TECHNISCHE MECHANIK, Band 18, Heft 3, (1998), 181-188
Manuskripteingang: 01. Dezember 1997

On the Calculation of Finite Plastic Strains in Shell Intersections with Finite Elements

R. Eberlein, P. Wriggers

The paper presents an appropriate parameterization concept (5/6–parameter) for the modelling of shell intersections. In addition to it, the classical displacement based 6–parameter concept is introduced. The algorithmic treatment of finite plastic strains is outlined with respect to a general 3–D material formulation that underlies the 6–parameter model. Furthermore, special aspects for its implementation into the proposed 5/6–parameter shell element are addressed. The discussion of numerical examples is split into two parts. At first, a purely elastic example shows the validity of the 5/6–parameter concept for the calculation of shell intersections. Hereby, the deficiencies of the 6–parameter model are outlined. The second example investigates the warping deformations of a cantilever beam made out of a commercial steel channel for purely elastic as well as elastic–plastic material behavior.

1 Introduction

The calculation of finite plastic strains in shells has been a major task for many researchers during recent years. A large variety of finite element formulations for shells has been presented in literature. For a current overview refer to Eberlein (1997). Within this context the discussion of different parameterization concepts was of special interest. The consitutive modelling, however, has successively been simplified to a standard description of 3–D material behavior. That fact holds for both, hyperelastic and finite plastic strains. Yet, in terms of shells with plane stress assumption, some further aspects have to be discussed in addition.

For shell intersections one has to think of non-smooth shell-like structures. A mathematical definition is provided by the fact that there does not have to exist an unique normal vector in any point of the shell midsurface, e.g. steel channels used in steel constructions. Therefore this article wants to present an appropriate parameterization concept that allows accurate calculations of shell intersections. To the authors' knowledge there exists no finite element formulation for shell intersections accounting for finite plastic strains in the literature so far.

For a correct finite element formulation of shell intersections, a 5/6–parameter concept is presented. Its main characteristic is the different parameterization of element nodes modelling smooth parts of a shell structure and element nodes, where an unique normal vector with respect to the shell midsurface does not exist. The need for this distinctive parameterization concept will be shown in an illustrative, purely elastic numerical example, where a standard displacement based parameterization (6–parameter) for smooth shells yields completely wrong results. Finally, the influence of finite plastic strains will be demonstrated by analysing the warping deformation of a commercial steel channel.

2 Kinematics – Parameterization

For a complete kinematic description of shells, the calculation of strain measures in shell space $\mathcal{S}$ is required. The definition of $\mathcal{S}$ can be given as follows:

$$\mathcal{S} := \{\mathbf{x} \in \mathrm{IR}^3 \,|\, \mathbf{x} = \varphi(\mathbf{X}(\xi^1, \xi^2, \xi^3), t)\} \tag{1}$$

Thus the position of a particle $X \in \mathcal{S}$ is uniquely determined by the mapping $\varphi(\mathbf{X}(\xi^1, \xi^2, \xi^3), t)$. The particle X is parameterized by a set of convective coordinates (ξ^1, ξ^2, ξ^3). Therefore the basic kinematic assumption underlying any shell theory is the form of the mapping $\varphi(\mathbf{X}, t)$.

In this paper we rely for all further derivations on the standard kinematic assumption, meaning a director field $\mathbf{d}$ is linearly interpolated accross the shell thickness (see Naghdi (1972)):

$$\mathbf{x}(\xi^1,\xi^2,\xi^3,t) = \boldsymbol{\phi}(\xi^1,\xi^2,t) + \xi\,\mathbf{d}(\xi^1,\xi^2,t) \qquad \xi = \xi^3 \tag{2}$$

The vector $\boldsymbol{\phi}$ represents the position vector of a particle with respect to the shell midsurface $\mathcal{M}$ ($\xi = 0$). With equation (2) the tangential covariant base vectors $\mathbf{g}_i$ in $\mathcal{S}$ can be determined:

$$\mathbf{g}_\alpha \;=\; \boldsymbol{\phi}_{,\alpha} + \xi\mathbf{d}_{,\alpha} \;=\; \mathbf{a}_\alpha + \xi\mathbf{d}_{,\alpha} \qquad \mathbf{g}_3 \;=\; \mathbf{d} \tag{3}$$

