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Chaotic Attitude Motion of a Class of Spacecraft on an Elliptic Orbit

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In this paper, the chaotic planar attitude motion of a spacecraft on an elliptic orbit in the gravitational field with air drag and internal damping is investigated. Based on the mathematical model of the spacecraft attitude motion, the necessary condition for chaos to occur is established by Melnikov's method. A numerical example is presented.

1 Introduction

Attitude dynamics of spacecraft is a research subject of great practical significance (Rimrott, 1989; Liu, 1995). As chaos is widely and deeply investigated, much attention is also paid to chaotic attitude motion of spacecraft. It not only provides a definite engineering approach for studying chaos, but also offers a new viewpoint for designing spacecraft. It is shown that there exists chaotic attitude motion in some models of spacecraft, such as spinning satellites in a circular orbit (Guran et al., 1991), gyrostat satellites in the gravitational field (Tong and Rimrott, 1993; Tong et al., 1995), and tethered satellites (Peng and Liu, 1996). However, the most often studied case is the planar libration of non-spinning spacecraft on an elliptic orbit in the gravitational field. Gulyer et al. (1989) researched the case without damping and found period-doubling bifurcations to chaos. In the same year Seisl and Steindl (1989) presented the necessary condition for chaos to occur for the case with atmospheric resistance. Tong and Rimrott (1991) numerically studied chaotic behaviour for the case with internal damping. Beletsky (1995) and Beletsky et al. (1996) have studied chaotic attitude motion of satellites in the magnetic field of the earth, in the gravitational field of two centers, or in solar wind. The present paper deals with the case of both atmospheric resistance and internal damping. The Melnikov method is applied to present analytic criteria to determine whether chaotic attitude motion might occur. Chaos is numerically demonstrated by the time history and the phase trajectory.

2 Differential Equation of Spacecraft Attitude Motion

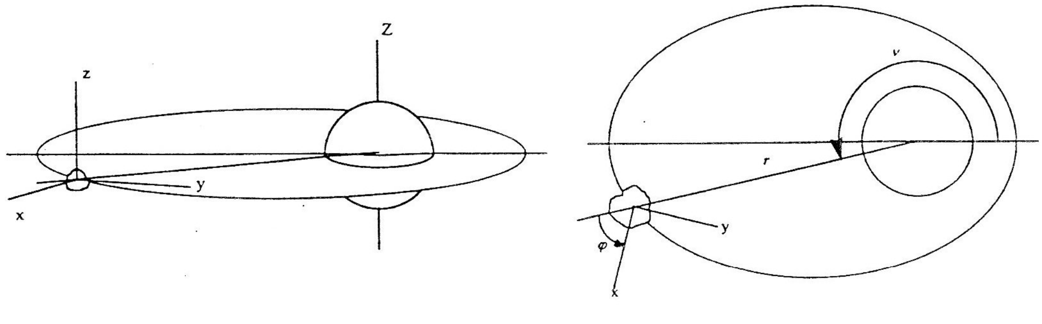

Figure 1 Planar Motion of Spacecraft in an Elliptic Orbit

As shown in Figure 1, an arbitrarily shaped spacecraft, whose principal inertia moments are A, B and C, moves in an elliptic orbit with one principal axis z normal to the orbital plane XY. Without loss of generality, suppose that $B>A$. Note that φ is the libration angle in the orbital plane as measured from the local vertical, ν is the position angle of the spacecraft in its orbit as measured from the perifocus, r is the distance between the spacecraft mass center and the earth mass center, μ is the gravitational attraction constant of the earth. Assume that the internal damping and the atmospheric resistance are proportional to the spacecraft's angular velocity and the square of angular velocity respectively, whose coefficients are γ and c. Considering the z-component of Euler's equations (Rimrott, 1989; Liu, 1995) leads to

$$C\frac{d}{dt}\left(\frac{d\nu}{dt}+\frac{d\varphi}{dt}\right) = \frac{3\mu}{r^3}(B-A)\cos\varphi(-\sin\varphi) - cC\left(\frac{d\varphi}{dt}\right)^2 - \gamma C\frac{d\varphi}{dt} \tag{1}$$

