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Homogenization Method in the Theory of Corrugated Plates

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The analysis of corrugated plates and shells is of significant practical value: a lot of such problems arise in machine design, civil engineering, etc.. The problems mentioned are usually solved using numerical methods such as finite element procedures. Nevertheless a numerical approach does not adequately fit the requirements of optimal structural design. Then approximate analytical expressions, accurate enough, will be of great practical advantage for these needs.

1 Introduction

Corrugated plates and shells are characterised by partial differential equations with rapidly varying coefficients, and their stress-strain state may be represented as sums of slowly and rapidly varying parts (Andreeva, 1981; Gibson, 1980; Reichhard, 1995). In many physical problems, some variables vary indeed slowly, others fast. It is natural to pose the question whether it is appropriate to have first studied a global structure under consideration, digressing from it´s local distinctive features, and then to investigate the system locally. It is the homogenization method which is aimed at a division of the fast and slow components of the solution. Without going into details of the method - the more because it has at present a lot of modifications - it will be noted only that it involves the introduction of „slow“ (macroscopic) and „rapid“ (microscopic) variables whose equations are separated and can be solved independently, or sequentially. This method was developed for and has gained wide use in celestial mechanics and in nonlinear oscillation theory. At present, the method is used to great advantage for solving variable-coefficient partial differential equations in such disciplines as the theory of composites (Bakhvalov and Panasenko, 1989; Sanchez-Palencia, 1980) or of the theory of reinforced, perforated, etc. shells (Bensoussan et al., 1978; Lions, 1982; Kalamkarov, 1992; Andrianov and Manevich, 1983; Andrianov et al., 1985; Andrianov et al., 1992).

An originally nonhomogeneous medium or structure is reduced to a homogeneous one (generally speaking to an anisotropic one) with some effective characteristics. The homogenization method allows not only to obtain effective characteristics but also to investigate the nonhomogeneous distribution of mechanical stresses in different materials and structures, which is of great significance for evaluating their strength. The approach presented below fills the substantial gap between numerical methods of thin shell theory, which methods lack generality and the possibility to grasp the common features of behaviour of structures concerned, and approximate design schemes, based on heuristic hypotheses. Methods proposed are wide-ranging in application and lead to simple and clear design formulas, useful for practical analyses. The aforesaid opens new prospects for the analysis of the new important problems arising in modern engineering and not yet solved fully and effectively enough.

Readers will probably ask the question: are asymptotic methods of any practical use at all when there are computers? Is it not simple to have programmed an original problem and have solved it using the universal numerical solutions available, for example, finite element procedures. The answer may be like this: Firstly the asymptotic methods are very useful in the preliminary stage of solving a problem even in cases where the eventual aim is to obtain numerical results. The asymptotic analysis makes it possible to choose the best numerical method and gain an understanding of a vast body of numerical material, often not properly arranged. Secondly the asymptotic methods are especially effective in those regions of parameter values where machine computations are faced with serious difficulties.

Laplace used to say, not without reason, that the asymptotic methods are „very accurate, therefore they are needed“. Moreover, the possibility exists of developing such algorithms wherein smooth portions of solutions are obtained numerically, and the asymptotic approaches are applied to those parameter value regions where these solutions change drastically, say, within boundary layers. Therefore, it would be more proper to consider the asymptotic and numerical methods not as competing, but as complementary.

The homogenization theory has been developed recently by many authors (Bakhvalov and Panasenko, 1989; Sanchez-Palencia, 1980; Bensoussan et al., 1978; Lions, 1982; Kalamkarov, 1992). The main problem in this field is the solving of the so-called cell (or local) problem. This problem is usually treated by numerical methods. We have used the asymptotic method for solving the cell problem and have constructed a special approach in this work. The work covers the following problems: the bending of circular and rectangular plates with periodic corrugations, and the eigenvalue problem for the circular plate. The execution of the above mentioned program turned out to be an extremely difficult task. We were faced with some difficulties of principle. The matter is that initial equations were written for the middle surface, but homogenized equations must be written for the middle plane. For overcoming this drawback we used projected initial equations on the middle plane.