In this way the components of the deformation gradient $\mathbf{F}$ can be respresented in terms of formula (3). Here they are split into constant [C] und linear [L] parts with respect to the thickness coordinate ξ:

$$\boxed{\begin{aligned} \mathbf{F} &= \mathbf{F}_{[C]} + \xi\,\mathbf{F}_{[L]} = \frac{\partial\mathbf{x}}{\partial\mathbf{X}} = \mathbf{g}_i \otimes \mathbf{G}^i \\ &\text{with} \\ \mathbf{F}_{[C]} &= \mathbf{a}_\alpha \otimes \mathbf{G}^\alpha + \mathbf{d} \otimes \mathbf{G}^3 \\ \mathbf{F}_{[L]} &= \mathbf{d}_{,\alpha} \otimes \mathbf{G}^\alpha \end{aligned}} \tag{4}$$

where capital letters refer to the undeformed configuration ($t = 0$). This completes the kinematic description of a shell. Indeed, all further strain measures can directly be obtained by exploiting equation (4) and additional algebraic operations. In contrast to classical shell theories, this allows a very efficient implementation of all required strain measures.

In order to gain explicit results for $\mathbf{F}$ in terms of equation (4), the position vector $\boldsymbol{\phi}$ and the director $\mathbf{d}$ from equation (2) have to be parameterized in an appropriate way. In classical shell theories the parameterization of a particle $X \in \mathcal{M}$ is subjected to the three components of the displacement vector $\mathbf{u} = \boldsymbol{\phi} - \boldsymbol{\Phi}$. For the director $\mathbf{d}$, however, one can think about various alternatives for its parameterization. Two variants are presented in detail here:

Variant I: Classical 6–parameter concept

By analogy with the position vector $\boldsymbol{\phi}$, a displacement vector $\mathbf{w}$ is introduced. In this way the normal vector $\mathbf{N}$ with respect to the undeformed configuration is related to the director $\mathbf{d}$ by $\mathbf{d} = \mathbf{N} + \mathbf{w}$ (see figure 1). In comparison to $\mathbf{N}$ the deformation of $\mathbf{d}$ is characterized by rotation and stretching and therefore $\mathbf{d}$ can be called an extensible director field. This can be expressed by $\|\mathbf{d}\| \neq 1$, which means that stretches in thickness direction of a shell are taken into account. Thus the 6–parameter concept allows the description of thick shell structures. Yet, it has to be mentioned that the parameterization of $\mathbf{d}$ via $\mathbf{w}$ does not consider a drilling rotation along the director axis. As will be shown subsequently, such a drilling rotation is mandatory for an accurate description of shell intersections. Nevertheless, the 6–parameter concept is defined by the three components of $\mathbf{u}$ and the three components of $\mathbf{w}$.

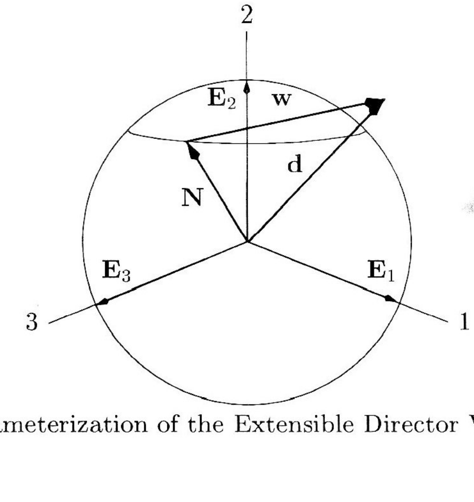


Figure 1. Parameterization of the Extensible Director Vector $\mathbf{d}$

Variant II: 5/6–parameter concept

This concept allows a reliable calculation of shell intersections. It was proposed by Hughes & Liu (1981) and further discussed by Simo (1993). Since it is restricted to thin shells only, it is based on an inextensible director field ($||\mathbf{d}|| = 1$). Thus thickness stretches are neglected a priori and the deformation of $\mathbf{d}$ can be represented by a pure rotation: $\mathbf{d} = \mathbf{R}\mathbf{N}$. For an inextensible director field holds:

$$\dot{\mathbf{d}} \cdot \mathbf{d} = 0 \qquad \Longleftrightarrow \qquad \dot{\mathbf{d}} = \boldsymbol{\omega} \times \mathbf{d} \tag{5}$$

