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Joint Work of Two Identical Synchronous Generators with Common RL Load

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A mathematical model for the joint work of two identical synchronous generators with common symmetric RL load is constructed. The static stability of the model is examined.

1 Introduction

On cosmic stations the autonomous electricity supply for different electrophysical devices and for keeping the system alive consists of two identical synchronous generators, the rotors of which are rotating in opposite directions. The last property is principal just for cosmic stations, because it ensures that the summarized angular momentum is equal to zero. In this paper we consider one case of the function of such generators with symmetrical RL load.

2 Mathematical Model

As a mathematical model for the synchronous generator (SG) we take equations (1) from Eirola et al. (1996). For simplification in the equations instead of the currents i_u and i_v we introduce the new variables x_u and x_v according to the formulas

$$\begin{aligned} i_u &= \tfrac{1}{\mu}\psi_u - b\alpha_{rf}x_u - u_f\cos\nu \\ i_v &= \tfrac{1}{\mu}\psi_v - b\alpha_{rf}x_v + u_f\sin\nu \end{aligned} \tag{1}$$

In the presence of damping circuits in the new variables the equations assume the form

$$\begin{aligned} \dot\psi_u &= \dot\gamma_k\psi_v - \alpha_s\psi_u - u_{uc} \\ \dot\psi_v &= -\dot\gamma_k\psi_u - \alpha_s\psi_v - u_{vc} \\ \dot x_u &= \dot\nu x_v - \alpha_{rf}x_u + \psi_u \\ \dot x_v &= -\dot\nu x_u - \alpha_{rf}x_v + \psi_v \\ \dot\theta &= s \qquad \nu = \gamma_k - \gamma = \gamma_k - \tau + \theta \\ \dot s &= -\delta[b\alpha_{rf}(\psi_v x_u - \psi_u x_v) + u_f(\psi_v\cos\nu + \psi_u\sin\nu) + m] \end{aligned} \tag{2}$$

u_{uc} and u_{vc} are the voltages on the RL load which are as mentioned above the same for both phases "u" and "v".

$$\begin{aligned} u_{uc} &= \dot\psi_{uc} - \dot\gamma_k\psi_{vc} + \varepsilon_c i_{uc} \\ u_{vc} &= \dot\psi_{vc} + \dot\gamma_k\psi_{uc} + \varepsilon_c i_{vc} \end{aligned} \tag{3}$$

As the RL load of all of the phases of the generators is the same, the first law of Kirchhoff must be satisfied.

$$i_{uc} = i_{u1} + i_{u2}$$
$$i_{vc} = i_{v1} + i_{v2} \tag{4}$$

The magnetic currents corresponding to the phases of the stators of SG are also summarized on the RL load. Assuming that these currents are directed on ferromagnetic cores, that is without loss, we can write them, according to an analog of Kirchhoff's first law

$$\Phi_{uc} = \Phi_{u1} + \Phi_{u2}$$
$$\Phi_{vc} = \Phi_{v1} + \Phi_{v2} \tag{5}$$

Multiplying equations (5) with the number of turns w_s of the stator windings of the SG we pass to flux linkages

$$\psi_{uc} = n(\psi_{u1} + \psi_{u2})$$
$$\psi_{vc} = n(\psi_{v1} + \psi_{v2}) \tag{6}$$

where $n = w_c/w_s$ (w_c is the number of turns in the inductivity coil of the RL load)

Substituting expressions (4) and (6) into expressions (3) and the last ones into the first two equations for each of the two identical SG, we get

