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Continuum Mechanics Description of Plastic Flow Produced by Micro-Shear Bands

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Physical motivation and continuum mechanics description of micro-shear banding effects are presented. Constitutive description of plastic flow accounting for micro-shear bands is given and a new method of numerical identification of the model is proposed.

1 Introduction

Experimental observations reveal that at the advanced stage of ductile fracture, while void coalescence takes place, the behaviour of the ligaments between the voids is controlled by the formation of micro-shear bands. Also in heavily deformed metals, in particular under highly constrained conditions which can appear in technological shaping operations, a multiscale hierarchy of shear banding occurs. The new mechanism of deformation replaces crystallographic multiple slip and twinning also at small strains, if they are preceded by the alteration of the scheme of straining. The change of deformation mode contributes to the development of strain induced anisotropy and modifies material properties. Therefore, continuum mechanics description of micro-shear banding is important for adequate constitutive modelling of inelastic strain and damage processes in metallic solids. Formulation of a complete theory based on the precise micro-to-macro transition remains an open and challenging question. An attempt to tackle the averaging procedure over the representative volume element (RVE) with micro-shear bands was presented earlier in Pecherski (1997), where the macroscopic measures of the rate of plastic deformation and plastic spin, necessary to formulate the constitutive description, were derived. Phenomenological model of plastic flow accounting for the effects of micro-shear bands, based on the double surface plasticity theory, was proposed by Pecherski (1998).

The aim of the paper is to formulate the approximate model with one limit surface and to study the possibilities of certain simplifications in relation to the experimental results presented recently in the literature. The simplified model shows that the contribution of active micro-shear bands with their characteristic geometric pattern transmitted to the macroscopic level, produce the non-coaxiality between principal directions of stress and rate of plastic deformations. The relation for plastic spin appears in a natural way as an effect of this non-coaxiality. It appears that depending on the contribution of the mechanisms involved in plastic flow, a fully active range, separated from the elastic range by a truly nonlinear zone called the partially active range, may exist. The idea of multiple potential surfaces forming a vertex on the smooth external surface is applied here to connect the fully active range and the partially active range with the definite geometric pattern of micro-shear bands. This leads to the new form of the hypoelastic version of J_2 deformation theory accounting for the effects of micro-shear banding. Consideration of the partially active range enables proper description of the unloading process. The preliminary results of the numerical identification of the model based on the available experimental data of Anand et al. (1992, 1994) are also presented.

2 Physical Motivation

The term micro-shear band is understood as a long and very thin (of order 0.1 μm) sheet-like region of concentrated plastic shear, crossing grain boundaries without deviation and forming a definite pattern in relation to the principal directions of strain. The experimental information about mechanical behaviour and related structural features is reviewed, e.g. in Hatherly and Malin (1984), Korbel (1992) and Pecherski (1991, 1992), where comprehensive lists of references are given. The metallographic observations reveal the hierarchy of plastic slip processes: from coplanar dislocation groups moving collectively along active slip systems, through slip lammellae and slip bands to coarse slip bands, which may further transform into transgranular micro-shear bands and form clusters of micro-shear bands of the thickness of the order of 10 to 100 μm. The clusters of micro-shear bands, produced for instance in rolling, form the planar structures, which are usually inclined by an angle φ of about $\pm 35°$ to the rolling plane and are orthogonal to the specimen lateral face. There can be, however, considerable deviations from this value within the 15° to 50° range. It is worth stressing that the problem of specifying the angle is complicated by the difficulty of distinguishing the most recently formed micro-shear bands from those that were formed earlier and subsequently rotated with the material towards the rolling plane. This is related to the important observation, stressed in Hatherly and Malin (1984), that a particular micro-shear band operates only once and develops rapidly to its full extent. The micro-shear bands, once formed, do not contribute further to the increase in plastic shear strain. Thus, it appears that successive generations of active micro-shear bands competing with the mechanism of multiple crystallographic slip are responsible for plastic strain in metals. The discussion of experimental observations concerning micro-shear bands geometry leads to the hypothesis, that it is typical of the clusters of active micro-shear bands that their planes are rotated relative to the respective planes of maximum shear stress by a certain angle β, which is usually of the order of 5° to 15°. This deviation angle plays an essential role in the phenomenological theory of plastic deformations accounting for the effects of micro-shear banding and will be considered as a statistically averaged micro-shear bands orientation parameter, transmitting to the macroscopic level the geometry of their spatial pattern. The experimental observations show that the spatial pattern of micro-shear bands does not change for loading conditions that deviate within limits from the proportional loading path, i.e. the load increments are confined to a certain cone the angle of which can be determined experimentally.

