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Thermal Anisotropy Inducing Brittle Damage

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This paper demonstrates the influence of brittle damage on the initiation and growth of mechanical and thermal anisotropy. It is shown that even a constant temperature may cause stresses which lead to brittle damage in an unconstrained continuum of sufficiently advanced level of damage. As example, a thick-walled sphere subjected to creep-damage under constant temperature is considered. Two independent formulations of the problem based on the Stress Equivalent Principle and the Energy Equivalent Principle are tested.

1 Introduction

The constitutive equations of the general, anisotropic thermo-elasticity fulfil the Duhamel-Neumann relationships:

$$\varepsilon_{ij} = \mathrm{E}^{-1}_{ijkl}\sigma_{kl} + \alpha_{ij}\theta$$

where the constitutive tensor $\mathbf{E}^{-1}$ and the tensor of thermal expansion $\boldsymbol{\alpha}$ exhibit certain groups of symmetry:

$$\mathrm{E}^{-1}_{ijkl} = \mathrm{E}^{-1}_{jikl} \quad \mathrm{E}^{-1}_{ijkl} = \mathrm{E}^{-1}_{ijlk} \quad \mathrm{E}^{-1}_{ijkl} = \mathrm{E}^{-1}_{klij} \quad \alpha_{ij} = \alpha_{ji}$$

Restricting our consideration, for a moment, to the case of a material which is mechanically anisotropic but thermally isotropic, the Duhamel-Neumann equations take a simpler form:

$$\varepsilon_{ij} = \mathrm{E}^{-1}_{ijkl}\sigma_{kl} + \alpha\theta\delta_{ij}$$

Supposed that each of the particles may freely expand, there are no stresses $\sigma_{kl} = 0$ and only thermal strains are observed $\varepsilon_{ij} = \varepsilon^{\mathrm{th}}_{ij} = \alpha\theta\delta_{ij}$. This particular case is subject of one of the fundamental theorems of thermo-elasticity which states that linear distribution of temperature does not produce stress in an isotropic continuum under the condition of absence of body forces and any constraints (cf. Fung, 1965). As a matter of fact, thermal strains have to satisfy the compatibility equations:

$$\theta_{,kl}\delta_{ij} + \theta_{,ij}\delta_{kl} - \theta_{,jl}\delta_{ik} - \theta_{,ik}\delta_{jl} = 0$$

so $\theta_{,ij} = 0$ or $\theta = a_0 + a_1x_1 + a_2x_2 + a_3x_3$. Referring to the case of general thermal anisotropy we convince ourselves that the crucial point of deciding whether any stresses may or may not appear, are therefore its thermal properties. Consider now the damage process when deterioration influences both constitutive and thermal expansion tensors. A structure, initially isotropic, acquires mechanical and thermal anisotropy. We will prove that in such a case even constant temperature produces stresses and consequently may lead the structure to brittle damage.

2 Basic Equations

2.1 Representation of Constitutive and Thermal Expansion Tensors for Damaged Material

Consider a damaged solid in a current configuration, the mechanical state of which is defined by the couple of external state variables $(\boldsymbol{\varepsilon}, \boldsymbol{\sigma})$, or equivalently in a fictive pseudo-undamaged configuration characterized by the effective state variables $(\tilde{\varepsilon}, \tilde{\sigma})$, linked by a constitutive law, the form of which depends on the damage equivalent principle used.

In case of the Stress Equivalence Principle (cf. Lemaitre and Chaboche, 1978), *The stress associated with a damage state under the applied strain ε is equivalent to the stress associated with the undamaged state under the effective strain $\tilde{\varepsilon}$* (Figure 1).Thus the following relationships hold:

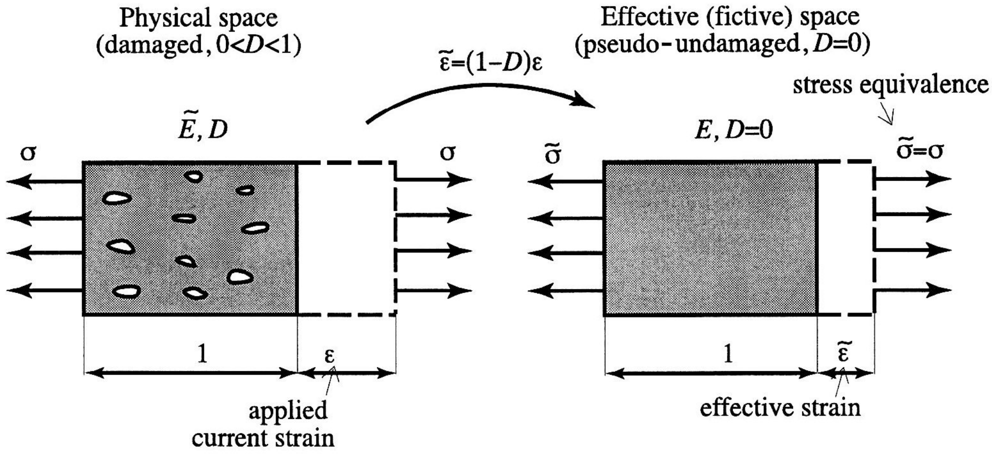

Figure 1: 1D Stress Equivalent Principle Visualization

$$\varepsilon = \mathbf{E}^{-1} : \tilde{\sigma} \quad \text{and} \quad \varepsilon =^{\text{SEP}} \tilde{\mathbf{E}}^{-1} : \sigma \tag{1}$$

whereas if the Energy Equivalence Principle is postulated (cf. Cordebois and Sidoroff, 1979), the respective relationships are valid:

$$\tilde{\varepsilon} = \mathbf{E}^{-1} : \tilde{\sigma} \quad \text{and} \quad \varepsilon =^{\text{EEP}} \tilde{\mathbf{E}}^{-1} : \sigma \tag{2}$$

where $\tilde{\mathbf{E}}$ denote the fourth-rank constitutive tensors modified by damage.

Suppose for a moment that stress and strain are caused by thermal expansion only (Figure 2), in such a case we have (cf. Ganczarski, 1999):

$$\tilde{\varepsilon} = \alpha\theta \quad \text{and} \quad \varepsilon = \tilde{\alpha}\theta \tag{3}$$

where $\tilde{\alpha}$ stands for the second-rank effective tensor of thermal expansion and $\theta = T - T_0$ is the difference between actual and reference temperatures.

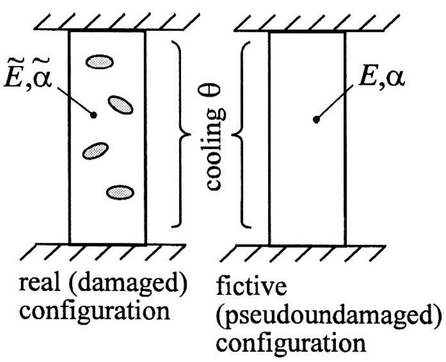

Figure 2: Concept of Real and Fictive Configurations in the Case of Pure Thermal Deformation

In order to develop the anisotropy caused by damage, the damage itself must be described by a tensor of sufficiently high rank, the simplest of which is the second-rank damage tensor $\mathbf{D}$ as defined by Murakami and Ohno (1981):

$$\mathbf{D} = \sum_{i=1}^{3} D_i \mathbf{n}_i \otimes \mathbf{n}_i \tag{4}$$

where D_i and $\mathbf{n}_i$ are principal values and the unit vectors of principal directions of the tensor $\mathbf{D}$.

Introducing so-called damage effect tensors, which map the second-rank damage tensor $\mathbf{D}$ or equivalently the second-rank continuity tensor $\boldsymbol{\Psi} = \mathbf{1} - \mathbf{D}$ to the fourth-rank tensor space (cf. Chen and Chow, 1995):

$$\mathbf{M}_1(\boldsymbol{\Psi}) = \begin{bmatrix} \Psi_{11} &&&&& \\ & \Psi_{22} &&&& \\ && \Psi_{33} &&& \\ &&& \dfrac{\Psi_{22}+\Psi_{33}}{2} && \\ &&&& \dfrac{\Psi_{11}+\Psi_{33}}{2} & \\ &&&&& \dfrac{\Psi_{11}+\Psi_{22}}{2} \end{bmatrix} \tag{5}$$

and (cf. Chaboche et all., 1995; Qi and Bertram, 1997)

$$\mathbf{M}_2(\boldsymbol{\Psi}) = \begin{bmatrix} \Psi_{11} &&&&& \\ & \Psi_{22} &&&& \\ && \Psi_{33} &&& \\ &&& \sqrt{\Psi_{22}\Psi_{33}} && \\ &&&& \sqrt{\Psi_{11}\Psi_{33}} & \\ &&&&& \sqrt{\Psi_{11}\Psi_{22}} \end{bmatrix} \tag{6}$$

and employing definitions equation (4) we may express the fourth-rank constitutive tensors equations (1, 2) in the following form:

$$^{\mathrm{SEP}}\widetilde{\mathbf{E}} = \frac{1}{2}\left[\mathbf{E} : \mathbf{M}_1(\boldsymbol{\Psi}) + \mathbf{M}_1(\boldsymbol{\Psi}) : \mathbf{E}\right] \tag{7}$$

or

$$^{\mathrm{EEP}}\widetilde{\mathbf{E}} = \mathbf{M}_2(\boldsymbol{\Psi}) : \mathbf{E} : \mathbf{M}_2(\boldsymbol{\Psi}) \tag{8}$$

