquad \omega(x,z,t) < \omega_*
\end{aligned}$$

For the calculation of the creep force N^{cr}, the creep moment M^{cr} as well as damage averages ω^z, ω^{zz} and ω^{zzz} in equations (9) the Gauss method with 9 integration points in the thickness direction is used. For the calculation of the matrices $\boldsymbol{R}^{mn}$ and the right parts of equations (12) the Simpson quadrature rule with N_s integration points in the beam axis x is used. The values of creep strain and damage for a current time step t are stored in all integration points along the beam axis and over the thickness direction for calculations in the next time step.

The accuracy of the numerical solution depends on the number of shape functions in equation (10), on the number of integration points and on the time step size. The sensitivity of creep solutions time step sizes has been previously studied by Altenbach and Naumenko (1997) on the plate bending problems. In the following numerical studies we will put our attention on the convergence of the time dependent solution with different number of shape functions in equation (9).

As a first example we consider the above discussed simply supported beam with a rectangular cross section $h \times b$. For the calculation we set $q = 60$ N/mm, $l = 1000$ mm, $h = 80$ mm, $b = 30$ mm and use the creep–damage material model (2) with material constants identified by Kowalewski et al. (1994) for an aluminium alloy: $A = 3.511 \cdot 10^{-31}$ MPa^{-n}/h^{m+1}, $B = 1.960 \cdot 10^{-23}$ MPa$^{-\chi}$/h^{m+1}, $\chi = 8.220$, $n = 11.034$, $\phi = 12.107$, $m = -0.3099$ and $E = 7.1 \cdot 10^4$ MPa, $\nu = 0.3$. For the first example we neglect the influence of creep damage on elasticity setting $\zeta = 0$ in equation (5). Further we assume the damage rate to be the same for tensile and compressive loading setting $\alpha = 0$ in equations (2). Both assumptions lead to a significant simplification of the numerical procedures. Using the first one we can set $\boldsymbol{R}^{uw} = \boldsymbol{R}^{wu} = 0$ in equation (12) and keep the matrices $\boldsymbol{R}^{ww}$, $\boldsymbol{R}^{uu}$ constant during the time step calculation. The second results in a symmetric stress redistribution across the z direction for the arbitrary time step which leads to $N^{cr} = 0$ and $\boldsymbol{f}^u = 0$. Consequently, the number of functions in the approximation for the displacement $u_0(x)$ has no influence on the numerical solutions.

Figure 1 shows time dependent solutions for maximum deflection and maximum stress obtained by different number of polynomial terms (11) in equations (10). The time step solutions are performed

