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Free Convection about a Vertical Wavy Surface with Prescribed Surface Heat Flux in a Micropolar Fluid

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An investigation on the free convection along a vertical wavy surface with prescribed surface heat flux in a micropolar fluid has been carried out. By applying a suitable transformation of the governing equations of continuity, momentum, microrotation and energy, we show that they can be reduced to a set of boundary layer equations for the free convection flow along a vertical flat plate. The transformed equations are then solved numerically using a very efficient finite-difference method known as Keller-box scheme. The results obtained for a Prandtl number Pr = 1 and various values of the parameters a (the amplitude of the wavy surface) and K (micropolar parameter) reveal the influence of these parameters on the flow and heat transfer behaviour.

1 Introduction

Interest in studying the convective phenomena of momentum and heat transfer between a moving fluid and a surface immersed in it stems from both theoretical and practical considerations. It is readily recognized that a wealth of information is now available for convective heat transfer of Newtonian fluids under the most general considerations of practical interest. Consequently, satisfactory means have evolved for the estimation of the macroscopic (such as drag and heat transfer) as well as microscopic (e.g. velocity and temperature) characteristic parameters in an envisaged application involving Newtonian fluids. Unfortunately, most fluids encountered in chemical and allied processing applications do not adhere to the classical Newtonian postulate and accordingly are known as non-Newtonian fluids. One particular class of materials of considerable practical interest is that which exhibits certain microscopic effects arising from the local structure and microrotations of the fluid elements known as microfluids, first introduced by Eringen (1966). As this model is not easily amenable the theoretical treatment a subclass, known as micropolar fluids, was further proposed by Eringen (1972). Such fluids contain dilute suspensions of rigid macromolecules with individual motions which support stress and body moments and are subject to skin inertia.

The theory of micropolar fluids may form suitable non-Newtonian fluid models which can be used to study the behaviour of lubricants, coloidal suspensions or polymeric additives, blood flow etc. This theory has generated a lot of interest and many problems have been studied (see, for example, Ariman et al., 1973, 1974; Jena and Mathur, 1981, 1982; Lien et al., 1986, 1990; Gorla and Takhar, 1987; Gorla, 1988, 1992; Wang and Kleinstreuer, 1988; Moulic, 1989; Gorla et al., 1995; Rees and Bassom, 1996).

One of the limitations of all the above investigations is that the surfaces were considered flat or regular. Relatively few studies have considered the effects of complex geometries such as wavy surfaces. Yao (1983) was probably the first who analysed the Newtonian free convection flow associated with a wavy surface. Recently, Chiu and Chou (1993, 1994) have used the transformation proposed by Yao (1983) to solve the problem of free convection along a vertical wavy surface with a constant wall temperature in a micropolar fluid.

In the present paper, we consider the problem of steady free convection in the boundary layer of a micropolar fluid along a vertical wavy surface with a constant heat flux rate q_w, which is often approximated in practical applications and is easier to measure in a laboratory. Of interest are the effects of the amplitude wavelength and micropolar parameter K on the velocity, temperature and microrotation fields as well as on the surface temperature. The formulation is valid for any wavy surface of small amplitude. Numerical results have been obtained for a sinusoidal wavy surface using the Keller-box scheme (see, Cebeci and Bradshaw, 1984). The results obtained show that the wall temperature varies periodically along the wavy surface and its amplitude gradually decreases downstream where the boundary layer grows thicker. As a check of the numerical method used in this paper, the known results of other authors are also reproduced. Comparison of these results shows excellent agreement.

2 Basic Equations

Consider a vertical wavy surface immersed in a micropolar fluid at the ambient temperature T_∞ as shown in Figure 1.

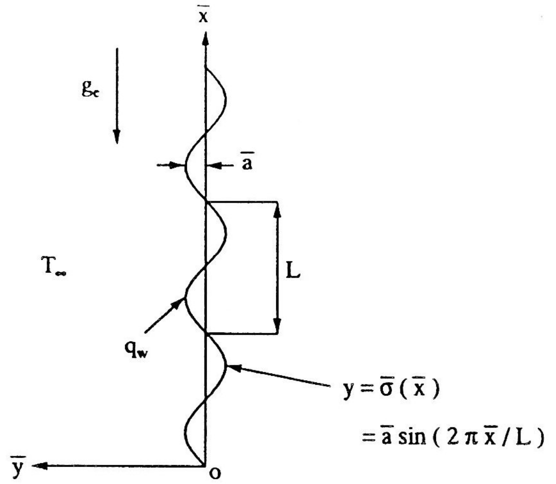

Figure 1. Physical Model and Coordinate System

We assume that the surface is described by

$$\bar{y} = \bar{\sigma}(\bar{x}) = \bar{a}\sin\left(\frac{2\pi\bar{x}}{L}\right) \tag{1}$$