3 Continuum Mechanics Description of Micro-Shear Banding Effects

The averaging procedure over the representative volume element (RVE) with micro-shear bands was presented earlier in Pecherski (1997), where the macroscopic measures of the rate of plastic deformation and plastic spin, necessary to formulate the constitutive description, were derived. The derivation was based on the physical model relating macroscopic shear strain rate with microstructural features of micro-shear bands, mathematical idealization of a system of active micro-shear bands as a singular surface of tangential velocity discontinuities, and the known averaging procedure applied to the new RVE, which is traversed by the discontinuity surface. For the sake of clarity, some basic results of this study will be presented here.

Consider a certain RVE containing the region of progressive shear banding. According to the physical motivation discussed above, at this level of observation the clusters of active micro-shear bands can be considered as elementary carriers of plastic strain. The cluster of micro-shear bands has the active zone of the thickness H_{ms} and the width L_{ms}, in which the passage of active micro-shear bands results in the local perturbation, Δ_{ms}, of the microscopic displacement field moving with the speed V_S as a distortion wave. Consider a set of N_{ms} active micro-shear bands of similar orientation and produced within the time period $\Delta\tau$, which can be considered as an infinitesimal increment of "time-like parameter" t in the macroscopic description. Such a system (cluster) of micro-shear bands produces the microscopic shear strain γ_{ms}, which is given by the following relation

$$\gamma_{ms} = \frac{\Delta_{ms}}{H_{ms}} \quad \text{with} \quad \Delta_{ms} = \frac{B_{ms} N_{ms}}{L_{ms}} \bar{x}_{ms} \quad \text{and} \quad \bar{x}_{ms} = \frac{1}{N_{ms}} \sum_{i}^{N_{ms}} x_i \qquad (1)$$

where B_{ms} is the total displacement produced by a single micro-shear band and $\bar{x}_{ms}$ denotes the average distance that N_{ms} micro-shear bands have moved in the active zone. The width of the active zone L_{ms} can be determined by the length of the path that micro-shear bands passed with an average speed υ_{ms} during the time period $\Delta\tau$. Assuming that the distance $\bar{x}_{ms}$ and the number of active micro-shear bands N_{ms} can change during the propagation of the active zone we have from equation (1)

$$\dot{\gamma}_{ms} = \frac{V_S}{H_{ms}} \quad \text{with} \quad V_S = \frac{B_{ms}}{L_{ms}} (N_{ms} \dot{\bar{x}}_{ms} + \bar{x}_{ms} \dot{N}_{ms}) \qquad (2)$$

where the dot denotes differentiation with respect to the "time-like parameter" t. Let us observe that under the simplifying assumption that the speed of micro-shear bands in the active zone of the cluster remains approximately the same, the rate $\dot{\bar{x}}_{ms}$ can be identified with the speed υ_{ms} of the head of a single micro-shear band, $\dot{\bar{x}}_{ms} \equiv \upsilon_{ms}$ (cf. Pecherski (1997), where the model of a single micro-shear band is studied). If the number of active micro-shear bands operating in the active zone of a single cluster can be assumed constant, equation (2) takes the form which is formally similar to the Orowan relation,

$$\dot{\gamma}_{ms} = B_{ms}\,\rho_{ms}\,\upsilon_{ms} \qquad \text{with} \qquad \rho_{ms} = \frac{N_{ms}}{L_{ms} H_{ms}} \tag{3}$$

where ρ_{ms} corresponds to the density of micro-shear bands operating within the active zone of the cluster. This is the number of active micro-shear bands that cut through a unit cross-sectional area.
The foregoing discussion of the physical nature of the micro-shear banding process gives rise the following hypothesis:

The passage of micro-shear bands within the active zone of the cluster results in the perturbation Δ_{ms} of the microscopic displacement field $\mathbf{u}_m = \mathbf{x}_m - \mathbf{X}_m$ travelling with the speed $\mathcal{V}_S$, which produces a discontinuity of the microscopic velocity field in the RVE it traverses. The progression of clusters of micro-shear bands can be idealized mathematically by means of a singular surface of order one propagating through the macro-element (RVE) of the continuum.

