TECHNISCHE MECHANIK, Band 19, Heft 4, (1999), 341-350
Manuskripteingang: 16. August 1999

A Continuum Model for Highly Porous Aluminium Foam

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The development of manufacturing highly porous metal foams by new and cheap production processes is nearly finished and ready for industrial usage. For further development of existing as well as new applications, it is necessary to have preliminary knowledge of the mechanical behaviour of the applications under external loads. By use of the Theory of Porous Media (TPM) it is possible to describe a gas-filled foam as a two-phase material within the framework of a macroscopic, continuum mechanical model. Numerical investigations are carried out by use of the finite element method.

1 Introduction

By the development of ecological and economical effective production processes for the manufacturing of metal foams, this material becomes more and more interesting for industrial usage. Especially, the mechanical behaviour opens a wide range of new possibilities for engineering applications. The low density and high stiffness, the special behaviour of plasticity under compression, the acoustic damping for open pored foams as well as the thermal isolation aspect of closed cell foams give a new solution potential for this kind of material.
In the further context, a foam is classified as a combined structure of fluid-filled cells, where neither the volume fraction of the matrix material nor the volume fraction of the fluid material is negligible. The foam is called 'closed cell foam' when the single cells are closed, whereas it is called 'open cell foam' when the cells are interconnected and the fluid has the possibility to flow through the matrix. Furthermore, if the cell matrix material consists of metal or metal alloys, the foam is called 'metal foam'.

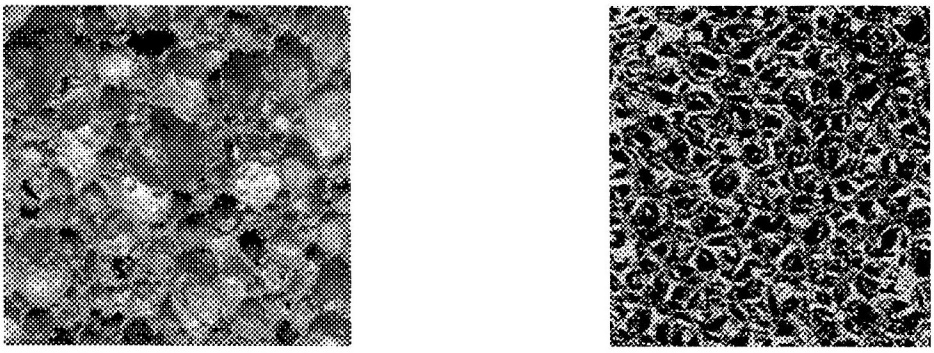

Figure 1. Closed cell (left) and open cell (right) aluminium foam.

There are four classes for the manufacturing processes of metallic foams: the pulver metalurgical process, the melt foaming process, the sputter technology and the electro chemical process (Weber, 1997). By the different processes, and, therefore, also by the different process controls, special material behaviour can be obtained by changing either the porosity or the geometry of the cells or by varying the homogeneity or the isotropy rates of the foam. These special requirements often increase the costs of the manufacturing process.
A detailed description of the manufacturing processes is given in e. g. Banhart (ed.) (1997). A lot of examinations have been made for the classification of the mechanical, acoustical, thermal and electrical behaviour. Gibson and Ashby (1997) presented an overview which allows for a comparison of metal foams with polymeric foams, which are for a longer time in practical use in real applications. Independent from the different foaming processes, a general macroscopic model is needed for the prediction of the material behaviour in components only depending on measurable macroscopic material parameters.

2 Theory of Porous Media (TPM)

Resulting from empirical examinations, it is nowadays possible to obtain detailed descriptions of the inner structure of a foam at the microscopic scale. The geometry of the individual pores and the cell walls depend on the used manufacturing process. On the one hand, new methods like the computer tomography (CT) allow for more precise classifications, but, on the other hand, even by increased computer power the calculation of real applications on the microscale is not feasible until now. Thus, statistical macroscopic continuum mechanical models are nessecary to successfully represent the behaviour of foam materials under external loads.
For a wide range of engineering aspects of multiphase materials, the continuum mechanically and thermodynamically based Theory of Porous Media (TPM) gives a general concept for a macroscopic description of the material behaviour. An overview of this theory can be found in Bowen (1980, 1982), de Boer and Ehlers (1986) and Ehlers (1989, 1993, 1996). In this paper, only a short summary of the TPM will be given.
The TPM is a phenomenological theory where the partial bodies of the mixture, the so-called phases or constituents φ^α, are in ideal disarrangement. This leads to a statistical model by a real or a virtual averaging process which results in the concept of superimposed and interacting continua.

