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One Way to Model the Influence of Heterogeneous Micro Stresses on Damage

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The stresses, which in continuum mechanics are attributed to a singular material point, are the averages of a distinctly non-homogeneous distribution of stresses within the neighbourhood of the point. This neighbourhood has to be both sufficiently small to still be called a point from the macroscopical point of view, and large enough to contain a statistically adequate number of different values, so that the process of averaging doesn't depend on the actual size of the neighbourhood. Such a neighbourhood is called a represenative volume element (RVE), the size of which is determined by a characteristic length differing for each material. The aforementioned homogenization by averaging has no ill consequences as long as the disregarded peaks are endured by the material in any case. But a model describing material behaviour in continuum damage mechanics (CDM) may risk ignoring the main reason for growth of damage if it were to apply the view of continuum mechanics without critical reflection.
In this paper, a model is proposed that pays attention to the roughness of stress distributions observable on a smaller scale by way of a scalar internal variable, which can be viewed as the 2nd moment of those distributions. In the case of homogeneous stress distributions the variable would vanish. The evolution of damage is designed to be only slightly connected with this variable, because it only influences the damage threshold. For the roughness variable itself a simple evolution equation is proposed. To give an example, the damage model is applied to a strip of fibre-enforced laminate exposed to a process of cyclic pure bending.

1 Background

In the recent endeavour to describe anisotropic behaviour of a damaged, originally isotropic material, as it is observed in experiments, there are several proposals as to how to define suitable internal variables in the form of tensors from 0th to 8th order. A detailed survey is found in Krajcinovic (1984). A systematic way of defining damage tensors is to develop the scalar-valued space- and orientation-dependent crack density into a generalized Fourier series of spherical functions. This method devised by Onat and Leckie (1988), also adopted by Lubarda and Krajcinovic (1993), provides for tensors of any order. However, in order to change a formerly isotropic linear elastic behaviour, governed by Young's modulus E and Poisson's ratio ν, into an orthotropic one, only seven further independent parameters are needed, all but one could be taken from a symmetric damage tensor of 2nd order $\boldsymbol{D}$.

Some models found in the literature within this narrowed scope are those which follow from a thermodynamic potential by derivation with respect to an associated force (Chaboche et. al., 1995). These models, like the ones already mentioned in Lubarda and Krajcinovic (1993), are not readily interpreted in a physical, or geometrical way. On the other hand, there are damage theories which take geometrical properties of the cracks into consideration, like the unit normal $\vec{n}$ to the crack surface A_k of kth crack of all cracks inside the volume V_{RVE} of the RVE multiplied either dyadically with the discontinuity vector $\vec{b}$ between the two faces A_k^+ and A_k^- ($A_k = A_k^+ + A_k^-$) of the crack, see Dragon (1995),

$$\boldsymbol{D} = \frac{1}{2V_{RVE}} \sum_k \int_{A_k^+} (\vec{b} \otimes \vec{n} + \vec{n} \otimes \vec{b})\,\mathrm{d}A \quad \text{or} \quad \boldsymbol{D} = \frac{3}{A} \sum_k \int_{A_k} \vec{n} \otimes \vec{n}\,\mathrm{d}A \quad A = \sum_k A_k \tag{1}$$

or with itself, respectively, see Murakami and Ohno (1981). The first theory has the drawback that it yields zero if all the cracks are closed. The latter relates one damage induced quantity to another, i.e. the oriented surface $\vec{n}\,\mathrm{d}A$ to the surface A itself. Instead, the damage quantity should be referred to the RVE observed.

The tensor-valued damage variable $\boldsymbol{D}$ in the present paper deviates in this point from Murakami's proposal. Furthermore, it is accompanied by a scalar-valued variable, which in other cases could be the specific void volume. Here, to be able to describe cracks of negligible volume, this variable is chosen to measure the heterogeneity of the stress distribution.