The necessary mathematical formalism of the theory of propagating singular surfaces is given e.g. in Kosinski (1986), where also a comprehensive list of references is provided. According to this study, there exists a jump discontinuity of derivatives of the function of motion χ_m, i.e. of the microscopic velocity field $[\dot{\chi}_m] \neq \mathbf{0}$ and the deformation gradient $[\mathbf{f}] \neq \mathbf{0}$, which are assumed smooth in each point of RVE for each instant of "time-like parameter" $t \in I \subset R$, excluding the discontinuity surface

$$[\dot{\chi}_m] = \dot{\chi}_m^+ - \dot{\chi}_m^- \neq \mathbf{0} \qquad [\mathbf{f}] = \mathbf{f}^+ - \mathbf{f}^- \neq \mathbf{0} \tag{4}$$

This determines the singular surface of order one, called also the surface of strong discontinuity. According to Kosinski (1986), the surface of strong discontinuity of the microscopic velocity field fulfills formally the properties of a non-material vortex sheet with the jump discontinuity of the first derivatives of χ_m given by

$$[\mathbf{v}_m] = \mathcal{V}_S\mathbf{s} \qquad [\mathbf{f}] = -\frac{\mathcal{V}_S}{U}\mathbf{s} \otimes \mathbf{n}\,\mathbf{f} \qquad \text{for} \qquad U \neq 0 \tag{5}$$

where $\mathbf{v}_m(\mathbf{x}_m, t) = \dot{\chi}_m\{\chi_m^{-1}(\mathbf{X}_m, t), t\}$, whereas **s** and **n** are, respectively, the unit tangent and the unit normal vectors to the discontinuity surface $S(t)$, while U corresponds to the local speed of propagation of $S(t)$.

According to the analysis in Pecherski (1997), the averaging procedure of the microscopic velocity field $\mathbf{v}_m$ over the macro-element V can be generalized for the macroscopic RVE traversed by the singular surface of tangential velocity discontinuities with the velocity jump of magnitude $\mathcal{V}_S$. Then, the macroscopic measure of velocity gradient $\mathcal{L}$, averaged over the macro-element V traversed by the discontinuity surface $S(t)$, is expressed by means of surface data in the following way

$$\mathcal{L} \equiv \frac{1}{V}\int_{\partial V - S(t)} \mathbf{v}_m \otimes \mathbf{v}\,dA = \frac{1}{V}\int_V \operatorname{grad}\mathbf{v}_m\,dV + \frac{1}{V}\int_{S(t)} \mathcal{V}_S\,\mathbf{s} \otimes \mathbf{n}\,dA \tag{6}$$

The averaging formula (6) enables us to account for the contribution of micro-shear banding in the macroscopic measure of the velocity gradient produced at finite elastic-plastic strain. According to equation (6), the velocity gradient $\mathcal{L}$ is decomposed as follows

$$\mathcal{L} = \mathbf{L} + \mathbf{L}_{MS} \qquad \mathbf{L} = \frac{1}{V}\int_V \operatorname{grad}\mathbf{v}_m\,dV \qquad \mathbf{L}_{MS} = \frac{1}{V}\int_{S(t)} \mathcal{V}_S\,\mathbf{s} \otimes \mathbf{n}\,dA \tag{7}$$

Assuming that the singular surface $S(t)$ forms a plane traversing volume V, with the unit vectors **s** and **n** held constant, equation $(7)_3$ results in $\mathbf{L}_{MS} = \dot{\gamma}_{MS}\,\mathbf{s} \otimes \mathbf{n}$, where the macroscopic shear strain rate $\dot{\gamma}_{MS}$ is determined according to equation (3) by the microscopic variables as

$$\dot{\gamma}_{MS} = B_{ms}\, \upsilon_{ms}\, \rho_{MS} \qquad \rho_{MS} = \frac{1}{V}\int_{S(t)} H_{ms}\, \rho_{ms}\, dA \tag{8}$$