where $\bar{x}$ and $\bar{y}$ are the Cartesian coordinates and $\bar{a}$ and L are the amplitude and the wavelength of the wavy surface, respectively. It is also assumed that the surface is subjected to a constant heat flux rate q_w normal to the surface. The flow is considered to be steady and the Boussinesq approximation is applied. Under these assumptions the governing equations (see Chiu and Chou, 1993) are given by

$$\frac{\partial\bar{u}}{\partial\bar{x}} + \frac{\partial\bar{v}}{\partial\bar{y}} = 0 \tag{2}$$

$$\rho\left(\bar{u}\frac{\partial\bar{u}}{\partial\bar{x}} + \bar{v}\frac{\partial\bar{u}}{\partial\bar{y}}\right) = -\frac{\partial\bar{p}}{\partial\bar{x}} + (\mu + \kappa)\nabla^2\bar{u} + \kappa\frac{\partial\bar{N}}{\partial\bar{y}} + \rho g_e\beta(T - T_\infty) \tag{3}$$

$$\rho\left(\bar{u}\frac{\partial\bar{v}}{\partial\bar{x}} + \bar{v}\frac{\partial\bar{v}}{\partial\bar{y}}\right) = -\frac{\partial\bar{p}}{\partial\bar{y}} + (\mu + \kappa)\nabla^2\bar{v} - \kappa\frac{\partial\bar{N}}{\partial\bar{x}} \tag{4}$$

$$\rho j\left(\bar{u}\frac{\partial\bar{N}}{\partial\bar{x}} + \bar{v}\frac{\partial\bar{N}}{\partial\bar{y}}\right) = \kappa\left(\frac{\partial\bar{v}}{\partial\bar{x}} - \frac{\partial\bar{u}}{\partial\bar{y}} - 2N\right) + \gamma\nabla^2 N \tag{5}$$

$$\bar{u}\frac{\partial T}{\partial\bar{x}} + \bar{v}\frac{\partial}{\partial\bar{y}} = \frac{\nu}{Pr}\nabla^2 T \tag{6}$$

where $\bar{u}$ and $\bar{v}$ are the velocity components along $\bar{x}$ and $\bar{y}$ axes, T is the temperature, $\bar{p}$ is the pressure, $\bar{N}$ is the microrotation, g_e is the acceleration due to gravity, Pr is the Prandtl number, ρ is the density, β is the thermal expansion coefficient, μ and ν are the dynamic and kinematic viscosity respectively, κ is the vortex viscosity, γ is the spin-gradient viscosity, j is the micro-inertia density and ∇^2 is the Laplacian operator. We assume that γ is given by (see Ahmadi, 1976)

$$\gamma = \left(\mu + \frac{\kappa}{2}\right)j \tag{7}$$

The boundary conditions appropriate for equations (2) to (6) are given by

$$\begin{aligned} &\bar{y} = \bar{\sigma}(\bar{x}): \quad \bar{u} = \bar{\upsilon} = 0 \quad \bar{\mathbf{n}}\cdot\nabla T = -\frac{q_w}{k} \\ &\bar{N} = \frac{1}{2}\left(\frac{\partial\bar{\upsilon}}{\partial\bar{x}} - \frac{\partial\bar{u}}{\partial\bar{y}}\right) \\ &\bar{y} \to \infty: \quad \bar{u} = \bar{\upsilon} = 0 \quad T = T_\infty \quad \bar{N} = 0 \quad \bar{p} = p_\infty \end{aligned} \tag{8}$$

where k is the thermal conductivity and $\bar{\mathbf{n}}$ is the unit vector normal to the wavy surface, which is given by

$$\bar{\mathbf{n}} = \left\{ -\frac{\bar{\sigma}_{\bar{x}}}{\left(1 + \bar{\sigma}_{\bar{x}}^2\right)^{\frac{1}{2}}}, \frac{1}{\left(1 + \bar{\sigma}_{\bar{x}}^2\right)^{\frac{1}{2}}} \right\} \tag{9}$$

and $\bar{\sigma}_{\bar{x}} = \dfrac{d\bar{\sigma}}{d\bar{x}}$.