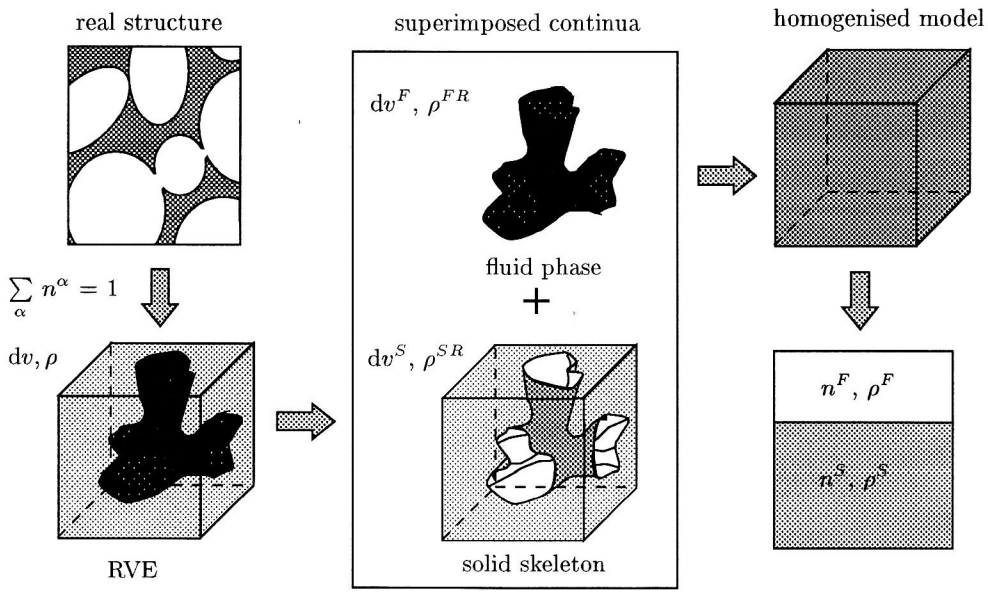

Figure 2. The theory of mixtures extended by the concept of volume fractions (TPM). The actual, saturated structure (volume dv, density ρ) is homogenised by an averaging process towards a statistical model with superimposed and interacting constituents φ^α (volume dv^α, effective density $\rho^{\alpha R}$).

Herein, the representative volume elements (RVE) are occupied by material points X^α of the involved phases φ^α, where the geometrical and physical characteristics of the partial bodies in each space point are defined by field functions. From the geometric relationships, the corresponding volume fractions of the phase φ^α occure as the local ratio of the volume element dv^α occupied by φ^α to the volume element dv of the mixture in the actual configuration:

$$n^\alpha = \frac{dv^\alpha}{dv} \tag{1}$$

Since in reality only saturated multiphase materials exist, the saturation condition has to be fulfilled:

$$\sum_\alpha n^\alpha = 1 \tag{2}$$

Based on the introduction of the volume fractions, two different density functions of the individual phases

exist. Thus, relating the mass element dm^α to the local volume dv^α leads to the effective or real density

$$\rho^{\alpha R} = \frac{dm^\alpha}{dv^\alpha} \tag{3}$$

whereas relating dm^α to the local volume dv of the mixture represents the partial density

$$\rho^\alpha = \frac{dm^\alpha}{dv} \tag{4}$$

The densities are coupled by the volume fractions:

$$\rho^\alpha = n^\alpha \rho^{\alpha R} \tag{5}$$

Following this concept, it is seen that in case of material incompressibility of the phase φ^α ($\rho^{\alpha R} = \text{const.}$) the partial density ρ^α in the control volume can vary through changes of the volume fraction n^α.

The TPM proceeds from the assumption that at each time t each spacial point $\mathbf{x}$ of the current configuration is simultaneously occupied by material points X^α of the involved phases φ^α, which originate from different reference positions $\mathbf{X}_\alpha$ at time $t_0 < t$. Thus, each constituent undergoes its own function of motion

$$\mathbf{x} = \chi_\alpha(\mathbf{X}_\alpha, t) \tag{6}$$

Presupposing a non-singular *Jacobi*an matrix, i. e. the existence of the inverse function of motion χ_α^{-1}, each material point X^α has assigned an one-to-one reference position at time t_0. In coupled solid-fluid problems, the motion of the solid is described in a *Lagrange*an form and the motion of the fluid is described relative to the deformed solid skeleton in a modified *Euler*ian setting. Considering the aluminium foam as a binary porous medium consisting of a materially incompressible solid phase φ^S saturated by a compressible gas (fluid phase) φ^F, the function of motion of the phase φ^α leads to a corresponding velocity and acceleration field for each constituent. Thus, one obtains in the *Lagrange*an form

$$\overset{\prime}{\mathbf{x}}_\alpha = \frac{\partial}{\partial t}\chi_\alpha(\mathbf{X}_\alpha, t) \qquad \overset{\prime\prime}{\mathbf{x}}_\alpha = (\mathbf{v}_\alpha)'_\alpha = \frac{\partial^2}{\partial t^2}\chi_\alpha(\mathbf{X}_\alpha, t) \tag{7}$$

