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Attitude Diagrams for the Description of the Attitude Drift of Torquefree Asymmetric Gyros

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A very instructive tool for the study of the behaviour of torquefree gyros (gyrosatellites) are attitude diagrams, which display the kinetic energy level, the three Mohr circles between which the tip of the angular velocity vector shuttles to and from, and the nutation angle between (constant) angular momentum vector and principal body axis of smallest inertia moment, as well as the maximum and minimum kinetic energy levels for a given (constant) angular momentum. Exact for rigid gyroscopes, attitude diagrams become indispensible for the study of the attitude drift due to internal energy dissipation for real gyroscopes, and the case of an asymmetric gyro is investigated in some detail in the present paper.

1 Introduction

Attitude diagrams (see Rimrott and Szczygielski, 1993) are a very instructive means for the description of the attitude drift of torquefree solid body gyros with internal energy dissipation. Strictly speaking an attitude diagram depicts data of a *rigid* body gyro. The attitude drift process of a deformable gyro of sufficient stiffness is, however, readily describable by means of an attitude diagram, in particular if one accepts the premise that the kinetic energy T of an associated rigid gyro equals the sum of kinetic energy T_e and elastic energy V_e of the real gyro (see Rimrott and Sperling, 1995), i. e.

$$T = T_e + V_e \tag{1}$$

Any loss rate of energy due to an internal energy dissipation rate $\dot{D}$ is then

$$\dot{T} = -\dot{D} \tag{2}$$

The energy loss is supplied by an change of the kinetic energy of the associated rigid body, a process that can only be brought about by a change of its attitude ν.

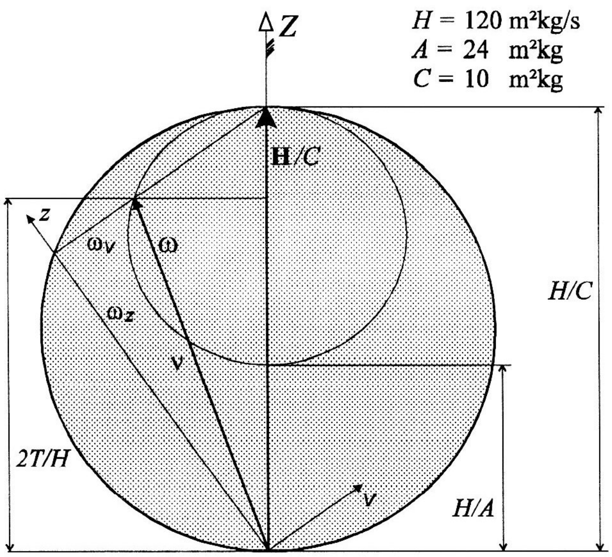

Figure 1. Attitude Diagram for an Axisymmetric Gyro

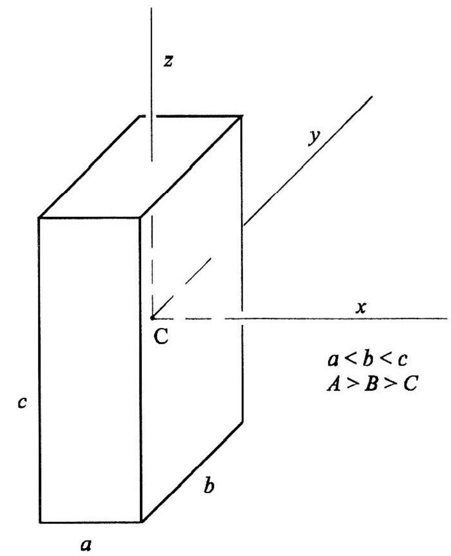

Figure 2. Model of an Asymmetric Gyro

Figure 1 depicts the attitude diagram of an *axisymmetric* gyro, with inertia moment $B = A > C$.
In Figure 2 depicts an asymmetric solid body gyro and Figure 3 shows the attitude diagram of an *asymmetric* rigid body gyro, for $A > B > C$.

2 Attitude Angle

The attitude (or nutation) angle ν assumes different values as the tip of the angular velocity vector ω shuttles between ω' and ω'', with

$$\sin \nu' = \sqrt{\frac{1 - \frac{2CT}{H^2}}{1 - \frac{C}{A}}} \tag{3}$$

and

$$\sin \nu'' = \sqrt{\frac{1 - \frac{2CT}{H^2}}{1 - \frac{C}{B}}} \tag{4}$$

as long as epicycloidal motion, with $T > H^2/2B$, prevails (Figure 3).
When pericycloidal motion, with $T < H^2/2B$, prevails, the tip of the angular velocity vector ω shuttles between ω' and ω''' (Figure 4). The associated values of the attitude angle are obtainable from

$$\sin \nu' = \sqrt{\frac{1 - \frac{2CT}{H^2}}{1 - \frac{C}{A}}} \tag{3}$$

and

$$\nu''' = 90^\circ$$

The fact that the angle ν' is given by one and the same equation (3) valid throughout the drift process, is seen as an invitation to use it as the basis for the description of the whole attitude drift process.

