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# Homogenization and Perturbation Procedures in the Theory of Ring-Stiffened Shells

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*We are concerned in this paper with the stress-strain problem for axisymmetric loading of a ring-stiffened cylindrical shell by means of the new asymptotic method. The basic idea of the method presented may be described as follows: First of all we use a homogenization procedure, then a perturbation approach and eventually Padé approximants for improving the perturbation series.*

## 1 Introduction

The stress-strain state of thin, reinforced plates and shells has been one of the most fundamental subjects in the study of structural members. Notable works in the literature survey of this subject are being given by Andrianov et al. (1985). Many approaches, such as variational, asymptotic, numerical are successfully employed for solving the present problem in case of homogeneous boundary conditions.

But the above mentioned procedures, as a rule, work poorly in the case when reinforcing elements have various rigidities. This case is the most complicated in this region. On the other hand, analyses of reinforced plates and shells for reinforcing elements with various rigidities represent a significant practical problem. A lot of problems arising in machine design, civil engineering etc., are reduced to similar ones. The problems mentioned are usually solved using numerical methods, such as finite element procedures. Nevertheless, a numerical approach does not adequately satisfy the requirements of optimal structural design or any other kind of optimal structural design ideology. Then approximate analytical expressions, provided they accurate enough, will be of great practical advantage for these needs.

## 2 Homogenization Procedure

We consider the axisymmetric loading of a ring-stiffened cylindrical shell (Figure 1). Equations of deformation of shell between rings and conditions of fit throughout the ring may be represented as follows:

$$\frac{d^4 w}{dx^4} + k_0 w = q \tag{1}$$

$$w^- = w^+ \qquad \frac{dw^-}{dw} = \frac{dw^+}{dx} \qquad \frac{d^2 w^-}{dx^2} = \frac{d^2 w^+}{dx^2}$$
$$\left(\frac{d^3 w}{dx^3}\right)^- - \left(\frac{d^3 w}{dx^3}\right)^+ = f(x) k_1 w \tag{2}$$

Without loss of generality we suppose the shell simply supported.

$$w = \frac{d^2 w}{dx^2} = 0 \qquad \text{for} \qquad x = 0, L \tag{3}$$

Let us suppose the parameter $\varepsilon = l / L$, with $l$ = step of rings, characterizing frequency of ring posing, is small, and $\varepsilon^{-1} k_1 \sim 1$ (this case is the most important from the practical point of view).

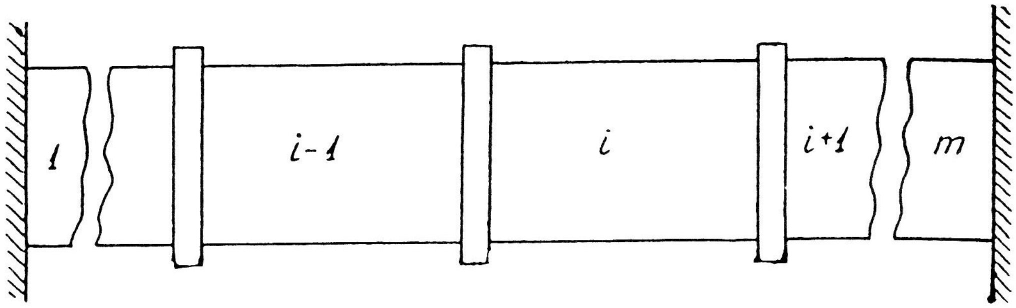


Figure 1. Ring-stiffened Shell

Then we may use for solving boundary value problem (1) to (3) the homogenization procedure given in Andrianov et al. (1985). The method used here is a variant of the multiscaling technique used in Obraztsov et al. (1991). Let us introduce a new „fast" variable $\xi = x/\varepsilon$ , then the derivative applied to a function ( ) has the form

$$\frac{d(\ )}{dx} = \frac{d(\ )}{dx} + \varepsilon^{-1}\frac{d(\ )}{d\xi} \tag{4}$$