2 Radially Axisymmetric Corrugated Circular Plate

First of all we shall formulate the basic equations describing a corrugated circular plate in a form suitable for later homogenization. The problem ist not a trivial one since the geometry of the corrugated plate is complicated. Projecting into the middle plane, equilibrium equations may be written in the form

$$\begin{aligned} &\frac{d}{dr}(rN_1) - N_2 = rq_1 \\ &\frac{d}{dr}(rQ_1) = rq_2 \\ &\frac{d}{dr}(rM_1) - M_2 - rQ_1 = 0 \end{aligned} \tag{1}$$

Physical and geometrical relations in projecting into the middle plane are

$$\begin{aligned} &\frac{N_1 + BQ_1}{A} = B(\varepsilon_1 + \nu\varepsilon_2) &\quad& N_2 = BA(\varepsilon_2 + \nu\varepsilon_1) \\ &M_1 - zN_1 = D(\chi_1 + \nu\chi_2) && M_2 - zN_2 = D(\chi_2 + \nu\chi_1) \\ &\varepsilon_1 = \frac{1}{A^2}\left(\frac{du}{dr} + \beta\frac{dw}{dr}\right) && \varepsilon_2 = \frac{u}{r} \\ &\chi_1 = \frac{1}{A}\frac{d}{dr}\left[\frac{1}{A^2}\left(\frac{dw}{dr} - \beta\frac{du}{dr}\right)\right] && \chi_2 = \frac{1}{rA^2}\left(\frac{dw}{dr} - \beta\frac{du}{dr}\right) \end{aligned} \tag{2}$$

Boundary conditions may be formalised as follows (plate clamped):

$$u = w = \frac{dw}{dr} = 0 \qquad \text{for} \quad r = r_0, R \tag{3}$$

The study of such problems is important from a theoretical as well as a numerical point of view. Because of the complicated structure of corrugated plate, any kind of calculation is difficult to perform. So, we would like to „approximate“ the given problem by a „homogenized“ problem. By the method of asymptotic development, the problem on a periodically corrugated plate is reduced to solving problems in the „basic cell“ for a plate without corrugations. The method used here is a variant of the multiscaling technique as used in the books by Bensoussan et al., (1978) (see also Bakhvalov and Panasenko, 1989) . We consider then $n \geq 1$ and introduce new „quick“ variables $\xi = nr$. Then

$$\frac{d}{dr} = \frac{\partial}{\partial r} + \varepsilon^{-1}\frac{\partial}{\partial \xi} \qquad (4)$$

The solution of the boundary value problems (1) to (3) we represent in the form of formal expansions

$$N_i = N_i^{(0)}(r,\xi) + \varepsilon N_i^{(1)}(r,\xi) + \ldots \qquad i = \overline{1,2}$$

$$M_i = M_i^{(0)}(r,\xi) + \varepsilon M_i^{(1)}(r,\xi) + \ldots$$

$$Q_i = Q_i^{(0)}(r,\xi) + \varepsilon Q_i^{(1)}(r,\xi) + \ldots \qquad (5)$$

$$u = u^{(0)}(r,\xi) + \varepsilon u^{(1)}(r,\xi) + \ldots$$

$$w = w^{(0)}(r,\xi) + \varepsilon^2 w^{(1)}(r,\xi) + \ldots$$

Changeability period $l = \varepsilon R$ in respect to variable x for function with index (j), $j = 1, 2, \ldots$, is admitted. Substituting series (5) into boundary value problem (1) to (3) and splitting it with respect to powers of ε , one obtains the recurrent sequence of boundary value problems

$$\frac{\partial N_1^{(0)}}{\partial \xi} = 0 \qquad \frac{\partial Q_1^{(0)}}{\partial \xi} = 0 \qquad \frac{\partial M_1^{(0)}}{\partial \xi} = 0$$

$$\frac{\partial w_1^{(0)}}{\partial \xi} = 0 \qquad \frac{\partial u_1^{(0)}}{\partial \xi} = 0 \qquad (6)$$

$$r\frac{\partial N_1^{(1)}}{\partial \xi} + \frac{\partial}{\partial r}\left(rN_1^{(0)}\right) - N_2^{(0)} = rq_1 \qquad r\frac{\partial Q_1^{(1)}}{\partial \xi} + \frac{\partial}{\partial r}\left(rQ_1^{(0)}\right) = rq_2$$

$$r\frac{\partial M_1^{(1)}}{\partial \xi} + \frac{\partial}{\partial r}\left(rM_1^{(0)}\right) - M_2^{(0)} - Q_1^{(0)} = 0 \qquad (7)$$

$$\frac{\partial u^{(1)}}{\partial \xi} + \frac{\partial u^{(0)}}{\partial r} = k_1\frac{N_1^{(0)}}{Eh} \qquad k_1 = A^{-1} + \frac{12z^2}{h^2}$$