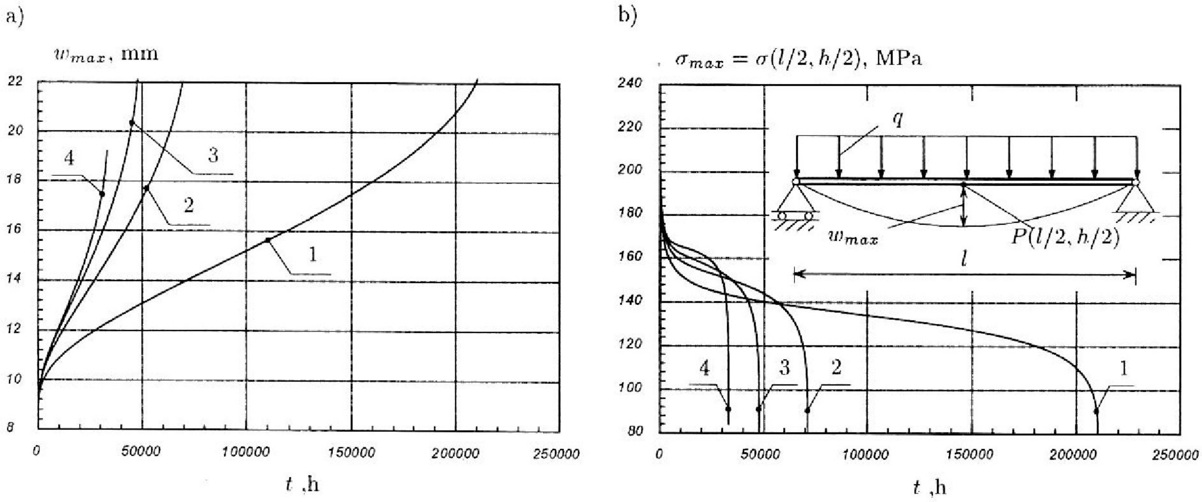

Figure 1. Solutions for a Bernoulli Beam basing on the Ritz Method using Functions (11): a) Time Variation of Maximum Deflection, b) Time Variation of Maximum Stress, 1 – Approximation using Elastic Deflection Function, 2 – $N = 1$, 3 – $N = 2$, 4 – $N = 8$

until the critical damage is achieved in one integration point. This condition of termination of the time step solution is $\omega(x_f, z_f, t_*) > 0.9$, where the integration point $F(x_f, z_f)$ can be specified as a point of failure initiation and the time step t_* as the time to failure initiation.

Since all approximations of the deflection function used for the Ritz method yield the exact elasticity solution, the life–time predictions are strongly sensitive to the number of shape functions, Figure 1. Even for the case of a statically determinate beam one can conclude that the approximation adjusted for the elastic solution cannot be used for the creep–damage predictions. The difference between the life–time predictions based on an approximation following from elasticity solution (fourth order polynomial), curves 1, and approximation which follows from the above discussed steady state creep solution (polynomial of the order 24, curves 4) is approximately six times. From the Figure 2 can be seen that the creep–damage solution converges with the increasing number of shape functions. The approximations with

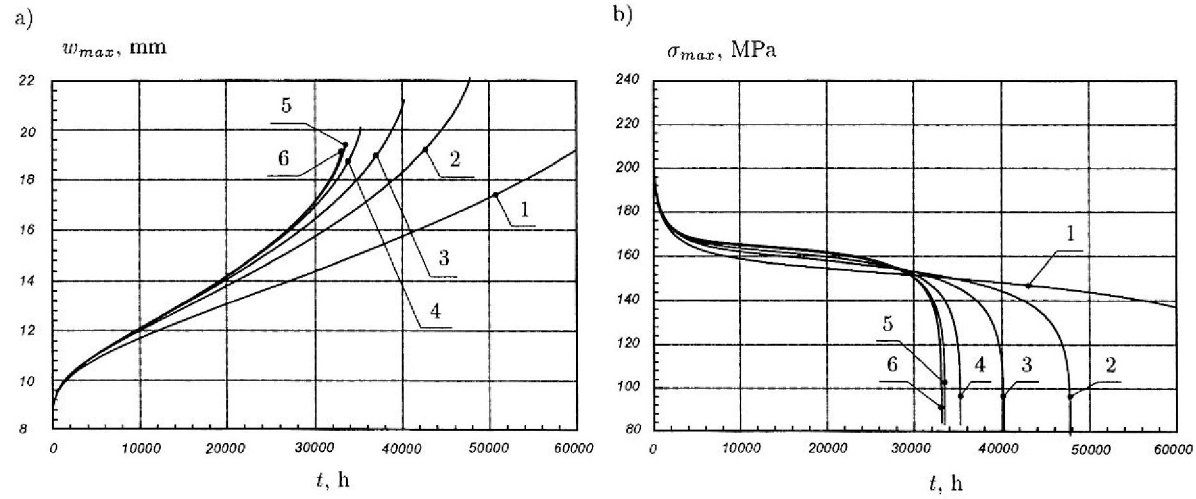

Figure 2. Convergence of Time Dependent Solution for a Bernoulli Beam using Functions (11): a) Time Variation of Maximum Deflection, b) Time Variation of Maximum Stress, 1 – $N = 1$, 2 – $N = 2$, 3 – $N = 3$, 4 – $N = 5$, 5 – $N = 7$, 6 – $N = 8$

$N = 5$, $N = 7$ and $N = 8$ yield the same solutions for the transition and steady state creep state but differ in the last stage. Since the difference between the cases 4 and 6 (approximately 17%) is not acceptable comparing with the accuracy of the available material description one can conclude that the order of approximation adjusted to the steady state creep solution can be enough for the numerical life–time predictions using continuum damage mechanics approach.