Orbital motion and attitude motion are assumed to be decoupled. Thus one has the Kepler motion (Rimrott, 1989; Liu, 1995)

$$\frac{d\nu}{dt} = \frac{\sqrt{\mu p}}{r^2} \qquad r = \frac{p}{1+e\cos\nu} \tag{2}$$

where p is the semi-parameter of the orbit, and e is the orbit eccentricity. Introducing the true anomaly ν as the independent variable, substituting equations (2) into equation (1) and noting

$$\frac{d}{dt} = \frac{\sqrt{\mu p}}{r^2}\frac{d}{d\nu} \qquad \frac{d^2}{dt^2} = \frac{\mu p}{r^4}\left(\frac{d^2}{d\nu^2} - \frac{2e\sin\nu}{1+e\cos\nu}\frac{d\nu}{dt}\right) \tag{3}$$

and letting $$K = \frac{3(B-A)}{2C} \qquad \dot{\varphi} = \frac{d\varphi}{d\nu} \qquad \ddot{\varphi} = \frac{d^2\varphi}{d\nu^2} \tag{4}$$

one obtains

$$\ddot{\varphi} - \frac{2e\sin\nu(1+\dot{\varphi})}{1+e\cos\nu} + \frac{K\sin 2\varphi}{1+e\cos\nu} + c\dot{\varphi}^2 + \frac{\gamma\dot{\varphi}}{(1+e\cos\nu)^2} = 0 \tag{5}$$

which is the differential equation desired.

3 Chaotic Attitude Motion of the Spacecraft

Since e, c, γ are all small, let $e = \varepsilon e_1, c = \varepsilon c_1, \gamma = \varepsilon\gamma_1 \quad (0<\varepsilon<<1)$. Neglecting higher order terms of ε in equation (5) leads to an integrable Hamiltonian system under small perturbations.

$$\ddot{\varphi} + K\sin 2\varphi = \varepsilon\left[2e_1\sin\nu(1+\dot{\varphi}) + e_1K\cos\nu\sin 2\varphi - c_1\dot{\varphi}^2 - \gamma_1\dot{\varphi}\right] \tag{6}$$

For $\varepsilon = 0$, the unperturbed planar Hamiltonian system (6) has first integrals of motion

$$\frac{1}{2}\dot{\varphi}^2 + K\sin^2\varphi = H \tag{7}$$

When $H = K$ the integrable system (7) has two hyperbolic saddle points $\left(\pm\frac{\pi}{2}, 0\right)$, whose unstable manifolds and stable manifolds constitute a heteroclinic cycle. The heteroclinic orbits $\Gamma^{\pm}$ starting at $\left(0, \pm\sqrt{2K}\right)$ are

$$(\varphi_{\pm}(\nu),\ \dot{\varphi}_{\pm}(\nu)) = \left(\pm\arcsin\left(\mathrm{th}\left(\sqrt{2K}\nu\right)\right),\ \pm\sqrt{2K}\mathrm{sech}\left(\sqrt{2K}\nu\right)\right) \tag{8}$$

For $\varepsilon \neq 0$, if the Melnikov functions

$$M_{\pm}(\nu_0) = \int_{-\infty}^{+\infty}\left[2e_1\sin(\nu+\nu_0)(1+\dot{\varphi}_{\pm}(\nu)) + e_1K\cos(\nu+\nu_0)\sin 2\varphi_{\pm}(\nu) - c_1\dot{\varphi}_{\pm}^2(\nu) - \gamma_1\dot{\varphi}_{\pm}(\nu)\dot{\varphi}_{\pm}(\nu)\right]d\nu \tag{9}$$

have a simple zero, then in the Poincaré map of equation (6) there exists a transverse heteroclinic cycle (Guckenheimer and Holmes, 1983). Substituting equations (8) into equations (9), one evaluates the Melnikov functions for the heteroclinic orbits Γ^+ and Γ^- to yield

$$M_+(\nu_0) = \frac{\pi}{2}e_1\left(4\operatorname{sech}\frac{\pi}{2\sqrt{2K}} + 3\operatorname{csch}\frac{\pi}{2\sqrt{2K}}\right)\sin\frac{\nu_0}{\sqrt{2K}} - \pi c_1 K - 4\gamma_1 K \tag{10}$$

$$M_-(\nu_0) = \frac{\pi}{2}e_1\left(-4\operatorname{sech}\frac{\pi}{2\sqrt{2K}} + 3\operatorname{csch}\frac{\pi}{2\sqrt{2K}}\right)\sin\frac{\nu_0}{\sqrt{2K}} + \pi c_1 K - 4\gamma_1 K \tag{11}$$