To transform equations (2) to (6), we first define the following dimensionless variables:

$$\begin{aligned} &x = \frac{\bar{x}}{L} \quad y = \left(\frac{\bar{y} - \bar{\sigma}(\bar{x})}{L}\right)Gr^{\frac{1}{5}} \quad u = \left(\frac{L}{\nu Gr^{\frac{2}{5}}}\right)\bar{u} \\ &\upsilon = \left(\bar{\upsilon} - \sigma_x \bar{u}\right)\left(\frac{L}{\nu Gr^{\frac{1}{5}}}\right) \quad T - T_\infty = \left(\frac{q_w L}{k\,Gr^{\frac{1}{5}}}\right)\theta \\ &p = \left(\bar{p} - p_\infty\right)\left(\frac{L^2}{\rho\nu^2 Gr^{\frac{1}{5}}}\right) \quad N = \left(\frac{L^2}{\nu Gr^{\frac{3}{5}}}\right)\bar{N} \\ &\sigma(x) = \frac{\bar{\sigma}(\bar{x})}{L} \quad a = \frac{\bar{a}}{L} \end{aligned} \tag{10}$$

where $Gr = g_e \beta\left(\dfrac{q_w}{k}\right)\dfrac{L^4}{\nu^2}$ is the Grashof number. When the variables (10) are substituted into equations (2) to (6) and terms of small order in negative powers of Gr are neglected, with the assumption of large values of Gr or boundary layer approximation, we obtain the following boundary layer equations for the problem under consideration:

$$\frac{\partial u}{\partial x} + \frac{\partial \upsilon}{\partial y} = 0 \tag{11}$$

$$u\frac{\partial u}{\partial x} + \upsilon\frac{\partial u}{\partial y} = -\frac{\partial p}{\partial x} + \sigma_x Gr^{\frac{1}{5}}\frac{\partial p}{\partial y} + \theta$$
$$+ (1 + K)\left(1 + \sigma_x^2\right)\frac{\partial^2 u}{\partial y^2} + K\frac{\partial N}{\partial y} \tag{12}$$

$$\sigma_{xx} u^2 + \sigma_x\left(u\frac{\partial u}{\partial x} + \upsilon\frac{\partial u}{\partial y}\right) = -Gr^{\frac{1}{5}}\frac{\partial p}{\partial y}$$
$$+ (1 + K)\sigma_x\left(1 + \sigma_x^2\right)\frac{\partial^2 u}{\partial y^2} + K\sigma_x\frac{\partial N}{\partial y} \tag{13}$$

$$u\frac{\partial N}{\partial x} + \upsilon\frac{\partial N}{\partial y} = (1 + K)\left(1 + \sigma_x^2\right)\frac{\partial^2 N}{\partial y^2} - K\left(\frac{\partial u}{\partial y} + 2N\right) \tag{14}$$

$$u\frac{\partial \theta}{\partial x} + \upsilon\frac{\partial \theta}{\partial y} = \frac{1 + \sigma_x^2}{Pr}\frac{\partial^2 \theta}{\partial y^2} \tag{15}$$

where $K = \frac{\kappa}{\mu}$ is the micropolar parameter. The boundary conditions (8) become

$$y = 0: \quad u = \upsilon = 0 \quad \left(1+\sigma_x^2\right)^{\frac{1}{2}}\frac{\partial \theta}{\partial y} = -1 \quad N = -\frac{1}{2}\left(1+\sigma_x\right)^{\frac{1}{2}}\frac{\partial u}{\partial y}$$
$$y = \infty: \quad u = 0 \quad \theta = 0 \quad N = 0 \quad p = 0 \tag{16}$$

which shows that equations (10) transform the wavy surface to a flat surface. Equation (12) indicates that $\frac{\partial p}{\partial y}$ is of order $Gr^{-\frac{1}{5}}$, which implies that the lowest order pressure gradient along the x axis is determined from the inviscid solution. However, for the present problem this gives $\frac{\partial p}{\partial x} = 0$. Further, in order to eliminate the term $Gr^{\frac{1}{5}}\frac{\partial p}{\partial y}$ from equations (12) and (13) we multiply equation (13) by σ_x and the resulting equation is added to equation (12). After some manipulation, we get

$$u\frac{\partial u}{\partial x} + \upsilon\frac{\partial u}{\partial y} + \frac{\sigma_x \sigma_{xx}}{1 + \sigma_x^2}u^2 = (1 + K)\left(1 + \sigma_x^2\right)\frac{\partial^2 u}{\partial y^2} + \frac{\theta}{1 + \sigma_x^2} + K\frac{\partial u}{\partial y} \tag{17}$$