5 Creep–Damage of Beams - Finite Element Solution

Using the ANSYS User Programmable Features (ANSYS User's Manual Volume I – IV , 1994) we have incorporated the material model (2) into the ANSYS finite element code by modifying a user creep material subroutine. For details of time integration and equilibrium iterations methods used in ANSYS for creep calculations we refer to ANSYS User's Manual Volume I – IV (1994) and Zienkiewicz and Taylor (1991).

For the testing of the implemented material law we introduce the beam considered below as the first example. For the meshing we use the 4 node shell element SHELL 43 available for creep computations. This element is based on the Reissner–Mindlin type shell theory and contains 2 × 2 Gauss points in the plane and five integration points for the thickness. The automatical time stepping feature with a minimum time step 0.1 h has been used. Figure 3 shows the solutions obtained with different meshes. The convergent solution (case 6) has been obtained with 200 elements and 96 time steps. This solution is in very good agreement with the solution obtained using the Ritz method, Figure 2. Similarly to the sensitivity of the solution to the degree of the polynomial discussed below the mesh sensitivity of the finite element solution can be observed. The mesh justified for the elastic solution (case 1) is not fine enough. The solutions in the cases 3–6 are approximately the same in the steady state and differ slightly in the last stage. Since for the large structures including stress singularities it is difficult to test the mesh sensitivity performing the whole creep damage calculations, one can conclude that the mesh adjusted to convergent solution for the steady state creep can provide an adequate (at least qualitatively correct) solution within the framework of continuum damage mechanics.

In the previous examples we assumed the damage evolution is the same for tensile or compressive loading. The next example illustrates the time dependent solutions of a beam setting damage rate to zero in the Gauss points with negative maximum principal stress. In order to compare the influence of the element type on the creep damage solutions we perform the calculations using the element SHELL 43 with 80 elements and the 4–node plane stress element PLANE 42 meshing the beam with 80 elements along the beam axis and 4 elements along the normal axis. Figure 4 shows the time dependent solutions for maximum deflections obtained with shell elements, plane stress elements and using the Ritz method.