The symbol ρ_{MS} denotes the macroscopic volume density of micro-shear bands that operate within the sequence of clusters sweeping the RVE. The above relations provide the following macroscopic measures of the rate of plastic deformations and plastic spin produced by active micro-shear bands:

$$\mathbf{D}^{p}_{MS} = \frac{1}{2}\left(\mathbf{L}_{MS} + \mathbf{L}^{T}_{MS}\right) \qquad \mathbf{W}^{p}_{MS} = \frac{1}{2}\left(\mathbf{L}_{MS} - \mathbf{L}^{T}_{MS}\right) \tag{9}$$

The averaging procedure over the RVE with the singular surface allows to account for the characteristic geometric pattern of micro-shear bands which is transmitted upwards through a multiscale hierarchy of observational levels.

4 Constitutive Description

The following kinematical relations can be derived, (Pecherski, 1997):

$$\mathscr{D} = \mathscr{D}^{e} + \mathscr{D}^{p} = \mathscr{D}^{e} + \mathbf{D}^{p} + \mathbf{D}^{p}_{MS} \qquad \mathscr{W} = \mathscr{W}^{e} + \mathscr{W}^{p} = \mathscr{W}^{e} + \mathbf{W}^{p} + \mathbf{W}^{p}_{MS} \tag{10}$$

where $\mathscr{D}$ and $\mathscr{W}$ correspond to the rate of deformation and the material spin, respectively, and the superscripts e and p refer to elastic and plastic, whereas $\mathbf{D}^{p}$ and $\mathbf{W}^{p}$ correspond, respectively, to the rate of plastic deformation and the plastic spin controlled by crystallographic multiple slip. If the contribution of micro-shear bands is negligible, we have, according to equation (6), $\mathbf{D}^{p}_{MS} = \mathbf{W}^{p}_{MS} = \mathbf{0}$. Further analysis will be confined to isothermal processes with small elastic strains. Then, the following approximate relations can be obtained:

$$\overset{\circ}{\tau} = \mathscr{L} : \mathscr{D}^{e} \qquad \overset{\circ}{\tau} = \dot{\tau} - \mathscr{W}^{e}\tau + \tau\,\mathscr{W}^{e} \qquad \mathscr{L} = \rho\frac{\partial^{2}\phi}{\partial\varepsilon\,\partial\varepsilon} \tag{11}$$

The symbol $\mathscr{L}$ denotes the fourth-order tensor of elastic moduli, ρ is the mass density and ϕ corresponds to the free energy function per unit mass.

According to Hill (1967), the macroscopic constitutive equations describing elastic-plastic deformations of polycrystalline aggregates are either thoroughly or partially incrementally nonlinear. Depending on the contribution of the mechanisms involved in plastic flow, a region of fully active loading, called also a fully active range, separated from the total unloading (elastic) range by a truly nonlinear zone corresponding to the partially active range, may exist. The connection of the fully active range and partially active range with the geometric pattern of micro-shear bands is necessary to specify the relation for the rate of plastic deformations for different loading paths. Due to the fact that multiple sources of plasticity are dealt with, the theory of multimechanisms with multiple plastic potentials can be considered. The concept of multiple potential surfaces forming a vertex on the smooth limit surface was studied earlier by Mroz (1963) within the framework of non-associated flow laws. In our case, the existence of the following plastic potentials related to the mechanisms responsible for plastic flow can be postulated (Pecherski, 1992 and 1998):

(i) the plastic potential g_0 that reproduces at the macroscopic level the crystallograhic multiple slips and is associated with the limit surface approximated by means of the Huber-Mises locus $\mathscr{F} = g_0$.

(ii) the non-associated plastic potentials g_1 and g_2 that approximate at the macroscopic level the multiplicity of plastic potential functions related with the clusters of active micro-shear bands.

