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Local CDM Based Approach to Fracture of Elastic-Brittle Structures

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This paper is devoted to the problem of mesh-dependence and regularization methods in the local approach to fracture analysis based on Continuum Damage Mechanics and the Finite Element Method. The causes of the mesh-dependence and some regularization methods are discussed first. Then the regularization technique is proposed and illustrated by a numerical example of damage evolution and crack propagation analysis in a 2D structure made of elastic-brittle-damage material.

1 Introduction

Modelling of damage is related to the scale. A commonly used classification of damage models is referred to three scales: the atomic scale (molecular dynamics, atomic bonds and vacancies, dislocations, etc.), the microscale (microcrack size, orientation, position and slip systems), and the macroscale (effective properties, damage variables, plastic strain tensor, etc.). On the atomic scale a material structure is not continuous, as represented by a configuration of atoms in the order of a crystalline lattice or molecular chains, bounded by the interatomic forces. The current configuration of the atomic bonds represents the damage state, while the breaking and re-establishing of them refer to the damage evolution. On the microscale a piece-wise discontinuous and heterogeneous material structure is observed, and the state of damage is determined by the topology and number of microcraks, their size and orientation. On the macroscale, when for a large number of microdefects in an element the exact position of the microcracks and interaction between them may be ignored, a concept of 'quasi-continuum' is introduced. In other words, the discontinuous and heterogeneous solid is approximated by the pseudo-undamaged continuum by the use of the couples of effective state variables. The definition of the effective state variables is usually based on the strain, the stress or the energy equivalence principles. In all cases of various equivalence principles it is assumed that in a quasi-continuum the true distribution of defects is smeared out and homogenized by properly defined internal variables that characterize damage: the scalar variables ω or D (Kachanov et al., 1958), the vector variables ω_α or D_α (Davison et al., 1973), the second rank tensor variables $\mathbf{\Omega}$, $\mathbf{D}$ (Vakulenko et al., 1971; Murakami et al., 1981), or the fourth-rank tensor variables, $\hat{\mathbf{D}}$ (Chaboche, 1982; Krajcinovic, 1989). The effective stiffness or compliance of solids that undergo damage may also be defined in terms of the actual damage state. This fully coupled CDM approach, when the damage evolution influences both the stress/strain state and also the elastic properties of the material (cf. Chaboche, 1977; Cordebois et al., 1979; Litewka, 1985; Murakami et al., 1997) leads to the concept of the fourth-rank elasticity tensors, stiffness $\tilde{\Lambda}$, or compliance $\tilde{\Lambda}^{-1}$ if the damage induced anisotropy is given by the so called fourth-rank damage effect tensor $\mathbf{M}(\mathbf{D})$. All these models are developed in the frame of the so called Local Approach, when the direct interaction between microcracks is ignored and the length over which they are correlated is negligibly small (an extensive and comprehensive survey of one- and three-dimensional damage models for elastic and inelastic solids, as well as examples of practical applications, was done by Skrzypek et al., 1999a).

A transition between the atomic- or microscale and the effective properties on the macroscale requires a proper selection of the Representative Volume Element (RVE), which maps a finite volume of the damaged piecewise-discontinuous and heterogeneous solid, onto a material point of the pseudo-undamaged quasi-continuum. This effective quasi-continuum method, also called CDM method, is based on the following assumptions (cf. Krajcinovic, 1995):

- each defect within the RVE is subjected to the same stress field derived from the external tractions applied at the boundary of this element,
- influence of all other defects within the RVE on the observed defect is measured through the change of effective stiffness or compliance.

In other words, the exact spatial correlation of the defects within the RVE has negligible influence on the effective properties defined within the element. The crucial problem is the proper selection of the linear length λ_{RVE} of the RVE. It must be large enough to include such a number of microdefects that is sufficient for the homogenization within the RVE. At the same time, the length λ_{RVE} of the RVE must be small enough for the stress to be considered as homogeneous. Hence, the magnitude of λ_{RVE} , and consequently the finite element size, depend on the material microstructure and on the stress nonhomogeneity. The existence of an RVE that allows the material to be considered statistically homogeneous within the element, is the condition for a local approach, in which there are no scale parameters involved. If, on the other hand, direct interaction of the correlated defects becomes significant for their growth and stability, it must be incorporated into the model by introducing the correlation length between neighbouring defects as a scale parameter. Local approach is no longer sufficient and a non-local damage model must be established (cf. Woo et al., 1993; Murakami et al., 1995).

When the local approach to fracture analysis by FEM is used, the crucial problem is the mesh-dependence and its regularization. This problem was examined by Murakami et al. (1995) when applied to creep-fracture analysis by the use of an elastic-creep material with isotropic damage, the Kachanov-Rabotnov theory, and a scalar Young's modulus drop caused by damage, all incorporated into UMAT of the FEM code ABAQUS. By the use of this simple model the assembly of fractured elements is considered as a crack, when the stress in an element is released, if the scalar damage variable in the element reaches the critical value D_{cr}, and the free surface is created. Hence, the crack width is governed by the size of the finite element, and it cannot develop in the direction transverse to the crack length. Strong mesh-dependence of both the crack length growth and the stress and damage concentration is observed. Possibilities of their regularization were examined in the frame of the local approach by use of the non-local damage variable (average over a neighbourhood of the crack tip), simplified stress limitation (ideal plasticity) and a modification of the damage evolution law.