whereas the second-rank effective tensor of thermal expansion equation (3) is defined as follows:

$$\widetilde{\alpha} = \alpha \boldsymbol{\Psi}^{-1} \tag{9}$$

2.2 Basic Equations for Spherical Symmetry

The objective of this paper is to show that even constant temperature may cause stresses in unconstrained, thermally anisotropic continuum. The natural way to do so is to consider a spherically-symmetric problem. Identifying direction 1 with the radial direction ρ and directions 2 and 3, which are equivalent, with the hoop direction ϑ, we may introduce the 2×2 matrices ($\widetilde{\mathrm{E}}_{ij} \neq \widetilde{\mathrm{E}}_{ji}$!) referring to the constitutive tensors equations (7, 8) of the following form:

$$^{\mathrm{SEP}}\widetilde{\mathrm{E}}_{ij} = \frac{E}{(1+\nu)(1-2\nu)} \begin{bmatrix} (1-\nu)\Psi_{\rho\rho} & 2\nu\dfrac{\Psi_{\rho\rho}+\Psi_{\vartheta\vartheta}}{2} \\ \nu\dfrac{\Psi_{\rho\rho}+\Psi_{\vartheta\vartheta}}{2} & \Psi_{\vartheta\vartheta} \end{bmatrix} \tag{10}$$

or

$$^{\mathrm{EEP}}\widetilde{\mathrm{E}}_{ij} = \frac{E}{(1+\nu)(1-2\nu)} \begin{bmatrix} (1-\nu)\Psi_{\rho\rho}^2 & 2\nu\Psi_{\rho\rho}\Psi_{\vartheta\vartheta} \\ \nu\Psi_{\rho\rho}\Psi_{\vartheta\vartheta} & \Psi_{\vartheta\vartheta}^2 \end{bmatrix} \tag{11}$$

and the effective tensor of thermal expansion equation (9) is given by:

$$\widetilde{\alpha}_{ij} = \alpha \begin{bmatrix} \Psi_{\rho\rho}^{-1} & 0 \\ 0 & \Psi_{\vartheta\vartheta}^{-1} \end{bmatrix} \tag{12}$$

Applying decomposition of small strains into elastic, creep an thermal parts: $\varepsilon = \varepsilon^{\mathrm{e}} + \varepsilon^{\mathrm{c}} + \varepsilon^{\mathrm{th}}$, the constitutive equations:

$$\begin{aligned} \sigma_{\rho\rho} &= \widetilde{\mathrm{E}}_{\rho\rho}\varepsilon_{\rho\rho} + 2\widetilde{\mathrm{E}}_{\rho\vartheta}\varepsilon_{\vartheta\vartheta} \\ \sigma_{\vartheta\vartheta} &= \widetilde{\mathrm{E}}_{\rho\vartheta}\varepsilon_{\rho\rho} + \widetilde{\mathrm{E}}_{\vartheta\vartheta}\varepsilon_{\vartheta\vartheta} \end{aligned} \tag{13}$$

are solved with respect to strains, which substituted into the compatibility equation:

$$\frac{\mathrm{d}\varepsilon_{\vartheta\vartheta}}{\mathrm{d}\rho} + \frac{\varepsilon_{\vartheta\vartheta} - \varepsilon_{\rho\rho}}{\rho} = 0 \tag{14}$$

finally yields the system of equations as follows:

$$\begin{aligned} &\mathcal{F}[f] = (\widetilde{\alpha}_{\rho\rho} - \widetilde{\alpha}_{\vartheta\vartheta})\theta && \text{for} \quad t = 0 \\ &\dot{\mathcal{F}}[f] = (\dot{\widetilde{\alpha}}_{\rho\rho} - \dot{\widetilde{\alpha}}_{\vartheta\vartheta})\theta - \rho\dot{\varepsilon}^{\mathrm{c}}_{\rho\rho} - \left(\dot{\varepsilon}^{\mathrm{c}}_{\vartheta\vartheta} - \dot{\varepsilon}^{\mathrm{c}}_{\rho\rho}\right) && \text{for} \quad t > 0 \end{aligned} \tag{15}$$