The plastic potential functions g_1 and g_2 display the geometry of the micro-shear bands systems considered and result in two separate planes that form in the space of principal stresses τ_k $(k = 1, 2, 3)$ a vertex at the loading point on the smooth Huber-Mises cylinder $\mathscr{F}$. The planes are defined by normals $\mathbf{N}_i$, which can be expressed in terms of the unit vectors $\mathbf{s}^{i}$, $\mathbf{n}^{i}$ $(i = 1, 2)$ defining the "i"th system (cluster) of micro-shear bands

$$\mathbf{N}_i = \frac{\sqrt{2}}{2}\left(\mathbf{s}^i \otimes \mathbf{n}^i + \mathbf{n}^i \otimes \mathbf{s}^i\right) \tag{12}$$

The normals $\mathbf{N}_i (i = 1, 2)$ can be expressed in terms of the unit normal $\boldsymbol{\mu}_F$ and the unit tangent $\mathbf{T}$ to the limit surface $\mathcal{F}(\boldsymbol{\tau}', \mathcal{K})$ at the loading point

$$\mathbf{N}_1 = \cos 2\beta\, \boldsymbol{\mu}_F + \sin 2\beta\, \mathbf{T} \qquad \mathbf{N}_2 = \cos 2\beta\, \boldsymbol{\mu}_F - \sin 2\beta\, \mathbf{T} \tag{13}$$

where $\boldsymbol{\mu}_F = \frac{1}{\sqrt{2}\mathcal{K}}\boldsymbol{\tau}'$. The tensor $\mathbf{T}$ is coaxial with the tangent to the limit surface $\mathcal{F}(\boldsymbol{\tau}', \mathcal{K})$

$$\mathcal{F}(\boldsymbol{\tau}, \mathcal{K}) = \frac{1}{2}(\boldsymbol{\tau} : \boldsymbol{\tau}) - \mathcal{K}^2 = 0 \qquad \dot{\mathcal{K}} = \mathcal{B}(\mathcal{K}_s - \mathcal{K})\dot{\bar{\gamma}} \qquad \mathcal{K}(0) = \mathcal{K}_o \tag{14}$$

in the deviatoric plane at the loading point

$$\mathbf{T} = \mathcal{N}\bar{\tau}\overline{\left(\frac{\boldsymbol{\tau}'}{\bar{\tau}}\right)}^{\circ} = \mathcal{N}\left[\overset{\circ}{\boldsymbol{\tau}}{}' - \left(\overset{\circ}{\boldsymbol{\tau}} : \boldsymbol{\mu}_F\right)\boldsymbol{\mu}_F\right] \quad \text{with} \quad \bar{\tau} = \left(\frac{1}{2}\boldsymbol{\tau}' : \boldsymbol{\tau}'\right)^{\frac{1}{2}} \tag{15}$$

while $\mathcal{K}$ is the "size" of the limit surface (i.e. $\mathcal{K} = \frac{1}{\sqrt{2}}\mathcal{R}$, where $\mathcal{R}$ is the radius of the limit surface), $\mathcal{K}_s$ is a material constant representing a saturation value for $\mathcal{K}$, and $\mathcal{B}$ is a material constant controlling the pace of saturation. Due to the pressure-insensitive Huber-Mises locus $\mathcal{F}(\boldsymbol{\tau}', \mathcal{K})$, $\overset{\circ}{\boldsymbol{\tau}}{}' : \boldsymbol{\mu}_F = \overset{\circ}{\boldsymbol{\tau}} : \boldsymbol{\mu}_F$ holds, whereas $\mathcal{N}$ is a normalization factor

$$\mathcal{N} = \left\| \overset{\circ}{\boldsymbol{\tau}}{}' - \left(\overset{\circ}{\boldsymbol{\tau}}{}' : \boldsymbol{\mu}_F\right)\boldsymbol{\mu}_F \right\|^{-1} \tag{16}$$

which due to

$$\left\| \overset{\circ}{\boldsymbol{\tau}}{}' - \left(\overset{\circ}{\boldsymbol{\tau}} : \boldsymbol{\mu}_F\right)\boldsymbol{\mu}_F \right\|^2 = \overset{\circ}{\boldsymbol{\tau}}{}' : \overset{\circ}{\boldsymbol{\tau}}{}' - \left(\overset{\circ}{\boldsymbol{\tau}}{}' : \boldsymbol{\mu}_F\right)^2 \qquad \overset{\circ}{\boldsymbol{\tau}}{}' : \boldsymbol{\mu}_F = \left\| \overset{\circ}{\boldsymbol{\tau}}{}' \right\| \cos\delta \tag{17}$$

is given by $\mathcal{N} = \left(\left\| \overset{\circ}{\boldsymbol{\tau}} \right\| \sin\delta\right)^{-1}$. Due to equations (6) to (8), $(10)_1$, and (12), the relation for the rate of plastic deformation takes form

$$\mathcal{D}^{\mathrm{p}} = \mathbf{D}^{\mathrm{p}} + \frac{\sqrt{2}}{2}\sum_{i=1}^{2}\dot{\gamma}^i_{\mathrm{MS}}\,\mathbf{N}_i \tag{18}$$

