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Model- and Parameteroptimization for a Constitutive Law Describing Deformation Induced Anisotropy

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A constitutive law is presented to describe the physical phenomenon of deformation induced anisotropy. Biaxial tension and biaxial compression test procedures for sheet metal are shown as possibilities to observe this phenomenon and to acquire data for optimisation. The constitutive model of Chan, Bodner and Lindholm is extended to deformation induced anisotropy. With a focus on the multi dimensionality of the test results and their fitting, the formulation of an objective function is shown. One main issue of the paper is the parameter fitting. Price's procedure is applied to fit the set of parameters of the system of ordinary differential equations (constitutive model) as good as possible to the material behaviour.

1 Introduction

In many technical areas it has become more and more important to know the behaviour of different materials under mechanical and thermal loading. This is important for the design and control of processing, working and deformation processes as well as for a safe and economical design of machine elements and constructions with respect to strength, stiffness and temperature resistance. For the minimisation of costs and to get more accurate stress results the numerical simulation plays an increasingly important part. In this context one crucial factor is the model describing the behaviour of the material. In this paper the procedure of testing the material, formulation of a constitutive law and optimisation of the parameters will be presented.

2 Physical Phenomena of Deformation Induced Anisotropy and Test Procedure of Biaxial Tension and Biaxial Compression

The inhomogeneous microstructure of poly-crystals leads to anisotropic elastic and inelastic material behaviour. As the elastic constants of a single crystal are not uniform in all directions, the not fully stochastic distribution of the orientations of the crystals in the body leads to a macroscopic anisotropy following a deformation. Plastic deformation of metals is bound up with the crystal lattice. Thus, an irregular distribution of crystal orientations, of embeddings, of lattice defects and of grain boundaries leads to plastic deformation of the metal, depending on the load directions. Since the micro structure of metals changes during deformation, e.g. by dislocation pileups, in general a change of the anisotropic property during plastic deformation is observable.

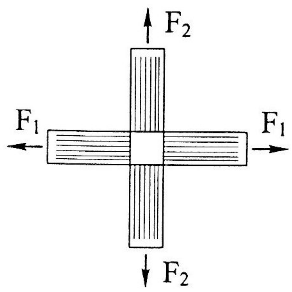

Figure 1. Cross Specimen

To investigate this mechanism, among others, biaxial tension and biaxial compression tests were executed. For that purpose cross specimens (Figure 1) were loaded with various load paths (e.g. Figure 2).

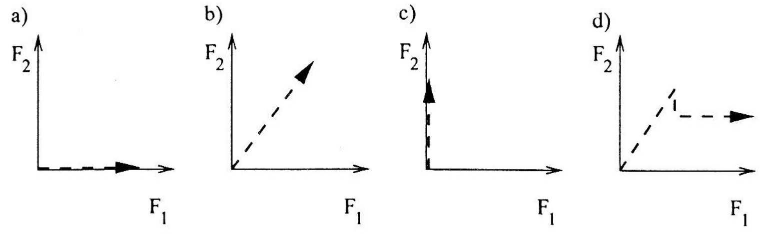

Figure 2. Possible Load Paths

Proportional load paths (F_1/F_2 = constant) are as well possible as any kind of a hook path (see Figure 2d). During the tests the stress–strain curves in the two principal axes were plotted. Furthermore, the deformation induced changes of the yield surface were determined. Various load paths are necessary to determine the correlation between the prescribed deformations and the change of physical properties.

To describe the measured results one special test shall be discussed exemplarily (for further discussion see Rost et. al., 1998). The deformation in the specimen body and the corresponding load are gauged in the 1– and the 2–direction (1–direction is the rolling direction of the sheet metal). From this data a stress–strain relation for each direction is obtained. Figure 3 shows the results of the biaxial tension tests. In this experiment the specimen was loaded with identical force rate in 1–and 2–direction ($\dot{F}_1 = \dot{F}_2$). After a predefined deformation (here: $\varepsilon_1 = 3\%, 5.8\%$ and 8.7%) the loading was stopped to detect the momentary yield surface (Figure 4). This is done in six directions in the tension-tension space and in three directions in the compression-compression space. These points characterize the limit of the elastic domain. The change of this shape describes the hardening of the material (isotropic, kinematic and formative). This behaviour shall be described as exactly as possible with the following constitutive law.