This problem does not have a similarity solution and to obtain a solution for all $x \geq 0$ the governing equations (11), (14), (15) and (17) have to be solved numerically. To obtain such a numerical solution we use the variables

$$\psi = (5x)^{\frac{4}{5}} f(x, \eta) \qquad \theta = (5x)^{\frac{1}{5}} h(x, \eta)$$
$$N = (5x)^{\frac{2}{5}} g(x, \eta) \qquad \eta = \frac{y}{(5x)^{\frac{1}{5}}} \tag{18}$$

where ψ is the stream function which is defined in the usual way $(u, v) = \left(\frac{\partial\psi}{\partial y}, -\frac{\partial\psi}{\partial x}\right)$. The above mentioned equations then become

$$(1 + K)(1 + \sigma_x^2)f''' + 4ff'' - \left(3 + \frac{5x\sigma_x\sigma_{xx}}{1 + \sigma_x^2}\right)(f')^2 + \frac{h}{1 + \sigma_x^2} + Kg' = (5x)\left(f'\frac{\partial f'}{\partial x} - f''\frac{\partial f}{\partial x}\right) \tag{19}$$

$$(1 + K)(1 + \sigma_x^2)g'' + 4fg' - 2f'g - K(5x)^{\frac{2}{5}}(f'' + 2g) = (5x)\left(f'\frac{\partial g}{\partial x} - g'\frac{\partial f}{\partial x}\right) \tag{20}$$

$$\frac{1 + \sigma_x^2}{Pr}h'' + 4fh' - f'h = (5x)\left(f'\frac{\partial h}{\partial x} - h'\frac{\partial f}{\partial x}\right) \tag{21}$$

subject to the boundary conditions

$$\eta = 0: \quad f = f' = 0 \quad (1 + \sigma_x^2)^{\frac{1}{2}}h' = -1$$

$$g = -\frac{1}{2}(1 + \sigma_x^2)f'' \tag{22}$$

$$\eta \to \infty: \quad f' = 0 \quad g = 0 \quad h = 0$$

where primes denote partial differentiation with respect to η.

3 Results and Discussion

The non-linear boundary value problem governed by equations (19) to (22) is solved numerically using the Keller-box method (see Cebeci and Bradshaw, 1984) and the numerical results have been obtained for amplitudes of the wavy surface $a = 0$ (flat plate), 0.1 and 0.2, Prandtl number $Pr = 1$ and the micropolar parameter $K = 0$ (Newtonian fluid), 1 and 5.

In order to assess the accuracy of the present method, we have applied it to the problem of free convection from a vertical flat plate ($a = 0$) with a constant heat flux in a Newtonian fluid ($K = 0$) when $Pr = 0.73$ (air) and 6.7 (water). The results obtained are compared with those reported by Mahajan and Gebhart (1978). This comparison is shown in Table 1 for the reduced wall temperature function $h_w(x) = h(x, 0)$ which is related to the temperature at the wavy surface T_w by the relation

$$\frac{T_w - T_\infty}{\left(\frac{\bar{x}q_w}{k}\right)}Gr^{\frac{1}{5}} = h_w(x) \tag{23}$$

where $Gr_x = \frac{g_e\beta\left(\frac{q_w}{k}\right)\bar{x}^4}{\nu^2}$ is the local Grashof number. The present results are found to be in excellent agreement with those of Mahajan and Gebhart (1978) and therefore we are confident that the present results are very accurate.