Assuming that $\mathbf{D}^{\mathrm{p}}$ can be expressed by means of the classical J_2 plasticity theory,

$$\mathbf{D}^{\mathrm{p}} = \frac{\sqrt{2}}{2}\dot{\bar{\gamma}}\boldsymbol{\mu}_F \qquad \dot{\bar{\gamma}} = \frac{\sqrt{2}}{2}\frac{\overset{\circ}{\boldsymbol{\tau}} : \boldsymbol{\mu}_F}{h} \tag{19}$$

Due to equations $(10)_1$ and (13), the relation for the rate of plastic deformation takes the form

$$\mathcal{D}^{\mathrm{p}} = \frac{\sqrt{2}}{2}\dot{\gamma}^*\boldsymbol{\mu}_F + \frac{\sqrt{2}}{2}\dot{\varepsilon}_{\mathrm{MS}}\mathbf{T} \tag{20}$$

where

$$\dot{\gamma}^* = \dot{\bar{\gamma}}_S + \dot{\gamma}_{MS} \qquad \dot{\gamma}_{MS} = \cos 2\beta\left(\dot{\gamma}_{MS}^{(1)} - \dot{\gamma}_{MS}^{(2)}\right) \qquad \dot{\varepsilon}_{MS} = \sin 2\beta\left(\dot{\gamma}_{MS}^{(1)} + \dot{\gamma}_{MS}^{(2)}\right) \tag{21}$$

Observe that only the normal component of the rate of plastic deformations contributes to the change of the radius of the limit surface $\mathcal{F}(\tau', \mathcal{K})$, and that the consistency condition, $\dot{\mathcal{F}} = 0$, yields

$$\dot{\gamma}^* = \frac{\sqrt{2}}{2}\frac{\overset{\circ}{\tau}:\mu_F}{H} \qquad \text{with} \qquad H = \mathcal{B}(\mathcal{K}_S - \mathcal{K}) \tag{22}$$

The following active micro-shear bands fractions, $f_{MS}^{(1)}$, $f_{MS}^{(2)}$, of the rate of plastic shearing $\dot{\gamma}^*$, are introduced, (cf. Pecherski 1992b):

$$\dot{\gamma}_{MS}^{(1)} \cos 2\beta = f_{MS}^{(1)}\, \dot{\gamma}^* \qquad -\dot{\gamma}_{MS}^{(2)} \cos 2\beta = f_{MS}^{(2)} \dot{\gamma}^* \tag{23}$$

where due to equation (21) and for $\dot{\gamma}^* > 0$ the following constraints hold:

$$\frac{\dot{\bar{\gamma}}_S}{\dot{\gamma}^*} + f_{MS}^{(1)} + f_{MS}^{(2)} = 1 \qquad f_{MS}^{(1)} + f_{MS}^{(2)} \in [0,\, 1] \qquad f_{MS}^{(1)}, f_{MS}^{(2)} \in [0,\, 1] \tag{24}$$

Based on the observation that the micro-shear bands can be active only in the case of continued plastic flow, i.e. when the loading condition is fulfilled, it is assumed that for $\dot{\gamma}^* = 0$, $f_{MS}^{(1)} = f_{MS}^{(2)} = 0$. The fractions $f_{MS}^{(1)}, f_{MS}^{(2)}$ display the contribution of the active micro-shear bands formation to plastic flow during the deformation process. According to equations (12) to (20), the rate of plastic deformation takes a form, which is formally similar to the hypoelastic version of J_2 deformation theory, studied earlier by Budianski (1959) and Stören and Rice (1975), (cf. Pecherski, 1998):

$$\mathcal{D}^p = \frac{\overset{\circ}{\tau}:\mu_F}{2H}\mu_F + \frac{1}{2H_1}\left[\overset{\circ}{\tau} - \left(\overset{\circ}{\tau}:\mu_F\right)\mu_F\right] \tag{25}$$

$$\frac{1}{H_1} = \begin{cases} \dfrac{\overset{\circ}{\tau}:\mu_F}{H} \dfrac{f_{MS} \tan 2\beta}{\left\|\overset{\circ}{\tau}'\right\| \sin\delta} & \delta \in \left(\delta_c,\, \dfrac{\pi}{2}\right] - \text{partially active range} \\[2ex] \dfrac{1}{H \tan\delta_c} f_{MS} \tan 2\beta & \delta \in [0,\, \delta_c] \; - \text{fully active range} \end{cases} \tag{26}$$