The solution is written in the form of formal expansions

$$w = w_0(x) + \varepsilon^4 w_1(x,\xi) + \ldots \tag{5}$$

A period $L$ with respect to the variables is assumed. Substituting series (5) into the boundary value problem (1) to (3), taking into account relation (4) and splitting it according to the powers of $\varepsilon$ , one obtains the recurrent sequence for the boundary value problem

$$\frac{d^4 w_1}{d\xi^4} + \frac{d^4 w_0}{dx^4} + k_0 w_0 = q \tag{6}$$

$$w = \frac{d^2 w}{dx^2} = 0 \qquad\qquad for\ x = 0, L \tag{7}$$

$$\left(w_1;\ \frac{dw_1}{d\xi};\ \frac{d^2 w_1}{d\xi^2}\right)_{\xi=0} = \left(w;\ \frac{dw_1}{d\xi};\ \frac{d^2 w_1}{d\xi^2}\right)_{\xi=L} \tag{8}$$

$$\left(\frac{d^3 w_1}{d\xi^3}\right)_{\xi=L} - \left(\frac{d^3 w_1}{d\xi^3}\right)_{\xi=0} = -f(x)\varepsilon^{-1}k_1 w_0 \tag{9}$$

It is easily obtained, from equation (6), that

$$w_1 = \left(q - \frac{d^4 w_0}{d^4 x} - k_0 w_0^4\right)\frac{\xi^4}{24} + C_1\xi^3 + C_2\xi^2 + C_3\xi \tag{10}$$

Conditions (8) give possibility to obtain functions $C_1$ to $C_3$ of „slow" variables and function $w_1$

$$w_1 = -\left(\frac{d^4 w}{dx^4} + k_0 w_0^2 - q\right)\xi^2(\xi - L)^2 / 24 \tag{11}$$

Equation (9) leads to the condition, coincident with the structural orthotropic theory equation

$$\frac{d\, w_0^4}{dx^4} + k\ (x) w_0 = q \tag{12}$$

where $k(x) = k_0 + f(x)\frac{k_1}{l}$

## 3 Perturbation Procedure

We now consider that ring rigidities may be represented as follows

$$k(x) = a + \varepsilon\, \varphi(x) \qquad a = const \qquad \varepsilon_1 << 1 \tag{13}$$

Solution of equation (12) may be written as a perturbation series

$$w_0 = \sum_{m=0}^{\infty} \varepsilon_1^m w_{0m} \tag{14}$$

It leads after a routine perturbation procedure to the recurrent system of equations

$$\frac{d^4 w_{00}}{dx^4} + a w_{00} = q \tag{15}$$

$$\frac{d^4 w_{0m}}{dx^4} + a\, w_{0m} = -\varphi\ (x) w_{0m-1} \qquad m = 1,2,\ldots \tag{16}$$

The solution of the system obtained may be produced using the Fourier method, splitting it into the trigonometric series

$$q = \sum_{n=1}^{\infty} q_n \sin(\alpha n x) \qquad \varphi = \sum_{n=1}^{\infty} \varphi_n \cos(\alpha n x) \qquad w_{0m} = \sum_{n=1}^{\infty} A_{mn} \sin(\alpha n x)$$

with $\alpha = 2\pi / L$, and easily obtaining (17)

$$w_0 = \sum_{n=1}^{\infty} \sum_{m=0}^{\infty} \varepsilon_1^m A_{mn} \sin(\alpha n x) \tag{18}$$

where

$$A_{0n} = \frac{q_n}{\left(n^4\alpha^4 + a\right)} \qquad A_{mn} = \frac{-aB_{m-1n}}{\left(n^4\alpha^4 + a\right)} \qquad B_{mn} = \sum_{r=1}^{\infty} \varphi_r \left(A_{mr+n} - A_{mr-n}\right)$$