The symbol $(..)'_\alpha$ denotes the first material time derivative and the symbol $(..)''_\alpha$ denotes the second material time derivative following the motion of the constituent φ^α.
Also a transformation relating the time derivative with respect to the motion of a constituent φ^β and that with respect to constituent φ^α can be formulated by

$$(..)'_\alpha = (..)'_\beta + \operatorname{grad}(..) \cdot (\overset{\prime}{\mathbf{x}}_\alpha - \overset{\prime}{\mathbf{x}}_\beta) \tag{8}$$

Following this, the corresponding total time derivatives of any scalar field $\alpha = \alpha(\mathbf{x}, t)$ or any vector field $\mathbf{a} = \mathbf{a}(\mathbf{x}, t)$ following the motion of φ^α are defined by

$$(\alpha)'_\alpha = \frac{\partial \alpha}{\partial t} + \operatorname{grad}\alpha \cdot \overset{\prime}{\mathbf{x}}_\alpha \qquad (\mathbf{a})'_\alpha = \frac{\partial \mathbf{a}}{\partial t} + (\operatorname{grad}\mathbf{a})\ \overset{\prime}{\mathbf{x}}_\alpha \tag{9}$$

Herein, the operator 'grad' means partial differentiation with respect to $\mathbf{x}$.
In the case of the solid skeleton φ^S, the *Lagrange*an description leads to the primary variables

$$\mathbf{u}_S = \mathbf{x} - \mathbf{X} \qquad \mathbf{v}_S = \overset{\prime}{\mathbf{x}}_S = (\mathbf{u}_S)'_S \tag{10}$$

where $\mathbf{u}_S$ is the displacement and $\mathbf{v}_S$ the velocity of the solid phase φ^S, respectively.

The deformation gradient of the solid skeleton and its inverse are given by

$$\mathbf{F}_S = \frac{\partial \mathbf{x}}{\partial \mathbf{X}_S} = \operatorname{Grad}_S \mathbf{x} \qquad \mathbf{F}_S^{-1} = \frac{\partial \mathbf{X}_S}{\partial \mathbf{x}} = \operatorname{grad}\mathbf{X}_S \tag{11}$$

where 'Grad$_S$' means partial differentiation with respect to $\mathbf{X}_S$.
The natural variable to describe the fluid motion relative to the deformed solid skeleton is the so called seepage velocity

$$\mathbf{w}_F = \overset{\prime}{\mathbf{x}}_F - \overset{\prime}{\mathbf{x}}_S \tag{12}$$

The acceleration of the fluid phase is expressed with respect to the deforming solid skeleton, with the equations (9) and (12), by

$$\overset{\prime\prime}{\mathbf{x}}_F = (\mathbf{v}_S + \mathbf{w}_F)'_S + \operatorname{grad}(\mathbf{v}_S + \mathbf{w}_F)\,\mathbf{w}_F \tag{13}$$

3 Balance Equations

By use of porous media theories, each constituent can be described by individual balance equations taking into account the necessary interactions between all constituents. The balance equations of the mixture are derived as the sum of the balance equations of all constituents. Furthermore, because the body does not know, whether it is a mixture or not (Truesdell, 1984), the mixture balance equations must have the same form as the respective balance equations of a single body. Thus, the properties of the mixture follow from the properties of the constituents without any additional assumption.
Excluding mass exchanges between the constituents, the balance of mass equation of a constituent φ^α is given by

$$(\rho^\alpha)'_\alpha + \rho^\alpha \operatorname{div} \overset{\prime}{\mathbf{x}}_\alpha = 0 \tag{14}$$

According to equation (5), the partial density can be formulated in terms of the volume fraction n^α of the constituent and the real density $\rho^{\alpha R}$ per unit volume. For incompressible constituents, the real density $\rho^{\alpha R}$ is constant. Therefore, the balance of mass equation (14) can be reduced to the balance equation for the volume fractions:

$$(n^\alpha)'_\alpha + n^\alpha \operatorname{div} \overset{\prime}{\mathbf{x}}_\alpha = 0 \tag{15}$$

For superimposed continua without mass exchanges, the balance of mass equation for the mixture generally can be derived simply by summing up the single balances:

$$\sum_\alpha [(\rho^\alpha)'_\alpha + \rho^\alpha \operatorname{div} \overset{\prime}{\mathbf{x}}_\alpha] = 0 \tag{16}$$