2 A Measure of Roughness

A lot of materials used in mechanical or structural engineering, e.g. fibre-enforced plastic or reinforced concrete, cannot be produced without being damaged right from the beginning. In both cases a decrease in volume, which accompanies the setting of the matrix-to-be, is counteracted by the inlying fibres or steel rods. This causes very high tension stresses in the matrix material, which even under the smallest of loads lead to a multitude of micro cracks, if not already at the time of production. For example, a composite of matrix and two equally distributed layers of fibre, orthogonal to each other, which occupy 20% of the volume, may be represented by a cube containing 4 rigid cylinders, pairwise parallel in two layers orthogonal to each other. The image of the whole structure being nothing other than the continuation of like cubes in every direction leads to simple boundary conditions on the sides of the cube (fixed at the bottom, inplane dislocations at the walls, normal dislocations at the top in the form of rigid body motion). An FE-calculation of the cube for the above conditions, elastic matrix behaviour and an assumed volume decrease of 3% gives a very heterogeneous distribution of stresses with peaks, which exceed the average by almost 70%, even along the most 'neutral' straight line from the middle of top to the middle of bottom, which keeps the greatest distance from all fibres.

Obviously, this example is a bit overdone, because there are no such things as rigid fibres and overall contact between fibre and matrix, as assumed. Also the shrinking starts when the matrix is still fluid and able to yield to forces, although decreasingly so. However, the argument applies only to the magnitude of the stress peaks but not to the phenomenon, which is well known to all producers of reinforced plastics.

The stresses attributed to a material point, which in continuum mechanics takes on the role of the related RVE, is nothing more or less than the average of the micro stresses $\boldsymbol{T}_\mu$ inside the volume.

$$\boldsymbol{T} = \frac{1}{V_{\mathrm{RVE}}} \int\limits_{V_{\mathrm{RVE}}} \boldsymbol{T}_\mu \,\mathrm{d}V$$

In the above-mentioned example, where the forces in matrix and fibre balance each other, $\boldsymbol{T}$ would be zero. A measure for the roughness of the micro stresses can be taken from the higher moments of the stress distribution, the already introduced average being the first of them. For the sake of simplicity the 2nd moment of the difference stresses should suffice:

$$r = \frac{1}{V_{\mathrm{RVE}}} \int\limits_{V_{\mathrm{RVE}}} (\boldsymbol{T}_\mu - \boldsymbol{T}) \cdot (\boldsymbol{T}_\mu - \boldsymbol{T}) \,\mathrm{d}V \tag{2}$$

(The dot denotes the scalar valued product between tensors of 2nd order.)

This quantity will be treated as an internal variable from now on. It will need an initial value as well as an evolution equation, the latter having to owe to the experience that early damage reduces peaks of stress and thus evens out heterogeneity or roughness. On the other hand, increasingly growing damage means the degradation of the material leading to greater heterogeneity.

3 Damage Model

The damage model (Schreiber, 1998) used here turns on the existence of a damage variable that could be calculated given the precise geometrical description of all the surfaces A_k of all the pores and cracks within a representative volume of known size ($l = V_{\mathrm{RVE}}^{1/3}$),

$$\boldsymbol{D} = \frac{1}{l^2} \int\limits_A \boldsymbol{n} \otimes \boldsymbol{n} \,\mathrm{d}A \qquad A = \sum_k A_k \tag{3}$$

This differs from Murakamin and Ohno (1981), ref. equation (1.b) in the reference quantity. There are more measures to be calculated in a like manner from the same information given hypothetically above, especially tensors of similar construction, but of odd order. They drop out of consideration because there are geometric forms of cracks to be thought of, e.g. penny-shaped ones with no crack volume at all. For these the calculation of damage tensors of odd order evens out to zero. Variables which do not register certain shapes of damage do not seem to be adequate.

As there is no possibility of getting all the information on damage as supposed above, the damage tensor $\boldsymbol{D}$ will be treated from now on as an internal variable. Given the availability of reasonable initial values, there will be an evolution equation.

The influence, the accumulated damage has on the stress-strain relation, is formulated following Lemaître's concept of equivalent strain. The strain which shows in the damaged material is that which could be observed in an undamaged material subjected to the effective stresses $\hat{\boldsymbol{T}}$, which are considerably higher than the nominal stresses $\boldsymbol{T}$, because damage reduces those parts of material still carrying load (Lemaître, 1992). The difference between both states of stress is induced by damage:

$$\boldsymbol{T} = \hat{\boldsymbol{T}} - \boldsymbol{T}_{\triangle}(\boldsymbol{D}) \tag{4}$$