## 4 Padé Approximants

Solution (18) has a very narrow area of applicability, but Padé approximants may be used for the extension of the area of applicability of a perturbation series (Nayfeh, 1973). Let us produce the PA (Padé Approximation) definition (Baker and Graves-Morris, 1981). For an expansion given by

$$F(\varepsilon)=\sum_{i=0}^{\infty} c_i\,\varepsilon^i \tag{19}$$

the fractional-rational function

$$F(\varepsilon)\left[\frac{m}{n}\right]=\left(\sum_{i=0}^{m} a_i\,\varepsilon^i\right)\left(\sum_{i=0}^{n} b_i\,\varepsilon^i\right)^{-1} \tag{20}$$

represents the PA of expansion (19), if the MacLaurin series of the $F(\varepsilon)$ expression shows the coincidence of its coefficients with corresponding ones of equation (20) up to the terms of $(m+n+1)$th order.
The features of the PA are: it possesses uniqueness while $m$ and $n$ are freely chosen; it performs a meromorphic continuation of a function; for its definition from the source expansion (20) a linear algebraic problem arises (Baker and Graves-Morris, 1981). In our case, after using Padé approximants for the coefficients of each harmonics of the expansions (14) and (17), one obtains

$$w_0=\sum_{n=1}^{\infty}\left(A_{on}A_{1n}+\varepsilon_1\left(A_{1n}^2-A_{0n}A_{2n}\right)\right)\left(A_{1n}-\varepsilon_1 A_{2n}\right)^{-1}\sin(\alpha n x) \tag{21}$$

## 5 Numerical Example

As an example we choose a shell, loaded by inner normal pressure, and suppose $k(x)=c(1-\varepsilon_1\cos 2x)$.
It gives the possibility to estimate effectiveness of this kind of support in comparsion with reinforcement by rings of equal rigidities. The coefficients of the solution (18) in this case are

$$A_{02n-1}=\frac{4q}{(2n-1)\pi\left((2n-1)^4+c\right)}\qquad A_{m2n-1}=\frac{c}{2}\frac{A_{m-12n-3}+A_{m-12n+1}}{(2n-1)^4+c}\qquad A_{m-1}=A_{m1}\qquad A_{m2n}=0$$

Some numerical results are displayed in Figure 2, where $\Delta=\varepsilon_1 A_{11}^2/\left((A_{11}-\varepsilon_1 A_{21})A_{01}\right)$

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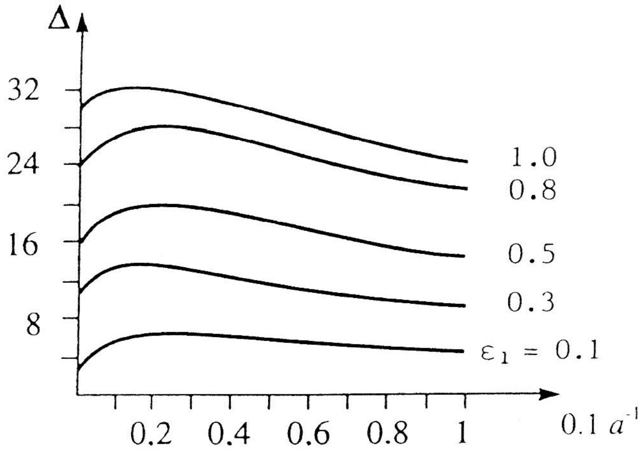


Figure 2. Stiffness versus Rigidity

## Literature

1. Andrianov, I. V.; Lesnichaya, V. A.; Manevich, L. I.: Homogenization Methods in Statics and Dynamics of Reinforced Shells, Moscow, Nauka, (1985)

2. Baker, G. A.; Graves-Morris, P.: Pade Approximants, Part 1: Basic Theory, Part 2: Extensions and Applications, New York, Addison-Wesley, (1981)

3. Bakhvalov, N.; Panasenko, G.: Averaging Processes in Periodic Media, Mathematical Problems in Mechanics of Composite Materials, Kluwer Academic Publishers, Dordrecht, (1989)

4. Bensoussan, A.; Lions, J.-L.; Papanicolaou, G.: Asymptotic Methods in Periodic Structures, New York, North-Holland, (1978)

5. Nayfeh, A. H.: Perturbation Methods, New York, John Wiley & Sons, (1973)

6. Obraztsov, I. F.; Nerubaylo, B. V.; Andrianov, I. V.: Asymptotic Methods in the Theory of Thin-Walled Structures, Moscow, Mashinostroyenie, (1991)

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