In the present paper the regularized local approach to damage evolution prior to fracture (precritical structure response) and the macrocrack propagation through the volume of the elastic-brittle metallic (steel) structure (postcritical structure response), is applied. The damage anisotropy is accounted for by the application of the Murakami and Ohno (1981) modified non-local second rank damage tensor, and the extended time-dependent elastic-brittle constitutive model originated by Litewka (1985), (1989). A non-local damage tensor is introduced in all constitutive equations, whereas the stress and strain tensors in the physical equations are treated locally (a similar approach was presented by Bazant et al. (1987), where only those variables that controll strain softening are subject to non-local treatment). The damage variable is averaged over a neighbourhood of a crack tip by introducing a modified, non-local and limited stress measure for the purpose of a damage evolution equation and a failure criterion. The failure criterion has the form of an isotropic scalar function of non-local stress and damage tensors, when the damage evolution law by Litewka and Hult (1989) is generalized by the use of an objective damage derivative to account for the rotation of the principal stress and the damage axes. Consequently, the shear effect on damage is included (cf. Skrzypek et al., 1998a).
The material model is put into the UMAT of ABAQUS, and the numerical example of a 2D plane stress structure is presented.

2 Mesh-dependence and Regularization Methods for the Local Approach to Fracture

A crack region is often understood as an assembly of elements in which the critical conditions of damage have been reached. The stress in an element that undergoes fracture is usually totally released after failure, so the stress on the free surface of the crack must vanish, and the crack cannot develop in the direction transverse to the crack length. Therefore the crack width is governed by the element size, and can be unlimitedly reduced by the mesh refinement. As a consequence of unlimited decrease of the predicted crack width the problem of damage localization appears.
Two different procedures to model the macrocrack penetration through the volume of the structure were proposed by Skrzypek et al. (1998b, 1999b). The first procedure is mostly the tension controlled crack length growth, if the anticipated crack is formed along the structure fixed edge. When the failure criterion is satisfied in a neighbouring (damaged) element, the appropriate kinematic boundary condition is released to allow for the crack opening on the free surface produced. The element disconnected from the rigid edge is left in the mesh and may carry the shear stress, although the tensile stress in the direction normal to the crack is released. Therefore the crack width is not limited by the element size. The second procedure is a combined tension/shear controlled crack branching mechanism that allows the crack to deviate from the primary direction along the fixed edge to the interior. In this case the neighbouring element which is led to failure is fully removed, and a crack of the width of the element is formed. Interaction of these two mechanisms, changing of the kinematic boundary condition on the element face and/or fully removing the element that has come to failure, models an unstable process of the structure under fracture, which leads to its fragmentation.
To mitigate the damage localization in the region preceding the crack tip several ways of regularization, referred to the damage variable, were classified by Murakami et al. (1995). First in the frame of non-local formulation:

I. non-local damage theory (Bazant, 1990; Bazant et al., 1987, 1988; Saanouni et al., 1989; Hall et al., 1991),

whereas the other in the frame of local approach:

II. averaging the damage process in the material by incorporating non-local damage measure,

III. using a moderately low value for the critical damage in the failure criterions of type $D \leq D_{cr}$ (if scalar damage variables are sufficient),

IV. mitigating stress sensitivity in the damage evolution equations.

Another problem in the local approach to fracture, when elastic-brittle structures are considered, is caused by the stress singularity around the crack tip, and the following two procedures have been proposed (Murakami et al., 1995) to obtain convergence of the solutions:

V. limitation of the stress level around the crack tip,

VI. relaxation of the stress concentration around the crack tip by incorporating plasticity to the elastic-plastic-damage models (Hayakawa et al., 1998).

In the present paper, where the elastic-brittle-damage model is used, the following regularization procedure is employed: the damage variable is subjected to non-local treatment, by introducing a non-local and limited stress measure in damage evolution equation and failure criterion. The stress concentration at the crack tip is limited according to the following formulae

$$\bar{\sigma}_{ij} = \begin{cases} \sigma_{ij}, & \sigma_{\mathrm{EQ}} \leq \sigma_{\mathrm{u}} \\ k\sigma_{ij}, & \sigma_{\mathrm{EQ}} > \sigma_{\mathrm{u}} \end{cases}$$

$$k\sqrt{\frac{3}{2}\mathbf{s}:\mathbf{s}} - \sigma_{\mathrm{u}} = 0 \tag{1}$$

where σ_{EQ} is the Huber-Mises-Hencky equivalent stress, $\mathbf{s}$ is the stress deviator, and the factor k is determined from the yield criterion.