$$(8)$$

$$\frac{\partial^2 w^{(1)}}{\partial \xi^2} + \frac{\partial^2 w^{(0)}}{\partial r^2} = A_{(\xi)}\frac{M_1^{(0)}}{D_1} \qquad D_1 = \frac{Eh^3}{12}$$

$$N_2^{(0)} = Eh\frac{A}{r}u^{(0)} \qquad M_2^{(0)} = D_1 k_1\frac{\partial w^{(0)}}{\partial r} \qquad (9)$$

$$N_2^{(1)} = Eh\frac{A}{r}u^{(1)} \qquad M_2^{(1)} = D_1 k_1\frac{\partial w^{(1)}}{\partial \xi} \qquad (10)$$

$$u^{(0)} = w^{(0)} = \frac{dw^{(0)}}{dr} = 0 \tag{11}$$

for $r = r_0, R$

$$u^{(1)} = w^{(1)} = \frac{dw^{(1)}}{dr} = 0 \tag{12}$$

It can be easily shown that

$$N_1^{(0)} = N_1^{(0)}(r) \qquad Q_1^{(0)} = Q_1^{(0)}(r) \qquad M_1^{(0)} = M_1^{(0)}(r)$$

$$u_1^{(0)} = u_1^{(0)}(r) \qquad w_1^{(0)} = w_1^{(0)}(r) \tag{13}$$

We consider the averaging operator defined upon the l-periodic function $\Phi(\xi)$

$$m = [\Phi(\xi)] = \frac{1}{l}\int_0^l \Phi(\xi)d\xi \tag{14}$$

The following is easily obtained from equations (7) and (8) by applying the averaging operator defined by equation (14):

$$\frac{d}{dr}\left(rN_1^{(0)}\right) - m\left(N_2^{(0)}\right) = rm(q_1) \qquad \frac{d}{dr}\left(rQ_1^{(0)}\right) = rm(q_2)$$

$$\frac{d}{dr}\left(rM_1^{(0)}\right) - m\left(M_2^{(0)}\right) - Q_1^{(0)} = 0$$

$$N_1^{(0)} = \frac{Eh}{m(k_1)}\frac{du^{(0)}}{dr} \qquad M_1^{(0)} = \frac{D_1}{m(A)}\frac{d^2w^{(0)}}{dr^2} \tag{15}$$

$$N_2^{(0)} = Eh\frac{m(A)}{r}u^{(0)} \qquad M_2^{(0)} = D_1 m(k_1)\frac{dw^{(0)}}{dr}$$

$$u^{(0)} = w^{(0)} = \frac{dw^{(0)}}{dr} = 0 \qquad \text{for} \qquad r = r_0, R$$

The homogenized boundary value problem (15) coincides with relations, which are obtained on the basis of physical assumptions (Andreeva, 1981). Otherwiese, equations (7) to (12) give us the possibility to obtain not only the homogenized boundary problem, but rapid oscillation terms of the solution too. Using equations (7) to (10) and taking into account equations (15) one obtains

$$r\frac{\partial N_1^{(1)}}{\partial \xi} = r[q_1 - m(q_1)] + N_2^{(0)} - m\left(N_2^{(0)}\right) \qquad r\frac{\partial Q_1^{(1)}}{\partial \xi} = q_2 - m(q_2)$$

$$\frac{\partial M_1^{(1)}}{\partial \xi} = \frac{1}{r}\left[M_2^{(0)} - m\left(M_2^{(0)}\right)\right] \tag{16}$$

$$\frac{\partial u^{(1)}}{\partial \xi} = [k_1 - m(k_1)]\frac{N_1^{(0)}}{Eh} \qquad \frac{\partial^2 w^{(1)}}{\partial \xi^2} = [A - m(A)]\frac{M_1^{(0)}}{D}$$

$$N_2^{(1)} = EhA\frac{u}{r} \qquad M_2^{(1)} = D_1(k_1 - m(k_1))\frac{\partial w^{(1)}}{\partial \xi}$$