In other words, $\boldsymbol{T}$ are the stresses to be observed in an experiment or those the balance of momentum deals with, while $\hat{\boldsymbol{T}}$ has its place in the stress-strain relation. The solutions found in the literature regarding the problem of how to expand Lemaître's one-dimensionally posed equation $\hat{\sigma} = \sigma(1-d)$ to 3-dimensionality sometimes involve symmetrizing tensor operations with a 2nd order damage tensor, ref. Cordebois and Sidoroff, 1982, or Chaboche, 1992; in some cases one uses a 4th order damage effect tensor $\boldsymbol{M}(\boldsymbol{D})$ (Chaboche, 1992). Deviating from these proposals, the present paper tries to focus on the damage induced difference $\boldsymbol{T}_{\triangle}$: given some non-zero $\boldsymbol{D}$ there are eigenvalues d_i and eigenvectors $\vec{\zeta}_i$ to be calculated from $\boldsymbol{D}$, which allow for $\hat{\boldsymbol{T}}$ and $\boldsymbol{D}$ being transformed to a base system in which both tensors take the form

$$\hat{\boldsymbol{T}} = \begin{pmatrix} \sigma_1 & \tau_{12} & \tau_{13} \\ & \sigma_2 & \tau_{23} \\ & & \sigma_3 \end{pmatrix}_{\mathrm{sym}} \qquad \boldsymbol{D} = \begin{pmatrix} d_1 & & \\ & d_2 & \\ & & d_3 \end{pmatrix}$$

With terms out of both, some switch functions are formulated using $i = 1, 2, 3$ to give the number of the eigenvalue:

$$h_i = d_i \langle \sigma_i \rangle^0 \qquad \text{MacCauley-brackets:} \quad \langle x \rangle^n = \begin{cases} x^n & \text{for} \quad x > 0 \\ 0 & \text{for} \quad x \leq 0 \end{cases} \tag{5}$$

These functions show up in the damage induced stress tensor $\boldsymbol{T}_{\triangle}$ in equation (4):

$$\boldsymbol{T}_{\triangle} = \begin{pmatrix} h_1\sigma_1 & \min(\frac{h_1+h_2}{2}, 1)\tau_{12} & \min(\frac{h_1+h_3}{2}, 1)\tau_{13} \\ & h_2\sigma_2 & \min(\frac{h_2+h_3}{2}, 1)\tau_{23} \\ & & h_3\sigma_3 \end{pmatrix}_{\mathrm{sym}} \tag{6}$$

The particular form of this relation refers to a certain interpretation of equation (3): a damage tensor that looks like

$$\boldsymbol{D} = \begin{pmatrix} d & & \\ & 0 & \\ & & 0 \end{pmatrix}$$

referring to some coordinate system, can only be the outcome of equation (3) if there is a distribution of cracks, the surface normal of all showing in the 1-direction of the coordinate system. A somewhat extreme example for this special case is a pack of cards, which endures forces of compression but not of tension in the 1-direction (therefore the MacCauley bracket in the switch functions). The other directions carry tension and compression forces as well. The plane normal to 1 doesn't carry shear forces either, i.e. in equation (6) all positions have to appear which cannot carry loads, in this case $[11, 12, 13]_{\mathrm{sym}}$. The model as described still has a lot of symmetries, e.g. taking simply the mean on the off-diagonal positions is as unfounded as the equivalence of the positions on the diagonal, but on the background of lacking experimental evidence, the proposed relation seems to be the simplest that meets the considerations.

4 Growth of Damage for Isotropic Materials

Realizing that there are lots of materials which have a prolonged alternating stress strength, a surface quite similar to the yield surface of plasticity is formulated in the stress space. A state of effective stress $\hat{\boldsymbol{T}}$ inside this surface, or damage threshold, implies no growth of damage. For an isotropic material this could be

$$S = \hat{\boldsymbol{T}} \cdot \hat{\boldsymbol{T}} - k_0 = 0$$

A very simple and obvious way to include the internal variable r, defined in equation (2), is to combine it with k_0 (square of the radius of the damage threshold), as it is of the same dimension. Proposing that always $0 \leq r \leq k_0$, the damage threshold for isotropic materials shall be:

$$S = \hat{\boldsymbol{T}} \cdot \hat{\boldsymbol{T}} - (k_0 - r) = 0 \tag{7}$$

When in the process of loading $\hat{\boldsymbol{T}}$ crosses the threshold the growth of damage shall start smoothly. So only that part $\overline{\boldsymbol{T}}$ of the effective stresses propels the evolution of damage, which overshoots the threshold:

$$\overline{\boldsymbol{T}} = \beta \hat{\boldsymbol{T}} \qquad \text{with } \beta = \langle 1 - \gamma \rangle^1 \qquad \text{and } \gamma = \sqrt{\frac{k_0 - r}{\hat{\boldsymbol{T}} \cdot \hat{\boldsymbol{T}}}} \qquad \begin{array}{lcl} \gamma > 1 & : & \hat{\boldsymbol{T}} \text{ below threshold} \\ \gamma = 1 & : & \hat{\boldsymbol{T}} \text{ meets threshold} \\ \gamma < 1 & : & \hat{\boldsymbol{T}} \text{ overshoots threshold} \end{array} \tag{8}$$