The balance of linear momentum of each constituent reads

$$\rho^\alpha \overset{\prime\prime}{\mathbf{x}}_\alpha = \operatorname{div} \mathbf{T}^\alpha + \rho^\alpha \mathbf{b} + \mathbf{p}^\alpha \tag{17}$$

where $\mathbf{T}^\alpha$ is the partial *Cauchy* stress tensor of φ^α, $\mathbf{b}$ is the body force density acting upon all constituents, and $\mathbf{p}^\alpha$ represents the momentum production term of φ^α, which can be interpreted, in the meaning of the two-phase aluminium foam description, as the interaction force per unit volume between the solid skeleton and the gas phase.
The balance equation of linear momentum of the mixture is given by the sum of the single momentum balance equations (17) including a constraint condition for the momentum productions between the phases. Thus,

$$\sum_\alpha \rho \overset{\prime\prime}{\mathbf{x}}_\alpha = \sum_\alpha [\operatorname{div} \mathbf{T}^\alpha + \rho^\alpha \mathbf{b} + \mathbf{p}^\alpha] \qquad \sum_\alpha \mathbf{p}^\alpha = \mathbf{0} \tag{18}$$

For numerical computations within the Finite Element Method (FEM) using the weak formulation of the balance equations, the advantage of equation (18) is that it is not necessary to separate the applied external loads into parts acting upon the solid and fluid consitutents.
Restricting to isothermal processes and assuming microscopically non-polar constituents (symmetric *Cauchy* stresses, $\mathbf{T}^\alpha = \mathbf{T}^{T\alpha}$), the balance of moment of momentum is preconditioned and the problem becomes independent of the balance of internal energy.

4 Constitutive Equations

The basic constitutive approach to binary models in porous media theories is sufficient for the determination of the respective sets of response functions, which cannot be chosen arbitrarily. Instead, they must be compatible with the second law of thermodynamics. So the constitutive equations must reflect the restrictions imposed on the different models by the dissipation principle. Thermodynamical restrictions for the two-phase model under consideration, namely the hybrid model consisting of an incompressible solid skeleton filled with a compressible fluid, result from the appropriate version of the entropy inequality combined with the basic constitutive assumptions for the respective material. Because the general procedure of deriving thermodynamical restrictions from the entropy inequality of the respective two-phase model proves to be rather laborious, producing lots of lengthy formulæ, only the main results related to the used simplified model are given here. The reader who is interested in further details is referred to Ehlers (1989a, 1989b, 1993a, 1993b).
Thus, in assuming an incompressible solid skeleton saturated by a compressible pore fluid, the partial stress tensor can be divided into a pressure depending part and an extra part:

$$\mathbf{T}^{\alpha} = -n^{\alpha} p \mathbf{I} + \mathbf{T}^{\alpha}_{E} \tag{19}$$

Therein, $\mathbf{I}$ is the second order identity tensor weighted by the effective fluid pressure p and the volume fraction n^{α} of phase φ^{α}, while the second term represents the extra stresses. These extra stresses, characterised by the additional index $(..)_E$, must be specified by additional constitutive relations.

In the following investigation, non-linear finite elasto-plastic behaviour of the solid skeleton is assumed. Proceeding from the multiplicative split of the deformation gradient $\mathbf{F}_S$ into an elastic and a plastic part, $\mathbf{F}_S = \mathbf{F}_{Se}\mathbf{F}_{Sp}$, a plastic intermediate configuration is implicitly introduced. A nonlinear elastic Neo-*Hooke* type law for porous materials (Eipper, 1998) modified by Mahnkopf (1999), leads to the elastic extra stresses formulated in terms of the *Kirchhoff* stress tensor $\boldsymbol{\tau}^{S}_{E} = (\det\mathbf{F}_S)\mathbf{T}^{S}_{E}$ in the following form:

$$\boldsymbol{\tau}^{S}_{E} = \mu^{S}(\mathbf{B}_{Se} - \mathbf{I}) + \lambda^{S}(1 - n^{S}_{p})^{2}\left(\frac{J_{Se}}{1 - n^{S}_{p}} - \frac{J_{Se}}{J_S - n^{S}_{p}}\right)\mathbf{I} \tag{20}$$

Therein, the elastic left *Cauchy-Green* strain tensor is defined by $\mathbf{B}_{Se} = \mathbf{F}_{Se}\mathbf{F}^{T}_{Se}$, $J_{Se} = \det\mathbf{F}_{Se}$ is the *Jacobi*an of the elastic part of the solid deformation and $n^{S}_{p} = n^{S}_{0S}(\det\mathbf{F}_{Sp})^{-1}$ the solid volume fraction with respect to the intermediate configuration; λ^S and μ^S are the *Lamé* constants of the solid skeleton. This formulation allows a correct representation of the point of compaction, where all pores are closed ($n^S = 1, n^F = 0$), see Section 6 (Figure 6).
In the following, it is assumed that in comparison with the interaction force, the friction effects depending on the fluid viscosity within the fluid phase are negligible. Thus, the extra stress is assumed to be zero, $\mathbf{T}^{F}_{E} \approx \mathbf{0}$. In this case, the partial fluid stresses only depend on the pore pressure and the actual volume fraction of the fluid, $\mathbf{T}^{F} = -n^{F} p \mathbf{I}$. For compressible fluids, the hydrostatic pressure is defined by an equation of state, and, under isothermal conditions, depends only on the real fluid density ρ^{FR}. In the simplest case it is given by the ideal gas law