Regularization of the local variation of the damage field $\mathbf{D}(\mathbf{x})$ is achieved by averaging a non-local stress variable $\hat{\boldsymbol{\sigma}}(\mathbf{x},\Omega_d)$ over the neighbourhood $\Omega_d(\xi)$ of $\mathbf{x}$

$$\hat{\sigma}_{ij}(\mathbf{x},\Omega_d) = \frac{\int_{\Omega_d} \bar{\sigma}_{ij}(\xi)\phi(\mathbf{x},\xi)\mathrm{d}\Omega_d(\xi)}{\int_{\Omega_d} \phi(\mathbf{x},\xi)\mathrm{d}\Omega_d(\xi)} \tag{2}$$

where $\mathbf{x}$, ξ, and $\bar{\sigma}_{ij}$ denote a material particle, a particle in the neighbourhood Ω_d of $\mathbf{x}$, and the local (but limited according to equation (1)) stress component of a particle ξ, respectively. Symbol ϕ denotes the weight function, which decreases with the distance from point $\mathbf{x}$. The simplest case is when $\phi(\mathbf{x},\xi)=1$ within a neighbourhood of $\mathbf{x}$, and vanishes outside. Then equation (2) provides mean values within specified domain (cf. Mróz et al., 1999). In the present paper $\phi(\mathbf{x},\xi)=\exp\left[-\left(\frac{d(\mathbf{x},\xi)}{d^*}\right)^2\right]$ was incorporated (cf. Murakami et al., 1995; Chaboche, 1999), where d and d^* are respectively the distance between $\mathbf{x}$ and ξ, and the characteristic distance on the scale of the material microstructure, for example the grain size (Chaboche, 1999).

3 Basic Equations of Anisotropic Elasticity Coupled with Damage

As damage variable the second-rank damage tensor $\mathbf{D}^*$ is applied, which in local formulation takes the form

$$\mathbf{D}^* = \sum_{i=1}^{3} D_i^* \mathbf{n}_i \otimes \mathbf{n}_i \tag{3}$$

The principal values of $\mathbf{D}^*$ are related to those representing the traditional Murakami-Ohno's tensor **D** through

$$D_i^* = \frac{D_i}{1-D_i}, \quad D_i \in \langle 0,1 \rangle, \; D_i^* \in \langle 0,\infty \rangle \tag{4}$$

The objective damage rate tensor $\overset{\nabla}{\mathbf{D}}$ corresponding to the rotation of principal stress axes from α_i to $\alpha_i + \mathrm{d}\alpha_i$, with time changing from t to t+dt, is obtained by the use of the Zaremba-Jaumann objective derivative (cf. Skrzypek et al., 1998)

$$\overset{\nabla}{\mathbf{D}} = \dot{\mathbf{D}} - \mathbf{D}^{\mathrm{T}}\mathbf{S} - \mathbf{S}^{\mathrm{T}}\mathbf{D} \tag{5}$$

where $\dot{\mathbf{D}}$ is the time derivative of the damage tensor, and $\mathbf{S}$ is the spin tensor.

The non-local non-objective damage rate $\dot{\hat{\mathbf{D}}}$ in current principal directions of the non-local stress tensor $\hat{\boldsymbol{\sigma}}$ (see equation (2) is calculated through the modified Litewka-Hult damage evolution equation (cf. Litewka et al., 1989)

$$\dot{\hat{\mathbf{D}}} = C\left[\frac{1-2\nu}{6E}\mathrm{Tr}^2\hat{\boldsymbol{\sigma}} + \frac{1+\nu}{2E}\mathrm{Tr}\left(\hat{\mathbf{s}}^2\right) + \frac{\hat{D}_{\mathrm{I}}^*}{2E\left(1+\hat{D}_{\mathrm{I}}^*\right)}\mathrm{Tr}\left(\hat{\boldsymbol{\sigma}}^2 : \hat{\mathbf{D}}^*\right)\right]^2 \hat{\boldsymbol{\sigma}}^* \tag{6}$$

Here C is a temperature-dependent material constant, $\hat{\mathbf{s}}$ - the non-local stress deviator, and the modified stress tensor $\hat{\boldsymbol{\sigma}}^*$ is employed to account for the limited damage evolution under compression

$$\hat{\boldsymbol{\sigma}}^* = \left\langle \hat{\boldsymbol{\sigma}} \right\rangle - \zeta \left\langle -\hat{\boldsymbol{\sigma}} \right\rangle, \quad \zeta \in [0,1] \tag{7}$$

where the symbol $\langle \; \rangle$ denotes Macauley brackets, and ζ is a material constant used to represent unilateral damage response. In the case when $\zeta = 0$ no damage growth under compressive stresses is allowed (cf. Litewka, 1985). On the other hand, $\zeta = 1$ means the same material response in the damage growth for compression and tension. In general, the parameter ζ should be determined from a tension/compression test (cf. Murakami et al., 1997). In the present paper $\zeta = 0$ is applied.