The eigenvalues $\bar{t}_i$ of this tensor are those of $\hat{\boldsymbol{T}}$ scaled by β. Referring to the principal axes of $\hat{\boldsymbol{T}}$ damage evolves with the positive eigenvalues of $\overline{\boldsymbol{T}}$ in form of a power law:

$$\dot{\boldsymbol{D}} = \begin{pmatrix} f(\bar{t}_1) & & \\ & f(\bar{t}_2) & \\ & & f(\bar{t}_3) \end{pmatrix} \qquad \begin{array}{rcl} f(t) & = & p \left\langle \frac{t}{E} \right\rangle^{\alpha}, \\ E & : & \text{reference value, e.g. Young's modulus} \\ p, \alpha & : & \text{material constants identified from} \\ & & \text{Wöhler-lines out of fatigue tests} \end{array} \tag{9}$$

This assumption is made for the sake of simplicity and will be taken to hold in the same way for orthotropic materials too. For a better fitting of the model to experimental evidence, additional versatility of its classical form can be reached by replacing the constants p and α by functions, e.g. of the state of damage or of stress rates, either both depending on a suitable invariant or each on the rate $\dot{t}_i$, which correlates with the eigenvalue t_i. Some further proposals can be found in Chaboche (1988). The necessity and complexity of such an endeavour can only be judged by suitable experimental data, not yet available.

5 Application to Fibre Enforced Laminates

Following the classical theory of laminates such a structure is built from thin layers of unidirectional fibres embedded in matrix material, each layer in ideal contact with the other. One unidirectional layer is seen as of transversally isotropic material, the x-direction along the fibres being the main direction of anisotropy. In Voigt's notation the stress-strain relation given in principal axes is:

$$\vec{t} = \begin{bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{xy} \\ \tau_{xz} \\ \tau_{yz} \end{bmatrix} = \begin{bmatrix} C_1 & C_2 & C_2 & & & \\ C_2 & C_3 & C_4 & & & \\ C_2 & C_4 & C_3 & & & \\ & & & C_5 & & \\ & & & & C_5 & \\ & & & & & \frac{1}{2}(C_3 - C_4) \end{bmatrix} \begin{bmatrix} \epsilon_x \\ \epsilon_y \\ \epsilon_z \\ \gamma_{xy} \\ \gamma_{xz} \\ \gamma_{yz} \end{bmatrix} = \underline{C}\vec{e} \tag{10}$$

(From here on the z-coordinate is assumed to be normal to the layers.)

In the same notation a transformation of $\vec{t}$ or $\vec{e}$ into coordinates, which differ by an angle of rotation α about z, turns out to be a matrix product, e.g. $\vec{\bar{t}} = \underline{Q}\vec{t}$, in which $\underline{Q}$ is composed of products of $\sin\alpha$ and $\cos\alpha$. This leads to a simple transformation formula for the stiffness matrix $\underline{C}$.

$$\vec{\bar{t}} = \underline{Q}\vec{t} = \underline{Q}\underline{C}\vec{e} = \underline{Q}\underline{C}\underline{Q}^T\vec{\bar{e}} = \underline{\bar{C}}\,\vec{\bar{e}}$$

Now the way is open for transforming the stiffness matrices of the single unidirectional layers into a common coordinate system, where they are added up to become the resulting stiffness matrix $\underline{K}$ of the laminate according to the rules of the theory of mixtures, i.e. each weighted by the ratio H_l/H, H_l being the thickness of the lth single layer of N layers.

$$\underline{K} = \sum_{l=1}^{N} \frac{H_l}{H}\underline{\bar{C}}_l \qquad H = \sum_{l=1}^{N} H_l$$