The axial vector $\boldsymbol{\omega}$ respresents the angular velocity of the director. The rotation tensor $\mathbf{R}$ can be parameterized in terms of $\boldsymbol{\omega}$ by applying the Rodrigues formula which is known from rigid body dynamics:

$$\begin{aligned} &\mathbf{R} = \cos\theta\,\mathbf{1} + \frac{\sin\theta}{\theta}\,\hat{\boldsymbol{\omega}} + \frac{1-\cos\theta}{\theta^2}\,\boldsymbol{\omega}\otimes\boldsymbol{\omega} \qquad \theta = ||\boldsymbol{\omega}|| \\ &\text{with} \quad \hat{\boldsymbol{\omega}}\,\mathbf{a} = \boldsymbol{\omega}\times\mathbf{a} \quad \forall \quad \mathbf{a} \qquad \Longrightarrow \qquad \hat{\boldsymbol{\omega}} = \dot{\mathbf{R}}\,\mathbf{R}^T \end{aligned} \tag{6}$$

For a detailed derivation of this formula refer to e.g. de Boer (1982). This parameterization for $\mathbf{d}$ is nonsingular like the classical, displacement based 6–parameter concept. The three vector components of $\boldsymbol{\omega}$ are used to parameterize $\mathbf{R}$ in equation (6) and furthermore $\mathbf{d}$. Together with the three displacement components of $\mathbf{u}$, which have the same definition here as in the classical 6–parameter concept, a 6–parameter model is obtained. This model includes drilling rotations along the director axis and is valid for modelling shell intersections. Nevertheless, it can be shown that drilling rotations in smooth shell structures with unique normal vectors on $\mathcal{M}$ must vanish (see e.g. Eberlein (1997)). In those cases the drilling degrees of freedom have to be eliminated. Otherwise the resulting equation system would become singular. This elimination is achieved by rotating the vector components of $\boldsymbol{\omega}$ into a local cartesian base system $\mathbf{E}^{loc}_{I\,i}$ in any element node where I indicates the node number and i the coordinate direction. Here the thickness direction (3–direction) serves as a fixed coordinate axis in any element node. Along these axes the drilling degrees of freedom are supposed to be zero by imposing boundary conditions. Equation (7) shows the distinct parameterization strategies for $\boldsymbol{\omega}$ in context:

$$\boxed{\begin{array}{llll} \text{Shell intersection:} & \boldsymbol{\omega} = \omega_{I\,i}\,\mathbf{E}_{I\,i} = \psi_{I\,i}\,\mathbf{E}^{loc}_{I\,i} & & \\ \text{Smooth shell:} & \boldsymbol{\omega} = \psi_{I\,\alpha}\,\mathbf{E}^{loc}_{I\,\alpha} \quad \text{and} & \psi_{I\,3} = 0 & \alpha = 1,2 \end{array}} \tag{7}$$

By doing that, the 6–parameter theory for shell intersections reduces to a 5–parameter concept when element nodes in smooth parts of the shell occur. As a final remark it should be noted that for thick shells with intersections, a 6/7–parameter theory could be derived in an analogous way. Only one additional parameter accounting for thickness changes in a shell must be considered. In detail this concept was presented by Betsch (1996) for hyperelastic shell elements.

3 Finite Plasticity

The constitutive description is based on arbitrarily large isotropic plastic strains. Originally, it can be found in Wriggers et al. (1996) and is outlined in its main parts here. As basic assumption the multiplicative decomposition of the deformation gradient $\mathbf{F}$ into elastic ($\mathbf{F}_e$) and plastic ($\mathbf{F}_p$) parts is introduced:

$$\mathbf{F} = \mathbf{F}_e\,\mathbf{F}_p \tag{8}$$

With equation (8) the elastic left Cauchy–Green tensor $\mathbf{b}_e$ is calculated by:

$$\mathbf{b}_e = \mathbf{F}_e\,\mathbf{F}_e^T = \mathbf{F}\,\mathbf{C}_p^{-1}\,\mathbf{F}^T \tag{9}$$

That means $\mathbf{b}_e$ can be expressed in terms of the inverse plastic right Cauchy–Green tensor $\mathbf{C}_p^{-1}$. Thus it is obvious to use the components of $\mathbf{C}_p^{-1}$ as history variables in order to determine the irreversible part of a deformation.

Next the dependence of the free energy $\Psi = \hat{\Psi}(\mathbf{b}_e, \alpha)$ of $\mathbf{b}_e$ and an internal variable α, which is the equivalent plastic strain, is assumed. Under the restriction of isotropy, the second law of thermodynamics then yields the constitutive relations for the Kirchhoff stresses $\boldsymbol{\tau}$ and the thermodynamic force q:

$$\boldsymbol{\tau} = 2\,\varrho_o \frac{\partial \Psi}{\partial \mathbf{b}_e}\,\mathbf{b}_e \qquad\qquad q = -\frac{\partial \Psi}{\partial \alpha} \tag{10}$$