The differential operator $\mathcal{F}[...]$ takes the form:

$$\mathcal{F}[...] = \frac{\widetilde{\mathrm{E}}^{-1}_{\vartheta\vartheta}}{2}\frac{\mathrm{d}^2...}{\mathrm{d}\rho^2} + \frac{1}{2}\frac{\mathrm{d}\widetilde{\mathrm{E}}^{-1}_{\vartheta\vartheta}}{\mathrm{d}\rho}\frac{\mathrm{d}...}{\mathrm{d}\rho} + \left(\frac{\mathrm{d}\widetilde{\mathrm{E}}^{-1}_{\vartheta\rho}}{\mathrm{d}\rho}\rho - \widetilde{\mathrm{E}}^{-1}_{\vartheta\rho} - \widetilde{\mathrm{E}}^{-1}_{\rho\rho}\right)\frac{...}{\rho} \tag{16}$$

and the stress components are defined by the stress function:

$$\sigma_{\rho\rho} = \frac{f}{\rho^2} \qquad \sigma_{\vartheta\vartheta} = \frac{1}{2\rho}\frac{\mathrm{d}f}{\mathrm{d}\rho} \tag{17}$$

In case of lack of damage ($\mathbf{D} = \mathbf{0}$) the first of the equations (15) has the elementary solution :

$$\sigma_{\rho\rho} = \frac{A}{\rho^3} + B \qquad \sigma_{\vartheta\vartheta} = -\frac{A}{2\rho^3} + B \tag{18}$$

where A and B stand for integration constants.

2.3 Constitutive Equations of Creep-damage Problem

The previously derived system of equations (15) requires the definition of constitutive equations for creep (cf. Skrzypek and Ganczarski, 1999).

Assuming the similarity of deviators based on the flow theory:

$$\dot{\varepsilon}^{\mathrm{c}}_{kl} = \frac{3}{2}\frac{\dot{\varepsilon}^{\mathrm{c}}_{\mathrm{eff}}}{\widetilde{\sigma}_{\mathrm{eff}}}\widetilde{s}_{kl} \tag{19}$$

and the time hardening hypothesis associated with the Kachanov orthotropic brittle rupture law (cf. Kachanov, 1986)

$$\begin{aligned} &\dot{\varepsilon}^{\mathrm{c}}_{\mathrm{eff}} = (\widetilde{\sigma}_{\mathrm{eff}})^m\,\dot{\mathrm{f}}\,(t) \\ &\dot{\Psi}_{ii} = -C_i\,\langle\widetilde{\sigma}_{ii}\rangle^{n_i} \quad \text{no sum over } i \end{aligned} \tag{20}$$

where $\mathsf{f}(t)$ is a given time function, $\langle\rangle$ denote Macauley brackets, the net stress deviator and the net stress and the strain rates are defined by the following formulae:

$$\begin{aligned} &\widetilde{s}_{kl} = \widetilde{\sigma}_{kl} - \frac{1}{3}\widetilde{\sigma}_{ii}\delta_{kl} \quad \dot{\varepsilon}^{\mathrm{c}}_{\mathrm{eff}} = \sqrt{\frac{2}{3}\dot{e}^{\mathrm{c}}_{kl}\dot{e}^{\mathrm{c}}_{kl}} \quad \widetilde{\sigma}_{\mathrm{eff}} = \sqrt{\frac{3}{2}\widetilde{s}_{kl}\widetilde{s}_{kl}} \\ &\widetilde{\sigma}_{ii} = \frac{\sigma_{ii}}{\Psi_{ii}} \quad \text{no summation over } i \end{aligned} \tag{21}$$

When incompressibility of creep is assumed we find:

$$\dot{\varepsilon}^{\mathrm{c}}_{\rho\rho} = (\widetilde{\sigma}_{\mathrm{eff}})^m \left(\frac{\sigma_{\rho\rho}}{\Psi_{\rho\rho}} - \frac{\sigma_{\vartheta\vartheta}}{\Psi_{\vartheta\vartheta}}\right)\dot{\mathsf{f}}(t) \quad \dot{\varepsilon}^{\mathrm{c}}_{\vartheta\vartheta} = (\widetilde{\sigma}_{\mathrm{eff}})^m \left(\frac{3}{2}\frac{\sigma_{\vartheta\vartheta}}{\Psi_{\vartheta\vartheta}} - \frac{1}{2}\frac{\sigma_{\rho\rho}}{\Psi_{\rho\rho}}\right)\dot{\mathsf{f}}(t) \tag{22}$$

In the above formulation the orthotropic damage law equation (20) contains material functions C_i, n_i, independent for each direction, which must be determined. Unfortunately, there are no data available concerning material orthotropy for creep rupture, so in the next application the material isotropy $C_\rho = C_\vartheta = C$ and $n_\rho = n_\vartheta = n$ is considered, but admitting an independent evolution of microcracks in both principal directions $\Psi_{\rho\rho}$, $\Psi_{\vartheta\vartheta}$.

3 Formulation of Boundary Problem

Let us consider a thick-wall sphere of inner and outer radii a and b, respectively, subjected to constant temperature $\theta = \mathrm{const}$ as it is shown in Figure 3.