The material to be discussed in the later example is assumed to consist of equally distributed layers of two directions, namely $\pm 30°$ from the common symmetry line, with enough layers to the RVE to meet the aforementioned conditions of continuum mechanics. For such a structured laminate the outcome of the procedure described above is the stiffness matrix $\underline{K}$,

$$\underline{K} = \frac{1}{16}\begin{bmatrix} K_1 & K_2 & K_3 & & & \\ K_2 & K_4 & K_5 & & & \\ K_3 & K_5 & K_6 & & & \\ & & & K_7 & & \\ & & & & K_8 & \\ & & & & & K_9 \end{bmatrix} \quad \begin{aligned} K_1 &= 9C_1 + 6C_2 + C_3 + 12C_5 \\ K_2 &= 3C_1 + 10C_2 + 3C_3 - 12C_5 \\ K_4 &= C_1 + 6C_2 + 9C_3 + 12C_5 \\ K_7 &= 3C_1 - 6C_2 + 3C_3 + 4C_5 \\ K_8 &= 2C_3 - 2C_4 + 12C_5 \\ K_9 &= 6C_3 - 6C_4 + 4C_5 \end{aligned} \quad \begin{aligned} K_3 &= 12C_2 + 4C_4 \\ K_5 &= 4C_2 + 12C_4 \\ K_6 &= 16C_3 \end{aligned}$$

an example of what is called orthotropic material.

The same path will be followed to construct an appropriate damage threshold for layered materials from single-layer criteria, now to be defined: in the same way as (7) was derived from an analogon in the theory of plasticity, the same can happen here because of a yield surface which Hill stipulated for transversally isotropic materials back in 1948. From his proposal a lot of different failure criteria for laminates were derived, on the merits and drawbacks of which a good survey is given in Puck (1996). The form of the damage threshold for a unidirectional laminate stipulated here follows the general lines of those models but distinguishes between the different mechanisms of laminate fracture under tension and compression by means of MacCauley-brackets:

$$S = \langle\frac{t_{xx}}{R_{\parallel}}\rangle^2 + \langle\frac{t_{xx}}{R_{\parallel}^-}\rangle^2 + \langle\frac{t_{yy}}{R_{\perp}}\rangle^2 + \langle\frac{t_{yy}}{R_{\perp}^-}\rangle^2 + \langle\frac{t_{zz}}{R_{\perp}}\rangle^2 + \langle\frac{t_{zz}}{R_{\perp}^-}\rangle^2 + \frac{t_{xy}^2 + t_{xz}^2}{R_{\perp\parallel}^2} + \frac{t_{yz}^2}{R_{\perp\perp}^2} - 1 = 0$$

The material constants $R_{\parallel}$, $R_{\parallel}^-$, $R_{\perp}$, $R_{\perp}^-$, $R_{\perp\parallel}$, and $R_{\perp\perp}$ are failure stresses resulting from different experiments to be carried out on specimens of unidirectional material. The tests are sketched in figure 1 below, each of them attributing to a certain mode of failure observable in laminates. Those on the right-hand side of the dashed line refer to modes of laminate failure which, for the sake of limited complexity, are ignored further in this paper. Apart from this, it remains to be discussed whether the above-mentioned failure stresses should be those at which an obvious breaking of material occurs or rather those stresses at which the first peak of crack events is detected by an acoustic measuring device.

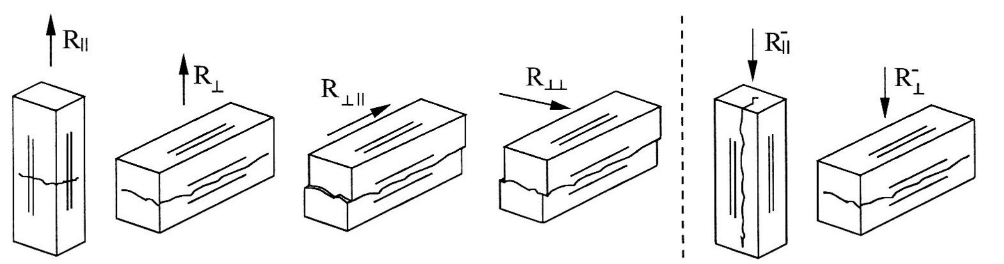

Figure 1. Six Different Experiments for Unidirectional Laminates

The most prominent of those failure stresses is $R_{||}$, which takes over the role of k_0 of the isotropic formulation (7). So, combined with the measure of roughness r, the damage threshold for one unidirectional layer takes the form