The non-local failure criterion is assumed as the three-parameter damage affected isotropic scalar function of stress and damage tensors

$$F\left(\hat{\boldsymbol{\sigma}}, \hat{\mathbf{D}}^*\right) = C_1 \mathrm{Tr}^2 \hat{\boldsymbol{\sigma}} + C_2 \left(\mathrm{Tr}\,\hat{\mathbf{s}}\right)^2 + C_3 \mathrm{Tr}\left(\hat{\boldsymbol{\sigma}}^2 : \hat{\mathbf{D}}^*\right) - \sigma_{\mathrm{u}}^2 = 0 \tag{8}$$

where C_1, C_2, and C_3 are material constants dependent on the state of damage growth process, and σ_{u} is the ultimate strength of the undamaged material. Constants C_1, C_2, and C_3 may be determined by applying equation (8) to three different states of stress: to uniaxial tension in the two perpendicular principal damage directions, and to equal biaxial tension in those directions (cf. Litewka et al., 1989).

It is usually assumed that failure occurs when at least one principal value of the damage tensor is equal to unity. This leads to stress sensitivity of the damage evolution equation when the damage variable is close to its maximum value 1.0 (cf. Murakami et al., 1995). According to the failure criterion (8) the onset of failure is observed when the material yield stress, continuously decreasing due to the damage growth, becomes equal to the stress actually applied. Equation (8), even if written in local stress and damage variables, leads to failure

when the principal damage tensor components reach their critical combination, for values usually considerably less than unity. Hence, there is no need to additionally reduce the critical damage values as it was proposed by Murakami et al. (1995). However, the failure criterion (8) itself is stress sensitive and leads to immediate failure when the equivalent stress at a crack tip exceeds the current yield stress for the damaged material. Therefore, non-local limited stress and non-local damage variables are applied.
The stress-strain relations of elasticity are written here for the local stress and strain variables, whereas the time-dependent fourth-rank anisotropy tensor $\hat{\tilde{\mathbf{\Lambda}}}\left(\hat{\mathbf{D}}^*\right)$ is treated non-locally

$$\boldsymbol{\varepsilon} = \hat{\tilde{\mathbf{\Lambda}}}^{-1}\left(\hat{\mathbf{D}}^*\right) : \boldsymbol{\sigma} \tag{9}$$

The representation of the elasticity tensor $\tilde{\Lambda}^{-1}_{ijkl}$ reduced to the form linear in damage, was derived by Litewka (1985). Introducing the extension to the non-local damage tensor $\hat{\mathbf{D}}^*$ we obtain

$$\hat{\tilde{\Lambda}}^{-1}_{ijkl} = -\frac{\nu}{E}\delta_{ij}\delta_{kl} + \frac{1+\nu}{2E}\left(\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk}\right) + \frac{\hat{D}^*_1}{4\left(1+\hat{D}^*_1\right)E}\left(\delta_{ik}\hat{D}^*_{jl} + \delta_{jl}\hat{D}^*_{ik} + \delta_{il}\hat{D}^*_{jk} + \delta_{jk}\hat{D}^*_{il}\right) \tag{10}$$

E and ν denote Young's modulus and Poisson's ratio of the undamaged material. Hence, the following equation of anisotropic elasticity coupled with damage is furnished

$$\boldsymbol{\varepsilon} = -\frac{\nu}{E}(\mathrm{Tr}\boldsymbol{\sigma})\mathbf{1} + \frac{1+\nu}{E}\boldsymbol{\sigma} + \frac{\hat{D}^*_1}{2\left(1+\hat{D}^*_1\right)E}\left(\boldsymbol{\sigma} : \hat{\mathbf{D}}^* + \hat{\mathbf{D}}^* : \boldsymbol{\sigma}\right) \tag{11}$$

The matrix form of constitutive equations (5) - (11), written in local variables, has been given in the paper by Skrzypek at al. (1998b).

4 Numerical Example for Mesh-dependence of the Non-regularized Solution

As a numerical example a cracked 2D structure subjected to in-plane uniform mechanical load is analysed (Figure 1). The ratio between width $2b$ and the initial crack length $2a$ is b/a=3. (cf. also Murakami et al., 1988).

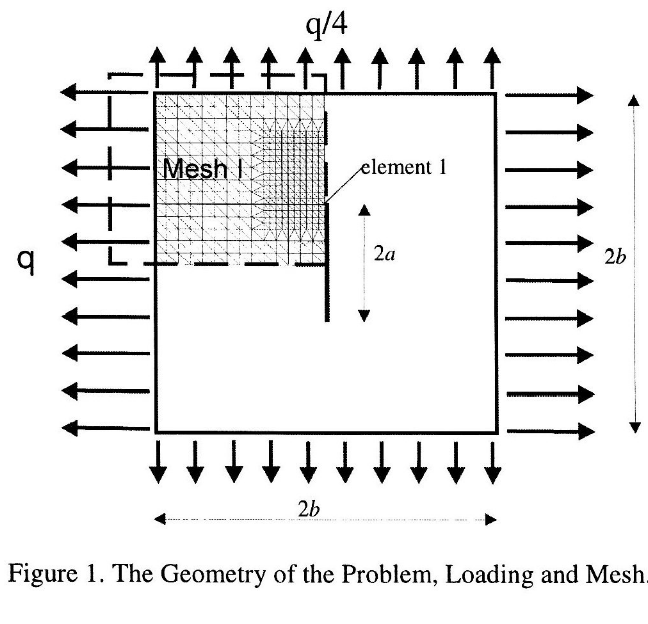

Figure 1. The Geometry of the Problem, Loading and Mesh.