Pr	Mahajan and Gebhart (1978)	Present results
0.733	1.4798	1.4797
6.7	0.8417	0.8424

Table 1. Values of $h_w(x)$ for $a = 0$ (flat plate) and $K = 0$ (Newtonian fluid)

Representative velocity components u and υ, temperature θ and microrotation N profiles are shown in Figures 2 to 8 exhibiting the effects of the parameters a and K at two positions $x = 1.5$ (node) and $x = 1.75$ (trough) for $Pr = 1$. It is worth mentioning that the behaviour between the profiles at $x = 1$ (node) and 1.5 (node) is similar and so is the case between the profiles at $x = 1.75$ (trough) and $x = 2.25$ (crest). For convenience therefore, only the case $x = 1.5$ and $x = 1{,}75$ are presented in this paper. These figures clearly show the effects of the parameters a and K on the velocity, temperature and microrotation profiles. Thus, it is seen from Figures 2 to 5 that u is higher at the trough than at the node; the reverse situation happens for υ. This behaviour is to be expected if we notice the presence of extra terms in equation (19) for the present problem in comparison with the case of a flat plate ($a = 0$). We also notice from Figures 8 and 9 that the microrotation profiles remain everywhere negative with a small overshoot from zero for higher values of K.

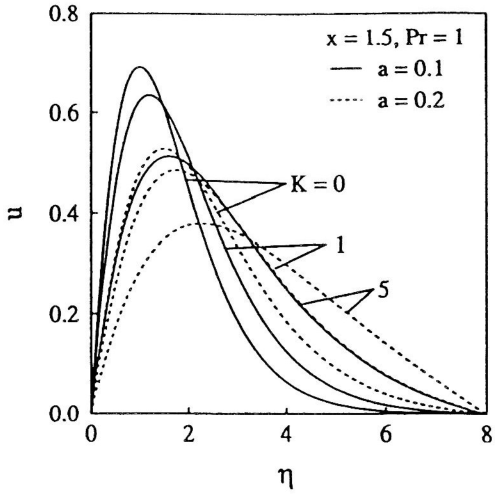

Figure 2. Axial Velocity Profiles at $x = 1.5$ for $Pr = 1$

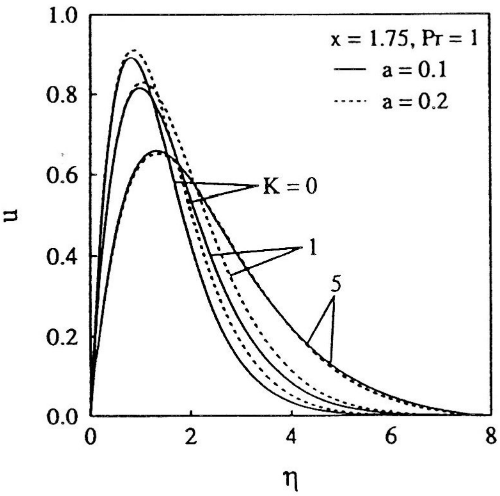

Figure 3. Axial Velocity Profiles at $x = 1.75$ for $Pr = 1$

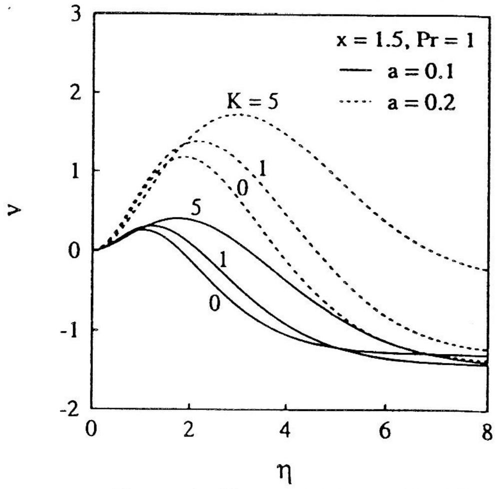

Figure 4. Normal Velocity Profile at $x = 1.5$ for $Pr = 1$

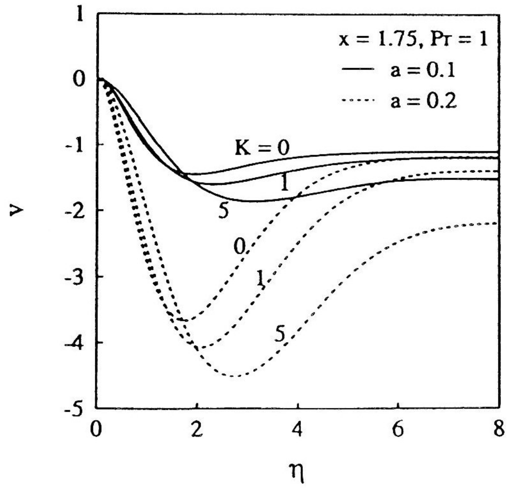

Figure 5. Normal Velocity Profiles at $x = 1.75$ for $Pr = 1$

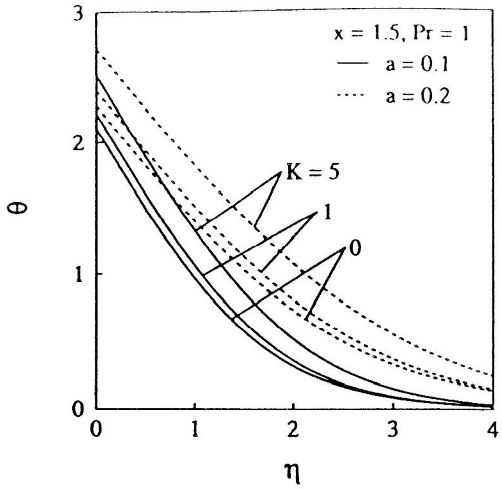

Figure 6. Temperature Profiles at $x = 1.5$ for $Pr = 1$

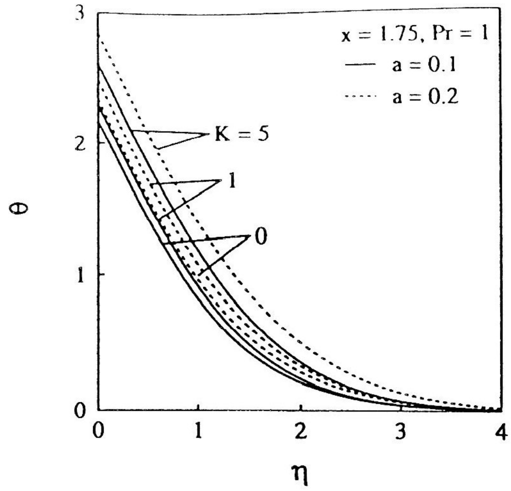