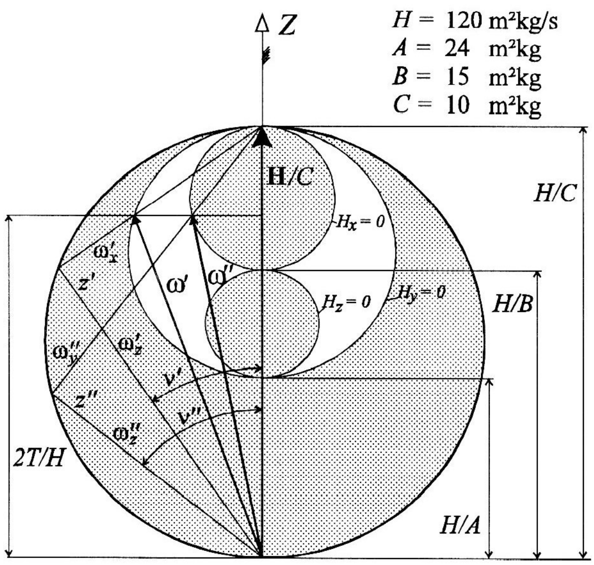

Figure 3. Attitude Diagram for an Asymmetric Gyro, Epicyclic Condition, $H^2/2B < T < H^2/2C$

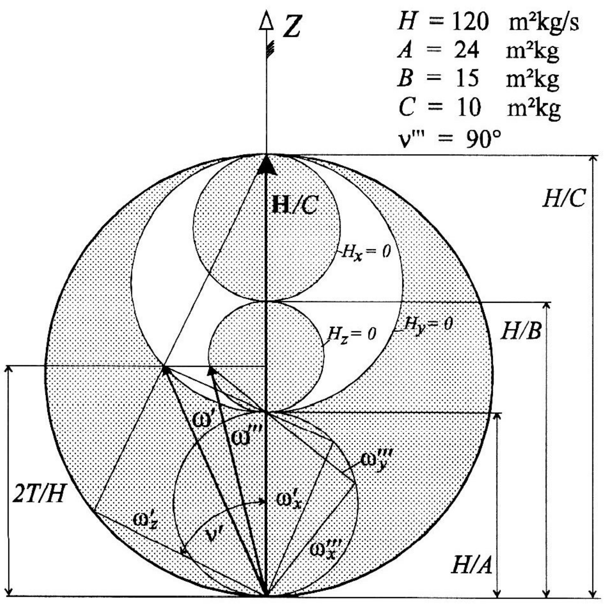

Figure 4. Attitude Diagram for an Asymmetric Gyro, Pericyclic Condition, $H^2/2A < T < H^2/2B$

3 Attitude Drift

Let us assume that each time when the tip of the angular velocity vector ω is at ω', the kinetic energy level is decreased by an amount

$$\Delta T = -2\pi\beta \frac{(A - C)}{A^3 C^3} H^5 \cos^3 \nu' \sin^2 \nu' \tag{6}$$

or, if equation (3) is made use of,

$$\Delta T = -2\pi\beta \frac{\left(2AT - H^2\right)\left(H^2 - 2CT\right)}{A^2 C^2 (A - C)} \sqrt{\frac{C\left(2AT - H^2\right)}{A - C}} \tag{7}$$

Thus at a given T level there is a drop ΔT which is proportional to the kinetic energy level. The quantity β is an internal damping coefficient in Ws^6. It should be noted that ΔT has been chosen in terms of the largest inertia moment, A, and the smallest inertia moment, C. The intermediate inertia moment, B, does not appear in equations (6) and (7). As the kinetic energy becomes smaller and smaller and eventually approaches $T = H^2 / 2A$, the attitude angle increases until it approaches $\nu' = 90^\circ$. Attitude angle ν' and kinetic energy level T are always related by equation (3).