The following dimensionless quantities have been considered: $\bar{q} = \frac{q}{\sigma_u \cdot 1[\mathrm{m}]} = 0.096$, Young's modulus $\bar{E} = 416.7$, Poisson's ratio ν=0.3, $\bar{C} = C\sigma_u \cdot 1[\mathrm{s}] = 6.81 \cdot 10^6$, and $\sigma_u = 288[\mathrm{MPa}]$. The material data correspond to carbon steel AISI at a temperature of 811 [K] (cf. Litewka, 1989). Due to the symmetry of the problem only one quarter is subject to numerical analysis. The structure is discretized by the 2D first-order isoparametric plane stress elements CPS4 and CPS3 (mesh I – see Figure 1).

The effect of element size on a process of damage growth and crack propagation was examined on three meshes of the same topology but different densities (see Figure 2).

Without regularization, the dimensionless crack incubation periods t_I of meshes II and III, referred to the crack incubation period of mesh I, are 0,062 and 0,002 respectively, whereas rupture times t_f of those meshes, referred to the rupture time of mesh I, are 0,015 and 0,0012 respectively. The differences between the results indicate very strong mesh size dependence of lifetime predictions. The pattern of macrocrack growth, on the other hand, is dependent rather on the finite element shape, because a crack always extends from the element at the current crack tip to the element that shares a segment with it. This results from the stress concentration at a crack tip.

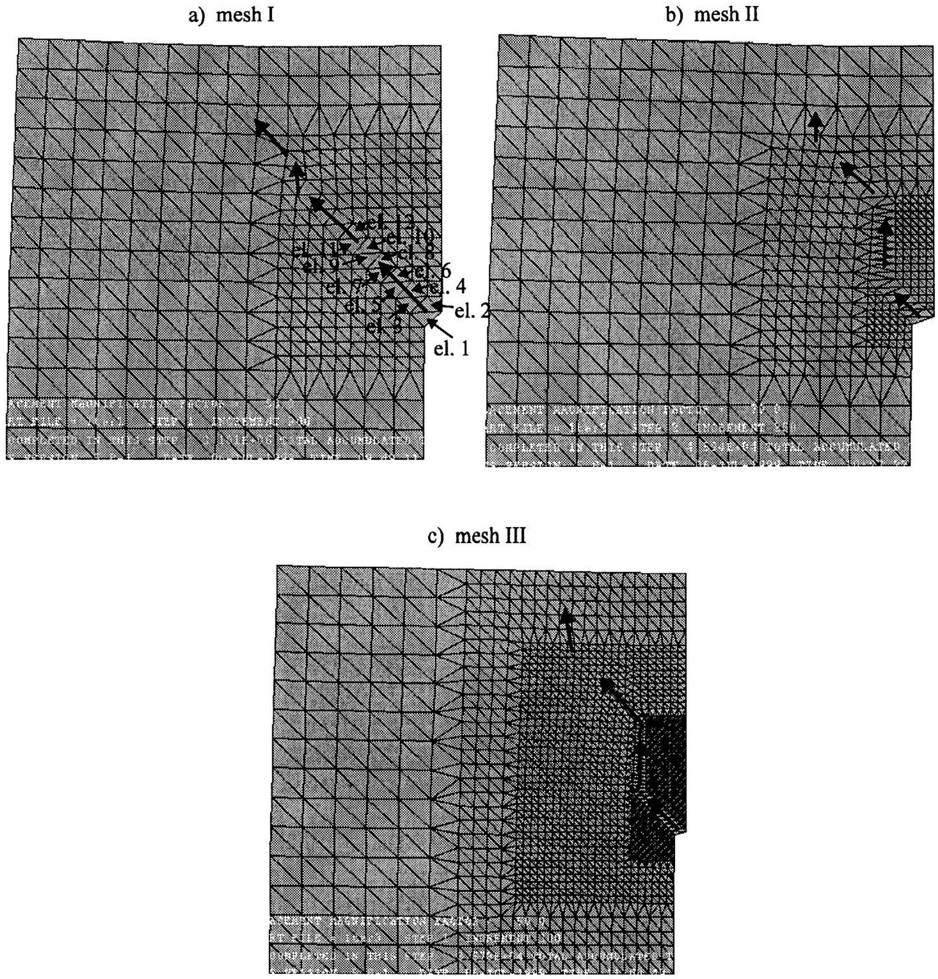

Figure 2. Mesh Discretization and Cracks Patterns.