$$S(\hat{\boldsymbol{T}}, r) = \langle \frac{t_{xx}}{R_{||}} \rangle^2 + \langle \frac{t_{yy}}{R_{\perp}} \rangle^2 + \langle \frac{t_{zz}}{R_{\perp}} \rangle^2 + \frac{t_{xy}^2 + t_{xz}^2}{R_{\perp ||}^2} + \frac{t_{yz}^2}{R_{\perp\perp}^2} - (1 - \rho) = 0 \qquad (11)$$

$$\rho = \frac{r}{R_{||}^2} \qquad 0 \le \rho \le 1 \qquad (12)$$

Of course, there are several layers to one material point (RVE), assumed to differ only in terms of fibre orientation. The layers, indexed by l, each have their own damage threshold according to the above definition (11). The averaging is done by transforming the strain tensor into the coordinate system of each layer l in order to determine the effective stresses $\hat{\boldsymbol{T}}_l$ in this layer by means of the unidirectional stress-strain relation (10). Suppose a fraction $\gamma_l \hat{\boldsymbol{T}}_l$ of $\hat{\boldsymbol{T}}_l$ fulfils the condition set by equation (11), then in the same way, it was shown in equation (8), the amount by which $\hat{\boldsymbol{T}}_l$ surpasses the layer's damage threshold can be calculated to be $\overline{\boldsymbol{T}}_l = \beta_l \hat{\boldsymbol{T}}_l$.

$$\beta_l = \langle 1 - \gamma_l \rangle^1 \qquad \text{with } \gamma_l = \sqrt{\frac{1-\rho}{\langle \frac{t_{lxx}}{R_{||}} \rangle^2 + \langle \frac{t_{lyy}}{R_{\perp}} \rangle^2 + \langle \frac{t_{lzz}}{R_{\perp}} \rangle^2 + \frac{t_{lxy}^2 + t_{lxz}^2}{R_{\perp ||}^2} + \frac{t_{lyz}^2}{R_{\perp\perp}^2}}} \qquad (13)$$

The contribution of layer l to the evolution of damage in the material point observed is (according to the principal axes of $\hat{\boldsymbol{T}}_l$) postulated to be:

$$\dot{\boldsymbol{D}}_l = \frac{H_l}{H} \begin{pmatrix} p\langle \frac{\beta_l t_1}{C_1} \rangle^\alpha & & \\ & p\langle \frac{\beta_l t_2}{C_1} \rangle^\alpha & \\ & & p\langle \frac{\beta_l t_3}{C_1} \rangle^\alpha \end{pmatrix} \qquad \dot{\boldsymbol{D}} = \sum_l \boldsymbol{Q}_l{}^T \dot{\boldsymbol{D}}_l \boldsymbol{Q}_l \qquad (14)$$

In this t_i are the principal values of $\hat{\boldsymbol{T}}_l$ which are scaled down by β_l to the extent to which they actually participate in the damaging process. $\boldsymbol{Q}_l$ are the tensors of transformation between the principal axes of $\hat{\boldsymbol{T}}_l$ and the common coordinate system. Young's modulus E in eq. (9), being a parameter of isotropy, is replaced as a reference magnitude by the more appropriate quantity C_1, the most prominent of the material parameters of the unidirectional stress-strain relation (10).

6 Process of Cyclic Pure Bending

A flat, thin, straight specimen of constant cross-section made of the previously mentioned material (layers in two equally distributed directions of $\pm 30°$), clamped at one end and subjected to a temporal changing of the angle of inclination about its broad axis y suffers a moment of bending, which is constant along x (long direction). The resulting deformation is therefore of constant curvature κ; Bernoulli's kinematics is applied.

The geometrical aspects of plate bending apply, i.e. there is no deformation in y-direction, and the small thickness in z-direction justifies the approximation that all stresses are zero except σ_x and σ_y. The usual conclusions are:

$$\begin{matrix} \sigma_z & = & \tau_{xy} & = & \tau_{xz} & = & \tau_{yz} & = & 0 \\ \epsilon_y & = & \epsilon_{xy} & = & \epsilon_{xz} & = & \epsilon_{yz} & = & 0 \end{matrix} \Rightarrow \epsilon_z = -\frac{K_3^2}{K_6} \epsilon_x$$