Then we can obtain displacements, forces and moments in the original corrugated plate using formulas

$$u_r = \frac{u + \beta w}{A} \qquad w_n = \frac{w + \beta u}{A} \qquad N_r = \frac{N_1 + \beta Q_1}{A}$$

$$N_0 = \frac{N_2}{A} \qquad M_r = M_1 - zN_1 \qquad M_0 = M_1 - zN_2$$

3 Numerical Results and Error Estimation

Now we examine the accuracy of our solution. Let us consider a clamped radially corrugated circular plate with a rigid central disk of radius r_{00} loaded by a uniformly distributed pressure q. Then

$$z = H\sin\left[n\left(r - r_{00}\right)\right]$$

We use the following geometrical and physical parameters:

$$R = 28.3 \cdot 10^{-3}\,\mathrm{m} \qquad r_0 = 1.9 \cdot 10^{-3}\,\mathrm{m} \qquad h = 0.22 \cdot 10^{-3}\,\mathrm{m} \qquad H = 0.75 \cdot 10^{-3}\,\mathrm{m}$$

$$E = 10^{-1}\,\mathrm{N/m^2} \qquad q = 10^{-8}\,\mathrm{N/m^2} \qquad \nu = 0.33 \qquad n = 4$$

Parameter ε equals 1/4, representing a bad case for our method (this „small" parameter is obviously not very small). The computed bending moments and stresses are shown in Figures 1 to 6.

The results, obtained by the numerical method (Biderman, 1977) are presented by curve 1, our results by curve 2. The discrepancy of asymptotic and numerical results for bending moments does not exceed 10 % which confirms an acceptable accuracy of the method presented.