3 Constitutive Law

The formulation of a constitutive law describing deformation induced anisotropy is oriented towards the model of Chan, Bodner and Lindholm (Chan et. al., 1988).First the assumption of the additive decomposition of the strain rate into an elastic and an inelastic part is used

$$\dot{\varepsilon} = \dot{\varepsilon}^e + \dot{\varepsilon}^p \tag{1}$$

The elastic part is described by Hooke's generalised law

$$\varepsilon^e := \boldsymbol{E}^{-1} \cdot\cdot \, \sigma \tag{2}$$

For the inelastic part of the strain rate $\dot{\varepsilon}$ an evolution law is formulated. To complete the model, evolution laws for the isotropic and the kinematic hardening K and $\boldsymbol{\beta}$, respectively, are formulated

$$\dot{\varepsilon}^p_{ij} := D_0 \exp\left\{ -\frac{1}{2}\left[\frac{(K+\beta_{kl}u_{kl})^2}{3J_2^\star}\right]^n \right\} \underset{\tilde{4}}{\mathcal{N}}\, \sigma \qquad i,j,k,l = 1,\dots,3 \tag{3}$$

$$\dot{K} := m_1\,[K_1 - K]\,\dot{W}^p \tag{4}$$

$$\dot{\beta}_{ij} := m_2\,[D_1 u_{ij} - \beta_{ij}]\,\dot{W}^p \qquad i,j = 1,\dots,3 \tag{5}$$

Here $\boldsymbol{s} := \boldsymbol{\sigma} - \frac{1}{3}\,\mathrm{tr}\boldsymbol{\sigma}\,\mathrm{I}$ is the deviatoric stress, $\dot{W}^p := \boldsymbol{\sigma} \cdot\cdot \dot{\varepsilon}^p$ is the rate of plastic work and u is defined as $\boldsymbol{u} := \frac{\boldsymbol{\sigma}}{\sqrt{\boldsymbol{\sigma}\cdot\cdot\boldsymbol{\sigma}}}$, where $\boldsymbol{\sigma} \cdot\cdot \boldsymbol{\sigma}$ is the inner product of the stress tensor.
To describe anisotropy in the inelastic deformation rate (3), $J_2^\star$ is defined in a different way as in the original model. It is set as

$$J_2^\star := \frac{1}{2}\sigma \cdot\cdot \underset{\tilde{4}}{\mathcal{N}}\, \sigma \tag{6}$$

For the forth order structure tensor $\underset{\tilde{4}}{\mathcal{N}}$ a definition of Spencer (1971) , with the application of Imatani