$$\dot{\psi}_{u1} = \dot{\gamma}_k\psi_{v1} - \alpha_s\psi_{u1} - n(\dot{\psi}_{u1} + \dot{\psi}_{u2}) + \dot{\gamma}_k n(\psi_{v1} + \psi_{v2}) -$$
$$-\varepsilon_c[\tfrac{1}{\mu}(\psi_{u1} + \psi_{u2}) - b\alpha_{rf}(x_{u1} + x_{u2}) - u_f(\cos\nu_1 + \cos\nu_2)]$$
$$\dot{\psi}_{v1} = -\dot{\gamma}_k\psi_{u1} - \alpha_s\psi_{v1} - n(\dot{\psi}_{v1} + \dot{\psi}_{v2}) - \dot{\gamma}_k n(\psi_{u1} + \psi_{u2}) -$$
$$-\varepsilon_c[\tfrac{1}{\mu}(\psi_{v1} + \psi_{v2}) - b\alpha_{rf}(x_{v1} + x_{v2}) + u_f(\sin\nu_1 + \sin\nu_2)]$$
$$\dot{\psi}_{u2} = \dot{\gamma}_k\psi_{v2} - \alpha_s\psi_{u2} - n(\dot{\psi}_{u1} + \dot{\psi}_{u2}) + \dot{\gamma}_k n(\psi_{v1} + \psi_{v2}) - \tag{7}$$
$$-\varepsilon_c[\tfrac{1}{\mu}(\psi_{u1} + \psi_{u2}) - b\alpha_{rf}(x_{u1} + x_{u2}) - u_f(\cos\nu_1 + \cos\nu_2)]$$
$$\dot{\psi}_{v2} = -\dot{\gamma}_k\psi_{u2} - \alpha_s\psi_{v2} - n(\dot{\psi}_{v1} + \dot{\psi}_{v2}) - \dot{\gamma}_k n(\psi_{u1} + \psi_{u2}) -$$
$$-\varepsilon_c[\tfrac{1}{\mu}(\psi_{v1} + \psi_{v2}) - b\alpha_{rf}(x_{v1} + x_{v2}) + u_f(\sin\nu_1 + \sin\nu_2)]$$

Forming the differences between the corresponding equations in equations (7) we get

$$\dot{\psi}_{u1} - \dot{\psi}_{u2} = \dot{\gamma}_k(\psi_{v1} - \psi_{v2}) - \alpha_s(\psi_{u1} - \psi_{u2})$$
$$\dot{\psi}_{v1} - \dot{\psi}_{v2} = -\dot{\gamma}_k(\psi_{u1} - \psi_{u2}) - \alpha_s(\psi_{v1} - \psi_{v2}) \tag{8}$$

3 Static Stability

With the notations $\psi_u = \psi_{u1} - \psi_{u2}$ and $\psi_v = \psi_{v1} - \psi_{v2}$, equations (8) become

$$\dot{\psi}_u = \dot{\gamma}_k\psi_v - \alpha_s\psi_u$$
$$\dot{\psi}_v = -\dot{\gamma}_k\psi_u - \alpha_s\psi_v$$

Clearly, in the steady-state regime of the system $\psi_u = 0$ and $\psi_v = 0$, and because we are interested in the static stability we assume $\psi_{u2} = \psi_{u1}$ and $\psi_{v2} = \psi_{v1}$. Using these conditions and neglecting small terms $\alpha_{rf} \sim 10^{-2}$ compared with terms of order $u_f \sim 1$ and introducing the notations

$$\varepsilon_c/\mu = \alpha_c \qquad \frac{\alpha_s + 2\alpha_c}{1+2n} = \alpha \qquad \frac{\varepsilon_c}{1+2n} = \varepsilon$$

the mathematical model assumes the following form:

$$\begin{aligned}
\dot{\psi}_{u1} &= \dot{\gamma}_k\psi_{v1} - \alpha\psi_{u1} + \varepsilon u_f(\cos\nu_1 + \cos\nu_2)\\
\dot{\psi}_{v1} &= -\dot{\gamma}_k\psi_{u1} - \alpha\psi_{v1} + \varepsilon u_f(\sin\nu_1 + \sin\nu_2)\\
\dot{x}_{u1} &= \dot{\nu}_1 x_{v1} - \alpha_{rf}x_{u1} + \psi_{u1}\\
\dot{x}_{v1} &= -\dot{\nu}_1 x_{u1} - \alpha_{rf}x_{v1} + \psi_{v1}\\
\dot{x}_{u2} &= \dot{\nu}_2 x_{v2} - \alpha_{rf}x_{u2} + \psi_{u1}\\
\dot{x}_{v2} &= -\dot{\nu}_2 x_{u2} - \alpha_{rf}x_{v2} + \psi_{v1}\\
\dot{\theta}_1 &= s_1\\
\dot{\theta}_2 &= s_2\\
\dot{s}_1 &= -\delta[b\alpha_{rf}(\psi_{v1}x_{u1} - \psi_{u1}x_{v1}) + u_f(\psi_{v1}\cos\nu_1 + \psi_{u1}\sin\nu_1) + m_1]\\
\dot{s}_2 &= -\delta[b\alpha_{rf}(\psi_{v1}x_{u2} - \psi_{u1}x_{v2}) + u_f(\psi_{v1}\cos\nu_2 + \psi_{u1}\sin\nu_2) + m_2]
\end{aligned} \tag{9}$$