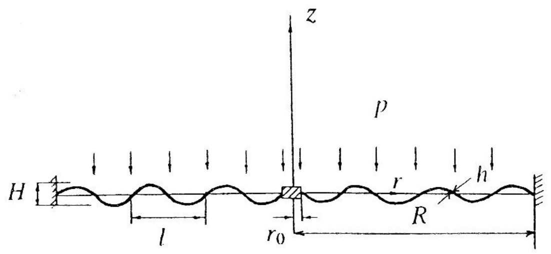

Figure 1. Circle Corrugated Plate

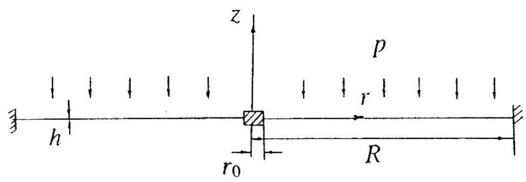

Figure 2. Equivalent Smooth Circle Plate

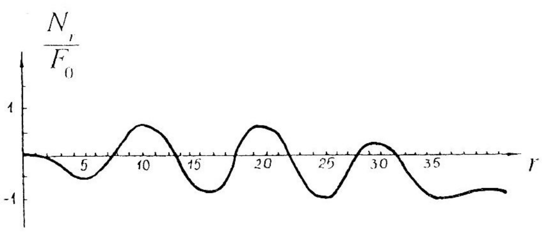

Figure 3. Stress N_r in Circle Corrugated Plate

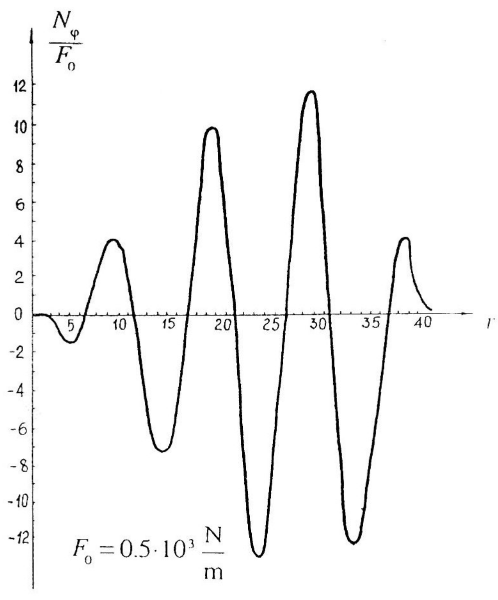

Figure 4. Stress N_φ in Circle Corrugated Plate

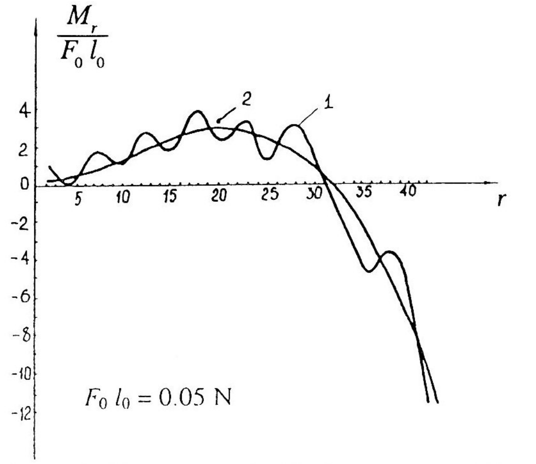

Figure 5. Moment M_r in Circle Corrugated Plate

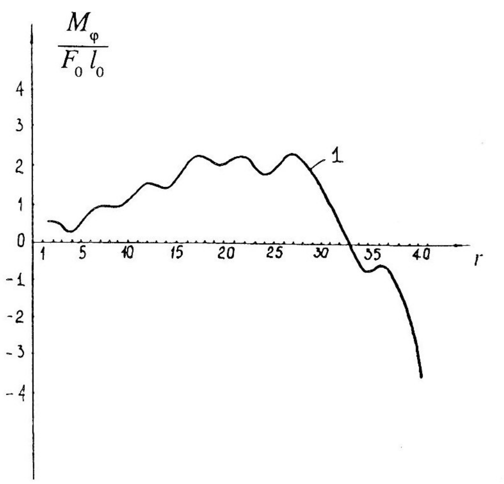

Figure 6. Moment M_φ in Circle Corrugated Plate

4 Eigenvalue Problem for Corrugated Plate

Using the notations introduced above we consider the subsequent eigenvalue problem, with $q_1 = 0$ and $q_2 = ph\omega^2 w_z$ in equations (1). Then for the modified equations (1) we may formulate boundary conditions (2). We represent moments and forces as in equation (5), and eigenvalue ω and eigenfunction w in the following form:

$$w_z = \left[w_0(r) + n^{-2}w_0(r) + \ldots\right] + \left[n^{-2}w_1(r,\xi) + n^{-4}w_2(r,\xi) + \ldots\right]$$
$$\omega^2 = \omega_0^2 + n^{-2}\omega_1^2 + \ldots \qquad (17)$$

Substituting expansions (5) and (17) into equations (1) and (2) and boundary conditions (3) and splitting the results according to the powers of ε, one obtains a recurrent systems of eigenvalue problems. The first step of solving is the same as that above.

The homogenised eigenvalue problem may be obtained by applying the averaging operator defined by equation (14)

$$D_0\left[w_{0,rrr} + \left(r^{-1}w_{0,rr}\right)_{,r} - k_1k_2\left(r^{-2}w_{0,r}\right)_{,r}\right] - \omega_0^2\rho hk_1^2 w_0 = 0 \qquad (18)$$

To this equation belong the homogenised boundary conditions (11). For the „slow" function $w(r)$ and the first-order term in the frequency expansion one obtains the equation

$$w_{01,rrr} + \left(r^{-1}w_{01,rr}\right)_{,r} + \left(k_1k_2r^{-2}w_{01,r}\right)_{,r} - \omega_0^2\rho hk_1^2D_0^{-1}w_{01} = k_1r^{-1}L_1(r) + \omega_0^2\rho hk_1D_0^{-1}r^{-1}L_2(r) + \omega_1^2\rho hk_1^2D_0^{-1}w_0$$

where

$$L_1(r) = \left[\int L_3\left[\int L_3(k_4)d\xi\right]d\xi\right]\left(r^{-1}w_{0,r}\right)_{,rrr} + \left[k_4\int Ad\xi\right]\left(r_{-1}w_{0,rr} + v^2r^{-2}w_{0,r}\right)_{,r}$$

$$L_2(r) = \left[\int L_3\left[\int L_3(A)d\xi\right]d\xi\right]\left(rw_0\right)_{,rrr} + \left[\iint L_4(A)d\xi d\xi\right]\left(rw_{0,rr} + v\,rw_{0,r}\right)_{,r}$$

$$L_3(\ldots) = (\ldots) - (\ldots) \qquad L_4(\ldots) = (\ldots)^{-1} - \left[(\ldots)\right]^{-1}$$

Conditions of secular term absence (Andrianov et al., 1992) produce the expressions

$$
\omega_1^2 = \frac{-D_0 \int_{r_0}^{R} r^{-1} L_1(r) dr + \omega_0^2 ph \int_{r_0}^{R} r^{-1} L_2(r) dr}{k_1 \int_{r_0}^{R} w_0^2 dr} \tag{19}
$$

For shallow corrugations, formula (19) may be reduced to a more simplified form

$$
\omega_1^2 = \frac{-D_0 \int_{r_0}^{R} \left\{ \left[\int L_3 \left[\int L_3 (k_4) d\xi \right] d\xi \right] r^{-2} w_{0,r} \right\} dr}{\int_{r_0}^{R} w_0^2 dr}
$$