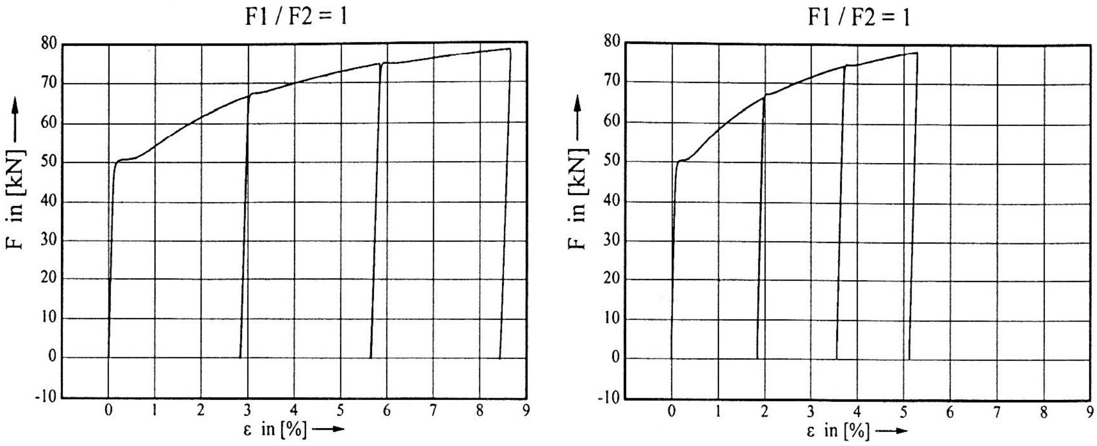

Figure 3. Force–Strain Relation in Biaxial Tension Tests

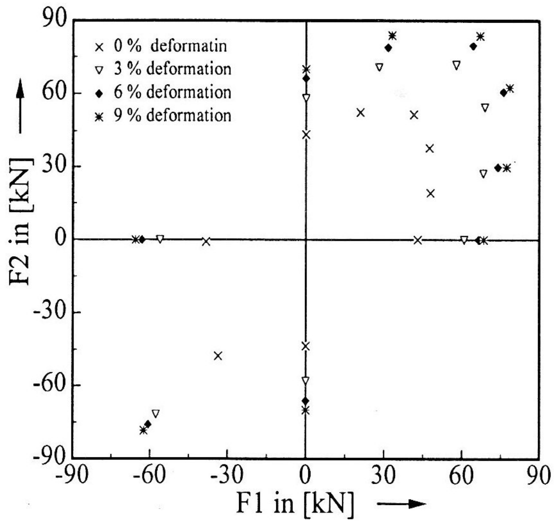

Figure 4. Evolution of Yield Surface

et. al. (1995), is used.

$$\underset{4}{\mathcal{N}} := \underset{4}{\mathcal{N}}\left(\boldsymbol{m}^{(1)}, \boldsymbol{m}^{(2)}, \boldsymbol{m}^{(3)}\right) \tag{7}$$

Spencer derived that tensor using invariants. It is built from three independent structure vectors $\boldsymbol{m}^1$, $\boldsymbol{m}^2$, $\boldsymbol{m}^3$. To describe deformation induced anisotropy evolution laws for these three vectors have to be formulated. They are given as

$$\dot{\boldsymbol{m}}^{(i)} := \dot{\varepsilon}^p \boldsymbol{m}^{(i)} \qquad i = 1, 2 \tag{8}$$

and $\boldsymbol{m}^{(3)}$ shall be perpendicular to the two others for planar anisotropy in sheet metal

$$\boldsymbol{m}^{(3)} := \frac{\boldsymbol{m}^{(1)} \times \boldsymbol{m}^{(2)}}{||\boldsymbol{m}^{(1)} \times \boldsymbol{m}^{(2)}||} \tag{9}$$

The 19 unknowns of the model are combined to a hyper-vector

$$\underline{y}(t) = \{\varepsilon^p_{11}, \ldots, \varepsilon^p_{13}, K, \beta_{11}, \ldots, \beta_{13}, m_{11}, m_{12}, m_{13}, m_{21}, m_{22}, m_{23}\}^T \tag{10}$$

$\underline{y}$ can be determined, if the evolution equation $\dot{\underline{y}}$ is integrated with a set of initial conditions $\underline{y}(t_0)$ and

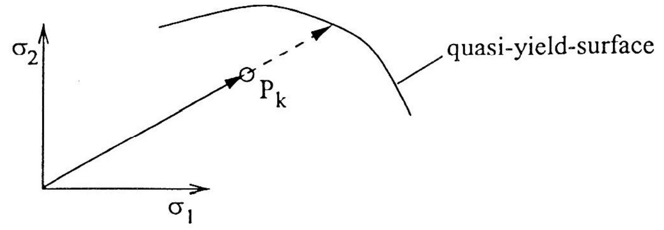

Figure 5. Rating of the Yield Surface

a load or deformation rate

$$\begin{cases} \underline{\dot{y}}(t) &= \underline{f}(t, \underline{y}_{(t)}) \\ \underline{y}(t_0) &= \underline{y}_{(0)} \end{cases} \tag{11}$$