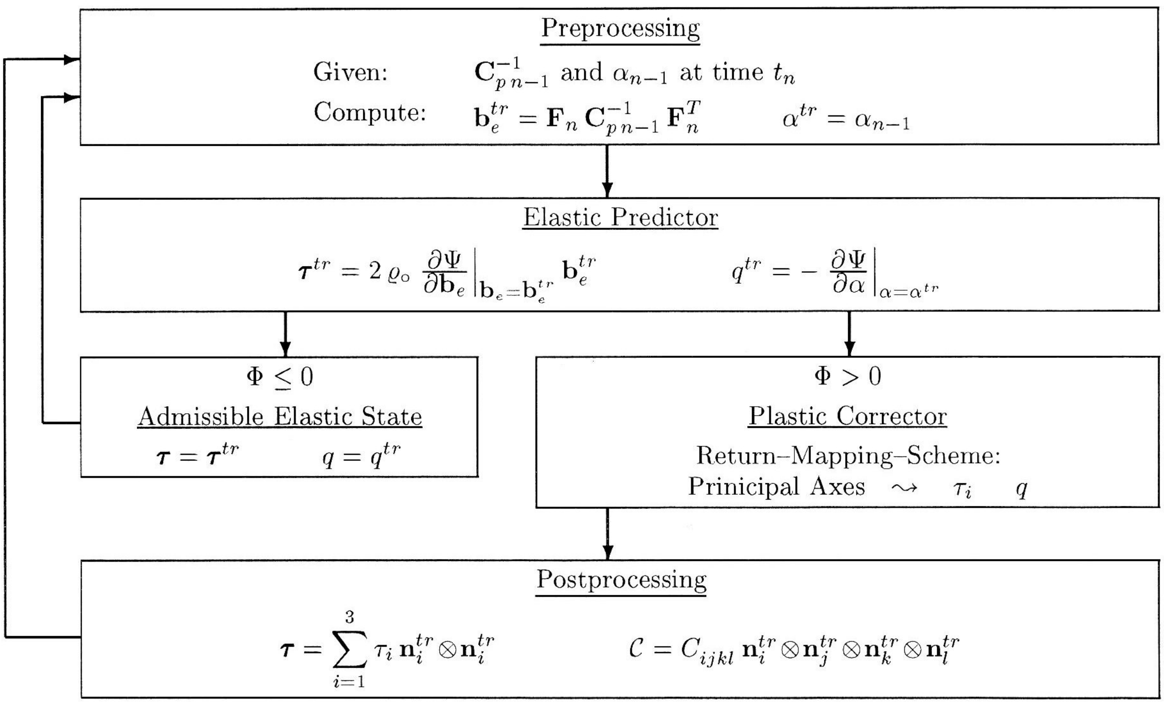
From the postulate of maximum dissipation the evolution equations (associative flow rules) are obtained (see Simo & Miehe (1992)):

$$-\frac{1}{2}\,\mathcal{L}_v \mathbf{b}_e = \gamma\,(\frac{\partial \Phi}{\partial \boldsymbol{\tau}}\,\mathbf{b}_e) \qquad\qquad \dot{\alpha} = \gamma\,(\frac{\partial \Phi}{\partial q}) \tag{11}$$

The Lie derivative $\mathcal{L}_v \mathbf{b}_e$ is referred to as $\mathcal{L}_v \mathbf{b}_e = \dot{\mathbf{b}}_e - \mathbf{l}\,\mathbf{b}_e - \mathbf{b}_e\,\mathbf{l}^T$ with the spatial velocity gradient $\mathbf{l} = \dot{\mathbf{F}}\,\mathbf{F}^{-1}$. In equation (11) the loading/unloading conditions in Kuhn–Tucker form must be fulfilled:

$$\gamma \geq 0 \qquad\qquad \Phi = \hat{\Phi}(\boldsymbol{\tau}, q) \leq 0 \qquad\qquad \gamma\,\Phi = 0 \tag{12}$$

In contrast to purely elastic material behavior, the plastic stresses are restricted by a yield criterion $\Phi \leq 0$. For numerical calculations $\mathbf{b}_e, \alpha, \boldsymbol{\tau}$ and Φ are required. They can be determined by applying the return mapping scheme for finite strains as proposed by Simo (1992). Without going into further detail, the tensorial stress update algorithm is presented in the following overview:



For the tensorial stress update the von Mises yield criterion, which is suitable to describe a wide range of problems in metal plasticity, with linear isotropic hardening is used:

$$\Phi_{\text{Mises}} = ||\text{dev}\,\boldsymbol{\tau}|| - \sqrt{\frac{2}{3}}\,(\tau_Y - q) \qquad\qquad q = -K\,\alpha \tag{13}$$

The parameters τ_Y and K indicate the yield stress and linear hardening parameter, respectively. All constitutive equations that have been shown so far are valid for a general 3–D continuum. They can also be applied to the classical 6–parameter shell theory without further modifications (see Eberlein (1997)).