Hence, in the case of triangular elements the crack growth direction is limited to the direction perpendicular to the initial crack, and left-and-upper direction. The mesh discretization in the region of element size change restricts the crack development to the direction parallel to the initial crack, and to left-and-lower direction, so there is a change of crack direction in that region (see Figure 2). Two other examples of mesh discretization: using quadratic elements, and triangular ones but of orientation different from the meshes I, II and III, and the corresponding crack patterns are shown in Figure 3.

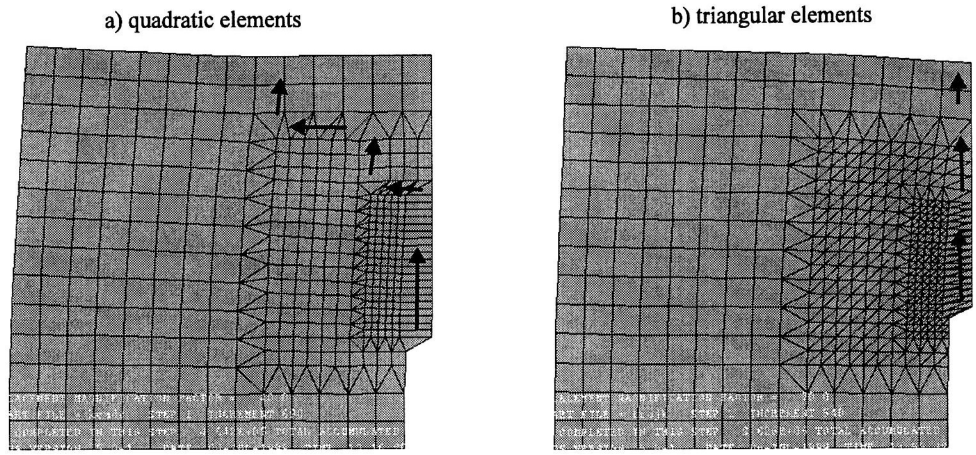

Figure 3. Effect of Element Size on Cracks Pattern.

The evolution with time of the equivalent stress (Figure 4a) in elements that constitute a macrocrack illustrates the following features of the stress field in the process of numerical analysis of damage evolution and crack propagation:

A) stress redistribution from the most deteriorated area to the neighbouring elements (due to coupling with damage in the physical equation (11)),
B) the avalanche character of the final stage of fracture,
C) the increase of the stress values at a crack tip with macrocrack development (due to effective structure width reduction),
D) stress drop to zero in the failed element,
E) discontinuous stress increase in the element at the current crack tip (due to releasing the stress level in previous element).

From Figure 4b) we can draw several conclusions referred to the damage field:

F) the critical damage values are less than 1.0, and they decrease with the stress increase at the crack tip,

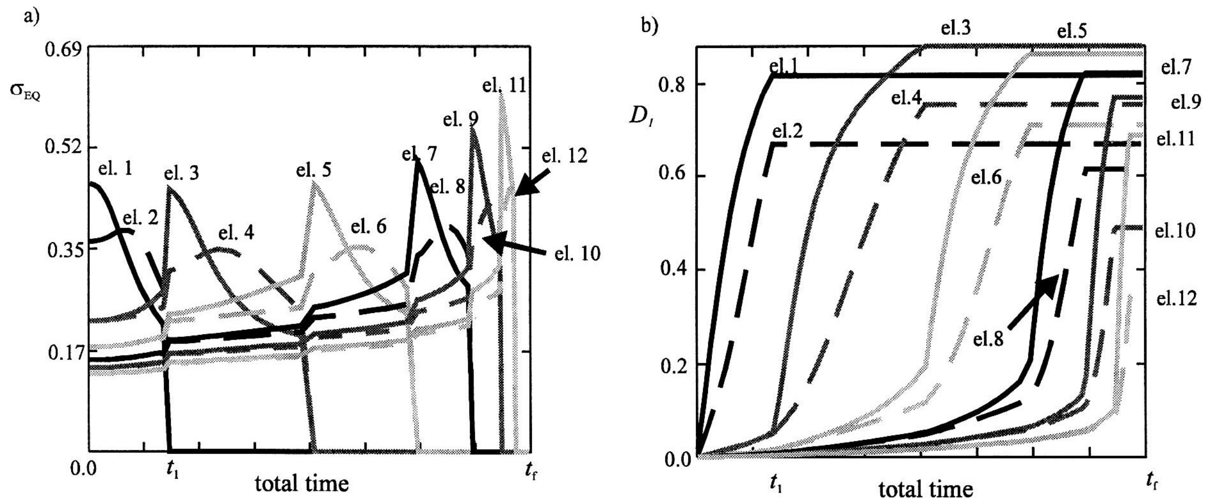

Figure 4. The Evolution in Time of a) Equivalent Stress and b) Maximum Principal Damage Component in the Elements along the Macrocrack (Mesh I).