5 Rectangular Plate

Only the final results of an asymptotic analysis are shown here. The governing equilibrium equations for a rectangular corrugated plate may be obtained as follows:

$$
\begin{aligned}
&N_{x,x} + N_{yx,y} = -q_x \qquad\qquad N_{xy,x} + N_{y,y} = -q_y \\
&Q_{x,x} + Q_{y,y} = -q_z \\
&M_{y,y} + M_{xy,x} + Q_y = zq_y \qquad\qquad M_{x,x} + M_{yx,y} + Q_x = zq_x
\end{aligned} \tag{20}
$$

Let us introduce the expansions

$$
\begin{aligned}
&N_x = \sum_{i=0}^{\infty} n^{-i} N_{xi}(x,\xi,y) \qquad\qquad N_y = \sum_{i=0}^{\infty} n^{-i} N_{yi}(x,\xi,y) \\
&N_x = (x, \xi + b, y) = N_x(x,\xi,y) \qquad\qquad \xi = nx
\end{aligned} \tag{21}
$$

Substituting expansion (21) into system (20) and splitting it with respect to powers of ε, one obtains cell boundary problems for the whole domain

$$
N_{x0,\xi} = N_{xy0,\xi} = Q_{x0,\xi} = M_{x0,\xi} = M_{xy0,\xi} = 0 \tag{22}
$$

$$
\begin{aligned}
&N_{x1,\xi} + N_{x0,x} + N_{yx0,y} = -q_x \qquad\qquad N_{xy1,\xi} + N_{xy0,x} + N_{y0,y} = -q_y \\
&Q_{x1,\xi} + Q_{x0,x} + Q_{y0,y} = -q_z \\
&M_{x1,\xi} + M_{x0,x} + M_{yx0,y} + Q_{x0} = 0 \qquad\qquad M_{xy1,\xi} + M_{xy0,x} + M_{y0,y} + Q_{y0} = 0 \\
&M_{x2,\xi} + M_{x1,x} + M_{yx1,y} + Q_{x1} = zq_x \qquad\qquad M_{xy2,\xi} + M_{xy1,x} + M_{y1,y} + Q_{y1} = zq_y
\end{aligned} \tag{23}
$$

From equations (22) we conclude that

$$N_{x0} = N_{x0}(x,y) \qquad N_{xy0} = N_{xy0}(x,y) \qquad Q_{x0} = Q_{x0}(x,y)$$

$$M_{x0} = M_{x0}(x,y) \qquad M_{xy0} = M_{xy0}(x,y)$$

Homogenized equations can be easily obtained by applying the averaging operator

$$N_{x0,x} + \left(\tilde{N}_{yx0}\right)_{,y} = -\tilde{q}_x \qquad N_{xy0,x} + \left(\tilde{N}_{y0}\right)_{,y} = -\tilde{q}_y$$

$$M_{x0,xx} + 2H_{0,xy} + \left(\tilde{M}_{y0}\right)_{,yy} = \tilde{q}_z \tag{24}$$

$$M_{x0,x} + \left(\tilde{M}_{yx0}\right)_{,y} + Q_{x0} = 0 \qquad M_{xy0,x} + \left(\tilde{M}_{y0}\right)_{,y} + Q_{y0} = 0$$

where

$$\widetilde{(\ldots)} = b^{-1}\int_0^b (\cdots)dx \qquad H_0 = 0{,}5\left[M_{xy0} + \tilde{M}_{yx0}\right]$$

Now let us represent displacements w, u, υ as sums of „bending“ (index f) and „stretch“ (index s) components

$$u^s = u_0^s(\xi,x,y) + n^{-1}u_1^s(\xi,x,y) + \ldots \qquad u^f = n^{-1}u_0^f(\xi,x,y) + n^{-2}u_1^f(\xi,x,y) + \ldots$$

$$w^s = n^{-1}w_0^s(\xi,x,y) + n^{-2}w_1^s(\xi,x,y) + \ldots \qquad w^f = w_0^f(\xi,x,y) + n^{-2}w_2^f(\xi,x,y) + \ldots \tag{25}$$

$$\upsilon^s = \upsilon_0^s(\xi,x,y) + n^{-1}\upsilon_1^s(\xi,x,y) + \ldots \qquad \upsilon^f = n^{-1}\upsilon_0^f(\xi,x,y) + n^{-2}\upsilon_1^f(\xi,x,y) + \ldots$$

where

$$\upsilon_0^f = nzw_{0,y}^f \qquad \upsilon_1^f = nzw_{1,y}^f \qquad u_i^{f(s)}((\xi+b),x,y) = u_i^{f(s)}(\xi,x,y)$$

$$\upsilon_i^{f(s)}((\xi+b),x,y) = u(\xi,x,y) \qquad w_i((\xi+b),x,y) = w_i(\xi,x,y)$$