In case of the 5/6–parameter model, however, a plane stress assumption has to be taken into account since through the thickness strains are neglected due to the inextensible director field. As long as small elastic strains are under consideration, the thickness stresses τ_{33} can be set to zero explicitly (see Wriggers et al. (1995)). Therefore the 5/6-parameter theory is restricted to small elastic but finite plastic strains here. For many applications in metal plasticity like deep drawing processes, this approach is proved to be sufficient.

In contrast to the classical 6–parameter concept, for the 5/6–parameter model a total Langrangian description is chosen. That means, in order to determine the trial logarithmic strains $\varepsilon_\alpha^{tr} = \ln \lambda_\alpha^{tr}$ needed for the return mapping scheme, the general eigenvalue problem

$$(\mathbf{C}_{p\,n-1}^{-1} - \lambda_\alpha^{tr^2}\,\mathbf{C}_n^{-1})\,\mathbf{N}_\alpha^{tr} = \mathbf{0} \qquad \alpha = 1,2 \tag{14}$$

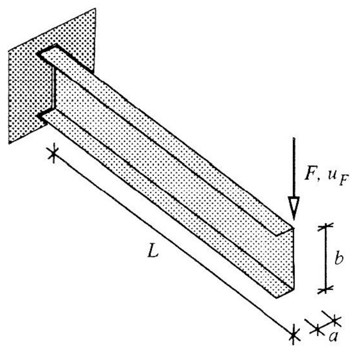
proposed by Ibrahimbegovic (1994) with respect to the undeformed configuration, has to be solved. One should note that the eigenvalue problem refers to the in–plane (membrane) strain components only, since the plane stress assumption is imposed. For a general 3–D material law a corresponding algorithm was recently presented by Miehe (1997).

The discussion of the finite element formulations would have to follow next. Here, only an overview can be given. For both parameterization strategies presented, quadrilateral 4–node mixed finite shell elements with bilinear shape functions are used. In order to avoid well known locking phenomena, the performance of the 6–parameter element is improved by means of the enhanced–assumed–strain method with respect to normal thickness strain and membrane strain components as well as the assumed–natural–strain method accounting for the transverse shear strains. For the 5/6–parameter element only the membrane strains are enhanced and the transverse shear strains are subjected to a reduced integration. Besides that, a penalty term (with penalty multiplier ε) enforces the transverse shear strains to become zero. Thus only in–plane strains occur and justify the plane stress assumption. For further details regarding the mixed variational approaches, their linearization and discretization refer to Gruttmann (1996), Eberlein (1997) and references therein.

4 Numerical Examples

This section presents the warping of angle irons in order to prove the validity of the 5/6–parameter concept and the failure of the 6–parameter model for the calculation of shell intersections. Furthermore, for a commercial steel channel the influence of finite plastic strains is discussed.

At first, a cantilever beam subjected to a point load is considered, as given in figure 2. The example was originally proposed by Chroscielewski et al. (1992), where purely elastic material behavior is assumed.



material data:	
$E = 1.0 \cdot 10^7$	$\nu = 0.33$
$\varepsilon = 1.0 \cdot 10^8$	(5/6–parameter model)
geometric data:	
$a = 2$	$b = 6$
$L = 36$	thickness: $H = 0.05$

Figure 2. Warping of an Elastic Steel Channel

Later on, this example was recalculated by Betsch (1996) who also presented a convergence test for his 6/7–parameter concept. In figure 3 it is shown that the load deflection curve obtained by the current 5/6–parameter model coincides very well with the converged solution proposed by Betsch (1996). For the computation a discretization of 20×72 elements is chosen.