Figure 7. Temperature Profiles at $x = 1.75$ for $Pr = 1$

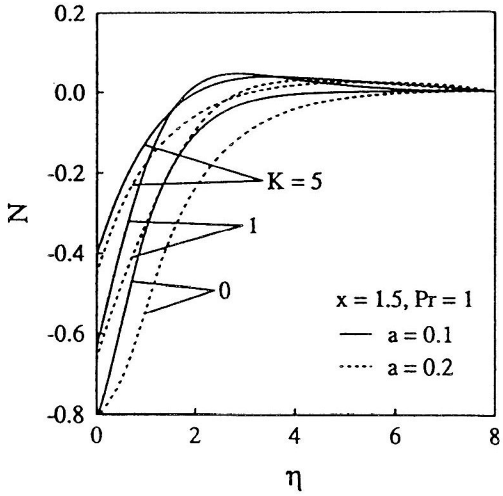

Figure 8. Microrotation Profiles at $x = 1.5$ for $Pr = 1$

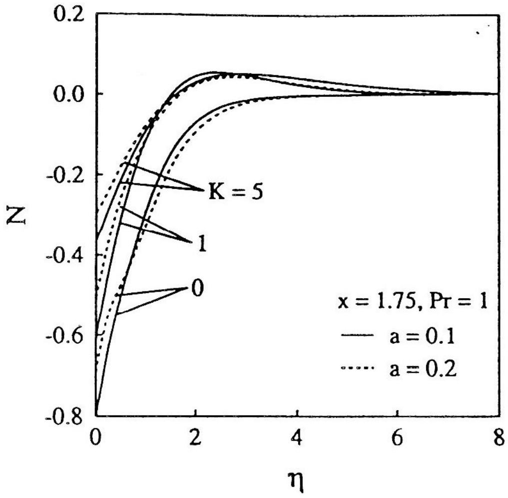

Figure 9. Microrotation Profiles at $x = 1.75$ for $Pr = 1$

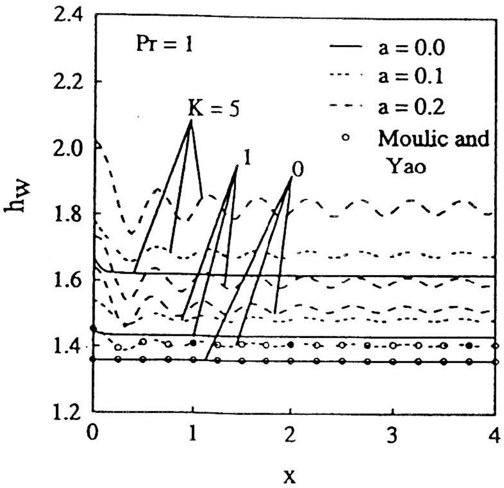

Figure 10. Variation of h_w with x for $Pr = 1$

Values of the wall temperature function $h_w(x)$ defined in equation (23) for the cases of a Newtonian fluid ($K = 0$) and a micropolar fluid ($K \neq 0$) are given in Table 2 for a flat plate ($a = 0$) and a wavy surface ($a \neq 0$), respectively, at different x positions. Also the variation with x of $h_w(x)$ is illustrated in Figure 10, where the results of Moulic and Yao (1989) for the case of a Newtonian fluid ($K = 0$) have also been included. We notice again an excellent agreement between the present results and those known from literature (Moulic and Yao, 1989). It is seen from Table 2 that the waviness of the surface increases the wall temperature, irrespective of whether the fluid is Newtonian or micropolar. Further, Figure 10 shows that $h_w(x)$ varies according to the slope of the wavy surface. However, the amplitude of $h_w(x)$ gradually decreases downstream where the boundary layer grows thick. This is due to (*i*) the effect of the centrifugal forces, the third term of equation (19), and (*ii*) the alignment of the buoyancy force with respect to the wavy surface, as indicated by the fourth term of equation (19). Finally, we notice from Table 2 and Figure 10 that values of $h_w(x)$ are greater for micropolar fluid ($K \neq 0$), than those for a Newtonian fluid ($K = 0$). This observation might be of considerable interest in several practical applications of micropolar fluids.

	Newtonian fluid (K=0)			K = 1			K = 5		