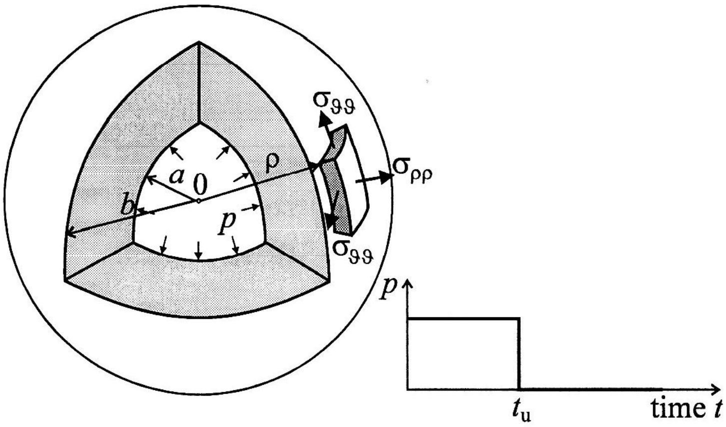

Figure 3: Thick-wall Sphere under Constant Temperature and Loading-unloading Cycle of Internal Pressure

Initially when there is no damage ($\mathbf{D} = \mathbf{0}$) the system of equations (15) together with homogeneous boundary conditions lead to the trivial solution $f \equiv 0$. In order to obtain a non-trivial solution we need to produce some state of damage ($\mathbf{D} \neq \mathbf{0}$). One natural way to do so is to consider two stages of loading. Suppose that in the first stage $t \in \langle 0, t_{\mathrm{u}})$ the sphere is subjected to constant inner pressure when corresponding boundary conditions take the form:

$$\begin{aligned} &\sigma_{\rho\rho}(a) = -p \quad \sigma_{\rho\rho}(b) = 0 \quad \text{for} \quad t = 0 \\ &\dot{\sigma}_{\rho\rho}(a) = 0 \quad\ \ \dot{\sigma}_{\rho\rho}(b) = 0 \quad \text{for} \quad t < t_{\mathrm{u}} \end{aligned} \tag{23}$$

Then, at the moment of time $t = t_{\mathrm{u}}$, the sphere is mechanically unloaded along the path referring, however not to the initial ($\widetilde{\mathbf{E}}(\mathbf{D} = \mathbf{0})$) but to the actual (deterioriated $\widetilde{\mathbf{E}}(\mathbf{D} \neq \mathbf{0})$) constitutive tensor:

$$\begin{aligned} &\sigma_{\rho\rho}(t_{\mathrm{u}}^{+}) = \sigma_{\rho\rho}(t_{\mathrm{u}}^{-}) - \sigma_{\rho\rho}(0, \widetilde{\mathbf{E}}(\mathbf{D})) \\ &\sigma_{\vartheta\vartheta}(t_{\mathrm{u}}^{+}) = \sigma_{\vartheta\vartheta}(t_{\mathrm{u}}^{-}) - \sigma_{\vartheta\vartheta}(0, \widetilde{\mathbf{E}}(\mathbf{D})) \end{aligned} \tag{24}$$

The second stage begins when only constant temperature $\theta = \text{const}$ acts:

$$\dot{\sigma}_{\rho\rho}(a) = 0 \quad \dot{\sigma}_{\rho\rho}(b) = 0 \quad \text{for} \quad t > t_{\mathrm{u}} \tag{25}$$

4 Numerical Algorithm for the Creep-damage Problem

To solve the initial-boundary value problem, we discretize the time by inserting N time intervals Δt_k, where $t_0 = 0$, $\Delta t_k = t_k - t_{k-1}$ and $t_N = t_{\mathrm{R}}$ (rupture) (cf. Ganczarski and Skrzypek, 1997; Skrzypek and Ganczarski, 1999). Hence, the initial-boundary value problem is reduced to a sequence of quasistatic boundary value problems, the solution of which determines unknown functions at given time t_k, e.g., $f(\rho, t_k) = f^k(\rho)$, $\mathbf{D}(\rho, t_k) = \mathbf{D}^k(\rho)$, etc. At each time step the Runge-Kutta II method is applied to yield updated functions f^k, $\mathbf{D}^k$, etc. To account for primary and tertiary creep regimes, a dynamically controlled time step Δt_k is required, the length of which is defined by the bounded maximum damage increment:

$$\Delta D^{\mathrm{lower}} \leq \max_{(\rho)} \left\{ \left[\dot{\mathbf{D}}^k(\rho) - \dot{\mathbf{D}}^{k-1}(\rho) \right] \Delta t_k \right\} \leq \Delta D^{\mathrm{upper}} \tag{26}$$

Discretizing also the radial coordinate ρ_j, by inserting an equal mesh $\Delta\rho = \rho_j - \rho_{j-1}$, we rewrite equation ($15_2$) for a time step t_k in terms of FDM with respect of ρ_j. The numerical procedure begins when the solution of the elastic problem is known. Assuming an initial components of the damage tensor $[\mathbf{D}]_j \equiv 0$, the elastic solution is given by equations (18, 23_1). Next, the program enters the creep damage loop which requires the vector of effective stress intensity, components of the damage tensor and strain rates $[\sigma_{\mathrm{eff}}^{\mathrm{net}}, \dot{\mathbf{D}}, \dot{\varepsilon}^{\mathrm{c}}]_j$ to be computed. The solution of the discretized creep-damage problem equations (15, 23_2) provides rates of the stress function $[\dot{f}]_j$. In the next time step the "new" stress function is computed $[f]_j$.