G) in the element at the current crack tip the increase of damage rate can be observed (due to E)), followed by the damage rate decrease (due to A)).

5 Effects of Regularization Technique

To suppress the element size and shape effect on the solution the following numerical algorithm has been applied: numerical calculations start with the elastic solution for a virgin material. Then the damage evolution process begins. The failure criterion (equation (8)) is checked at the beginning of each time increment, after the non-local damage and stress tensors are transformed to the damage principal directions. In the case of non-advanced damage the non-local stress and damage tensors are transformed to the principal directions of the non-local stress tensor. The non-objective (equation (6)) and the objective (equation (5)) derivatives of the non-local damage tensor components are calculated next, and the current damage tensor in the new principal stress directions is obtained. If, on the contrary, the failure criterion has already been satisfied, the macrocrack modelling is activated, by reducing the stiffness in the cracked element to zero. Then the elastic response again takes place, but after the elastic compliance matrix (according to equation) has been updated to account for the current damage and macrocrack development state.

Incorporating the regularization technique described above allows to mitigate the stress singularity at a crack tip. Figure 5 shows the distributions of the equivalent stress at initial time t_0 in the area around the initial crack tip. Without any additional regularizations the elastic solution exhibits unlimited stress increase and localization with the fineness of the mesh (Figure 5a), 5c)). The results of regularization on the initial equivalent stress field are shown in Figures 5b) and 5d). Figure 6a) shows the function of the dimensionless equivalent stress (referred to the yield stress of a virgin material) versus decreasing element size in element 1 (see Figure 2a)). The limitation of stresses in a current element together with averaging the stress tensor components over the neighbourhood of a current integration point lead to converging results that differ by no more than 8%. The convergence of the equivalent stresses with mesh fineness results in improving the predictions of the crack incubation period. The function of the dimensionless crack incubation period (referred to the crack incubation period of mesh I without regularization) is shown in Figure 6b). The regularization described above also seems to mitigate the influence of the element shape on a crack pattern (see Figure 7).