x	a = 0.0	a = 0.1	a = 0.2	a = 0.0	a = 0.1	a = 0.2	a = 0.0	a = 0.1	a = 0.2
1.500 (node)	1.3585	1.4083	1.5217	1.4320	1.4831	1.5982	1.6185	1.6771	1.8117
1.625	1.3585	1.4121	1.5307	1.4319	1.4907	1.6189	1.6184	1.6916	1.8543
1.750 (trough)	1.3585	1.4061	1.5137	1.4318	1.4846	1.6022	1.6184	1.6864	1.8396
1.875	1.3585	1.4018	1.5009	1.4318	1.4764	1.5763	1.6183	1.6719	1.7938
2.000 (node)	1.3585	1.4078	1.5201	1.4317	1.4825	1.5970	1.6183	1.6771	1.8121
2.125	1.3585	1.4112	1.5279	1.4317	1.4894	1.6154	1.6183	1.6901	1.8502
2.250 (crest)	1.3585	1.4066	1.5147	1.4317	1.4848	1.6028	1.6182	1.6866	1.8403
2.375	1.3585	1.4025	1.5029	1.4316	1.4772	1.5791	1.6182	1.6735	1.7987
2.500 (node)	1.3585	1.4074	1.5191	1.4316	1.4822	1.5964	1.6182	1.6772	1.8128
2.625	1.3585	1.4106	1.5261	1.4316	1.4886	1.6132	1.6182	1.6891	1.8475
2.750 (trough)	1.3585	1.4069	1.5154	1.4315	1.4849	1.6033	1.6182	1.6868	1.8408
2.875	1.3585	1.4030	1.5043	1.4315	1.4779	1.5812	1.6181	1.6747	1.8023
3.000 (node)	1.3585	1.4072	1.5184	1.4315	1.4820	1.5960	1.6181	1.6775	1.8137

Table 2. Values of $h_w(x)$ for $Pr = 1$

4 Conclusions

In summary, the present paper describes the free convection along a vertical wavy surface with a constant heat flux heating in a micropolar fluid. New variables to transform the complex geometry into a simple shape were proposed and a very efficient implicit finite-difference (Keller-box) scheme was employed to solve the boundary layer equations. Based on the numerical results, we have the following conclusions in the range of the parameters:

1. An increasing wave amplitude parameter a leads to an increase of wall temperature $h_w(x)$, irrespective of whether the fluid is Newtonian or micropolar.

2. The influence of the micropolar parameter K is most pronounced for the flow field and microrotation profiles.

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