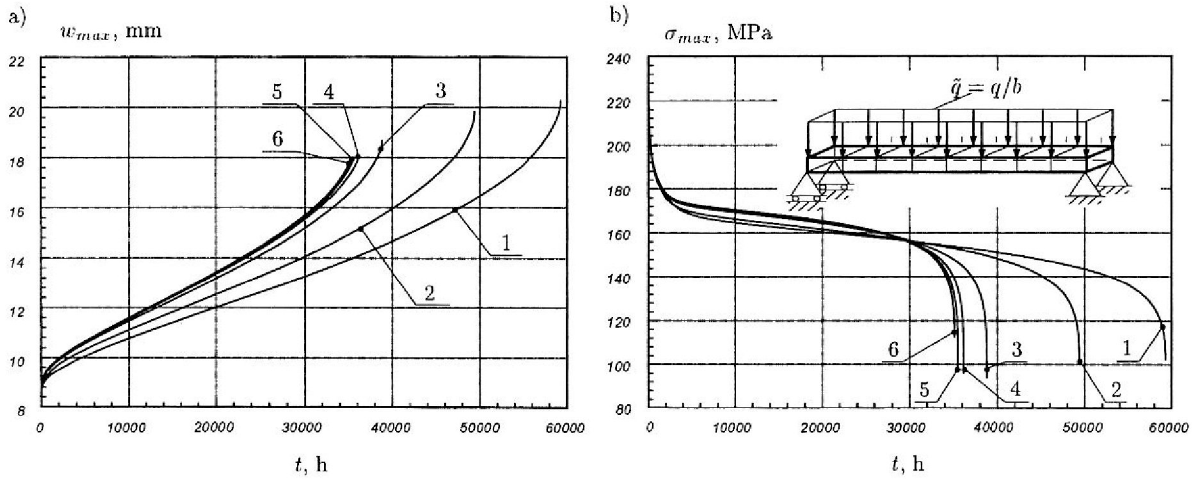

Figure 3. Solutions for a Bernoulli Beam using the ANSYS Code with Plastic Shell Elements SHELL 43: a) Time Variation of Maximum Deflection, b) Time Variation of Maximum Stress, 1 – 8 Elements, 2 – 10 Elements, 3 – 20 Elements, 4 – 40 Elements, 5 – 80 Elements, 6 – 200 Elements

It can be seen that if the sensitivity of the material damage to the kind of stress state is taken into account results obtained from shell and plane elements differ substantially. The difference in the life-time prediction is approximately 30%. For the comparison two convergent solutions based on the Ritz method applied for a beam with the Bernoulli assumptions are plotted. The first solution is obtained with 5 Gauss points for the thickness integration (curves 3), and the second with 9 Gauss points (curves 4). Both solutions are obtained with the same number of the shape functions, $N = 8$, $M = 8$ in equations (10). The solution with 9 Gauss points is in better agreement with the plane stress solution, which is seen particularly on the time variation of maximum negative stress, Figure 5, b). The solution with 5 Gauss points agrees better with the solution from the shell element. Since the element SHELL 43 contains 5 Gauss points for the thickness integration (see ANSYS User's Manual Volume I – IV , 1994), one can conclude that more accurate thickness integration should be performed using shell elements in creep damage computations.

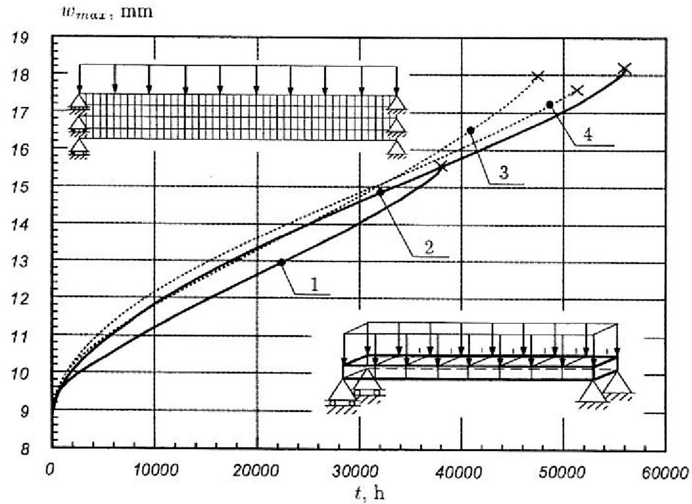

Figure 4. Time Dependent Deflection of a Beam: 1 – SHELL 43, 2 – PLANE 42, 3 – Ritz Method with 5 Gauss Points for Thickness Integration, 4 – Ritz Method with 9 Gauss Points for Thickness Integration