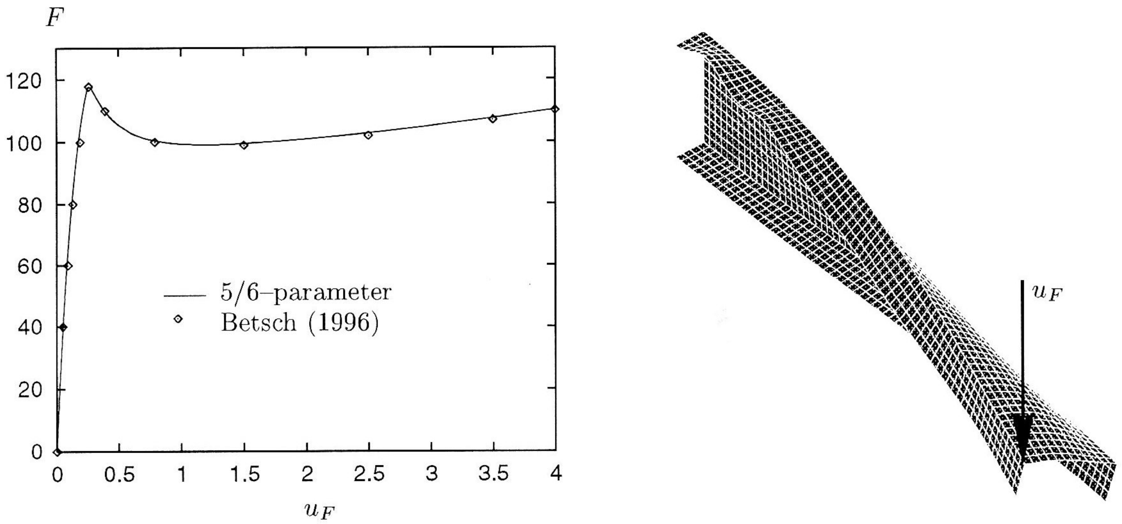


Figure 3. 5/6–Parameter Model; Load Deflection Curves; Deformed Configuration for $u_F = 4$

The deformed configuration for $u_F = 4$ is also depicted in figure 3. Buckling of the upper flange can be observed in the vicinity of the clamped end, whereas the free end of the beam is twisted. This is due to the fact that the external load F does not act in the shear center.

For the 6–parameter concept the corresponding results are shown in figure 4. The same 20×72 elements discretization is applied as before. It turns out that the 6–parameter element behaves much too stiff. As for the 5/6–parameter element the computation is performed with non unique initial normal vectors in the intersections of the steel channel. Thus the only reason for the poor results can be the neglect of drilling degrees of freedom, because for smooth shells, where drilling rotations cannot exist a priori, the 6–parameter concept proved its applicability (see Eberlein (1997)). Eventually, the deformed configuration for $u_F = 4$ shows qualitatively a completely different behavior in comparison with the results obtained by the 5/6–parameter model (see figure 3).

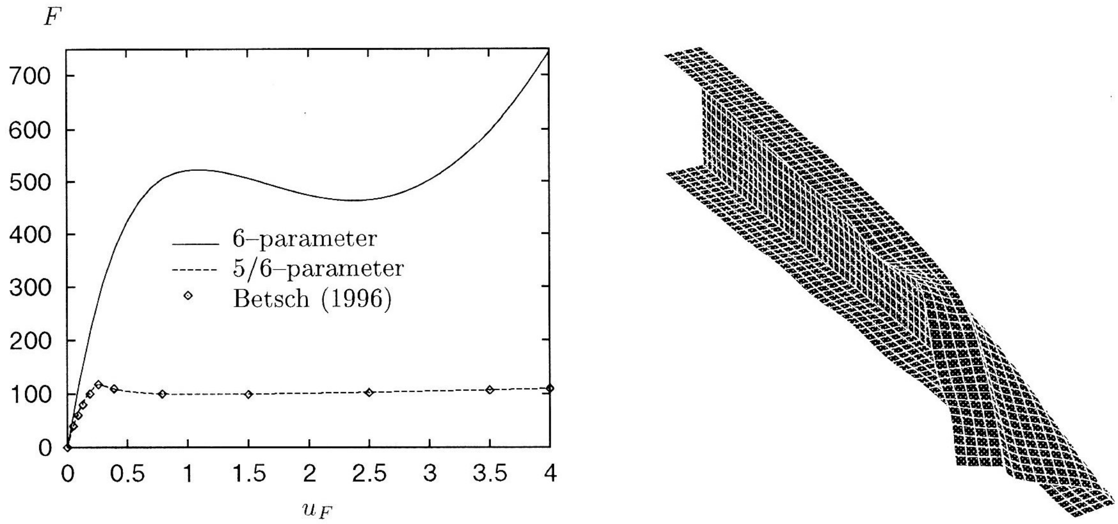
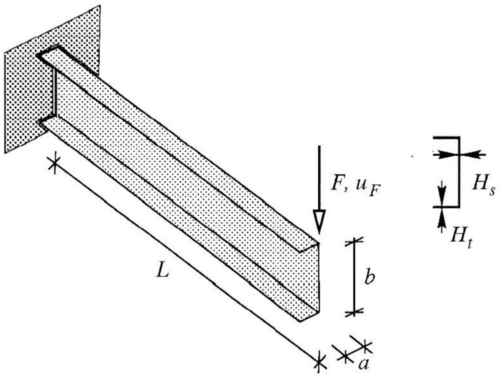