With this and equation (10) all $\hat{\boldsymbol{T}}_l (l = 1, 2)$ are determined, because the deformation ϵ_x follows from

$$\epsilon_x = \kappa(t)\, z = \kappa_{\max}\, z\, f(t) \qquad f(\tau = t - nT) = \begin{cases} \frac{4}{T}\tau & ; \quad 0 < \tau \le \frac{T}{4} \\ 2 - \frac{4}{T}\tau & ; \quad \frac{T}{4} < \tau \le \frac{3T}{4} \\ -5 + \frac{4}{T}\tau & ; \quad \frac{3T}{4} < \tau \le T \end{cases} \qquad n = 1, 2, \ldots$$

to be a given function of position and time. Now for any orientation of any layer the condition of further damage growth (13) can be examined. It yields

$$\gamma_1 = \gamma_2 = \gamma = K(C_1, \ldots, C_5; R_{||}, R_\perp, R_{\perp||}) \frac{\sqrt{1-\rho}}{\epsilon_x(z,t)} \qquad \beta_1 = \beta_2 = \beta = \langle 1-\gamma \rangle^1 \tag{15}$$

in which K is a constant resulting from almost all the material parameters. There is no growth of damage if $\epsilon_x < 0$ and if $\beta = 0$. The latter is a condition which it is always possible to check for any given z.

If there is damage growth, the biggest component of $\dot{\boldsymbol{D}}$ is supposed to be the one indexed $_{11}$. It is on this component $d = d_{11}$ that the attention is focused from now on.

$$\begin{aligned}(14),(10),(15) \rightarrow \dot{d} &= p\left[\frac{(1-\gamma)\, t_1}{C_1}\right]^\alpha \\ &= q\left(Q\zeta f(t) - \sqrt{1-\rho}\right)^\alpha \end{aligned} \tag{16}$$

Again, as far as possible, all constants are bundled in q and Q; ζ is z normalized by the thickness H of the specimen. As $f(t)$ is given, and the assumption is sound that r changes so slowly that it can be viewed as constant during one cycle, the growth of damage Δd during this cycle can be calculated for any position $\zeta = z/H$.

$$\Delta d(\zeta) = \frac{qQ^\alpha}{2(\alpha+1)}\zeta^\alpha \left(1 - \frac{\sqrt{1-\rho}}{Q\zeta}\right)^{\alpha+1} T$$

This growth per cycle gives an opportunity to define an evolution law for d (in this specific process) on a slower time scale. The superposed $\circ$ denotes a rate, which is the average of the rate denoted by a dot. The thus defined evolution law is complemented by another evolution law for the normalized roughness variable ρ of the micro stresses, left open until now.

$$\overset{\circ}{d} := \frac{\Delta d}{T} = \overline{q}\zeta^\alpha \left(1 - \frac{\sqrt{1-\rho}}{Q\zeta}\right)^{\alpha+1} \tag{17}$$

$$\overset{\circ}{\rho} := a(1-\rho)d - b(1-d)\rho \tag{18}$$

The latter gives form to the requirements outlined in section 2.

7 Results and Conclusions

All the following results are based on selected, not experimentally determined values for the dimensionless parameters, i.e. $\overline{q} = 10^{-5}$, $Q = 6.25$, $\alpha = 2$, $a = 2 \cdot 10^{-5}$ and $b = 10^{-5}$. The initial values in each position ζ_j, for which numerical calculations were run, were $\rho = 0.5$ and $d = 0$. The results shown refer to 7 different positions z_j, equally distributed over half of the specimen's thickness. All calculations were continued as long as the damage in the outmost layer didn't reach $d = 1$.

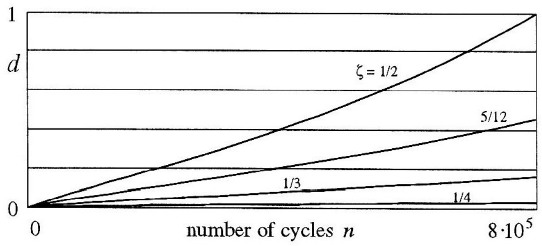

Figure 2. Damage at Various Heights in Specimen

Figure 2 shows the evolution of the damage component d plotted against the number of cycles n. $\zeta = 1/2$ denotes the outer surface where damage grows most rapidly; $\zeta = 0$ means the middle surface of the specimen. A significant aspect of these curves is the over-proportional growth of damage, which without the quantity r would turn out to be strictly linear.