$$p = \rho^{FR} R^{F} \Theta \tag{21}$$

where the multiplication of the specific gas constant R^F, the constant temperature Θ and the real fluid density ρ^{FR} is defining the hydrostatic pressure p of the pore-fluid.
As was assumed, the solid skeleton is fully saturated by the pore-gas, and constitutive relations must be given for the partial stresses $\mathbf{T}^{\alpha}$ and the interaction force $\mathbf{p}^{\alpha}$ of the fluid phase depending on the primary variables and their derivatives. Two different velocity effects for open pored structures ($\mathbf{w}_F \neq \mathbf{0}$) or closed pored structures ($\mathbf{w}_F = \mathbf{0}$) can be modelled by the interaction force leading to the *Darcy* law:

$$\mathbf{p}^{F} = p\,\mathrm{grad}\,n^{F} - \frac{(n^{F})^{2}\gamma^{FR}}{k^{F}}\mathbf{w}_F \tag{22}$$

Therein, γ^{FR} is the effective specific weight of the pore fluid and k^F is the *Darcy* permeability coefficient, which, as a macroscopic quantity, does not only depend on the viscosity of the pore-fluid but also on

both the size and the structure of the pore space.
In the present work, the considerations are based on the assumption of an isotropic pore structure and closed cells, i. e. ($\mathbf{w}_F = \mathbf{0}$).

5 Plasticity

The general response of the elasto-plastic model is affected by the solid deformation gradient $\mathbf{F}_S$. The description of finite elasto-plasticity for the solid constituent is based on the multiplicative decomposition of $\mathbf{F}_S$ into an elastic and a plastic part

$$\mathbf{F}_S = \mathbf{F}_{Se}\mathbf{F}_{Sp} \tag{23}$$

In addition, as is known from several publications on continuum theories for single continua, the concept of the multiplicative decomposition is connected with the assumption of a stress-free intermediate configuration (Figure 3) which, in the case of the general deformation process, is incompatible with the existence of partial derivatives when they are considered as field functions.

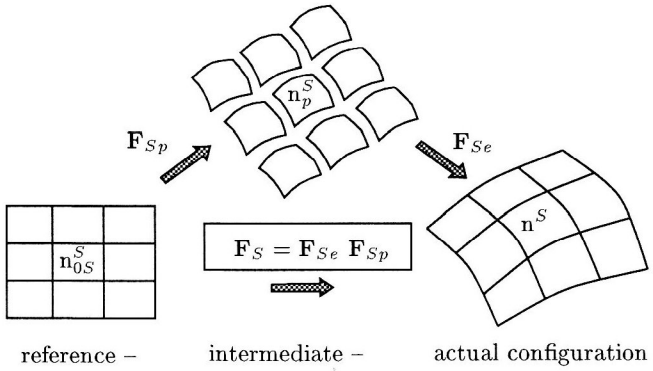

Figure 3. Sketch of the multiplicative split of $\mathbf{F}_S$.

In Figure 3, n_{0S}^S represents the volume fraction of the solid in the reference configuration ($t = t_0$), n_p^S is the volume fraction of the solid with respect to the intermediate configuration (see also equation (20)) and n^S is the actual volume fraction of the solid. The quantities

$$\mathbf{F}_{Se} = \mathrm{grad}_{Sz}\mathbf{x} \qquad \mathbf{F}_{Sp} = \mathrm{Grad}_S\mathbf{x}_{Sz} \tag{24}$$

are point functions, only defined in the subregions of the intersected intermediate configuration. The position of particles X^S in the intermediate configuration of φ^S are defined by $\mathbf{x}_{Sz}$, whereby grad_{Sz} means partial differentiation with respect to $\mathbf{x}_{Sz}$. In the case of homogeneous deformations, the field character of relation (24) generally holds. The multiplicative decomposition is not unique but can only be determined, up to an arbitrary rotation (Haupt, 1985, Ehlers, 1991). Here, quantities belonging to this intermediate configuration are indicated by the topscript ($\hat{\cdot}$). The multiplicative decomposition of the deformation gradients implies the existence of only two proper spatial velocity gradients, $\mathbf{L}_S$ and $\hat{\mathbf{L}}_{Sp}$, the former relative to the actual and the latter relative to the intermediate configuration of φ^S, see Ehlers (1989a/b, 1991):

$$\mathbf{L}_S = (\mathbf{F}_S)'_S\mathbf{F}_S^{-1} \qquad \hat{\mathbf{L}}_S = (\mathbf{F}_{Sp})'_S\mathbf{F}_{Sp}^{-1} \tag{25}$$

Their symmetric parts are

$$\mathbf{D}_S = \frac{1}{2}(\mathbf{L}_S + \mathbf{L}_S^T) \qquad \hat{\mathbf{D}}_{Sp} = \frac{1}{2}(\hat{\mathbf{L}}_{Sp} + \hat{\mathbf{L}}_{Sp}^T) \tag{26}$$

where $\mathbf{D}_S$ characterises the solid deformation rate of the spatial configuration and $\hat{\mathbf{D}}_{Sp}$ the purely plastic deformation rate of the intermediate configuration.