Figure 4. 6–Parameter Model; Load Deflection Curves; Deformed Configuration for $u_F = 4$

It should be noted that the performance of the proposed 6–parameter model could be considerably improved by using averaged initial normal vectors as originally proposed for the degenerated solid approach (see e.g. Ramm (1976)). However, this averaging procedure may cause severe ill–conditioning of the global stiffness matrix for refined meshes and in those cases means the loss of practical applicability.

In order to show the influence of finite plastic strains, the warping of a commercial steel channel (U 300 according DIN 1026) is investigated. Only the 5/6–parameter element is used for computations here.

The problem definition is given in figure 5. As in the previous example, 20×72 elements are applied to discretize the steel channel.



material data:	
$E = 2.1 \cdot 10^4$	$\nu = 0.33$
$\varepsilon = 1.0 \cdot 10^5$	
$\tau_Y = 24$	$K = 0$
geometric data:	
$a = 10$	$b = 30$
$L = 180$	
$H_t = 1.6$	$H_s = 1.0$

Figure 5. Warping of a Commercial Steel Channel

Perfectly plastic material behavior is assumed. The load deflection curves in figure 6 show the results for a purely elastic calculation (elastic constants from figure 5). In case of the elastic–plastic material with perfectly plastic behavior, one observes considerable softening as soon as plastic strains occur.

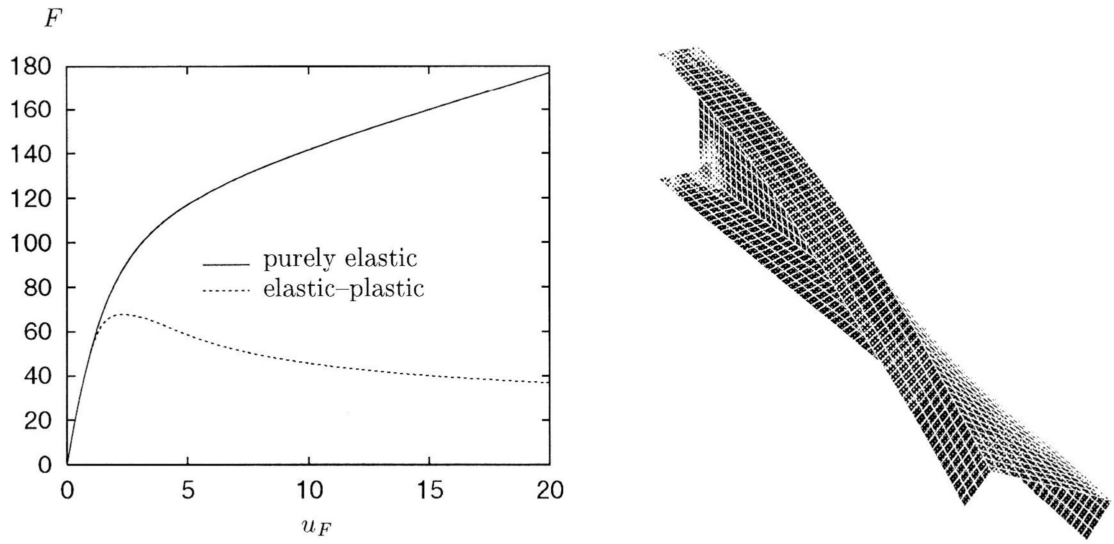


Figure 6. Load Deflection Curves; Deformed Configuration for $u_F = 20$ (Elastic–Plastic) Including Plot of the Eqivalent Plastic Strain

Finally, figure 6 also shows a plot of the equivalent plastic strain for the deformed configuration ($u_F = 20$). As could be expected, there is a maximum of plastic deformation in the lower and upper flange at the clamped end of the profile. In contrast to the previous example, there is no buckling phenomenon in the upper flange. However, this is not due to the plastic strains but the altered geometric data, instead. Indeed, for the purely elastic calculation, buckling of the upper flange could not be observed in this example, either. This can also be perceived from the fact that there is no limit point for the elastic steel channel in figure 6 but in figure 3 there is.