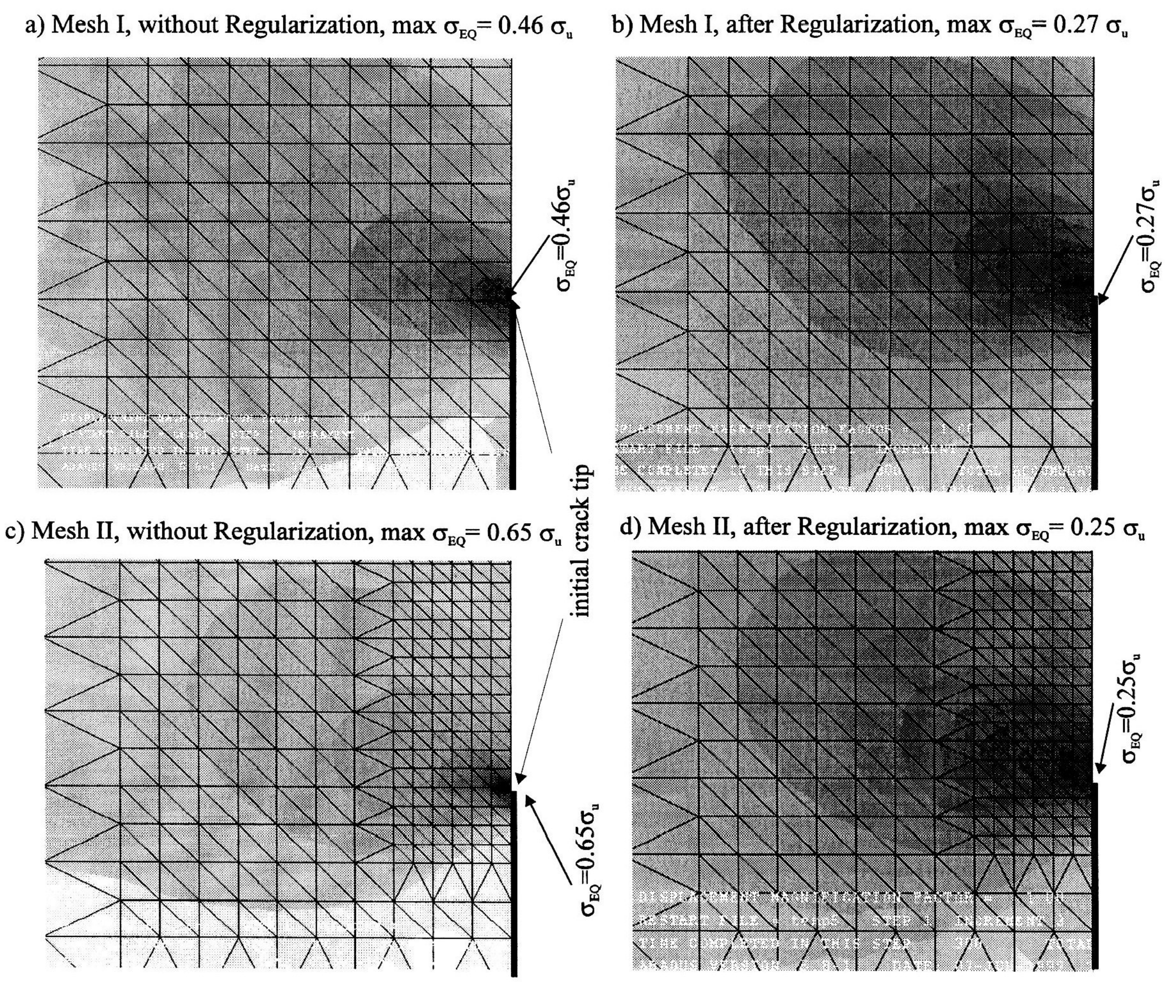

Figure 5. Distribution of Equivalent Stress at Time t_0.

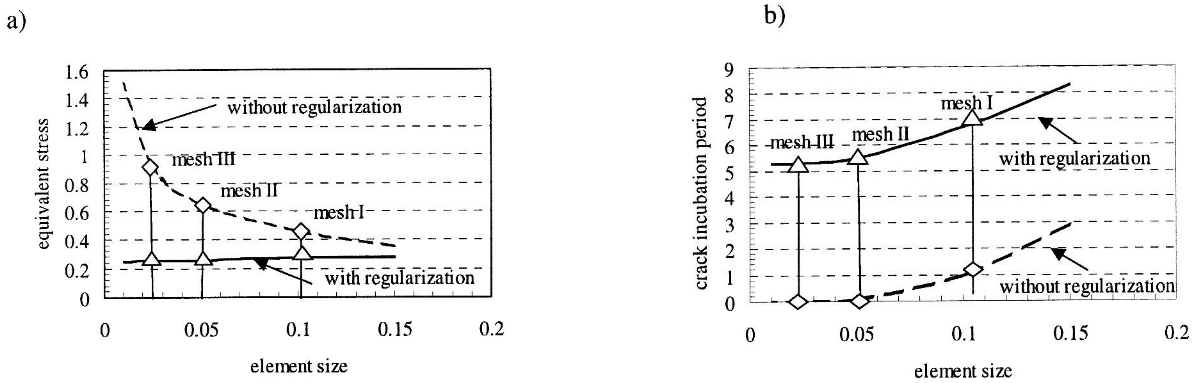

Figure 6. a) Equivalent Stress; b) Crack Incubation Period versus Element Size.

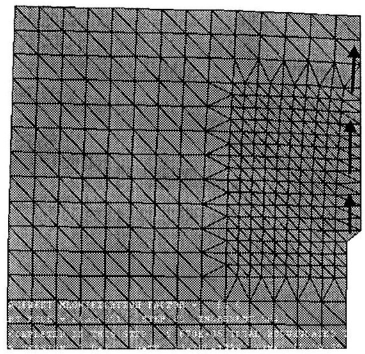

Figure 7. Crack Pattern in Mesh I after Regularization.

6 Conclusions

- The use of a non-local stress measure in the damage evolution equation, while local stress and strain variables are considered in the physical equation, leads to convergence of time predictions with the fineness of the mesh.
- Proper selection of the characteristic distance d^* is a crucial point of this non-local approach. Once d^* is established, there is also a problem of proper element size. As it can be seen from Figure 8, the regularization has a negligible effect on the results if the linear size of the finite elements at a crack tip becomes greater than the characteristic distance d^*.

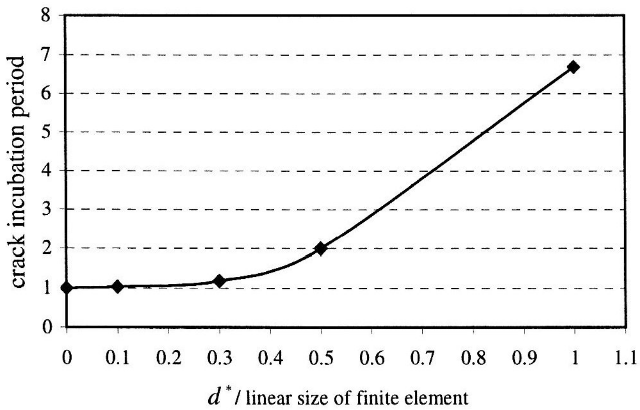

Figure 8. Crack Incubation Period versus Characteristic Length d^* (Referred to the Linear Size of Finite Element).

- Since the non-local damage measure allows to integrate defects over several finite elements, the minimal finite element size is no longer restricted to the size of the Representative Volume Element. On the other hand, the characteristic distance d^*, when correlated with the RVE size, enforces the use of very fine meshes, where the elements around the crack tip are smaller than the Representative Volume Element for a certain material.
- The regularization technique described above seems to mitigate the dependence of the crack pattern on an element shape, which results in different crack patterns in comparison to the basic solution (see Figure 7). However, this problem needs further investigations.

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