The cell problems have been solved on the basis of the approach presented above. Then one can express projections of moments and stresses upon the projection of displacement

$$N_{x0} = Ehk_3^{-1}\left(u_{0,x}^s + u_{1,\xi}^s + \nu\upsilon_{0,y}^s\right) \qquad N_{y0} = EhA\left[\upsilon_{0,y}^s + k_4^{-1}A^{-1}\left(u_{0,x}^s + u_{1,\xi}^s\right)\right]$$

$$M_{x0} = D_0A^{-1}\left(w_{0,xx}^f + w_{2,\xi\xi}^f + \nu w_{0,yy}^f\right) \qquad M_{y0} = D_0k_4\left[w_{0,yy}^f + \nu A^{-1}\left(w_{0,xx}^f + w_{2,\xi\xi}^f\right)\right]$$

$$N_{xy0} = GA^{-1}\left(u_{0,y}^s + \upsilon_{0,x}^s + \upsilon_{1,\xi}^s\right) \qquad N_{yx0} = GA\left(u_{0,y}^s + \upsilon_{0,x}^s + \upsilon_{1,\xi}^s\right) \tag{26}$$

$$M_{xy0} = D_1k_5\left(w_{1,\xi y}^f + w_{0,xy}^f\right) \qquad M_{yx0} = D_1k_6\left(w_{1,\xi y}^f + w_{0,xy}^f\right) \qquad M_{xy1} = D_1k_5\left(w_{2,\xi y}^f + w_{1,xy}^f\right)$$

where

$$k_5 = A^{-1}\left[1 + 12(z/h)^2\right] \qquad k_6 = A^{-2}k_5 \qquad D_1 = D(1-\nu) \qquad D_0 = Eh^3/12$$

One obtains from equations (26)

$$u_1^s = \left[\int \left(k_3 k_1^{-1} - 1\right) d\xi\right]\left(u_{0,x}^s + \nu \upsilon_{0,y}^s\right) + u_{10}^s(x,y)$$

$$\upsilon_1^s = \left[\int \left(A k_2^{-1} - 1\right) d\xi\right]\left(u_{0,y}^s + \nu \upsilon_{0,x}^s\right) + u_{10}^s(x,y) \qquad (27)$$

$$w_{i,y}^s = \left\{\int\left[\left(k_5 \left(\tilde{k}_{12}^{-1}\right)\right)^{-1} - 1\right] d\xi\right\} w_{0,xy}^f + w_{10,y}^f(x,y)$$

$$w_2^f = \left[\iint \left(A k_2^{-1} - 1\right) d\xi\right]\left(w_{0,xx}^f + \nu w_{0,yy}^f\right) + w_{20}^f(x,y)$$

Taking into account equations (26), the expressions for stresses N_{x0} and N_{xy0} and moments M_{x0} and M_{xy0} may be reduced to the form

$$N_{x0} = Ehk_1^{-1}\left(u_{0,x}^s + \nu \upsilon_{0,y}^s\right) \qquad N_{xy0} = Ghk_2^{-1}\left(u_{0,y}^s + \upsilon_{0,x}^s\right) \qquad (28)$$

$$M_{x0} = D_0 k_2^{-1}\left(w_{0,xx}^f + \nu w_{0,yy}^f\right) \qquad M_{xy0} = \frac{Gh^3}{6}\tilde{k}_5^{-1} w_{0,xy}^f$$

Equations for u_0^s, v_0^s, w_0^s have been obtained on the basis of the approach presented above

$$Eh\left[u_{0,xx}^p + (\nu + k_7)\upsilon_{0,xy} + k_7 u_{0,yy}^p\right] = -k_1 \tilde{q}_x$$

$$Eh\left[k_1 k_2 \upsilon_{0,yy}^s + (\nu + k_7) u_{0,xy}^s + k_7 \upsilon_{0,xx}^s\right] = -k_1 \tilde{q}_y \qquad (29)$$

$$D_0\left[w_{0,xx}^f + \left(2\nu + k_2 k_8 G E^{-1}\right) w_{0,xxyy}^f + k_1 k_2 w_{0,yyyy}^f\right] = k_2 \tilde{q}_z$$

where

$$k_7 = \frac{k_1 G}{k_2 E} \qquad k_8 = \left[\tilde{k}_5^{-1}\right]\left(1 + \tilde{k}_3^{-1}\tilde{k}_6\right)$$