Several integration methods are used to solve this set of 19 coupled ordinary differential equations (11). Explicit and implicit Euler methods are used as well as the very efficient generalised Runge-Kutta method (Kaps and Rentrop, 1979). In the evolution equations and the initial conditions twelve parameters occur

$$\underline{\theta}_s = \{D_0, K_{(0)}, K_1, D_1, m_1, m_2, n, \alpha_1, \ldots, \alpha_5\}^T \tag{12}$$

Five of it $\alpha_1, \ldots, \alpha_5$ are anisotropy parameters from the structure tensor.

4 Formulation of an Objective Function

To be able to find the set of parameters for the constitutive law, which reflects the test results in an optimal way, a measure for the quality of the fit must be found. This quality function Q (in the following called objective function) assesses the conformity of measured values with corresponding simulation data. These simulation data are obtained by integration of the constitutive law using the parameter set $\underline{\theta}_s$ and will be called the model answer in the following .

As a quality statement the sum of weighted and normalised distances between the measured values and the model answer is used. The objective function is defined as

$$\begin{aligned} Q_s := \quad & a_1 \sum_{l=1}^{m} w_l^{\star} \left[\frac{1}{\sum_{i=1}^{p} w_i^{\star\star}} \sum_{i=1}^{p} w_i^{\star\star} \frac{\left\| \varepsilon_{Exp\,(i)} - \varepsilon_{Sim(\underline{\theta}_s)\,(i)} \right\|}{\left\| \varepsilon_{Exp\,(i)} \right\|} \right] \\ & + a_2 \sum_{l=1}^{m} w_l^{\star} \left[\frac{1}{\sum_{j=1}^{n} w_j^{\star\star\star}} \sum_{j=1}^{n} w_j^{\star\star\star} \left(\frac{1}{\sum_{k=1}^{q} w_k^{\star\star\star\star}} \sum_{k=1}^{q} w_k^{\star\star\star\star} \left| (a_k - 1) \right| \right) \right] \end{aligned} \tag{13}$$

Where m is the number of stress-strain tests, n is the number of detected yield surfaces, q is the number of detected points per yield surface, p is the number of calculated stress points per stress-strain simulation and $w_i^{\star}$, $w_i^{\star\star}$, $w_i^{\star\star\star}$, $w_i^{\star\star\star\star}$ and $a_1 + a_2 = 1$ are weights.

The first term of the sum assigns the value to the fitting quality of the stress–strain proportion (Figure 3). For that reason the gap, using a weighted error amount, is calculated.

The second term of equation (13) assigns the value to the fitting quality of the yield surfaces after various deformation steps (Figure 4). A direction dependent scalar is defined to measure the error. Direction dependent in this context means, that with the origin of the main stress space and one special, measured yield point a direction is given (Figure 5).

The distance a_k in the direction between the yield point and the quasi yield surface of the model answer is summed up with weights. With this background a criterion for the model must be formulated for the occurence of inelastic deformation. Starting from the rate of plastic work $\dot{W}^p$, the quasi yield surface shall be reached when the equivalent inelastic strain rate becomes equal to a critical value.

$$\varepsilon^p_{eff} \;=\; \varepsilon^p_{crit} \tag{14}$$

ε^p_{eff} is defined as

$$\varepsilon^p_{eff} \;:=\; \sqrt{\frac{2}{3}\varepsilon^p \cdot\cdot\, \varepsilon^p} \tag{15}$$

Integrating the evolution laws ((3) - (5), (8)) it is possible to calculate the stress σ_{Sim} when the quasi yield surface is reached. a_k for equation(13) is defined as

$$a_k \;:=\; \frac{||\sigma_{Sim}||}{||\sigma_{Exp}||} \tag{16}$$

The identification of the optimal set of parameters will be done by the minimisation of the objective function Q (13) with regard to the set of parameters.