5 Results

5.1 Material Data

The numerical result presented in this paper deals with a sphere made of ASTM-321 stainless steel (rolled 18Cr 8Ni 0.45Si 0.4Mn 0.1C Ti/Nb) stabilized austenitic annealed at 1070°C with the following mechanical properties at temperature (cf. Odqvist, 1974): $T = \theta = 500$°C, $E = 180$ GPa, $\sigma_{0.2} = 120$ MPa, $\nu = 0.3$, $\alpha = 1.85 \times 10^{-5}$, $m = 5.6$, $n = 3.9$, $\sigma_{c_B}^5 = 210$ MPa. The magnitude of initial pressure is equal to $p = 0.2 \times \sigma_{0.2}$.

5.2 Examples

Minimal Level of Damage Advance

As it is mentioned in Section 3, the first stage of loading described by the time t_{u}, or equivalently by the level of damage growth $\max_\rho D_{ij}(t_{\mathrm{u}})$, controlling the actual level of mechanical and thermal orthotropy, decides which of two processes is dominant: either stress relaxation or damage accumulation. In the first case, an infinite time to rupture $t_{\mathrm{R}} \rightarrow \infty$ is observed, whereas in the other case, the structure is subjected to rupture in a finite time t_{R}. It turns out, for the presented geometry of structure and material data, that there exists a threshold of time $t_{\mathrm{u}}^{\mathrm{treshold}}$ referring to a minimal level of damage growth $\max_\rho D_{ij} = 19\%$ common for both applied hypotheses of equivalence, the reaching of which guarantees a finite time to rupture t_{R}.

Evolution of Anisotropy

A typical distribution of the dominant damage component $\Psi_{\vartheta\vartheta}$ is shown in Figure 4. The damage accumulation concentrates along the inner edge, whereas the center of the structure exhibits a rather

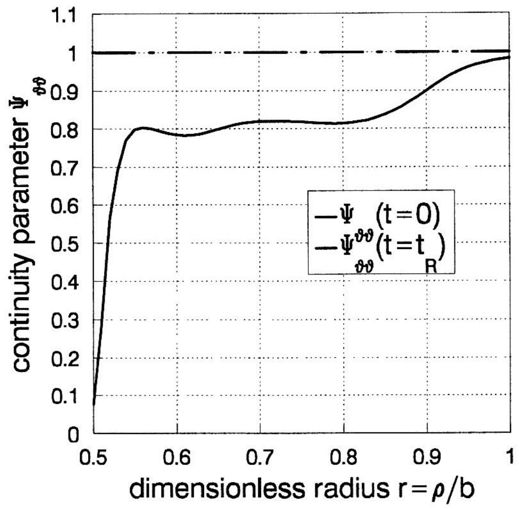

Figure 4: Distribution of the Dominant Component of the Continuity Tensor $\Psi_{\vartheta\vartheta}$ at the Moment of Rupture

homogeneous and low level of damage growth $\Psi_{\vartheta\vartheta} = 80\%$. Thermal anisotropy, which is inversely proportional to the continuity tensor equations (9, 12), exhibits a similar tendency. The mechanical properties in equations (7, 8, 10, 11) remain generally unchanged saving their initial elastic magnitudes except for the last period of tertiary creep, directly preceding the moment of rupture. At this moment $\widetilde{\mathrm{E}}_{\vartheta\vartheta}^{-1}$ and $\widetilde{\mathrm{E}}_{\rho\vartheta}^{-1}$, which essentially contribute to the differential operator in equation (15), exhibit a rapid increase. Theoretically they go to $\pm\infty$, respectively (Figure 5).