4 Time Steps

The time period between two successive ω' positions is

$$\Delta t = 4K\left(k^2\right)\sqrt{\frac{ABC}{(B - C)\left(2AT - H^2\right)}} \tag{8}$$

where k is the modulus, with

$$k^2 = \frac{A - B}{B - C}\,\frac{H^2 - 2CT}{2AT - H^2} \tag{9}$$

and $K\left(k^2\right)$ is the complete elliptic integral of the first kind. Equations (8) and (9) are valid for epicycloidal motion, i. e. motion when $T > H^2 / 2B$.

For pericycloidal motion, i. e. when $T < H^2 / 2B$, the corresponding equations are

$$\Delta t = 4K\left(k^2\right)\sqrt{\frac{ABC}{(A - B)\left(H^2 - 2CT\right)}} \tag{10}$$

with

$$k^2 = \frac{B - C}{A - B}\,\frac{2AT - H^2}{H^2 - 2CT} \tag{11}$$

5 Kinetic Energy and Attitude versus Time

With the help of equations (8) and (7), and then equations (10) and (7) the kinetic energy T can be plotted as function of time. This has been done in Figure 5.
By means of equation (3) the attitude angle ν' can also be plotted as function of time, as has been done in Figure 6.
A close inspection of Figure 5 will indicate that the slope of the T versus t curve is somewhat reduced around the T = 480 J level for the example chosen. A simular observation applies at the $\nu' = 49{,}1^\circ$ level of Figure 6.
The reason is, that at $T = \dfrac{H^2}{2B} = 480\,\mathrm{J}$ the time Δt, as given by equations (8) or (10), approaches infinity. In practical terms, it becomes large, such that $\Delta \nu' / \Delta t$ becomes small, as is depicted in Figure 7.