As it is known from theories of constrained materials, the plastic behaviour only depends on the extra stresses. Following this, a yield function applicable for a wide range of materials including isotropic

hardening porous materials can be introduced by the single surface yield condition by Ehlers (1993):

$$\hat{F}(\hat{I},\hat{II}^D,\hat{III}^D)=\sqrt{\hat{II}^D\left(1+\gamma\frac{\hat{III}^D}{(\hat{II}^D)^{3/2}}\right)^m}+\frac{1}{2}\alpha\hat{I}^2+\delta^2\hat{I}^4+\beta\hat{I}+\varepsilon\hat{I}^2-\kappa=0 \tag{27}$$

Therein, $\hat{I}=\hat{\boldsymbol{\tau}}_E^S\cdot\mathbf{I}$ is the first invariant of $\hat{\boldsymbol{\tau}}_E^S$, $\hat{II}^D=\frac{1}{2}(\hat{\boldsymbol{\tau}}_E^S)^D\cdot(\hat{\boldsymbol{\tau}}_E^S)^D$ is the second deviatoric invariant of $(\hat{\boldsymbol{\tau}}_E^S)^D$ and $\hat{III}^D=\frac{1}{3}(\hat{\boldsymbol{\tau}}_E^S)^D\cdot(\hat{\boldsymbol{\tau}}_E^S)^D(\hat{\boldsymbol{\tau}}_E^S)^D$ is the third deviatoric invariant of $(\hat{\boldsymbol{\tau}}_E^S)^D$, given in terms of the intermediate configuration.
The parameters $\alpha,\beta,\gamma,\delta,\varepsilon,\kappa$ and m are material parameters which, in the frame of isotropic hardening, may depend on the plastic deformation history through the plastic strain paths $\mathbf{E}_{Sp}(t)$. In general: $\Phi=\Phi(\mathbf{E}_{Sp}(t))$ with $\Phi=\{\alpha,\beta,\gamma,\delta,\varepsilon,\kappa,m\}$.
The yield function (27) must be understood as a very general condition suitable to describe the onset of plastic deformation effects of both compression and extension. By setting $\gamma=0$ and $m=1$, the triangular shape of the deviatoric plane canges to a circular form. Setting additionally β, δ and ε to zero, the *Green*'s yield condition for ductile media (Green, 1972) representing an ellipsoid in stress space results. With $\alpha=\delta=\varepsilon=0$, the well-known *Drucker-Prager* criterion can be modelled or with $\alpha=\beta=\delta=\varepsilon=0$ the *von Mises* yield function is represented. In the principal stress space, the general yield function $\hat{F}=0$ represents the single surface yield criterion, Figure 4.

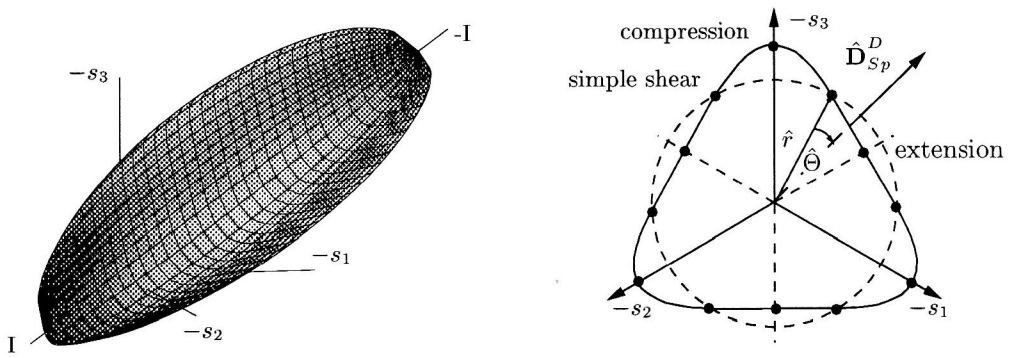

Figure 4. Single surface yield function in the principal stress space (left) and the deviatoric plane (right).