The corresponding relation for plastic spin in the case of micro-shear banding reads (Pecherski, 1992 and 1998)

$$\mathcal{W}^p = \frac{f_{MS}\, \chi(\delta)}{4\bar{\tau} H \cos 2\beta}\left(\overset{\circ}{\tau}\tau - \tau\overset{\circ}{\tau}\right) \qquad \chi(\delta) = \begin{cases} \cot\delta, \; \delta \in \left(\delta_c,\, \dfrac{\pi}{2}\right] \\ \cot\delta_c, \; \delta \in [0,\, \delta_c] \end{cases} \tag{27}$$

Observe that accounting for the partially active range facilitates a proper description of the unloading process. The second term in equation (25) is responsible for the non-coaxiality between the principal directions of stress and rate of plastic deformations and H_1 plays the role of the non-coaxiality modulus. The symbol $f_{MS} = f_{MS}^{(1)} - f_{MS}^{(2)}$, $f_{MS} \in [-1,\, 1]$, denotes the net fraction of the active micro-shear bands that contribute to the total rate of plastic shearing. The fraction f_{MS} is the controlling parameter of the non-coaxiality modulus

H_1. If $f_{MS} = 0$, equation (25) transforms into the J_2 flow law. According to equations (23) and (24), the magnitude of f_{MS} can fluctuate within two limits, $f_{MS} \in [-1,\ 1]$.

5 Identification of the Model

In order to make the model presented applicable, possible simplifications and the identification of the material parameters should be studied. The experimental results of Anand et al. (1992, 1994), obtained in simple compression test and channel-die test for polycrystalline Copper have been applied to identify the micro-shear bands fraction f_{MS}. Numerical calculations were made under the simplified assumption that only one system of active micro-shear bands operates. In such a case, we take $f_{MS} = f_{MS}^{(1)}$. The channel-die test corresponds to proportional loading conditions (plane strain compression), under which the second term in equation (25) vanishes and the effect of micro-shear banding is hidden in the hardening modulus H, which can be expressed, according to equation (19), (21), and (22) with use of the plastic hardening modulus h available from the simple compression test:

$$H = (1 - f_{MS})h \qquad f_{MS} \in [0,\ 1) \tag{28}$$

The comparison of the computational results with experimental measurements are shown in Figure 1.