5 Parameter Optimization

In principle it is possible to use two families of nonlinear optimisation strategies for solving this kind of minimisation problems. They can be separated in a class that searchs deterministically, and one using stochastic elements. Deterministic methods are fast but they can stick to a local minimum and, therefore, then will not find the global minimum. Stochastic search methods have the advantage that they do not stop in local minima, but they need a large amount of CPU time.

In this paper the very efficient stochastic method of Price (1978) is applied to identify the global minimum of the objective function Q (see equation (13)). The procedure begins with the generation of a start cluster (N sets of parametersets) in parameter space Ω. For that the Monte Carlo method is used.

$$\underline{x}^{(1,0)}, \ldots, \underline{x}^{(N,0)} \in \Omega \tag{17}$$

Then the worst point (set of parameters) of the cluster has to be found

$$i_0 \;:=\; \max\{\, i \in \{1,\ldots,N\} \,|\, \forall\, k \in \{1,\ldots,N\} \setminus \{i\} : Q(\underline{x}^{(i,j-1)}) \geq Q(\underline{x}^{(k,j-1)})\} \tag{18}$$

Following this the first phase (outer search) of the procedure starts. A random selection of n points of the cluster creates a subset P. Then the mean position $\underline{g}$ of this subset is calculated

$$\underline{g} \;:=\; \frac{1}{n}\sum_{k=1}^{n} \underline{x}^{(i_k,j-1)} \tag{19}$$

A new point $\tilde{\underline{x}}^{(j)}$ (set of parameters) is generated by making a point $\underline{x}$ of the cluster symmetric with respect to the mean position $\underline{g}$

$$\tilde{\underline{x}}^{(j)} \;:=\; 2\underline{g} - \underline{x}^{(i_{n+1},j-1)} \tag{20}$$

After a check whether the new point is included in the search area $\tilde{\underline{x}}^{(j)} \in \Omega$, the objective function Q (13) has to be evaluated. If the quality value is better (that means smaller) than the worst one of the old cluster

$$Q(\tilde{\underline{x}}^{(j)}) \geq Q(\underline{x}^{(i_0,j-1)}) \tag{21}$$

the new point is sorted in and the worst is discarded. Otherwise the new set will be thrown away and the outer search starts again. This outer search continues until the rate of success of finding new better points is getting too small.

Following this, the second phase, the inner search is started. Again a subcluster is defined. But, different to the outer search, the new point is generated by finding the average of the mean position $\underline{g}$ of the subcluster and one other point $\underline{x}^{(i_n+1,j-1)}$ of the rest of the cluster

$$\tilde{\underline{z}}^{(j)} := \frac{1}{2}\left(\underline{g} + \underline{x}^{(i_n+1,j-1)}\right) \tag{22}$$

If the quality value of the new generated point is better then worst one of the old cluster

$$Q(\tilde{\underline{z}}^{(j)}) \geq Q(\underline{x}^{(i_0,j-1)}) \tag{23}$$

the new point is added to the cluster and the former worst point is removed. If the new $\tilde{\underline{z}}$ is not better it is put aside. This procedure continues until an iteration condition is fulfilled. Such a condition can be that the number of evolution of the objective function is bigger then a predefined number, that the N points of the cluster being sufficiently close to each other or that the computing time reaches a given value.

Seibert (1996) and Lesche (1996) showed the efficiency of Price's procedure for this kind of minimization problems, compared with other stochastic optimization strategies. The reason why the procedure is so efficient, is that it takes into account the structure of the parameter space and does not search completely randomly like others.

With this procedure the course of material testing, formulation of a constitutive law and parameter fitting is completed. More detailed information about testing results and the parameter optimization will be given in a forthcoming paper.

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