The terms in equation (27) depending of γ and m govern the typical triangular shape of the yield surface in the deviatoric plane, whereas $\alpha,\beta,\delta,\varepsilon,\kappa$ describe the ellipse-like yield function in the hydrostatic plane. These parameter set must be fitted to experimental results from triaxial and biaxial tests, see Ehlers & Müllerschön (1999). Furthermore, the concept of associated flow leads to unrealistic results for frictional materials such as porous metal foams or granular porous materials. Thus, a general non-associated flow rule must be introduced to govern the plastic rate of the deformation tensor $\hat{\mathbf{D}}_{Sp}$:

$$\hat{\mathbf{D}}_{Sp}=\Lambda\frac{\partial\hat{G}}{\partial(\hat{\boldsymbol{\tau}}_E^S)} \tag{28}$$

Herein, the plastic potential

$$\hat{G}(\hat{I},\hat{II}^D)=\sqrt{\hat{II}^D+\frac{1}{2}\alpha\hat{I}^2+\delta^2\hat{I}^4}+\beta\hat{I}+\varepsilon\hat{I}^2+\hat{g}(\hat{I}) \tag{29}$$

and a proportional factor Λ are used. The function $g(\hat{I})$ describes the difference between the measured dilatation under compression and the values resulting from an associated flow rule.
Following this concept, it is easily concluded from the preceeding results that the plastic response for brittle aluminium foam proves to yield the respective response for ductile materials if γ, m and $\hat{g}(\hat{I})$ are zero. Furthermore, the flow rule can be transformed from the plastic intermediate configuration of the solid to the actual configuration or the reference configuration of φ^S by the well known push-forward or pull-back operations.
Finally, with respect to numerical computations, an algorithmic predictor-corrector procedure is used for elasto-plastic computations. If in the predictor step $\hat{F}(\hat{I},\hat{II}^D,\hat{III}^D)$ is less than zero, the material

remains in the elastic domain. If $\hat{F}$ is greater or equal zero, plastic behaviour is taken into account by solving the nonlinear differential algebraic equation (DAE) system where $\hat{F} = 0$ is conditioned. For more details of the numerical handling, see Ellsiepen (1999).

6 Examples

The first example shows the effects resulting from the plateau stress and the point of compaction in a simple one dimensional test under compression within a defined closed system in the nonlinear elasto-plastic range (Mahnkopf, 1999) and the pure nonlinear elastic range (Eipper, 1998), see Figure 5.

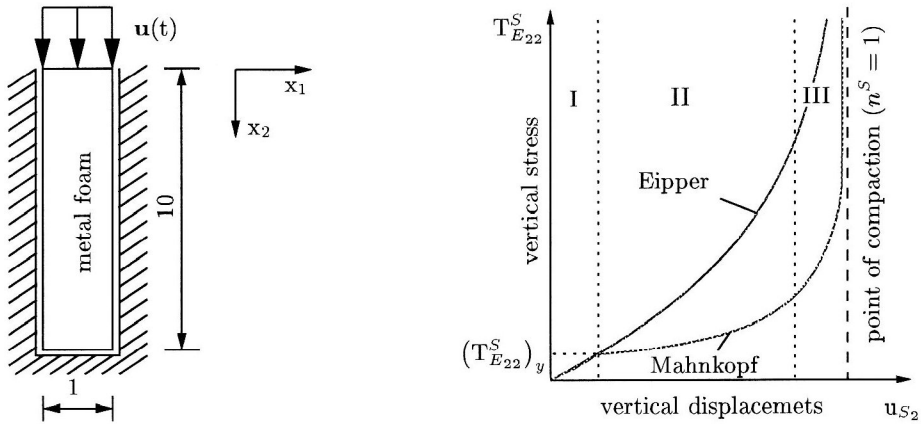

Figure 5. 1-d compression, purely elastic (Eipper) and elasto-plastic (Mahnkopf) with plateau stress and point of compaction with PANDAS (qualitative sketch)

After a short and steep so-called elastic range (I), an average yield stress $(\mathbf{T}^S_{E_{22}})_y$ of the elasto-plastic approach is reached and the typical plateau stress (II) is obtained, until the global densification of the material starts at large strains (III). The compression stress increases very fast up to the full compacted solid material. At this point, the volume fraction of the fluid vanishes and the material has to be treated as a single-phase solid material without porosity. With this initial boundary value problem, the main physical aspects of foam material can be simulated.

The second example shows the stiffening effect for a basically hollow structure, see Figure 6. The simulation of a shear loaded empty box (1), stiffened with inlays (1+2) and stiffened with foam (1+3) is presented. The main task is to verify that the global mass in all three cases is the same. Therefore, the thicknesses δ of the box and/or the amount of inlays are varied, while the length l, the hight h and the depth d are constant.