5 Conclusions

In the current paper a proper parameterization strategy for accurate modelling of shell intersections is presented. Within this context, the influence of drilling degrees of freedom is discussed in detail. Furthermore, the importance of the deformation gradient, which serves as kinematic basis, is particularly emphasized. The constitutive description of finite plastic strains is derived for a 3D–continuum. The resulting equations can be applied to the 6–parameter concept for smooth shell structures without further modifications. In case of the 5/6–parameter model a plane stress state is assumed. Due to the total Lagrangian description for this theory, the corresponding eigenvalue problem with respect to the undeformed configuration for the calculation of principal trial stretches is introduced. The numerical examples show the applicability of the 5/6–parameter element for the calculation of shell intersections. Furthermore, the influence of finite plastic strains is shown in this context. One aspect of future work could be the implementation of a 6/7–parameter element accounting for thickness strains. Such an element would allow the calculation of thick shells with intersections including finite plastic strains. However, for most practical applications, the 5/6–parameter element shows an absolutely satisfying performance.

Literature

1. Betsch, P.: Statische und dynamische Berechnungen von Schalen endlicher elastischer Deformationen mit gemischten Finiten Elementen. Dissertation. Institut für Baumechanik und numerische Mechanik der Universität Hannover. Bericht Nr. F 96/4. (1996)
2. Chroscielewski, J.; Makowski, J.; Stumpf, H.: Genuinely Resultant Shell Finite Elements Accounting for Geometric and Material Non–Linearity. Int. J. Num. Meth. Eng., 35, (1992), 63–94.
3. de Boer, R.: Vektor– und Tensorrechnung für Ingenieure. Springer–Verlag. Berlin. (1982)
4. Eberlein, R.: Finite–Elemente–Konzepte für Schalen mit großen elastischen und plastischen Verzerrungen. Dissertation. Institut für Mechanik IV. Technische Hochschule Darmstadt. (1997)
5. Gruttmann, F.: Theorie und Numerik dünnwandiger Faserverbundstrukturen. Habilitation. Institut für Baumechanik und numerische Mechanik der Universität Hannover. Bericht Nr. F 96/1. (1996)
6. Hughes, T. J. R.; Liu, W. K.: Nonlinear Finite Element Analysis of Shells: Part I. Threedimensional Shells. Comp. Meth. Appl. Mech. Eng., 26, (1981), 331–362.
7. Ibrahimbegovic, A.: Finite Elastoplastic Deformations of Space–Curved Membranes. Comp. Meth. Appl. Mech. Eng., 119, (1994), 371–394.
8. Miehe, C.: A Formulation of Finite Elastoplasticity in Shells based on Dual Co– and Contra–Variant Eigenvectors Normalized with respect to a Plastic Metric. In 'Computational Plasticity, Fundamentals and Applications'. Owen, D. R. J.; Onate, E.; Hinton, E. (Eds.). CIMNE. Barcelona. (1997), 1898–1903.
9. Naghdi, P. M.: The Theory of Shells. Vol. VIa/2. Handbuch der Physik. Mechanics of Solids II. Springer–Verlag. Berlin. (1972)
10. Ramm, E.: Geometrisch nichtlineare Elastostatik und Finite Elemente. Habilitationsschrift. Institut für Baustatik. Universität Stuttgart. (1976)
11. Simo, J. C.: Algorithms for Static and Dynamic Multiplicative Plasticity that Preserve the Classical Return Mapping Schemes of the Infinitesimal Theory. Comp. Meth. Appl. Mech. Eng., 99, (1992), 61–112.
12. Simo, J. C.: On a Stress Resultant Geometrically Exact Shell Model: Part VII. Shell Intersections with 5/6–Dof Finite Element Formulations. Comp. Meth. Appl. Mech. Eng., 108, (1993), 319–339.
13. Simo, J. C.; Miehe, C.: Coupled Associative Thermoplasticity at Finite Strains. Formulation, Numerical Analysis and Implementation. Comp. Meth. Appl. Mech. Eng., 98, (1992), 41–104.
14. Wriggers, P.; Eberlein, R.; Gruttmann, F.: An Axisymmetrical Quasi–Kirchhoff–Type Shell Element for Large Plastic Deformations. Arch. Appl. Mech., 65, (1995), 465–477.
15. Wriggers, P.; Eberlein, R.; Reese, S.: A Comparison of Three–Dimensional Continuum and Shell Elements for Finite Plasticity. Int. J. Sol. Struct., 33, (1996), 3309–3326.

Address: Dr.-Ing. Robert Eberlein and Prof. Dr.-Ing. Peter Wriggers, Institut für Mechanik IV, Technische Universität Darmstadt, Hochschulstr. 1, D-64289 Darmstadt