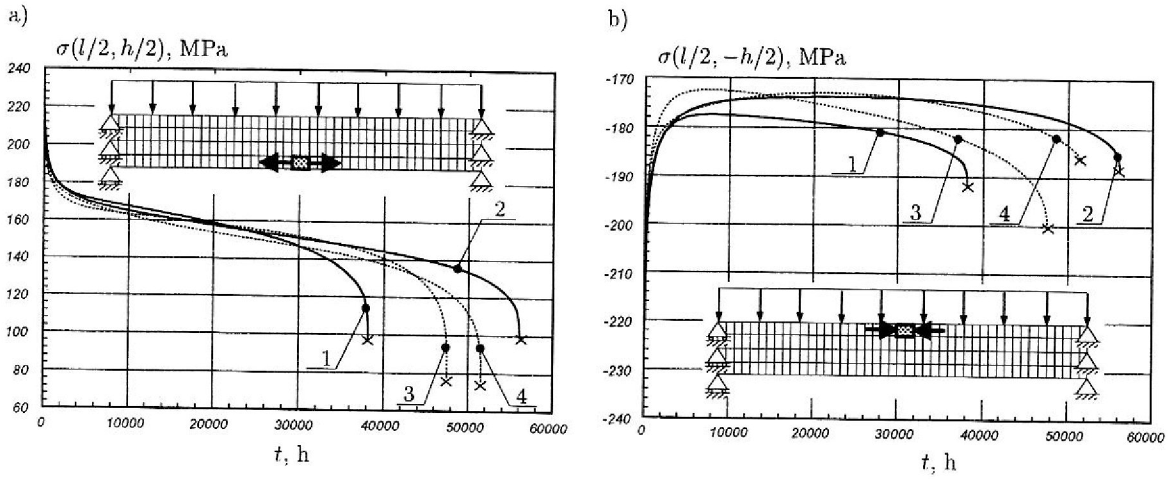

Figure 5. Time Dependent Stresses for a Beam: a) Positive Stress on the Bottom Side, b) Negative Stress on the Top Side, Curve Symbols see Figure 4

6 Creep–Damage of Plates in Bending

As the second mechanical model for the verification of user material subroutine let us introduce the thin plate problem. Analogously to the equations formulated for a beam we use the governing equations for a thin plate using the Kirchhoff's assumptions and including geometrical nonlinear terms (von Kármán's plate theory). The solution procedure for a plate in bending is based on the Ritz method and is similar to that formulated for a beam. Details are presented in Altenbach and Naumenko (1997) and Altenbach et al. (1997a)

In Figure 6 the time–dependent maximum deflection for a clamped square plate transversely loaded by $q = 10$ MPa uniformly distributed on a square area is presented. The plate length is $l = 800$ mm and

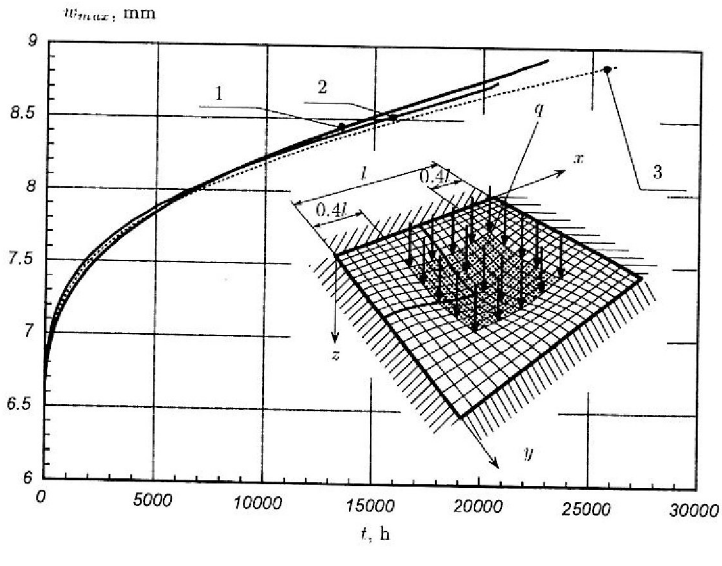

Figure 6. Time Dependent Maximum Deflection of a Clamped Square Plate: 1 – SHELL 43, 2 – SOLID 95, 3 – Solution Basing on the Ritz Method (Kirchhoff's Theory)

the plate thickness - $h = 27$ mm. The material constants are the same as in the foregoing examples for beams. In the first example we set $\alpha = 0$ in equations (2) assuming the damage evolution to be dependent on the von Mises stress only. This dependency assumes the same damage rate for the tensile and compressive loading. The first solution (Figure 6, curve 1) has been obtained using the rectangular shell elements and a 20 × 20–element mesh for a quarter of the plate. The second solution (Figure 6, curve 2) is based on 20–nodes solid element (3D) and 20 × 20–elements for a quarter of the plate and 3 elements in the thickness direction. The third solution is obtained using the Ritz solution technique. All solutions are in good agreement. In addition, the good agreement of these three solutions can be seen on Figure 7 for time variation of the von Mises stress plotted in two Gauss points (in the middle of the plate, bottom side and in the middle of the clamped edge, top side).