Letting

$$R^+\left(K, \frac{\gamma}{c}\right) = \frac{2K\left(1+\dfrac{4}{\pi}\dfrac{\gamma}{c}\right)}{4\operatorname{sech}\dfrac{\pi}{2\sqrt{2K}} + 3\operatorname{csch}\dfrac{\pi}{2\sqrt{2K}}} \tag{12}$$

$$R^-\left(K, \frac{\gamma}{c}\right) = \frac{2K\left|1-\dfrac{4}{\pi}\dfrac{\gamma}{c}\right|}{\left|4\operatorname{sech}\dfrac{\pi}{2\sqrt{2K}} - 3\operatorname{csch}\dfrac{\pi}{2\sqrt{2K}}\right|} \tag{13}$$

then from equations (10), (12), and equations (11), (13), one separately knows that if

$$\frac{e}{c} > \max\left\{R^+\left(K, \frac{\gamma}{c}\right), R^-\left(K, \frac{\gamma}{c}\right)\right\} \tag{14}$$

both $M_+(\nu_0)$ and $M_-(\nu_0)$ have simple zeros. Thus there exist transverse heteroclinics in the Poincaré map of the system (6). The existence of such cycles implies that Smales horseshoe occurs and chaotic behavior may result.

4 A Numerical Example

For $K = 0.75$, $\gamma = 0.05$, $c = 0.04$, equation (12) and equation (13) respectively yield $R^+ = 1.00635$ and $R^- = 3.44505$. Hence the condition (14) becomes $e > 0.137802$. Let $e = 0.14$. In this case, chaotic motion occurs. The chaotic time history and the chaotic phase trajectory are separately shown in Figure 2 and Figure 3.

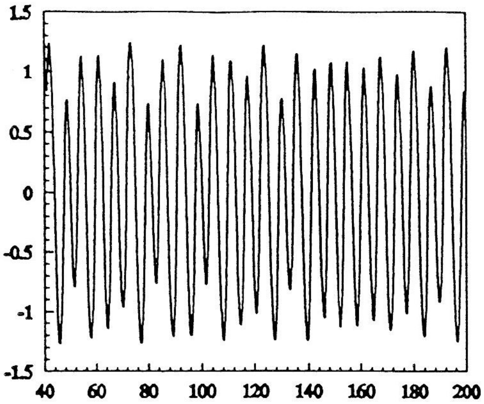

Figure 2. The Chaotic Time History

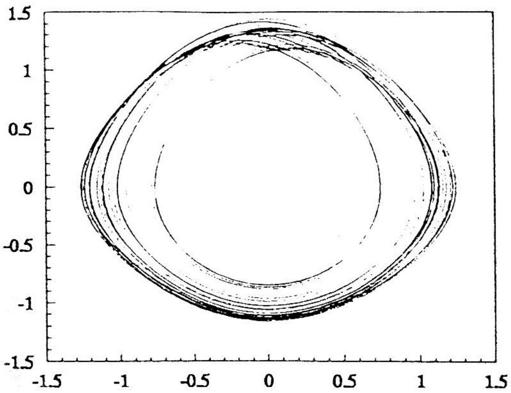

Figure 3. The Chaotic Phase Trajectory

5 Conclusion

The planar libration of non-spinning spacecraft on an elliptic orbit in the gravitational field with both atmospheric resistance and internal damping is described by the differential equation (4). The necessary condition for chaos to occur in equation (4) is equation (14). If the condition is satisfied, the time history and the phase numerically demonstrate the existence of chaos.

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