The homogenized boundary value problem (29) coincides with relations, which were obtained on the basis of physical assumptions (Andreeva, 1981) (except the twisting moment). Otherwise, equations (27) give us the possibility to obtain not only the homogenized boundary problem, but rapid oscillation terms of the solutions as well. Using equations (29) and taking into account equations (26) to (28) one obtains

$$N_{x1,\xi} = \left(\tilde{N}_{yx0} - N_{yx0}\right)_{,y} + \tilde{q}_x - q_x \qquad N_{xy1,\xi} = \left(\tilde{N}_{y0} - N_{y0}\right)_{,y} + \tilde{q}_y - q_y$$

$$M_{x1,\xi} = \left(\tilde{M}_{yx0} - M_{yx0}\right)_{,y}$$

$$N_{y1} = EhA\left(\upsilon_{1,y}^p - nzw_{0,y}^u\right) \qquad M_{y1} = D_0 k_5 w_{1,yy}^u \qquad M_{xy1} = D(1-\nu)k_8\left(w_{2,\xi y}^u - w_{1,xy}^u\right)$$

As a numerical example let us consider the bending of a rectangular plate simply supported along edges $x = 0$, b, and clamped along edges $y = \pm b/4$, under its own gravity with $z = 12h\sin\left(\frac{6\pi x}{b}\right)$; $\nu = 0.3$.

The results obtained by the asymptotic method are presented in Figures 7, 8 (here γ is specific gravity).

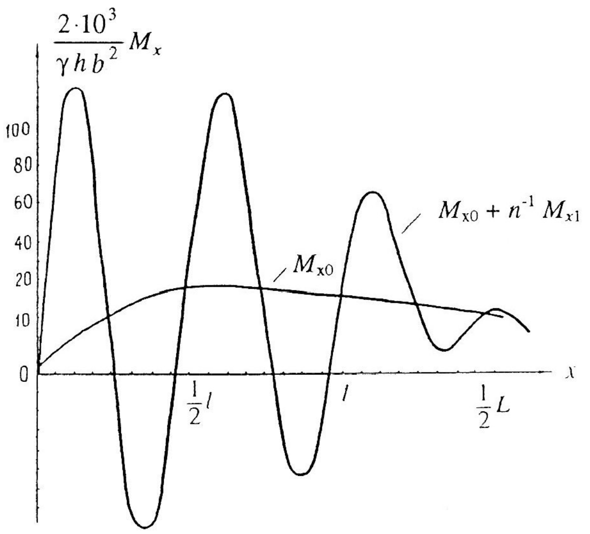

Figure 7. Comparison of Homogenized and Real Value of M_x Moments for Rectangular Plate

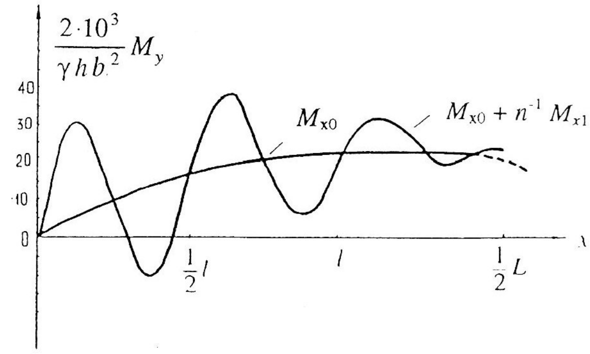

Figure 8. Comparison of Homogenized and Real Value of M_y Moments for Rectangular Plate

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Symbols

u	initial radial displacement
w	initial normal displacement
u, w	projecting of displacements on the middle plane
ν	Poisson's ratio
E	modulus of elasticity
$B = Eh/(1-\nu)$	
$D = Eh^3/(12(1-\nu^2))$	
n	number of corrugations
ε_1, ε_2	membrane deformations
ε_{12}	shear deformation
l_1, l_2, l_{12}	projection of deformations on the middle plane
χ_1, χ_2	curvatures
N_1 (N_2)	membrane stress
M_1 (M_2)	bending moment
H	torsion moment
Q_1 (Q_2)	shearing force
q_1	normal load
q_2	radial load
q_x (q_y)	tangential load in x (y) direction
$\varepsilon = 1/n$	
r	polar radius
$z(r)$	function defining corrugation geometry
h	plate thickness
$A = (1 + b)$	
a, b	lengths of rectangular plate sides
t	time
ρ	density of plate material
ω	frequency

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