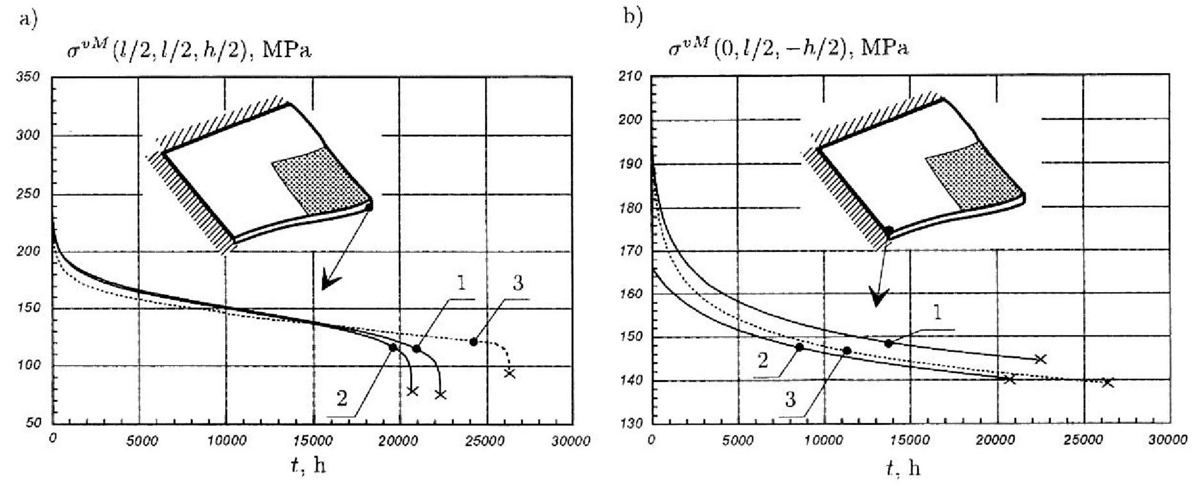

Figure 7. Time Variations of the von Mises Stress in Two Points: a) Midpoint of the Bottom Side, b) Midpoint of the Clamped Edge (Top Side), Curve Symbols see Figure 6

7 Conclusions

The standard FE-code ANSYS which allows elastic and inelastic analysis of thin–walled structures like beams, plates and shells was used for the long–term predictions with respect to the creep behaviour of materials at elevated temperatures. Due to the necessity of the simulation of the tertiary creep the Norton creep law for secondary creep was extended with a scalar damage variable. The damage was included in Kachanov's and Rabotnov's sense. This model is incorporated into the ANSYS code by developing a user creep material subroutine.

The accuracy of the creep–damage finite element predictions in thin–walled structures was estimated comparing with solutions based on the Ritz method. Various numerical tests show the sensitivity of the solutions to the mesh sizes and the element types. Solving the creep damage problem the mesh should be adjusted using steady state creep solutions rather than elastic solutions. However, if the material behaviour depends on the kind of loading (or the kind of stress state), the structure mechanics equations for beams, plates and shells have to be based on refined cross section kinematics (e.g., shear deformable theories).

Further investigations can be directed towards the use of enhanced finite elements for plates and shells. In addition, the material model must be enlarged taking into account the damage induced anisotropy.

Acknowledgement

We wish to acknowledge the support from the German Research Foundation (DFG) under Grant AL341/15-1.

Literature

1. Altenbach, H.: Classical and non-classical creep models. In: *Creep and Damage in Materials and Structures* (edited by Altenbach, H.; Skrzypek, J.), pages 45–94, Wien et al.: Springer–Verlag (1999).