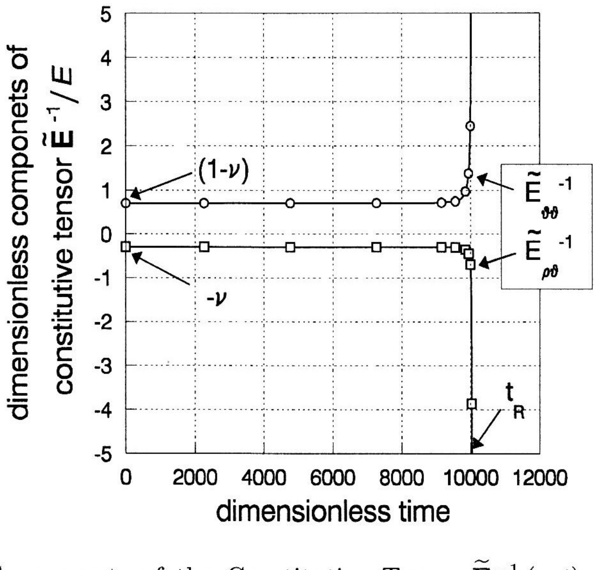

Figure 5: Evolution of Components of the Constitutive Tensor $\widetilde{\mathbf{E}}^{-1}(a,t)$ during the Creep-damage Process

Stress versus Energy Equivalent Principle

In fact, the essential difference between the Stress and Energy Equivalent Principles causes that in the first case the local stiffness drop deals with a local Young's modulus only, whereas in the other case the microcrack growth influences both, Young's modulus and Poisson's ratio. In the presented example no qualitative differences between both approaches are observed. For a better visualization of damage a map of its dominant component $D_{\vartheta\vartheta}$ is shown in Figure 6.

Corresponding to the strong damage accumulation along the inneredge, the hoop stress $\sigma_{\vartheta\vartheta}$ relaxation is not fast enough to prevent the structure from collapse due to the increase of the thermal term in equation (15_2). Residual stresses at the moment of rupture $t_{\rm R}$ versus initial stress distributions are presented in Figure 7, whereas its evolution in time as well as the evolution of the dominant continuity parameter are shown in Figure 8. The quantitative difference between the Stress and the Energy

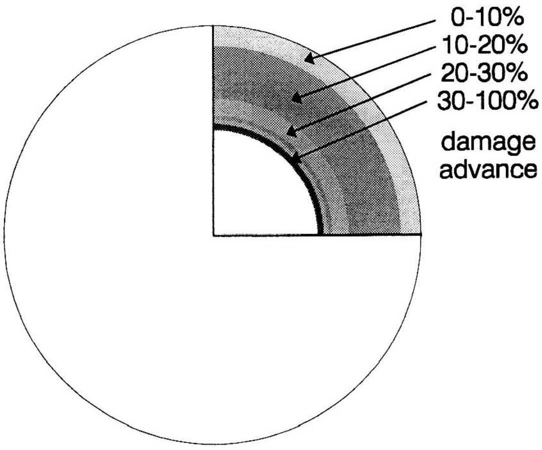

Figure 6: Visualization of the Dominant Component of the Damage Tensor $D_{\vartheta\vartheta}$ at the Moment of Rupture

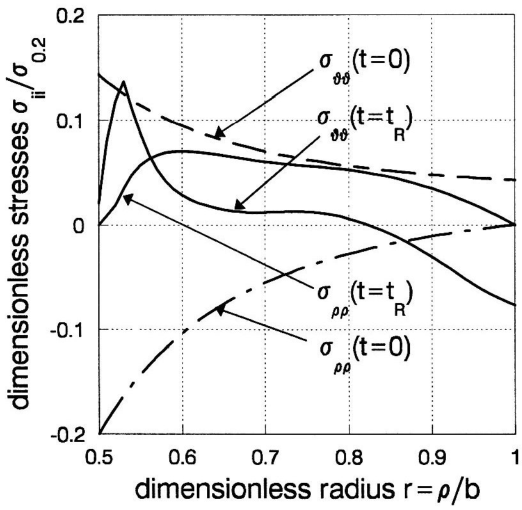

Figure 7: Residual Stresses $\sigma_{ii}\,(\rho, t_{\rm R})$ versus Initial Stresses $\sigma_{ii}\,(\rho, 0)$ in a Sphere during the Creep Damage Process under Constant Temperature

Equivalent Principles is observed when comparing times to rupture. They are shown in Table 1 for case of the common threshold time $t_{\rm u}^{\rm threshold}$ referring to the minimal level of damage growth $\max_\rho D_{ij} = 19\%$. For longer times $t_{\rm u} > t_{\rm u}^{\rm threshold}$ differences in times to rupture of both approaches decrease.