The distribution of damage along ζ is qualitatively correct, but far from what experimental observations (although in metallic specimens) demand. Owing to the peculiar form of $\dot{\boldsymbol{D}}$, according to equation (9), the profile $d(\zeta)$ is approximately similar to ζ^α, see equation (17), in other words much too shallowly curved. The slope of d should stay near zero in wide inner regions and then take a steep increase on the last μm towards the outer surface of the specimen.

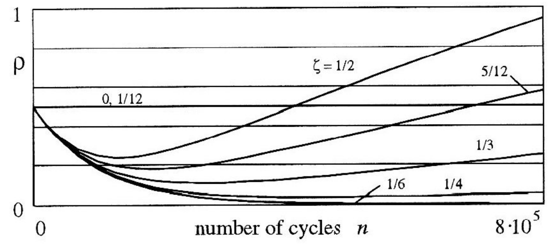

Figure 3. Normalized Roughness of Micro Stresses

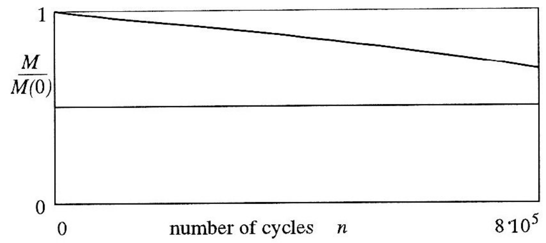

Figure 4. Moment of Bending

In Figure 3 the evolution of the normalized roughness ρ, ref. equation (12), of the micro stresses is depicted. The homogenizing effect of primary damage, as well the increase of heterogeneity towards higher values of damage is quite evidently to be seen.

The diagram in Figure 4 shows a quantity which, contrary to the two above, can be measured in a real experiment, i.e. the resultant moment of bending. Of this quantity it is well known, at least for metallic specimens, that its value stays constant for a great number of cycles before it decreases dramatically. The curve shown resembles the real behaviour (of metals) to some extent, but still not in a satisfactory way. The ability of the proposed model to reflect the real behaviour of laminates and other materials will be the object of future work in theoretical and experimental research.

Literature

1. Chaboche, J.L.: Continuum Damage Mechanics: Part II – Damage Growth, Crack Initiation, and Crack Growth, J.Appl.Mech., 55, (1988), 65-72.
2. Chaboche, J.L.: Damage Induced Anisotropy: On the Difficulties Associated with the Active/Passive Unilateral Condition, Int.J.Dam.Mech., 1, (1992), 148-171
3. Chaboche, J.L./ Lesne, P.M./ Maire, J.F.: Continuum Damage Mechanics, Anisotropy and Damage Deactivation for Brittle Material Like Concrete and Ceramic Composites, Int.J.Dam.Mech., 4, (1995), 5-22
4. Cordebois, J.P./ Sidoroff, F.: Damage Induced Elastic Anisotropy. In: Boehler, J.P. (ed.): Mechanical Behaviour of Anisotropic Solids, Martinus Nijhoff, The Hague, (1982), 761-774
5. Dragon, A.: Plasticity and Ductile Fracture Damage: Study of Void Growth in Metals, Eng.Frac. Mech., 21(4), (1985), 875-885
6. Krajcinovic, D.: Continuous Damage Mechanics, Appl.Mech.Rev., 37(1), (1984), 1-6,
7. Lemaître, J.: A Course in Damage Mechanics. Springer Verlag, Berlin, (1992)
8. Lubarda, V.A./ Krajcinovic, D.: Damage Tensors and the Crack Density Distribution, Int.J.Solids Struct., 30(20), (1993), 2859-2877
9. Murakami, S./ Ohno, N.: A Continuum Theory of Creep and Creep Damage, In: Ponter, A.R.S./ Hayhurst, D.R. (eds.): Creep in Structures, Springer-Verlag, New York, (1981), 422-434
10. Onat, E.T./ Leckie, F.A.: Representation of Mechanical Behavior in the Presence of Changing Internal Structure, J.Appl.Mech., 55, (1988), 1-10
11. Puck, A.: Festigkeitsanalyse von Faser-Matrix-Laminaten. Modelle für die Praxis, C. Hanser Verlag, München, (1996)
12. Schreiber, L.: Skizze eines tensoriellen Schädigungsmodells. In: Hartmann/Tsakmakis (Ed.): Aspekte der Kontinuumsmechanik und Materialtheorie. Festschrift zum 60. Geburtstag von Professor Peter Haupt. Berichte des Instituts für Mechanik, 1, (1998), 175-182

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