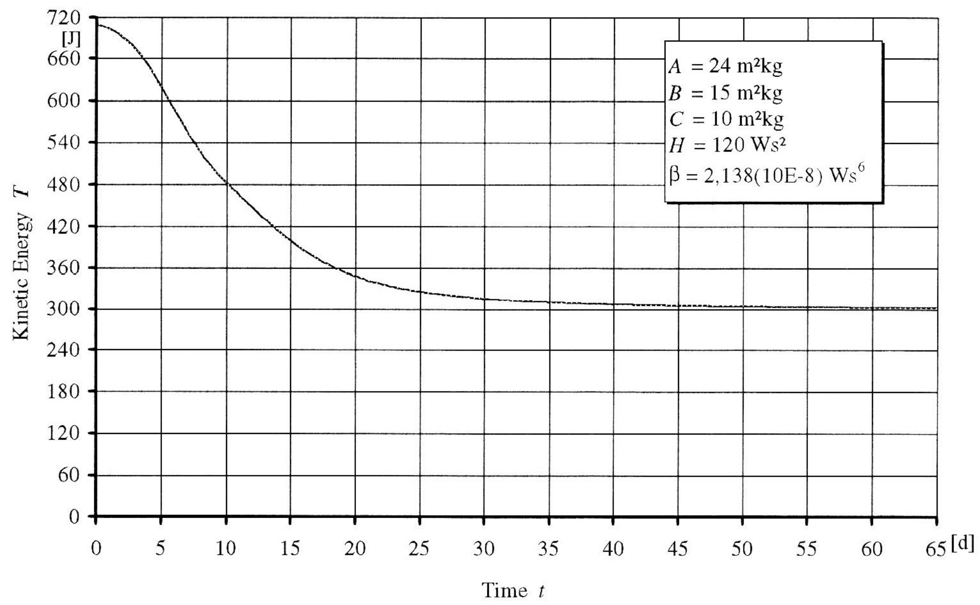

Figure 5. Decay of Kinetic Energy to its Minimum Value of $T = H^2 / 2A$

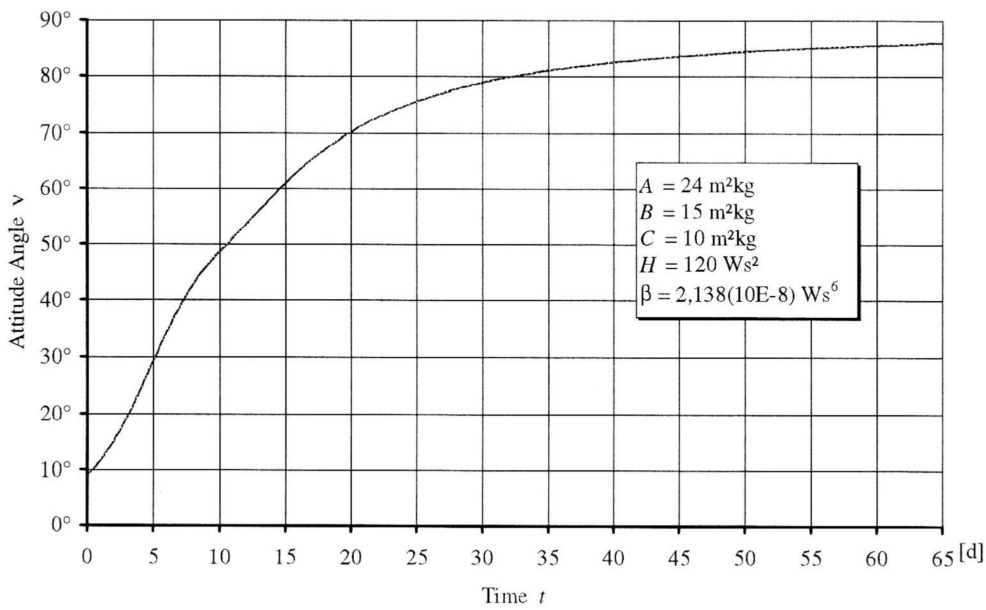

Figure 6. Increase of Attitude Angle towards its Final Value of $\nu' = 90^\circ$

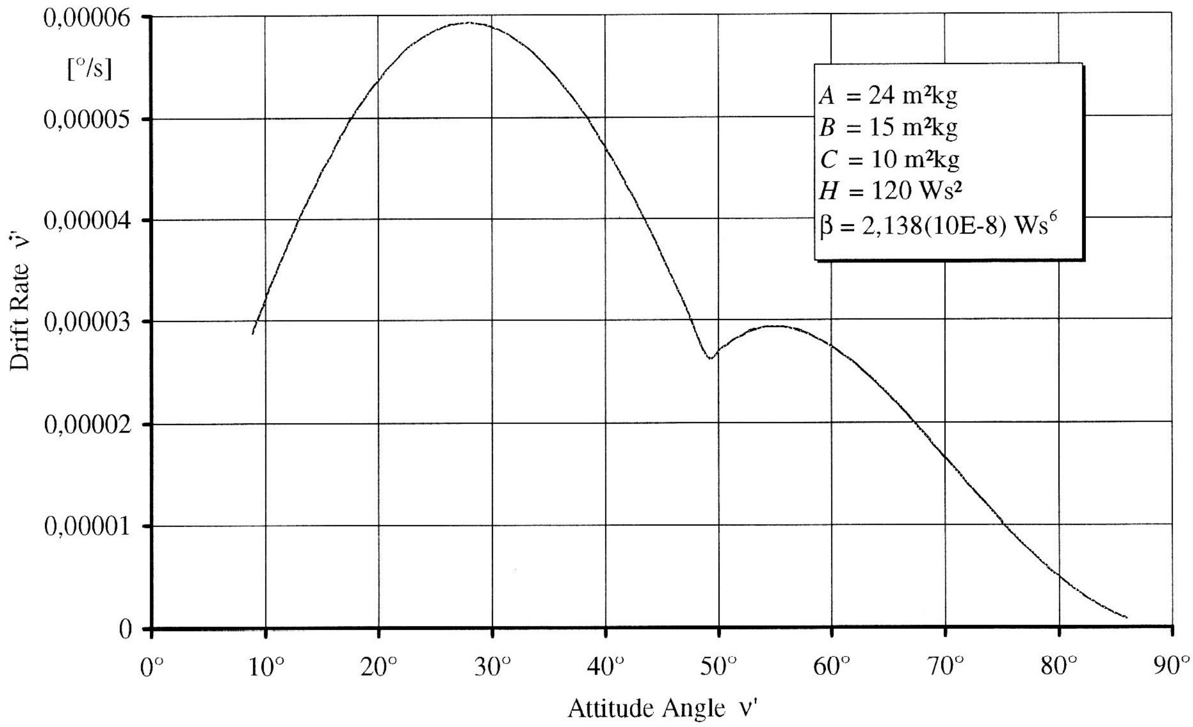

Figure 7. Attitude Drift Rate $\dot{\nu}'$ as Function of Attitude Angle
($\nu' = 49{,}1^\circ$ corresponding to $T = H^2/2B = 480$ J)

6 Conclusion

In order to obtain a relatively simple global description of a typical drift process of a torquefree asymmetric gyro, a stepwise reduction of the gyro's kinetic energy is assumed every time the gyro reaches an attitude where the intermediate angular momentum component vanishes. This assures a uniform description of the whole attitude drift process, at the expense of ignoring the effect of the magnitude of the intermediate inertia moment except in the latter's influence on the time interval between energy reduction steps, which happens to become very large near energy levels between epicyclic and pericyclic motion.

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