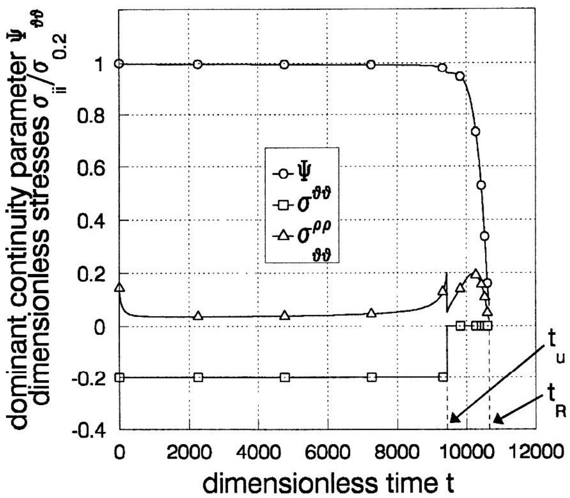

Figure 8: Dominant Continuity Parameter $\Psi_{\vartheta\vartheta}(a,t)$ and Dimensionless Stress Components $\sigma_{ii}(a,t)/\sigma_{0.2}$ versus Dimensionless Time

Stress Equivalence Principle	Energy Equivalence Principle
$t_{\mathrm{R}}^{\mathrm{SEP}}$	$t_{\mathrm{R}}^{\mathrm{EEP}} = 1.06 \times t_{\mathrm{R}}^{\mathrm{SEP}}$

Table 1: Comparison of Times to Rupture in Case of Stress and Energy Equivalence Principles

Acknowledgments

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Literature

1. Chaboche, J.–L.; Lesne, P. M.; Moire, J. F.: Continuum damage mechanics, anisotropy and damage deactivation for brittle materials like concrete and ceramic composites, Int. J. Damage Mech., 4, (1995), 5 – 22.
2. Chen, X. F.; Chow, C. L.: On damage strain energy release rate Y, Int. J. Damage Mech., 4, 3, (1995), 251 – 263.
3. Cordebois, J. P.; Sidoroff, F.: Damage induced elastic anisotropy, Coll. Euromech 115, Villard de Lans, 1979, also in Mechanical Behavior of Anisotropic Solids, ed. Boehler, J. P., Martinus Nijhoff, Boston, (1983), 761 – 774.
4. Fung, Y. C.: Foundations of Solid Mechanics, Prentice-Hall, (1965).
5. Ganczarski, A.: Effect of brittle damage on thermal expansion tensor, Proc. of Thermal Stresses'99, (1999), 267 – 270.
6. Ganczarski, A.; Skrzypek, J.: Optimal design of rotationally-symmetric disks in thermo-damage coupling conditions, Technische Mechanik, **17**, 4, (1997), 365 – 378.
7. Kachanov, L. M.: Introduction to Continuum Damage Mechanics, Martius Nijhoff, (1986).
8. Lemaitre, J.; Chaboche, J.–L.: Aspect phenomenologique de la rupture par endommagement, J. de Méchanique Applique, 2, (1978), 317 – 365.
9. Murakami, S.; Ohno, N.: A continuum theory of creep and creep damage, in Creep in Structures, ed. Ponter, A. R. S. and Hayhurst, D. R., Springer, Berlin, (1981), 442 – 444.

10. Odqvist, F. K. G.: Mathematical Theory of Creep and Creep Rupture, Clarendon Press Oxford, (1974).

11. Qi, W.; Bertram, A.: Anisotropic creep damage modeling of single crystal superalloys, Techn. Mechanik, 17, 4, (1997), 313 – 332.

12. Skrzypek, J.; Ganczarski, A.: Modeling of Material Damage and Failure of Structures, Theory and Applications, Springer-Verlag Berlin Heidelberg, (1999).

Symbols

Mechanical Quantities	
a, b	radii of sphere
C_i, n_i	constants in damage law
$\mathbf{D}$	second-order damage tensor
δ_{ij}	Kronecker's symbol
E, ν	Young's modulus, Poisson's ratio
$\mathbf{E}$	fourth-order constitutive tensor
$\mathbf{e}, \boldsymbol{\varepsilon}, \varepsilon_{\mathrm{eff}}$	strain deviator, tensor and intensity
$\mathsf{f}(t)$	given time function
m	exponent in creep law
$\mathbf{M}$	fourth-order damage effect tensor
p	internal pressure
$\boldsymbol{\Psi}$	second-order continuity tensor
ρ, ϑ	spherical coordinates
$\mathbf{s}, \boldsymbol{\sigma}, \sigma_{\mathrm{eff}}$	stress deviator, tensor and intensity
t	time
t_{R}	rupture time
t_{u}	time of unloading
$\mathbf{1}$	second-order unit tensor
Thermal Quantities	
$\alpha, \boldsymbol{\alpha}$	coefficient and tensor of thermal expansion
T	actual temperature
T_o	reference temperature
θ	change of temperature

Additionally, following superscripts stand for quantities

e	elastic
c	inelastic (creep)
th	thermal
SEP	Stress Equivalence Principle
EEP	Energy Equivalence Principle

respectively. Also, a tilde over a symbol refers to quantity with deterioration taken into account, a dot over quantity denotes derivative with respect to time.

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