2. Altenbach, H.; Morachkovsky, O.; Naumenko, K.; Sychov, A.: Geometrically nonlinear bending of thin–walled shells and plates under creep–damage conditions. Arch. Appl. Mech., 67, (1997a), 339–352.

3. Altenbach, H.; Naumenko, K.: Creep bending of thin–walled shells and plates by consideration of finite deflections. Comp. Mech., 19, (1997), 490–495.

4. Altenbach, J.; Altenbach, H.; Naumenko, K.: Lebensdauerabschätzung dünnwandiger Flächentragwerke auf der Grundlage phänomenologischer Materialmodelle für Kriechen und Schädigung. Technische Mechanik, 17, 4, (1997b), 353–364.

5. ANSYS User's Manual Volume I – IV: Swanson Analysis Systems, Inc. (1994).

6. Becker, A. A.; Hyde, T. H.; Xia, L.: Numerical analysis of creep in components. J. of Strain Analysis, 29, 3, (1994), 27–34.

7. Boyle, J. T.; Spence, J.: Stress analysis for creep. London: Butterworths (1983).

8. Fleig, T.: Lebensdaueranalyse unter Berücksichtigung viskoplastischer Verformung und Schädigung mit der Methode der Finite Elemente. Dissertation, Universität Karlsruhe (1996).

9. Hayhurst, D. R.: The use of continuum damage mechanics in creep analysis for design. J. of Strain Analysis, 25, 3, (1994), 233–241.

10. Hyde, T. H.; Xia, L.; Becker, A. A.: Prediction of creep failure in aeroengine materials under multi–axial stress states. Int. J. Mech. Sci., 38, 4, (1996), 385–403.

11. Kachanov, L.: Introduction to Continuum Damage Mechanics. Dordrecht et al.: Martinus Nijhoff Publishers (1986).

12. Kowalewski, Z. L.; Hayhurst, D. R.; Dyson, B. F.: Mechanism-based creep constitutive equations for an aluminium alloy. J. of Strain Anal., 29, 4, (1994), 309–316.

13. Leckie, F.; Hayhurst, D.: Constitutive Equations for Creep Rupture. Acta Metall., 25, (1977), 1059 – 1070.

14. Lemaitre, J.; Chaboche, J. L.: Mechanics of Solid Materials. Cambridge et al.: Cambridge University Press (1990).

15. Murakami, S.; Liu, Y.: Mesh–dependence in local approach to creep fracture. Int. J. of Damage Mech., 4, (1995), 230–250.

16. Othman, A. M.; Dyson, B. F.; Hayhurst, D. R.; Lin, J.: Continuum damage mechanics modelling of circumferentially notched tension bars undergoing tertiary creep with physically-based constitutive equations. Acta Metall. Mater., 42, 3, (1994), 597–611.

17. Pilkey, W.; Wunderlich, W.: Mechanics of Structures - Variational and Computational Methods. Boca Raton et al.: CRC Press (1994).

18. Rabotnov, Y. N.: Creep Problems in Structural Members. Amsterdam: North Holland (1969).

19. Roche, R. L.; Townley, C. H. A.; Regis, V.; Hübel, H.: Structural Analysis and Available Knowledge. In: *High Temperature Structural Design* (edited by Larson, L. H.), pages 161–180, London: Mechanical Engineering Publications (1992).

20. Saanouni, K.; Chaboche, J. L.; Lense, P. M.: On the creep crack–growth prediction by a non–local damage formulation. Eur. J. Mech., A Solids, 8, 6, (1989), 437–459.

21. Zienkiewicz, O. C.; Taylor, R. L.: The Finite Element Method. London et al.: McGraw–Hill (1991).

Addresses: Prof. Dr.-Ing. Holm Altenbach, Dr.-Ing. Konstantin Naumenko, Fachbereich Ingenieurwissenschaften, Lehrstuhl für Technische Mechanik, Martin-Luther-Universität Halle-Wittenberg, D-06099 Halle, Dr.-Ing. Georgi Kolarow, Lehrstuhl für Festigkeitslehre, TU Sofia, BG-1756 Sofia, Bulgarien