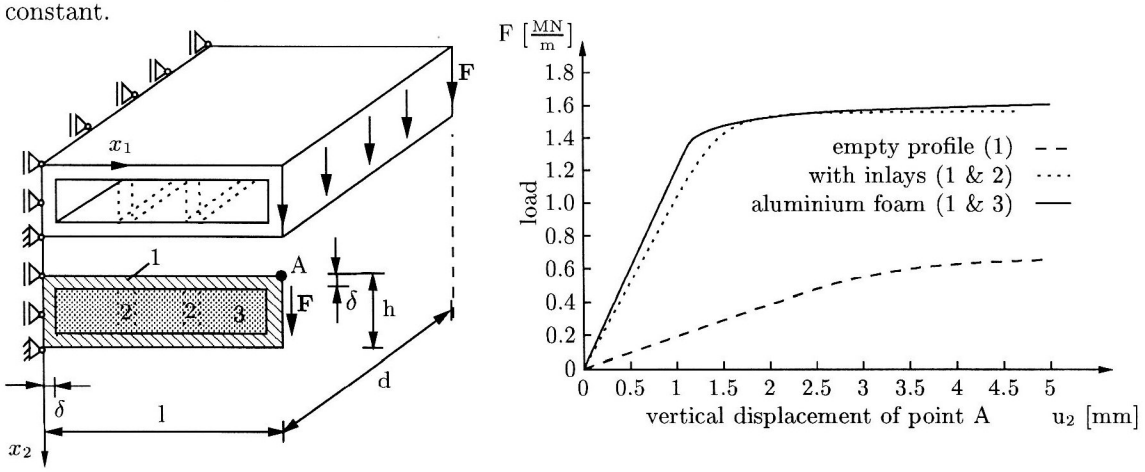

Figure 6. Stiffening of a hollow structure with inlays and foam computed with PANDAS.

As shown in Figure 6, the stiffening effect from the inlays can also be reached by the foam material, which is an economic and also ecologic alternative material for such applications.

The third example shows the local plastification zone of a partially loaded foam brick, see Figure 7. Because of the fact that the influence of the fluid pressure in relation to the solid stresses is negligible, the presented model can be simplified by a porous single phase material if no thermal or acoustical influences should be considered.

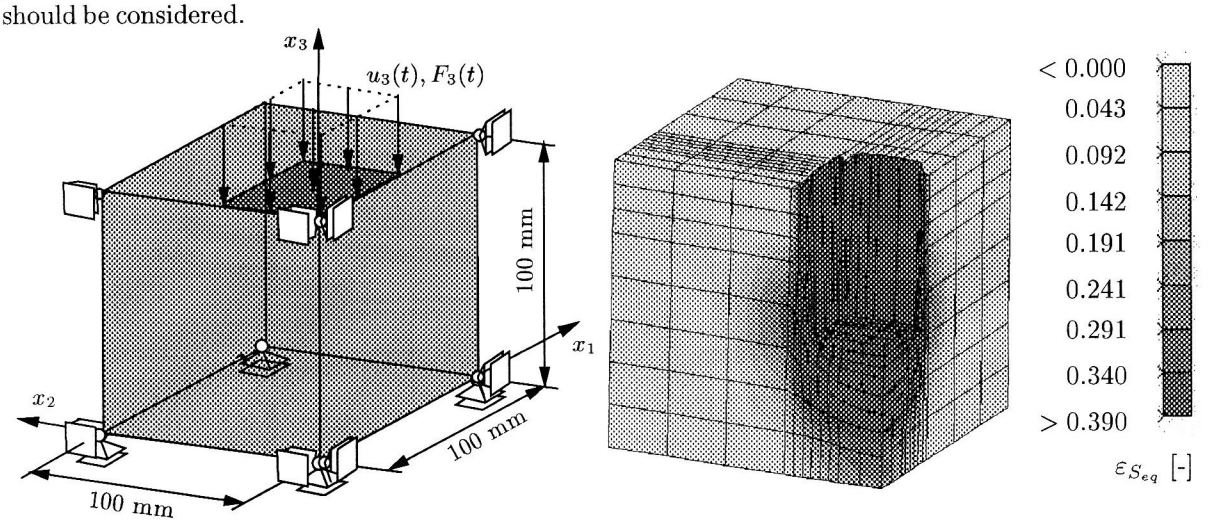

Figure 7. Indentation of a foam brick computed with LS-DYNA3D.

The equivalent plastic stesses $\varepsilon_{S_{eq}} = \sqrt{\mathbf{E}_{Sp} \cdot \mathbf{E}_{Sp}}$, with $\mathbf{E}_{Sp}$ as the plastic part of the finite *Lagrange*an strain tensor, are localised in the load area, as regarded in real experiments of this kind of boundary value problems.

Acknowledgements

The research work has been carried out within the framework of the European Commission's "Industrial and Materials Technologies" (Brite EuRam III) multiannual (94-98) programme for R & D. The authors gratefully acknowledge the collaboration of all partners of the METEOR project "Light-weight Metal Foam Components for the Transport Industry" (Contract BRPR-CT96-0215, project BE96-3018).

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