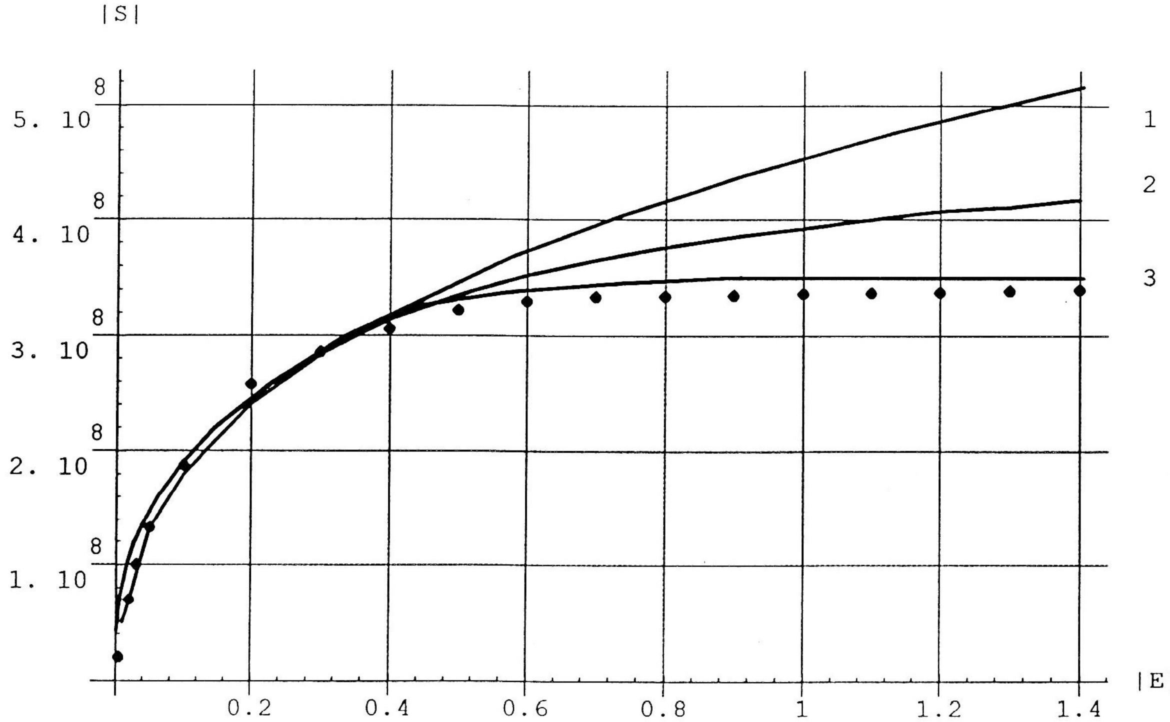

Figure 1. Stress $|\sigma_{33}|$ against logarithmic strain $|\varepsilon_{33}|$ response for plane strain compression in channel-die test: the points correspond to the experimental results and curve 2 represents the numerical results of Anand et al. (1992, 1994), whereas curve 3 displays the results of numerical computations with use of the model presented, confronted for comparison with the results obtained for J_2 flow law - curve 1.

The required consistency of the computational results with the experimental points has been provided by means of the suitable adjustment of the relation for micro-shear bands fraction f_{MS} versus absolute value of true strain $|\varepsilon_{33}|$, which can be approximated by means of the following logistic curve:

$$f_{MS} = \frac{f_o}{1 + \exp(a - b|\varepsilon_{33}|)} \qquad f_o = 0.95 \qquad a = 7.5 \qquad b = 13.6 \tag{29}$$

shown in Figure 2.

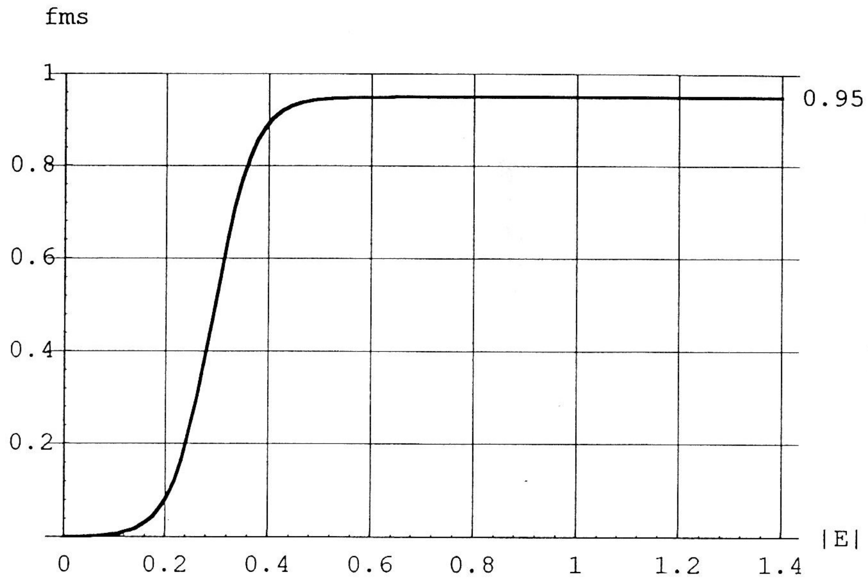

Figure 2. The dependence of micro-shear bands fraction f_{MS} against the logarithmic strain $|\varepsilon_{33}|$ satisfying the required consistency of the numerical results with experiment.

As we can observe, such a procedure provides numerical identification of the controlling parameter f_{MS} of the model presented in equations (25) to (27) and opens the possibility of reliable numerical simulation of metal forming operations and ductile fracture processes.

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