We now write system (9) in the projections on the *d*-*q*-axis rigidly tied to the rotor of the first SG. We have to set $\nu_1 = 0$. Then $\gamma_k = \gamma_1 = \tau - \theta_1$ and $\nu_2 = \gamma_k - \tau + \theta_2 = \theta_2 - \theta_1$. The first two equations of this system assume the form

$$\begin{aligned}
\dot{\psi}_{d1} &= (1 - s_1)\psi_{q1} - \alpha\psi_{d1} + \varepsilon u_f(1 + \cos\theta)\\
\dot{\psi}_{q1} &= -(1 - s_1)\psi_{d1} - \alpha\psi_{q1} + \varepsilon u_f \sin\theta
\end{aligned}$$

As s_1 is close to zero we will neglect it compared with unity. Then the order of the system can be decreased introducing the new variables $\theta = \theta_1 - \theta_2$ and $s = s_1 - s_2$. Finally the system assumes the following form (ψ_{d1}, ψ_{q1} we denote by ψ_d, ψ_q and introduce $\triangle m = m_1 - m_2$):

$$\begin{aligned}
\dot{\psi}_d &= \psi_q - \alpha\psi_d + \varepsilon u_f(1 + \cos\theta)\\
\dot{\psi}_q &= -\psi_d - \alpha\psi_q + \varepsilon u_f \sin\theta\\
\dot{x}_{d1} &= -\alpha_{rf}x_{d1} + \psi_d\\
\dot{x}_{q1} &= -\alpha_{rf}x_{q1} + \psi_q\\
\dot{x}_{d2} &= -sx_{q2} - \alpha_{rf}x_{d2} + \psi_d\\
\dot{x}_{q2} &= sx_{d2} - \alpha_{rf}x_{q2} + \psi_q\\
\dot{\theta} &= s\\
\dot{s} &= -\delta\{b\alpha_{rf}[\psi_q(x_{d1} - x_{d2}) - \psi_d(x_{q1} - x_{q2})] + u_f[\psi_d\sin\theta + \psi_q(1 - \cos\theta)] + \triangle m\}
\end{aligned} \tag{10}$$

Determining the equilibrium we get the equation

$$\sin\theta_0 = -\frac{\Delta m}{2\alpha\beta u_f} \tag{11}$$

where $\beta = \varepsilon u_f/(1+\alpha^2)$.

From this equality we get the necessary conditions for stability.

$$\sin\theta_0 \le 0 \qquad \Delta m \le 2\alpha\beta u_f \tag{12}$$

Because $\delta \sim 10^{-3}$ to 10^{-4}, the electrical variables are slow compared with the mechanical variables θ and s. Clearly the fast subsystem has a unique asymptotically stable solution (in it the slow variables are assumed constant). The exponentially decreasing terms do not affect averaging the slow subsystem. Therefore to examine the stability of the equilibria of equation (11) it is only necessary to substitute into it the partial solution of the "fast" subsystem (assuming the derivates of the "fast" electrical variables to be zero). After examining the stability of the whole system by elementary calculations we get the second order system

$$\dot\theta = s$$

$$\dot s = -\delta[2b\alpha_{rf}(1+\alpha^2)\beta^2\frac{s}{\alpha_{rf}^2+s^2}(1+\cos\theta) + 2\alpha\beta u_f\sin\theta + \Delta m]$$

The characteristic equation of the corresponding linearized equation has the form

$$\lambda^2 + 2\frac{\delta b\beta^2}{\alpha_{rf}}(1+\alpha^2)(1+\cos\theta_0)\lambda + 2\delta\alpha\beta u_f\cos\theta_0 = 0$$

From the equation it follows that

$$\cos\theta_0 > 0$$

is a necessary and sufficient condition for the stability of the system. Finally, using also condition (12) we conclude that for stable equilibrium the angle θ_0 is in the fourth quadrant.

$$\frac{3\pi}{2} \le \theta_0 < 2\pi$$

Thereby the second of the conditions (12) must be satisfied.

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Literature

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